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GATE IN · 2024

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GATE IN 2024 Solved Paper

INSIDE

- All 65 questions, in the order of the paper
- The official answer to every question
- A step-by-step solution to every question
- The answer key on one page

Question paper and official answer key: IISc Bengaluru

Online, with the questions linked to their topic notes: **iaeacademy.in/study/papers/in-2024**

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About this paper

65 questions, 100 marks. Question types: 33 MCQ · 1 MSQ · 31 NAT.

Official answer key: IISc Bengaluru · https://gate2024.iisc.ac.in/wp-content/uploads/2024/INFinalAnswerKey.pdf

We placed each question under a subject and topic ourselves; the exam paper does not label them.

This paper online, with the questions linked to their topic notes:
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Answer key

Q	TYPE	MARKS	ANSWER
Q.1	MCQ	1	**A**
Q.2	MCQ	1	**B**
Q.3	MCQ	1	**D**
Q.4	MCQ	1	**A**
Q.5	MCQ	1	**B**
Q.6	MCQ	2	**D**
Q.7	MCQ	2	**B**
Q.8	MCQ	2	**C**
Q.9	MCQ	2	**A**
Q.10	MCQ	2	**A**
Q.11	MCQ	1	**B**
Q.12	MCQ	1	**B**
Q.13	MCQ	1	**D**
Q.14	MCQ	1	**A**
Q.15	MCQ	1	**A**
Q.16	MCQ	1	**A**
Q.17	MCQ	1	**B**
Q.18	MCQ	1	**A**
Q.19	MCQ	1	**C**
Q.20	MCQ	1	**A**
Q.21	NAT	1	**0.15 to 0.17**
Q.22	NAT	1	**0.70 to 0.80**
Q.23	NAT	1	**1.6**
Q.24	NAT	1	**498 to 502**
Q.25	NAT	1	**32**
Q.26	NAT	1	**-2**
Q.27	NAT	1	**400**
Q.28	NAT	1	**22**
Q.29	NAT	1	**-3**
Q.30	NAT	1	**200**
Q.31	NAT	1	**27.0 to 30.0**
Q.32	NAT	1	**10.1 to 10.3**
Q.33	NAT	1	**15**
Q.34	NAT	1	**1.10 to 1.12**
Q.35	NAT	1	**0.5**
Q.36	MCQ	2	**A**
Q.37	MCQ	2	**A**
Q.38	MCQ	2	**C**
Q.39	MCQ	2	**C**
Q.40	MCQ	2	**B**
Q.41	MCQ	2	**A**
Q.42	MCQ	2	**A**
Q.43	MCQ	2	**A**
Q.44	MCQ	2	**A**
Q.45	MCQ	2	**A**
Q.46	MCQ	2	**A**
Q.47	MCQ	2	**C**
Q.48	MCQ	2	**A**
Q.49	MSQ	2	**A, C**
Q.50	NAT	2	**0.450 to 0.550**
Q.51	NAT	2	**4**
Q.52	NAT	2	**6.00 to 6.30**
Q.53	NAT	2	**2**
Q.54	NAT	2	**58 to 62**
Q.55	NAT	2	**2**
Q.56	NAT	2	**2.81 to 2.85**
Q.57	NAT	2	**9.5 to 10.5**
Q.58	NAT	2	**3**
Q.59	NAT	2	**-2.6 to -2.4**
Q.60	NAT	2	**0**
Q.61	NAT	2	**95.2 to 95.4**
Q.62	NAT	2	**17.00 to 17.30**
Q.63	NAT	2	**0.25**
Q.64	NAT	2	**33**
Q.65	NAT	2	**0.130 to 0.140**

General Aptitude Q.1 to Q.10

Q.1 MCQ · 1 MARK

General Aptitude › Grammar & vocabulary

If '→' denotes increasing order of intensity, then the meaning of the words [drizzle → rain → downpour] is analogous to [_____ → quarrel → feud]. Which one of the given options is appropriate to fill the blank?

(A) bicker

(B) bog

(C) dither

(D) dodge

Official answer: **A**

SOLUTION

1. Work out the pattern in the given chain. A drizzle is light rain, rain is ordinary rain and a downpour is heavy rain. All three words name the same thing, and each one is more intense than the one before it.
2. The second chain must follow the same pattern. A quarrel is an angry argument. A feud is a bitter quarrel that lasts a long time, often between families or groups. So the blank needs a word for a disagreement that is milder than a quarrel.
3. (A) bicker: to argue about small, unimportant things. This is a petty argument, milder than a quarrel, so it fits.
4. (B) bog: wet, muddy ground, or (in 'bog down') to get stuck. It has nothing to do with arguing.
5. (C) dither: to be unable to make up one's mind. It is about indecision, not disagreement.
6. (D) dodge: to avoid something by moving quickly or by trickery. It has nothing to do with arguing.
7. Only 'bicker' completes the chain bicker → quarrel → feud, which matches drizzle → rain → downpour. The answer is (A).

KEY IDEA

In an intensity analogy, every word in a chain names the same kind of thing at a different strength. First find what the other two words have in common (here, disagreement). Then pick the option that belongs to that same kind and sits at the weakest end, below 'quarrel'.

WATCH OUT FOR

Choosing a word that merely sounds like conflict (such as 'dodge', which is about avoiding something) or that suggests weakness without being about arguing (such as 'dither', which is about indecision). The blank has to be the same kind of thing as 'quarrel' and 'feud', only milder.

Q.2 MCQ · 1 MARK

General Aptitude › Reasoning

Statements:

1. All heroes are winners.
2. All winners are lucky people.

Inferences:

I. All lucky people are heroes.
II. Some lucky people are heroes.
III. Some winners are heroes.

Which of the above inferences can be logically deduced from statements 1 and 2?

(A) Only I and II

(B) Only II and III

(C) Only I and III

(D) Only III

Official answer: **B**

SOLUTION

1. Combine the two statements. "All heroes are winners" and "All winners are lucky people" are both universal affirmatives, and they chain to give: All heroes are lucky people. So $\text{Heroes} \subseteq \text{Winners} \subseteq \text{Lucky people}$.
2. Draw it as a Venn diagram: the Heroes circle sits inside the Winners circle, which sits inside the Lucky-people circle. As usual in GATE syllogism questions, each class named in a statement is taken to have at least one member, so the Heroes circle is not empty.
3. Inference I, "All lucky people are heroes", is the converse of a universal statement. A universal affirmative cannot be converted simply: the Lucky-people circle may contain people outside the Heroes circle. So I does not follow.
4. Inference III, "Some winners are heroes": "All heroes are winners" converts by limitation (A to I) into "Some winners are heroes". The Heroes circle is non-empty and lies inside the Winners circle, so at least one winner is a hero. III follows.
5. Inference II, "Some lucky people are heroes": the derived conclusion "All heroes are lucky people" also converts by limitation, giving "Some lucky people are heroes". Equivalently, the heroes who are winners (from III) are all lucky people, by statement 2. II follows.
6. Only II and III follow, which is option (B).

KEY IDEA

Chain the two universal statements into Heroes $\subseteq$ Winners $\subseteq$ Lucky people. An "All A are B" statement can never be reversed to "All B are A". It can be converted by limitation to "Some B are A", since GATE treats every named class as non-empty.

WATCH OUT FOR

Students go wrong in two ways. Some reverse "All heroes are lucky people" into "All lucky people are heroes" and accept I, which leads to (A) or (C). Others argue that "All" never implies "Some" because the Heroes set might be empty, and choose (D) or reject II. GATE follows the traditional convention that universal statements are about classes with members, so both II and III follow.

Q.3 MCQ · 1 MARK

General Aptitude › Numerical

A student was supposed to **multiply** a positive real number p with another positive real number q. Instead, the student **divided** p by q. If the percentage error in the student's answer is 80%, the value of q is

(A) 5

(B) $\sqrt{2}$

(C) 2

(D) $\sqrt{5}$

Official answer: **D**

SOLUTION

1. The correct answer is pq. The student's answer is $\frac{p}{q}$.
2. Percentage error is measured against the correct value:
$$\text{Error} = \frac{\left|pq - \frac{p}{q}\right|}{pq} \times 100 = \left|1 - \frac{1}{q^2}\right| \times 100$$
3. Set this equal to 80%: $\left|1 - \frac{1}{q^2}\right| = 0.8$.

4. Case 1: $1 - \frac{1}{q^2} = 0.8 \Rightarrow \frac{1}{q^2} = 0.2 \Rightarrow q^2 = 5 \Rightarrow q = \sqrt{5}$, since $q > 0$.
5. Case 2: $1 - \frac{1}{q^2} = -0.8 \Rightarrow \frac{1}{q^2} = 1.8 \Rightarrow q^2 = \frac{5}{9} \Rightarrow q = \frac{\sqrt{5}}{3}$. This is not one of the options.
6. Check: with $q = \sqrt{5}$, the correct value is $\sqrt{5}\,p$ and the student's value is $\frac{p}{\sqrt{5}} = \frac{\sqrt{5}\,p}{5}$, which is 20% of the correct value. So the error is 80%, and the answer is (D).

KEY IDEA

Percentage error is always taken relative to the correct value. Here the ratio of the student's answer to the correct answer is $\frac{p/q}{pq} = \frac{1}{q^2}$, and p cancels out. An 80% error means the student got 20% of the correct value, so $\frac{1}{q^2} = 0.2$ and $q^2 = 5$.

WATCH OUT FOR

Dividing by the student's (wrong) answer $\frac{p}{q}$ instead of the correct answer pq. That gives $|q^2 - 1| = 0.8$, so $q^2 = 1.8$ or 0.2, and neither matches an option. Some students also read an 80% error as "the student got 80% of the correct value", which gives $q^2 = 1.25$. It actually means the answer is off by 80%, so the student got 20%.

Q.4 MCQ · 1 MARK

General Aptitude › Numerical

If the sum of the first 20 consecutive positive odd numbers is divided by 20^2, the result is

(A) 1

(B) 20

(C) 2

(D) 1/2

Official answer: **A**

SOLUTION

1. The first 20 consecutive positive odd numbers are $1, 3, 5, \ldots, 39$. The n-th positive odd number is $2n - 1$, so the 20th is $2(20) - 1 = 39$.
2. These numbers form an arithmetic progression with common difference 2. Sum of an arithmetic progression = (number of terms) $\times$ (first term + last term) / 2:
$$S = \frac{20}{2}(1 + 39) = 10 \times 40 = 400$$
3. This agrees with the standard result that the sum of the first n positive odd numbers is n^2: $S = 20^2 = 400$.
4. Divide by 20^2:
$$\frac{S}{20^2} = \frac{400}{400} = 1$$
5. The result is 1, which is option (A).

KEY IDEA

The sum of the first n positive odd numbers is always n^2, because $1 + 3 + \cdots + (2n - 1) = \frac{n}{2}(1 + (2n - 1)) = n^2$. Here $n = 20$, so the sum is exactly 20^2 and the ratio is 1.

WATCH OUT FOR

Taking the 20th odd number to be 41 (confusing $2n + 1$ with $2n - 1$) or including 0, which gives a wrong sum. Another slip is using the formula for the sum of the first n natural numbers, $\frac{n(n+1)}{2} = 210$, in place of the sum of odd numbers. Both lead to a ratio that is not 1.

Q.5 MCQ · 1 MARK

General Aptitude › Numerical

The ratio of the number of girls to boys in class VIII is the same as the ratio of the number of boys to girls in class IX. The total number of students (boys and girls) in classes VIII and IX is 450 and 360, respectively. If the number of girls in classes VIII and IX is the same, then the number of girls in each class is

(A) 150

(B) 200

(C) 250

(D) 175

Official answer: **B**

SOLUTION

1. Let g be the number of girls in each class. The question says the number is the same in class VIII and class IX.
2. Boys = total minus girls. Class VIII has $450 - g$ boys and class IX has $360 - g$ boys.
3. The question says (girls : boys in VIII) = (boys : girls in IX), so
$$\frac{g}{450 - g} = \frac{360 - g}{g}$$
4. Cross-multiply:
$$g^2 = (450 - g)(360 - g) = 162000 - 450g - 360g + g^2 = 162000 - 810g + g^2$$
5. The g^2 terms cancel and a linear equation is left: $810g = 162000$, so $g = \frac{162000}{810} = 200$.
6. Check: class VIII has 200 girls and $450 - 200 = 250$ boys, so girls : boys $= 200 : 250 = 4 : 5$. Class IX has $360 - 200 = 160$ boys and 200 girls, so boys : girls $= 160 : 200 = 4 : 5$. The two ratios are equal, so the answer is (B) 200.

KEY IDEA

Use one variable for the girls, since their number is the same in both classes. Write each class's boys as total minus girls and set the two ratios equal. Because the ratios are reversed (girls : boys against boys : girls), the g^2 terms cancel when you cross-multiply, and what is left is a linear equation.

WATCH OUT FOR

Reading both ratios as girls : boys and writing $\frac{g}{450-g} = \frac{g}{360-g}$. That has no positive solution, because it forces $450 = 360$. Another slip is expanding $(450 - g)(360 - g)$ with the wrong sign on the $810g$ term. Testing the options is a fast check: with 200 girls, class VIII is $200 : 250$ and class IX is $160 : 200$, and both are $4 : 5$.

Q.6 MCQ · 2 MARKS

General Aptitude › Grammar & vocabulary

In the given text, the blanks are numbered (i)–(iv). Select the best match for all the blanks.

Yoko Roi stands _____ (i) as an author for standing _____ (ii) as an honorary fellow, after she stood _____ (iii) her writings that stand _____ (iv) the freedom of speech.

(A) (i) out (ii) down (iii) in (iv) for

(B) (i) down (ii) out (iii) by (iv) in

(C) (i) down (ii) out (iii) for (iv) in

(D) (i) out (ii) down (iii) by (iv) for

Official answer: **D**

SOLUTION

1. Every blank completes a phrasal verb built on *stand*, so start by fixing what each particle means. *Stand out* means to be noticeable or distinguished. *Stand down* means to resign from or give up a post. *Stand*

by means to keep supporting something (here, one's own words). *Stand for* means to represent or uphold.

2. Blank (ii): someone stands _____ **as an honorary fellow**. An honorary fellowship is a position, and the phrasal verb for leaving a position is *stand down*. So (ii) = down. That rules out (B) and (C), which both give (ii) = out. Both of them also give (iv) = in, and *writings that stand in the freedom of speech* makes no sense, so they fail twice.
3. Blank (i): *Yoko Roi stands _____ as an author*. She is being praised as a distinguished author because she gave up a fellowship on principle, so she *stands out*. So (i) = out, which agrees with (A) and (D).
4. Blank (iii): *she stood _____ her writings*. Refusing to take back what you wrote is *standing by* it. *Stood in her writings* means nothing here (*stand in* means to act as a substitute). So (iii) = by, which rules out (A).
5. Blank (iv): *writings that stand _____ the freedom of speech*. Writings that champion a value *stand for* it. So (iv) = for.
6. The completed sentence reads: "Yoko Roi stands **out** as an author for standing **down** as an honorary fellow, after she stood **by** her writings that stand **for** the freedom of speech." The sequence out, down, by, for is option (D).

KEY IDEA

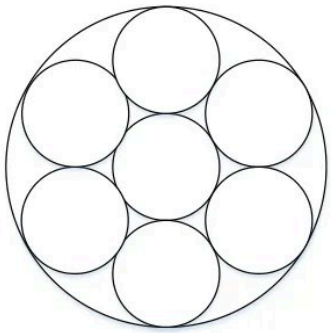
All four blanks are phrasal verbs built on *stand*, and the particle changes the meaning completely: *stand out* (be distinguished), *stand down* (resign a post), *stand by* (keep supporting one's words), *stand for* (represent or uphold). Pin down the blank with the strongest clue first. Here that is (ii), because a fellowship is a post you step down from. Then eliminate options one blank at a time.

WATCH OUT FOR

Picking (A) because (i), (ii) and (iv) are all right, and missing that *stood in her writings* means nothing. *Stand in* means to act as a substitute. The idiom for refusing to take back what you wrote is *stand by*. A second trap is swapping *out* and *down* for (i) and (ii). *Stands down as an author* would mean she quit writing, which contradicts a sentence praising her as an author.

Q.7 MCQ · 2 MARKS

General Aptitude › Geometry

Q.7	Seven identical cylindrical chalk-sticks are fitted tightly in a cylindrical container. The figure below shows the arrangement of the chalk-sticks inside the cylinder.  The length of the container is equal to the length of the chalk-sticks. The ratio of the occupied space to the empty space of the container is
(A)	5/2
(B)	7/2
(C)	9/2
(D)	3

Official answer: **B**

SOLUTION

1. Let each chalk-stick have radius r and let the common length be h. The container has the same length h, so each volume is (cross-sectional area) $\times\, h$, and h cancels in any ratio. A ratio of volumes is therefore the same as a ratio of cross-sectional areas.
2. Two equal circles of radius r that touch have centres $2r$ apart. Each of the six outer circles touches the central one, so every outer centre is $2r$ from the centre of the container. (The six outer centres form a

regular hexagon with side $2r$, so neighbouring outer circles touch each other too, which agrees with the tight fit in the figure.)

3. Each outer circle also touches the container wall, so the wall is $2r + r = 3r$ from the centre. The container's radius is $R = 3r$.
4. Container cross-section: $\pi R^2 = \pi(3r)^2 = 9\pi r^2$.
5. Occupied cross-section (7 chalk-sticks): $7 \times \pi r^2 = 7\pi r^2$.
6. Empty cross-section: $9\pi r^2 - 7\pi r^2 = 2\pi r^2$.
7. Ratio of occupied space to empty space:

$$\frac{7\pi r^2}{2\pi r^2} = \frac{7}{2}$$

This is option (B).

KEY IDEA

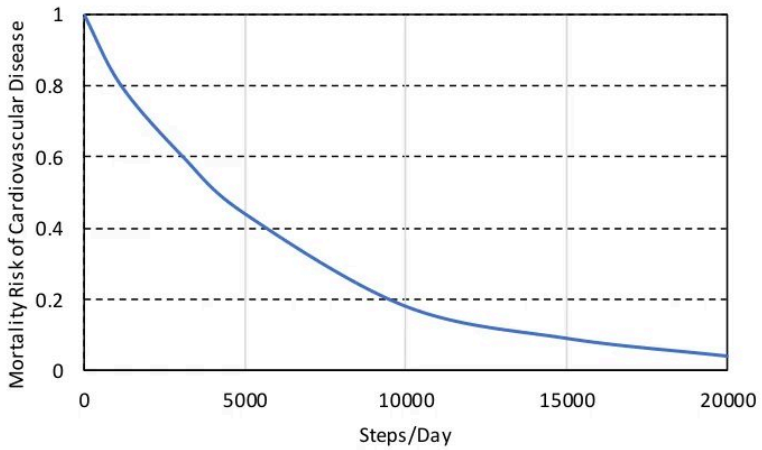
The container and the sticks have the same length, so the volume ratio equals the ratio of cross-sectional areas. In the hexagonal packing of seven equal circles, the outer centres are 2r from the middle, so the enclosing circle has radius 3r. The area is then 9 small-circle areas: 7 are filled and 2 are empty.

WATCH OUT FOR

Answering 9/2 (C) or 7/9, which is occupied to total space rather than occupied to empty space. A second slip is taking the container's radius as 2r, the distance to the outer centres, and forgetting to add the outer stick's own radius.

Q.8 MCQ · 2 MARKS

General Aptitude › Data interpretation

Q.8	The plot below shows the relationship between the mortality risk of cardiovascular disease and the number of steps a person walks per day. Based on the data, which one of the following options is true? 
(A)	The risk reduction on increasing the steps/day from 0 to 10000 is less than the risk reduction on increasing the steps/day from 10000 to 20000.
(B)	The risk reduction on increasing the steps/day from 0 to 5000 is less than the risk reduction on increasing the steps/day from 15000 to 20000.
(C)	For any 5000 increment in steps/day the largest risk reduction occurs on going from 0 to 5000.
(D)	For any 5000 increment in steps/day the largest risk reduction occurs on going from 15000 to 20000.

Official answer: **C**

SOLUTION

1. Read the curve where it meets the vertical gridlines: the risk is about 1.0 at 0, 0.45 at 5000, 0.18 at 10000, 0.10 at 15000 and 0.04 at 20000 steps/day.
2. The risk reduction over an interval is how far the curve falls across it: $R(\text{start}) - R(\text{end})$.
3. Reduction in each 5000-step block: $0 \to 5000$: $1.0 - 0.45 \approx 0.55$; $5000 \to 10000$: $0.45 - 0.18 \approx 0.27$; $10000 \to 15000$: $0.18 - 0.10 \approx 0.08$; $15000 \to 20000$: $0.10 - 0.04 \approx 0.06$.

4. (A): $0 \to 10000$ gives $1.0 - 0.18 \approx 0.82$ and $10000 \to 20000$ gives $0.18 - 0.04 \approx 0.14$. The first reduction is the larger one, so (A) is false.
5. (B): the reduction from $0 \to 5000$ (≈ 0.55) is much larger than the one from $15000 \to 20000$ (≈ 0.06), so (B) is false.
6. (C) and (D): from step 3, $0.55 > 0.27 > 0.08 > 0.06$. Each 5000-step block gives a smaller drop than the one before it, because the curve falls steeply at first and then flattens. The largest reduction is $0 \to 5000$ and the smallest is $15000 \to 20000$, so (C) is true and (D) is false.
7. The exact readings do not change the answer. The curve starts at 1 and is already below 0.5 at 5000 steps/day, so the first block drops by more than 0.5. Every later block lies between 5000 and 20000, so its drop is at most $R(5000) - R(20000) \approx 0.45 - 0.04 = 0.41$, which is smaller.

KEY IDEA

The risk reduction over an interval is the vertical drop of the curve across it. This curve falls steeply at first and then flattens, so equal 5000-step intervals give smaller and smaller drops. The first interval, 0 to 5000, gives by far the largest one.

WATCH OUT FOR

Confusing the risk itself with the risk reduction. The risk is lowest between 15000 and 20000, but the curve is nearly flat there, so that interval gives the smallest reduction and (D) is wrong. The other mistake is comparing the curve's heights at the two ends of an interval instead of the drop across it.

Q.9 MCQ · 2 MARKS

General Aptitude › Spatial

Q.9 Five cubes of identical size and another smaller cube are assembled as shown in Figure A. If viewed from direction X, the planar image of the assembly appears as Figure B.

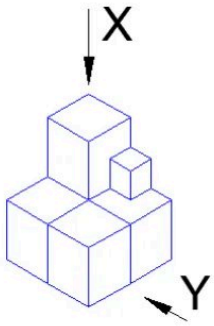


Figure A

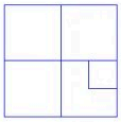
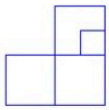
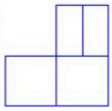
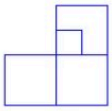


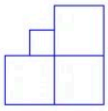
Figure B

If viewed from direction Y, the planar image of the assembly (Figure A) will appear as

(A) 

(B) 

(C) 

(D) 

Official answer: **A**

SOLUTION

1. **Read the solid from Figure A.** The bottom layer is a 2×2 block of unit cubes: front, left, right and rear. The fifth unit cube stands on the rear cube. The small cube has edge $\frac{1}{2}$ and rests on the right-hand bottom cube, in the corner of its top face that touches the upper cube and the outer (back-right) edge of the block. Figure B, the top view, agrees: the small square sits in the corner of the cell next to the upper cube's cell, touching the edge the two cells share and the outer edge.

2. **Work out what view Y shows.** Arrow Y is perpendicular to the lower-right vertical face of the block (the face made by the front and right cubes). So view Y is a straight-on elevation of that face: width runs along the face (two cubes wide) and height is height. The viewer stands on the same side as in Figure A, so left and right stay as drawn. The front corner is on the left and the back-right corner is on the right.
3. **Draw the outline.** Each column of the view shows the tallest part along that line of sight. Left column (the front cube, with the left cube behind it): height 1, so one square at bottom left. Right column (the right cube, with the rear cube and the upper cube behind it): height 2, so a square on top of a square at the right. This gives the L-shape that every option shares.
4. **Place the small cube vertically.** It rests on top of a bottom cube, so its base is on the line between the two rows. Its edge is $\frac{1}{2}$, so it fills only the lower half of the upper row. It is in front of the upper cube (nearer the viewer), so its outline shows inside the upper square.
5. **Place the small cube sideways.** Its outer face lies in the block's back-right face, and in view Y that face is the right-hand edge of the right column. It is $\frac{1}{2}$ wide, so it covers the right half of that column. The small square therefore sits in the **lower-right corner** of the upper square.
6. **Eliminate the other options.** (B) has a half-width strip that runs the full height of the upper square. That would need something a whole cube tall in front of the upper cube, but the small cube is only half as tall. (C) is the mirror image, with the small cube on the inner side. (D) puts the small cube above the left column, outside the upper square. That is impossible, because the small cube lies in the same width band as the tall column. Only (A) matches.

KEY IDEA

An orthographic view flattens the solid along the line of sight. Each column of the image is as tall as the tallest part behind it, and a nearer object's outline appears inside the outline of a farther object it overlaps. Fix left and right from where the viewer stands. Facing the lower-right face, the front corner is on your left and the outer back-right edge is on your right.

WATCH OUT FOR

Getting left and right the wrong way round, as if the face were seen from behind, gives the mirror image (C). Confusing view Y with the view of the other vertical face puts the small cube outside the tall column (D). Forgetting that the small cube is only half as tall draws it as a full-height strip (B).

Q.10 MCQ · 2 MARKS

General Aptitude › Spatial

Visualize a cube that is held with one of the four body diagonals aligned to the vertical axis. Rotate the cube about this axis such that its view remains unchanged. The magnitude of the minimum angle of rotation is

(A) 120°

(B) 60°

(C) 90°

(D) 180°

Official answer: **A**

SOLUTION

1. A body diagonal joins two opposite vertices, say P (top) and Q (bottom). Stand the cube on Q so that PQ is vertical. With coordinates, take $Q = (0, 0, 0)$ and $P = (1, 1, 1)$, so the axis is the line through $(1, 1, 1)$.
2. Height along the axis is proportional to $x + y + z$. The three neighbours of P, $(0, 1, 1)$, $(1, 0, 1)$ and $(1, 1, 0)$, all have $x + y + z = 2$, so they sit at height $\dfrac{2}{3}$ of the diagonal (the upper level). The three neighbours of Q, $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$, have $x + y + z = 1$, so they sit at height $\dfrac{1}{3}$ (the lower level). No other vertex is at either height.

3. Seen from above, each level is an equilateral triangle, so its three vertices are $\frac{360^\circ}{3} = 120^\circ$ apart around the axis. The lower triangle points the opposite way to the upper one: the two are offset by 60°, and together they make the familiar zig-zag ring of 6 vertices.
4. Turning about the vertical axis never changes a point's height. So if the cube is to look exactly the same, the three upper-level vertices (the only vertices at height $\frac{2}{3}$) must land on one another. Because they are 120° apart, the angle of rotation must be a multiple of 120°.
5. 120° works. The rotation by 120° about this diagonal is the cyclic swap $(x, y, z) \to (z, x, y)$. It sends every corner of the cube to another corner, for example $(1, 0, 0) \to (0, 1, 0) \to (0, 0, 1) \to (1, 0, 0)$, so the cube lands exactly on itself and the view does not change.
6. 60° fails. It turns an upper-level vertex to the direction of a lower-level vertex while keeping it at height $\frac{2}{3}$, and there is no corner there. $180^\circ = 120^\circ + 60^\circ$ fails for the same reason. 90° fails because it is not a multiple of 120°: the three edges at P, which are 120° apart, would not land on one another.
7. So the body diagonal is a 3-fold symmetry axis of the cube, and the smallest rotation that leaves the view unchanged is $\frac{360^\circ}{3} = 120^\circ$. Answer: (A).

KEY IDEA

Three edges meet at each end of a body diagonal, and the six remaining corners form two equilateral triangles at heights $\frac{1}{3}$ and $\frac{2}{3}$ of the diagonal. A turn about a vertical axis cannot change a corner's height, so each triangle has to land on itself. That means the angle must be a multiple of 120°, and 120° itself works: it is the cyclic swap $(x, y, z) \to (z, x, y)$.

WATCH OUT FOR

Choosing 60° because the six middle corners look like a regular hexagon from above. They are not all at the same height: alternate corners sit at $\frac{1}{3}$ and $\frac{2}{3}$ of the diagonal, so a 60° turn puts a corner where the cube has none. Choosing 90° mixes up this axis with the face-centre axis, which is 4-fold.

Instrumentation Engineering Q.11 to Q.65

Q.11 MCQ · 1 MARK

Engineering Mathematics › Complex analysis

Let $z = x + iy$ be a complex variable and $\bar{z}$ be its complex conjugate. The equation $\bar{z}^2 + z^2 = 2$ represents a

(A) parabola

(B) hyperbola

(C) ellipse

(D) circle

Official answer: **B**

SOLUTION

1. Write $z = x + iy$ and $\bar{z} = x - iy$, then expand both squares: $z^2 = x^2 - y^2 + 2ixy$ and $\bar{z}^2 = x^2 - y^2 - 2ixy$.
2. Add them. The imaginary parts $\pm 2ixy$ cancel, which is expected because $\bar{z}^2$ is the conjugate of z^2, so $z^2 + \bar{z}^2 = 2\,\mathrm{Re}(z^2)$:
$$z^2 + \bar{z}^2 = 2(x^2 - y^2)$$
3. Set this equal to 2: $2(x^2 - y^2) = 2 \;\Rightarrow\; x^2 - y^2 = 1$.
4. Compare with the standard form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Here $a = b = 1$, so the curve is a rectangular hyperbola with vertices at $(\pm 1, 0)$ and asymptotes $y = \pm x$.
5. So the equation represents a hyperbola, which is option (B).

KEY IDEA

Substitute $z = x + iy$ and turn the complex equation into a real equation in x and y. A sum of conjugates is real: $z^2 + \bar{z}^2 = 2\,\mathrm{Re}(z^2) = 2(x^2 - y^2)$. Then read off the conic from its Cartesian form: $x^2 - y^2 = 1$ is a hyperbola.

WATCH OUT FOR

Treating $z^2 + \bar{z}^2$ as if it were $z\bar{z} = |z|^2 = x^2 + y^2$. That gives $x^2 + y^2 = 1$ and the wrong answer, circle (D). Another slip is to expand z^2 as $x^2 + y^2$, forgetting that $(iy)^2 = -y^2$. Squaring flips the sign of y^2, and that sign is what makes the curve a hyperbola and not a circle or an ellipse.

Q.12 MCQ · 1 MARK

Control Systems › Process control (IN)

The pressure drop across a control valve is constant. The control valve with inherent characteristic has decreasing sensitivity. If x represents the fraction of maximum stem position of the control valve, then the function $f(x)$ representing the fraction of maximum flow is

(A) α^{x-1}, where α is constant

(B) $\sqrt{x}$

(C) x

(D) x^2

Official answer: **B**

SOLUTION

1. The pressure drop across the valve is constant, so the valve behaves according to its **inherent** characteristic: the flow fraction $f(x) = \frac{q}{q_{max}}$ depends only on the stem fraction x, with $0 \le x \le 1$ and $f(1) = 1$.

2. Valve sensitivity is the slope of this characteristic, $S(x) = \dfrac{df}{dx}$: the change in flow fraction per unit change in stem fraction. 'Decreasing sensitivity' means S falls as the valve opens, i.e. $\dfrac{dS}{dx} = \dfrac{d^2 f}{dx^2} < 0$ (a concave curve).
3. (A) Equal percentage, $f = \alpha^{x-1}$: $S = (\ln \alpha)\, \alpha^{x-1}$ and $\dfrac{dS}{dx} = (\ln \alpha)^2\, \alpha^{x-1} > 0$ for any constant $\alpha > 0,\ \alpha \neq 1$. (Physically $\alpha > 1$, since $f(0) = 1/\alpha$ must be less than 1.) Sensitivity **increases**.
4. (B) Quick opening, $f = \sqrt{x}$: $S = \dfrac{1}{2\sqrt{x}}$ and $\dfrac{dS}{dx} = -\dfrac{1}{4} x^{-3/2} < 0$ for $x > 0$. Sensitivity is very high near closure and **decreases** as the valve opens.
5. (C) Linear, $f = x$: $S = 1$, so sensitivity is **constant**.
6. (D) $f = x^2$: $S = 2x$ and $\dfrac{dS}{dx} = 2 > 0$, so sensitivity **increases**.
7. Only $f(x) = \sqrt{x}$ (the quick-opening characteristic) has decreasing sensitivity. Answer: **(B)**.

KEY IDEA

Sensitivity is the slope df/dx of the inherent characteristic, so 'decreasing sensitivity' means a concave curve ($d^2 f/dx^2 < 0$). Quick opening ($\sqrt{x}$) is concave, linear (x) has constant slope, and equal percentage (α^{x-1}) and x^2 are convex, so their sensitivity increases.

WATCH OUT FOR

Picking equal percentage (A) because each equal increment of stem gives an equal *percentage* change in flow. Read as a relative change, $\dfrac{1}{f}\dfrac{df}{dx} = \ln \alpha$, that looks constant or 'gentle', but the absolute slope $\dfrac{df}{dx} = (\ln \alpha)\, \alpha^{x-1}$ grows with opening. The other trap is mixing up which curve is concave: quick opening rises steeply and then flattens, so its sensitivity is highest near closure.

Q.13 MCQ · 1 MARK

Signals and Systems › LTI & convolution

A discrete-time sequence is given by $x[n] = [1, 2, 3, 4]$ for $0 \leq n \leq 3$. The zero lag auto-correlation value of $x[n]$ is

(A) 1
(B) 10
(C) 20
(D) 30

Official answer: **D**

SOLUTION

1. For a real energy sequence, the autocorrelation at lag l is
$$R_{xx}[l] = \sum_{n=-\infty}^{\infty} x[n]\, x[n-l]$$
2. Set the lag to zero, $l = 0$:
$$R_{xx}[0] = \sum_{n=-\infty}^{\infty} x^2[n]$$
This is exactly the energy of the sequence.
3. The sequence is non-zero only for $0 \leq n \leq 3$, so the sum has four terms:
$$R_{xx}[0] = 1^2 + 2^2 + 3^2 + 4^2 = 1 + 4 + 9 + 16 = 30$$
4. The zero-lag autocorrelation is therefore 30, which is option (D).

KEY IDEA

At zero lag the sequence is multiplied by an unshifted copy of itself, so $R_{xx}[0] = \sum_n x^2[n]$, which is the signal's energy. It is also the largest value the autocorrelation takes at any lag: $R_{xx}[0] \geq |R_{xx}[l]|$.

WATCH OUT FOR

Adding the samples instead of their squares gives $1 + 2 + 3 + 4 = 10$, option (B), which is a trap. Dividing by the number of samples (average power, $30/4 = 7.5$) is also wrong here: the question asks for the autocorrelation of an energy signal, not a time-averaged one.

Q.14 MCQ · 1 MARK

Sensors and Industrial Instrumentation › Sensing principles

Q.14 Match the following measuring devices with their principle of measurement.

Measuring Device	Principle of Measurement
(P) Optical pyrometer	(I) Variation in mutual inductance
(Q) Thermocouple	(II) Change in resistance
(R) Strain gauge	(III) Wavelength of radiated energy
(S) Linear variable differential transformer	(IV) Electromotive force generated by two dissimilar metals

(A) (P) – (III), (Q) – (IV), (R) – (II), (S) – (I)

(B) (P) – (IV), (Q) – (III), (R) – (II), (S) – (I)

(C) (P) – (III), (Q) – (I), (R) – (IV), (S) – (II)

(D) (P) – (II), (Q) – (IV), (R) – (I), (S) – (III)

Official answer: **A**

SOLUTION

1. Optical pyrometer (P): a non-contact temperature instrument that compares the brightness or colour (wavelength) of the radiation from a hot body with that of a calibrated reference filament. So (P)

matches (III), wavelength of radiated energy.

2. Thermocouple (Q): the Seebeck effect. Two dissimilar metals joined at a junction generate an electromotive force that depends on the temperature difference between the measuring junction and the reference junction. So (Q) matches (IV).
3. Strain gauge (R): a strain ε changes the resistance of the gauge wire, with gauge factor $G = \dfrac{\Delta R/R}{\varepsilon}$. So (R) matches (II), change in resistance.
4. Linear variable differential transformer (S): the core position changes the mutual inductance between the primary and the two secondary windings, so the differential output voltage is proportional to the core displacement. So (S) matches (I).
5. The pairing is (P)-(III), (Q)-(IV), (R)-(II), (S)-(I), which is option (A). Option (B) puts (P) with (IV) and (Q) with (III). Option (C) puts (Q) with (I) and (R) with (IV). Option (D) puts (P) with (II), (R) with (I) and (S) with (III). Each of these breaks at least one pairing above, so all three are wrong.

KEY IDEA

Match each sensor to the physical quantity it actually converts. A pyrometer reads radiated energy, a thermocouple produces a Seebeck EMF, a strain gauge changes its resistance, and an LVDT changes the mutual inductance between its windings.

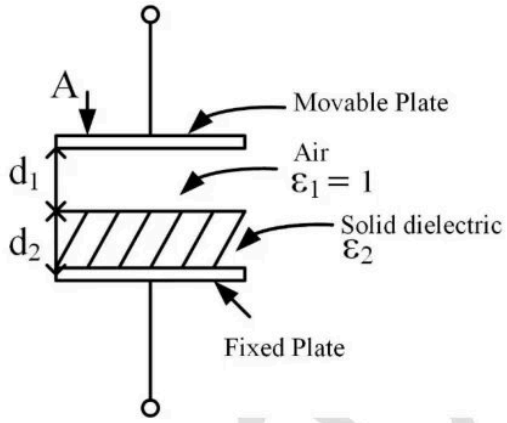
WATCH OUT FOR

Pairing the thermocouple with a wavelength or the pyrometer with an EMF, because both are temperature sensors. Another slip is pairing the strain gauge with mutual inductance, when it is a resistive sensor and the LVDT is the inductive one.

Q.15 MCQ · 1 MARK

Electromagnetics › Electrostatics

Q.15 The capacitor shown in the figure has parallel plates, with each plate having an area A. The thickness of the dielectric materials are d_1 and d_2 and their relative permittivities are ε_1 and ε_2, respectively. Assume that the fringing field effects are negligible and ε_0 is the permittivity of free space.



If d_1 is decreased by δd_1, the resultant capacitance becomes

(A) $\dfrac{\varepsilon_0 A}{d_1 - \delta d_1 + \dfrac{d_2}{\varepsilon_2}}$

(B) $\dfrac{\varepsilon_0 A}{d_2 + \dfrac{d_1}{\varepsilon_2}}$

(C) $\dfrac{\varepsilon_0 A}{d_2 - \delta d_2 + \dfrac{d_1}{\varepsilon_2}}$

(D) $\dfrac{\varepsilon_0 A}{d_1 + \delta d_1 + \dfrac{d_2}{\varepsilon_2}}$

Official answer: **A**

SOLUTION

1. **Identify the connection.** The air layer and the solid dielectric are stacked between the plates, and each covers the full plate area A. With fringing neglected, the same electric flux density $\mathbf{D}$ passes through both layers, so they behave as **two capacitors in series**.
2. **Capacitance of each layer** (parallel plate, $C = \dfrac{\varepsilon_0 \varepsilon_r A}{d}$, with $\varepsilon_0 \approx 8.854 \times 10^{-12}\ \mathrm{F/m}$). The air layer has $\varepsilon_1 = 1$:

$$C_1 = \frac{\varepsilon_0(1)A}{d_1}, \qquad C_2 = \frac{\varepsilon_0\varepsilon_2 A}{d_2}$$

3. **Series combination** $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$:
$$\frac{1}{C} = \frac{d_1}{\varepsilon_0 A} + \frac{d_2}{\varepsilon_0\varepsilon_2 A} \Rightarrow C = \frac{\varepsilon_0 A}{d_1 + \frac{d_2}{\varepsilon_2}}$$
4. **Apply the change.** Only the movable plate moves, so only the air gap shrinks: $d_1 \to d_1 - \delta d_1$. The slab resting on the fixed plate keeps its thickness d_2 and relative permittivity ε_2.
5. **Resulting capacitance:**
$$C' = \frac{\varepsilon_0 A}{d_1 - \delta d_1 + \frac{d_2}{\varepsilon_2}}$$

This is option (A). As a check, a smaller air gap makes the denominator smaller and so gives a larger capacitance, as it should.

KEY IDEA

When dielectric layers are stacked between the plates and each covers the full plate area, they are capacitors in series. The total capacitance is $C = \frac{\varepsilon_0 A}{\sum_i d_i/\varepsilon_{ri}}$: each layer adds its "equivalent air thickness" d_i/ε_{ri}. Moving the plate changes only the air-gap term.

WATCH OUT FOR

Dividing the wrong thickness by ε_2: the air gap ($\varepsilon_1 = 1$) enters as plain d_1 and only the slab enters as d_2/ε_2, so options (B) and (C) have the roles reversed. Another slip is writing $d_1 + \delta d_1$ (option D), which is an increase in the gap, not a decrease. A third is treating the stacked layers as capacitors in parallel.

Q.16 MCQ · 1 MARK

Digital Electronics › Boolean logic

Among the given options, the simplified form of the Boolean function $F = (A + \overline{A}.B) + \overline{A}.(A + \overline{B}).C$ is

(A) $A + B + C$

(B) $A.B.C$

(C) $B + \overline{A}.C$

(D) $\overline{A} + B.C$

Official answer: **A**

SOLUTION

1. Notation: $\overline{X}$ is the complement of X, and a dot (or two letters side by side) means AND. Start with the first bracket. The redundancy theorem $X + \overline{X}Y = X + Y$ with $X = A$, $Y = B$ gives
$$A + \overline{A}.B = A + B$$
2. Expand the second term with the distributive law:
$$\overline{A}.(A + \overline{B}).C = \overline{A}AC + \overline{A}\,\overline{B}C$$
3. Complement law: $\overline{A}A = 0$. So the second term is just $\overline{A}\,\overline{B}C$.
4. Put the two parts together:
$$F = A + B + \overline{A}\,\overline{B}C$$
5. Apply the redundancy theorem again, with $X = A$ and $Y = \overline{B}C$: $A + \overline{A}(\overline{B}C) = A + \overline{B}C$. So $F = A + B + \overline{B}C$.
6. Apply it once more, with $X = B$ and $Y = C$: $B + \overline{B}C = B + C$. Therefore
$$F = A + B + C$$

7. Check: $F = A + B + \overline{A}\,\overline{B}C$ is 0 only when $A = 0$, $B = 0$ and $\overline{A}\,\overline{B}C = 0$. With $A = B = 0$ the last term is just C, so $C = 0$ as well. F is therefore 0 only at $ABC = 000$, which is exactly $A + B + C$. The other options fail at a single input. At $ABC = 001$ we have $F = 1$, but (B) $A.B.C = 0$. At $ABC = 100$ we have $F = 1$, but (C) $B + \overline{A}.C = 0$ and (D) $\overline{A} + B.C = 0$. The answer is option (A).

KEY IDEA

Use the redundancy theorem $X + \overline{X}Y = X + Y$ again and again. It first turns $A + \overline{A}B$ into $A + B$. After $\overline{A}A = 0$ removes one product, it takes $A + B + \overline{A}\,\overline{B}C$ down to $A + B + C$. A quick way to check: F is 0 only when every input is 0, and that function is the OR of all the inputs.

WATCH OUT FOR

Losing the leading A partway through. Redundancy on B alone correctly gives $B + \overline{A}\,\overline{B}C = B + \overline{A}C$, which is option (C). But F still has the A term, and $A + \overline{A}C = A + C$ turns it into $A + B + C$. Other mistakes are keeping $\overline{A}AC$ as a real term instead of setting it to 0, or thinking the whole second product is absorbed just because it contains A.

Q.17 MCQ · 1 MARK

Control Systems › State space

Consider the state-space representation of a system

$$\dot{x} = Ax + Bu$$

where x is the state vector, u is the input, A is the system matrix and B is the input matrix. Choose the matrix A from the following options such that the system has a pole at the origin.

(A) $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & -1.5 \\ -2 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1.5 \\ 2 & -3 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$

Official answer: **B**

SOLUTION

1. **Poles are the eigenvalues of A.** They are the roots of the characteristic equation $|sI - A| = 0$. For a 2×2 matrix this is $s^2 - (\operatorname{tr} A)\, s + \det A = 0$.
2. **Condition for a pole at the origin.** Put $s = 0$ in $|sI - A| = 0$: you get $\det(-A) = 0$. For a 2×2 matrix $\det(-A) = (-1)^2 \det A = \det A$, so the condition is $\det A = 0$. In other words, A must be singular. (Equivalently, the product of the eigenvalues equals $\det A$, so one eigenvalue is zero exactly when $\det A = 0$.)
3. **(A)** $\det A = (0)(-3) - (1)(-2) = 2 \neq 0$. The characteristic equation is $s^2 + 3s + 2 = 0$, so the poles are -1 and -2. No pole at the origin.
4. **(B)** $\det A = (1)(3) - (-1.5)(-2) = 3 - 3 = 0$. This matrix is singular.
5. **(C)** $\det A = (1)(-3) - (1.5)(2) = -3 - 3 = -6 \neq 0$. No pole at the origin.
6. **(D)** $\det A = (0)(3) - (1)(-2) = 2 \neq 0$. The characteristic equation is $s^2 - 3s + 2 = 0$, so the poles are 1 and 2. No pole at the origin.
7. **Check (B).** $\operatorname{tr} A = 1 + 3 = 4$ and $\det A = 0$, so $s^2 - 4s = s(s - 4) = 0$. The poles are $s = 0$ and $s = 4$, so one pole is at the origin. The answer is (B).

KEY IDEA

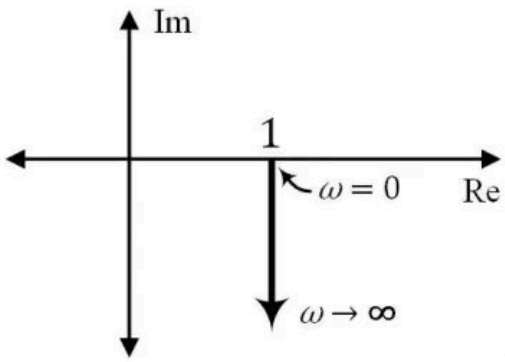
The poles of $\dot{x} = Ax + Bu$ are the eigenvalues of A. The product of the eigenvalues equals $\det A$, so the system has a pole at the origin exactly when $\det A = 0$. You only need to find the one singular matrix, and you never have to solve a quadratic.

WATCH OUT FOR

One mistake is picking (A) or (D) because they have a zero on the diagonal. A zero entry says nothing about the eigenvalues; both of those matrices have $\det A = 2$. The other is a sign slip in (B): $(-1.5)(-2) = +3$, not -3. With that slip you get $\det A = 6$ and throw away the correct option.

Q.18 MCQ · 1 MARK

Control Systems › Bode & margins

Q.18	The sinusoidal transfer function corresponding to the polar plot shown in the figure, for $T > 0$, is
	
(A)	$1 - j\omega T$
(B)	$\dfrac{1 - j\omega T}{1 + j\omega T}$
(C)	$1 + j\omega T$
(D)	$\dfrac{1}{1 + j\omega T}$

Official answer: **A**

SOLUTION

1. Read the plot. The real part stays at 1 for every ω. The imaginary part starts at 0 when $\omega = 0$ and falls without limit as $\omega \to \infty$. So we need $\mathrm{Re}\, G(j\omega) = 1$ for all ω and $\mathrm{Im}\, G(j\omega)$ decreasing from 0 towards $-\infty$.
2. Option (A), $G(j\omega) = 1 - j\omega T$: $\mathrm{Re} = 1$ and $\mathrm{Im} = -\omega T$. With $T > 0$ it starts at $1 + j0$ for $\omega = 0$ and moves straight down to $-j\infty$ as $\omega \to \infty$. This matches the plot exactly.

3. Option (C), $G(j\omega) = 1 + j\omega T$: it lies on the same line $\mathrm{Re} = 1$, but $\mathrm{Im} = +\omega T$ goes to $+j\infty$, so the locus would run upward. Rejected.
4. Option (B), all-pass: $|G| = \dfrac{\sqrt{1+\omega^2T^2}}{\sqrt{1+\omega^2T^2}} = 1$ for every ω, and $\angle G = -2\tan^{-1}\omega T$ goes from 0° to -180°. That traces the lower half of the unit circle from 1 to -1, not a vertical line. Rejected.
5. Option (D), first-order lag: $|G| = \dfrac{1}{\sqrt{1+\omega^2T^2}} \to 0$ and $\angle G = -\tan^{-1}\omega T \to -90^\circ$. That traces a lower semicircle of diameter 1 from 1 to the origin, not a line going to infinity. Rejected.
6. So the plot is $G(j\omega) = 1 - j\omega T$, which is $G(s) = 1 - sT$: a single zero at $s = +1/T$ in the right half-plane (non-minimum phase). Its magnitude $\sqrt{1+\omega^2T^2}$ grows without limit while its phase $-\tan^{-1}\omega T$ goes to -90°, which is why the locus heads straight down. The answer is (A).

KEY IDEA

Split $G(j\omega)$ into real and imaginary parts and match them to the plot. A vertical line at $\mathrm{Re} = 1$ means the real part does not depend on ω, so G must be 1 plus a purely imaginary term. The direction the line runs (down to $-j\infty$) fixes the sign of that term as $-j\omega T$. Neither rational option can do this: they stay bounded, tracing the unit circle (all-pass) or a semicircle into the origin (lag).

WATCH OUT FOR

Picking (C), $1 + j\omega T$, because it gives the same vertical line. It runs upward to $+j\infty$, while the plot runs down. The other trap is choosing the all-pass (B) because it also has a right-half-plane zero and starts at 1. Its magnitude stays at 1, so it traces a semicircle ending at -1 and never heads off to infinity.

Q.19 MCQ · 1 MARK

Engineering Mathematics › Linear algebra

A matrix M is constructed by stacking three column vectors v_1, v_2, v_3 as

$$M = [v_1 \quad v_2 \quad v_3].$$

Choose the set of vectors from the following options such that $rank(M) = 3$.

(A) $v_1 = \begin{bmatrix}1\\0\\1\end{bmatrix}$, $v_2 = \begin{bmatrix}0\\-1\\0\end{bmatrix}$, $v_3 = \begin{bmatrix}1\\-1\\1\end{bmatrix}$

(B) $v_1 = \begin{bmatrix}1\\1\\1\end{bmatrix}$, $v_2 = \begin{bmatrix}-1\\0\\1\end{bmatrix}$, $v_3 = \begin{bmatrix}0\\0\\0\end{bmatrix}$

(C) $v_1 = \begin{bmatrix}1\\0\\1\end{bmatrix}$, $v_2 = \begin{bmatrix}-1\\0\\1\end{bmatrix}$, $v_3 = \begin{bmatrix}1\\-1\\1\end{bmatrix}$

(D) $v_1 = \begin{bmatrix}1\\1\\1\end{bmatrix}$, $v_2 = \begin{bmatrix}-1\\1\\-1\end{bmatrix}$, $v_3 = \begin{bmatrix}0\\-1\\0\end{bmatrix}$

Official answer: **C**

SOLUTION

1. A 3×3 matrix has $rank(M) = 3$ exactly when its three columns are linearly independent, which is the same as $\det M \neq 0$. So we look for the option in which no column is a combination of the others.
2. Option (A): $v_1 + v_2 = \begin{bmatrix}1\\-1\\1\end{bmatrix} = v_3$. The columns are dependent. Since v_1 and v_2 are not multiples of each other, the rank is 2.

3. Option (B): v_3 is the zero vector, and any set containing the zero vector is linearly dependent. The rank is therefore at most 2 (it is exactly 2, because v_1 and v_2 are not parallel).
4. Option (D): $v_1 + v_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} = -2\,v_3$. The columns are dependent. Since v_1 and v_2 are not parallel, the rank is 2.
5. Option (C): $M = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$. Expand the determinant along row 2. Its only nonzero entry is $m_{23} = -1$, with cofactor sign $(-1)^{2+3} = -1$, and deleting row 2 and column 3 leaves the minor $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$:

$$\det M = (-1) \cdot (-1)^5 \cdot \det \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = (1)\,(1 \cdot 1 - (-1) \cdot 1) = 2 \neq 0.$$

6. Because $\det M = 2 \neq 0$, the three columns in (C) are linearly independent and $rank(M) = 3$. The answer is (C).

KEY IDEA

A square $n \times n$ matrix has full rank n exactly when its columns are linearly independent, i.e. when its determinant is nonzero. To rule out an option quickly, look for an obvious dependence: a zero column, or one column equal to a sum or multiple of the others. Then confirm the remaining option with a determinant, expanding along the row or column with the most zeros.

WATCH OUT FOR

Seeing that no two columns are parallel and concluding the rank is 3. Pairwise non-parallel columns are not enough: in (A) and (D) no column is a multiple of another, yet v_3 is a combination of v_1 and v_2, so the rank is only 2. A second slip is dropping the cofactor sign $(-1)^{i+j}$ in a row expansion. Here it does not change whether $\det M$ is zero, but it does give the wrong determinant.

Q.20 MCQ · 1 MARK

Electromagnetics › Electrostatics

The capacitance formed between two concentric spherical metal shells having radii x and y with $y > x$ is

Note: ϵ is the permittivity of the medium between the shells.

(A) $4\pi\epsilon\left(\dfrac{xy}{y-x}\right)$

(B) $4\pi\epsilon\left(\dfrac{x^2}{y-x}\right)$

(C) $4\pi\epsilon\left(\dfrac{y^2}{y-x}\right)$

(D) $4\pi\epsilon\left(\dfrac{y^2-xy}{x}\right)$

Official answer: **A**

SOLUTION

1. Put charge $+Q$ on the inner shell (radius x) and $-Q$ on the outer shell (radius y). The charges sit on the facing surfaces, so the field exists only in the gap $x < r < y$. Inside the inner shell no charge is enclosed, and outside the outer shell the net enclosed charge is zero.
2. Apply Gauss's law over a concentric sphere of radius r in the gap: $\epsilon E_r\,(4\pi r^2) = Q$, so

$$\mathbf{E} = \frac{Q}{4\pi\epsilon r^2}\,\hat{a}_r, \quad x < r < y.$$

3. Find the potential of the inner shell relative to the outer one, $V_{xy} = -\int_y^x E_r\,dr$:

$$V_{xy} = -\frac{Q}{4\pi\epsilon}\int_{y}^{x}\frac{dr}{r^2} = \frac{Q}{4\pi\epsilon}\left(\frac{1}{x}-\frac{1}{y}\right) = \frac{Q}{4\pi\epsilon}\cdot\frac{y-x}{xy}.$$

This is positive because $y > x$.

4. Capacitance is $C = Q/V_{xy}$:

$$C = 4\pi\epsilon\left(\frac{xy}{y-x}\right).$$

5. Check the limit. As $y \to \infty$, $\dfrac{xy}{y-x} \to x$, so $C \to 4\pi\epsilon x$, which is the capacitance of an isolated sphere of radius x. Option (B) goes to 0 in this limit, and options (C) and (D) grow without bound, so none of them reduces to the isolated sphere. Answer: (A).

KEY IDEA

Assume charges $\pm Q$ on the two shells. Gauss's law gives $E = Q/(4\pi\epsilon r^2)$ in the gap. Integrate from x to y to get $V = \dfrac{Q}{4\pi\epsilon}\left(\dfrac{1}{x}-\dfrac{1}{y}\right)$, then $C = Q/V = 4\pi\epsilon\,\dfrac{xy}{y-x}$. To check the answer quickly, let the outer radius go to infinity: the result must become the isolated-sphere value $4\pi\epsilon x$.

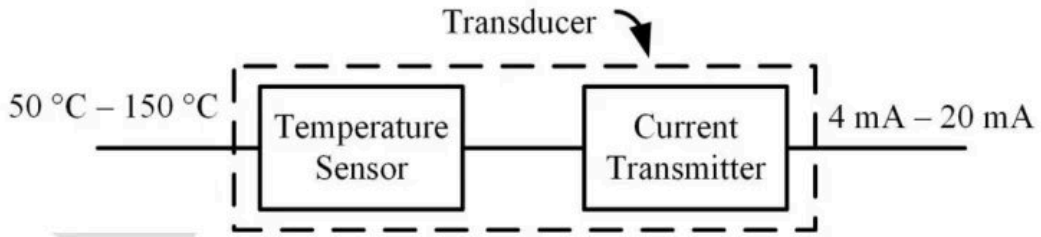
WATCH OUT FOR

Students often slip when combining $\dfrac{1}{x}-\dfrac{1}{y}$ into one fraction. Some write $\dfrac{x-y}{xy}$, which is negative, or $\dfrac{y-x}{y^2}$, and then pick (C) or (B). Others use the coaxial-cable result $2\pi\epsilon L/\ln(y/x)$, which applies to cylinders, not spheres. Putting $y \to \infty$ exposes all of these errors at once.

Q.21 NAT · 1 MARK

Sensors and Industrial Instrumentation › 4-20 mA & networks

Q.21 A linear transducer is calibrated for the ranges shown in the figure. The gain of the transducer is _____ mA/°C (**rounded off to two decimal places**).



Official answer: **0.15 to 0.17**

SOLUTION

1. The dashed block (temperature sensor followed by current transmitter) is one linear transducer. Its gain is the slope of output against input: $K = \dfrac{\Delta\text{output}}{\Delta\text{input}}$.
2. Input span: $150\,^{\circ}\text{C} - 50\,^{\circ}\text{C} = 100\,^{\circ}\text{C}$.
3. Output span: $20\,\text{mA} - 4\,\text{mA} = 16\,\text{mA}$. The 4 mA offset at 50 °C is a zero shift and does not enter the gain.
4. $$K = \frac{16\ \text{mA}}{100\ ^{\circ}\text{C}} = 0.16\ \text{mA}/^{\circ}\text{C}$$
5. The gain is therefore $0.16\ \text{mA}/^{\circ}\text{C}$.

KEY IDEA
For a linear transducer, gain is the slope: output span divided by input span. The calibrated ranges give the spans directly, and the offset (4 mA at 50 °C) does not affect the slope.

WATCH OUT FOR
Dividing the end values (20 mA / 150 °C or 4 mA / 50 °C) instead of the spans, or counting the 4 mA live-zero offset as part of the gain.

Q.22 NAT · 1 MARK

Signals and Systems › FIR & IIR filters

Consider a filter defined by the difference equation

$$y[n] - 0.5\,y[n-2] = a\,x[n-4]$$

where $x[n]$ and $y[n]$ represent the input and output, respectively. If the magnitude response of the filter at $\omega = \frac{\pi}{2}$ is $\left|H\left(\frac{\pi}{2}\right)\right| = 0.5$, the value of a is _____ (rounded off to two decimal places).

Official answer: **0.70 to 0.80**

SOLUTION

1. Take the DTFT of both sides, using the time-shift property $x[n-k] \leftrightarrow e^{-j\omega k}X(e^{j\omega})$:
$$Y(e^{j\omega})\left(1 - 0.5\,e^{-j2\omega}\right) = a\,e^{-j4\omega}X(e^{j\omega})$$
2. Frequency response:
$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{a\,e^{-j4\omega}}{1 - 0.5\,e^{-j2\omega}}$$
(The equivalent $H(z) = \dfrac{a\,z^{-4}}{1 - 0.5\,z^{-2}}$ has poles at $z = \pm\dfrac{1}{\sqrt{2}}$, inside the unit circle, so the causal filter is stable and its frequency response exists.)
3. At $\omega = \frac{\pi}{2}$: $e^{-j2\omega} = e^{-j\pi} = -1$ and $e^{-j4\omega} = e^{-j2\pi} = 1$.
4. So
$$H\left(\frac{\pi}{2}\right) = \frac{a \cdot 1}{1 - 0.5(-1)} = \frac{a}{1.5}$$
5. Magnitude condition:
$$\left|H\left(\frac{\pi}{2}\right)\right| = \frac{|a|}{1.5} = 0.5 \;\Rightarrow\; |a| = 0.75$$
6. The magnitude fixes only $|a|$; the value asked for is the positive gain, $a = 0.75$. (A gain of -0.75 would give the same magnitude, since it only adds a phase of π.)

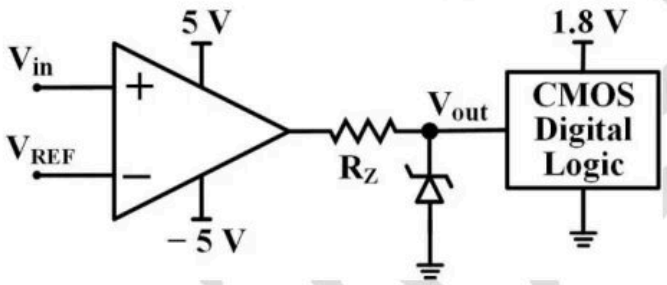
KEY IDEA
Delays in the difference equation become factors $e^{-j\omega k}$ in the frequency response. At $\omega = \pi/2$, $e^{-j2\omega} = -1$ and $e^{-j4\omega} = 1$, so the input delay $x[n-4]$ has magnitude 1 and drops out, and the denominator becomes $1 + 0.5 = 1.5$.

WATCH OUT FOR
Taking $e^{-j2\omega}$ as $+1$ at $\omega = \pi/2$ (it is $e^{-j\pi} = -1$). That gives a denominator of 0.5 and the wrong answer $a = 0.25$. Another mistake is thinking the delay $x[n-4]$ changes the magnitude. It only adds phase, because $|e^{-j4\omega}| = 1$ at every frequency.

Q.23 NAT · 1 MARK

Analog Electronics › Diode circuits

Q.23 Consider the circuit shown in the figure.



The CMOS digital logic circuit has infinite input impedance. Assume the opamp is ideal. A 1.8 V Zener diode with a minimum Zener current of 2 mA is used. The corresponding maximum value of resistance R_Z is _____ kΩ. **(rounded off to one decimal place).**

Official answer: **1.6**

SOLUTION

1. **What the op-amp does.** There is no feedback, so the ideal op-amp works as a comparator. Its output goes to one supply rail or the other: $+5$ V when $V_{in} > V_{REF}$, and -5 V when $V_{in} < V_{REF}$. An ideal op-amp saturates exactly at its rails, so $V_{sat} = 5$ V.
2. **Which state sets the limit.** The Zener cathode is at V_{out} and its anode is at ground. When the output is high ($+5$ V), the Zener is reverse-biased and in breakdown it holds $V_{out} = V_Z = 1.8$ V, the logic-high level the 1.8 V CMOS gate needs. This is the only state where the Zener has to regulate. When the output is -5 V, the Zener is forward-biased and clamps V_{out} near -0.7 V, so the breakdown current condition does not apply there.
3. **KCL at the V_{out} node.** The CMOS input impedance is infinite, so the logic gate draws no current. All the current in R_Z therefore flows into the Zener: $I_Z = I_{R_Z}$.
4. **Ohm's law across R_Z** (output high, Zener in breakdown):
$$I_Z = \frac{V_{sat} - V_Z}{R_Z} = \frac{5 - 1.8}{R_Z} = \frac{3.2\ \text{V}}{R_Z}$$
5. **Keep the Zener in breakdown.** I_Z falls as R_Z rises, so the condition $I_Z \geq I_{Z,\min} = 2$ mA puts an upper limit on R_Z:
$$\frac{3.2}{R_Z} \geq 2 \times 10^{-3} \;\Rightarrow\; R_Z \leq \frac{3.2}{2 \times 10^{-3}} = 1600\ \Omega$$
6. **Result.** $R_{Z,\max} = 1.6$ kΩ. With any larger resistance the Zener current drops below 2 mA, and V_{out} is no longer held at 1.8 V.

KEY IDEA

The open-loop op-amp is a comparator, so its high output is the $+5$ V rail. The CMOS input draws no current, so all of $I_{R_Z} = (5 - 1.8)/R_Z$ goes into the Zener. Setting this current to at least $I_{Z,\min} = 2\ \text{mA}$ gives an upper limit on R_Z.

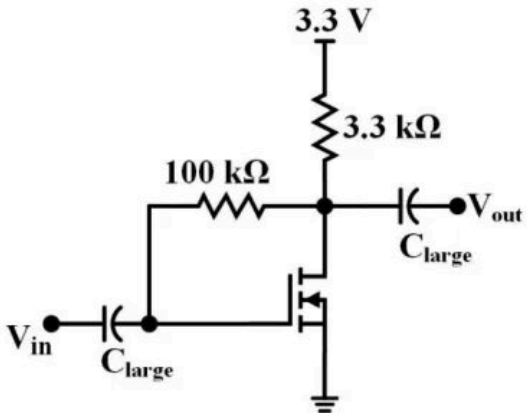
WATCH OUT FOR

Using the Zener voltage alone, $1.8/2\ \text{mA} = 0.9\ \text{k}\Omega$, when the drop across R_Z is actually $5 - 1.8 = 3.2$ V. A second slip is to treat the -5 V state as the limiting case, or to subtract load current into the CMOS gate, which draws none.

Q.24 NAT · 1 MARK

Analog Electronics › Biasing

Q.24 Figure shows an amplifier using an NMOS transistor. Assume that the transistor is in saturation with device parameters, $\mu_n C_{ox} = 250\ \mu\text{A/V}^2$, threshold voltage $V_T = 0.65$ V and W/L = 4. Ignore the channel length modulation effect. The drain current of the transistor at the operating point is _____ μA **(rounded off to nearest integer)**.



Official answer: **498 to 502**

SOLUTION

1. DC analysis: both C_{large} capacitors are open circuits at DC. What remains is the 3.3 V supply, the 3.3 kΩ drain resistor, the 100 kΩ drain-to-gate resistor and the MOSFET, whose source is grounded.
2. The MOSFET gate draws no DC current, so no current flows in the 100 kΩ resistor and there is no voltage across it. So $V_G = V_D$, which means $V_{GS} = V_{DS}$ because the source is grounded. Since $V_{DS} = V_{GS} > V_{GS} - V_T$, the device is indeed in saturation, as the question says.
3. Saturation square law, with channel-length modulation neglected:
$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_T)^2 = \frac{1}{2}(250\ \mu\text{A/V}^2)(4)(V_{GS} - 0.65)^2 = 500\ \mu\text{A/V}^2\,(V_{GS} - 0.65)^2$$
4. KVL from the supply through the drain resistor. All of I_D flows in the 3.3 kΩ resistor, because the feedback resistor carries none:
$$V_{GS} = V_D = 3.3\ \text{V} - (3.3\ \text{k}\Omega)\,I_D$$
5. Let $x = V_{GS} - 0.65$ (in volts). Substituting the square law into the KVL equation:
$$3.3 - 0.65 - x = (3.3\ \text{k}\Omega)(500\ \mu\text{A/V}^2)\,x^2 = 1.65\,x^2 \ \Rightarrow\ 1.65x^2 + x - 2.65 = 0$$
6. Quadratic formula:
$$x = \frac{-1 \pm \sqrt{1 + 4(1.65)(2.65)}}{2(1.65)} = \frac{-1 \pm \sqrt{18.49}}{3.3} = \frac{-1 \pm 4.3}{3.3}$$
The negative root $x \approx -1.61$ V would make $V_{GS} < V_T$. The transistor would then be off and the square law would not apply, so that root is rejected. That leaves $x = 1$ V, so $V_{GS} = 1.65$ V.

7. $I_D = 500\ \mu\text{A/V}^2 \times (1\ \text{V})^2 = 500\ \mu\text{A}$. Check with the drain resistor: $(3.3 - 1.65)\ \text{V}/3.3\ \text{k}\Omega = 0.5\ \text{mA} = 500\ \mu\text{A}$, which agrees.

KEY IDEA

The gate draws no DC current, so the 100 kΩ feedback resistor has no voltage across it. This forces $V_{GS} = V_{DS}$, which keeps the transistor in saturation (a diode-connected bias). Solve the square law and the drain-resistor KVL together, then keep only the root with $V_{GS} > V_T$.

WATCH OUT FOR

One common slip is including a voltage drop across the 100 kΩ resistor, or letting it share current with the drain resistor, when it carries no DC current. Another is forgetting the factor $\frac{1}{2}$ in the square law, which gives $k = 1000\ \mu\text{A/V}^2$ and the wrong current. A third is keeping the negative root of the quadratic, which puts V_{GS} below V_T where the square law does not hold.

Q.25 NAT · 1 MARK

Signals and Systems › DTFT & DFT

The number of complex multiplications required for computing a 16-point DFT using the decimation-in-time radix-2 FFT is _____ (in integer).

Official answer: **32**

SOLUTION

1. **Number of stages (radix-2 FFT).** $N = 16 = 2^4$, so the DIT FFT has $\log_2 N = 4$ stages.
2. **Butterflies per stage.** Each stage combines the N points in pairs, so there are $\dfrac{N}{2} = 8$ butterflies per stage.
3. **Cost of one DIT butterfly.** The butterfly computes $A' = A + W_N^k B$ and $B' = A - W_N^k B$. The product $W_N^k B$ is formed once and used in both outputs, so each butterfly needs **1** complex multiplication and 2 complex additions. (The standard count treats every twiddle-factor product as one complex multiplication, even when W_N^k is 1 or $-j$.)
4. **Total multiplications.**
$$\text{Complex multiplications} = \frac{N}{2}\log_2 N = 8 \times 4 = 32$$
5. **Check against the direct DFT.** Computing the DFT directly needs $N^2 = 256$ complex multiplications, so the FFT cuts the count by a factor of $256/32 = 8$. The FFT's complex additions number $N \log_2 N = 64$.

KEY IDEA

A radix-2 FFT of length $N = 2^m$ has $\log_2 N$ stages, each with $N/2$ butterflies. Each DIT butterfly needs one complex multiplication, by the twiddle factor W_N^k, and two complex additions. That gives $\frac{N}{2}\log_2 N$ multiplications and $N \log_2 N$ additions in total.

WATCH OUT FOR

There are three usual slips. The first is counting two multiplications per butterfly (one for each output), which gives $N \log_2 N = 64$, but the product $W_N^k B$ is computed once and shared by both outputs. The second is answering with the direct-DFT count $N^2 = 256$. The third is counting only the non-trivial twiddle products, skipping every W_N^k equal to 1 or $-j$. That gives a smaller number, but it is not the standard count the question asks for.

Q.26 NAT · 1 MARK

Engineering Mathematics › Linear algebra

A 3×3 matrix P with all real elements has eigenvalues $\frac{1}{4}$, 1, and -2. The value of $|P^{-1}|$ is _____ (rounded off to nearest integer).

Official answer: **-2**

SOLUTION

1. The determinant of a square matrix equals the product of its eigenvalues (counted with multiplicity). A 3×3 matrix has exactly three eigenvalues, and here they are $\frac{1}{4}$, 1 and -2, so
$$|P| = \lambda_1\lambda_2\lambda_3 = \frac{1}{4} \times 1 \times (-2) = -\frac{1}{2}.$$
2. No eigenvalue is zero, so $|P| \neq 0$ and P is invertible.
3. If $P\mathbf{x} = \lambda\mathbf{x}$ with $\lambda \neq 0$, then $\mathbf{x} = \lambda P^{-1}\mathbf{x}$, so $P^{-1}\mathbf{x} = \frac{1}{\lambda}\mathbf{x}$. The eigenvalues of P^{-1} are therefore the reciprocals: $4,\ 1,\ -\frac{1}{2}$.
4. Hence
$$|P^{-1}| = 4 \times 1 \times \left(-\frac{1}{2}\right) = -2.$$
The same result follows from $|P^{-1}| = \dfrac{1}{|P|} = \dfrac{1}{-1/2} = -2.$
5. Here $|P^{-1}|$ is determinant notation (not absolute value), so the value is -2. It is already an integer, so rounding changes nothing.

KEY IDEA

$|P|$ is the product of the eigenvalues, and the eigenvalues of P^{-1} are their reciprocals. So $|P^{-1}| = \dfrac{1}{|P|} = \dfrac{1}{\frac{1}{4} \cdot 1 \cdot (-2)} = -2.$

WATCH OUT FOR

Reading $|P^{-1}|$ as an absolute value and answering 2 (it is the determinant, and it is negative here). Another slip is to give $|P| = -\frac{1}{2}$ and round it to 0 or -1, forgetting that the question asks for the determinant of the inverse.

Q.27 NAT · 1 MARK

Signals and Systems › Sampling

The Nyquist sampling frequency for $x(t) = 10\sin^2(200\pi t)$ is _____ Hz (**rounded off to nearest integer**).

Official answer: **400**

SOLUTION

1. Write the square as a sum of sinusoids using $\sin^2\theta = \dfrac{1 - \cos 2\theta}{2}$ with $\theta = 200\pi t$:
$$x(t) = 10\sin^2(200\pi t) = 10 \cdot \frac{1 - \cos(400\pi t)}{2} = 5 - 5\cos(400\pi t)$$
2. The signal now has only two components: a DC term at 0 Hz and a cosine with $\omega = 400\pi$ rad/s, which is $f = \dfrac{400\pi}{2\pi} = 200$ Hz.
3. So the signal is band-limited, and its highest frequency is $f_{max} = 200$ Hz. It is not 100 Hz, because squaring a sinusoid doubles its frequency.
4. By the sampling theorem, the Nyquist sampling frequency (Nyquist rate) is
$$f_s = 2f_{max} = 2 \times 200 = 400 \text{ Hz}$$

KEY IDEA

Before you apply $f_s = 2f_{max}$ to a non-linear expression, first rewrite it as a sum of pure sinusoids. The identity $\sin^2\theta = \frac{1-\cos 2\theta}{2}$ turns $10\sin^2(200\pi t)$ into a DC term plus a cosine at 400π rad/s (200 Hz), so $f_{max} = 200$ Hz and the Nyquist rate is 400 Hz.

WATCH OUT FOR

Reading f_{max} straight off the argument $200\pi t$ gives 100 Hz and the wrong answer 200 Hz, because it misses that squaring doubles the frequency. A second slip is dividing by 2π incorrectly and treating 400π as 400 Hz, which gives 800 Hz.

Q.28 NAT · 1 MARK

Electrical and Electronic Measurements › Errors & standards

The resistance of a 20 kΩ resistor is measured six consecutive times using an LCR meter. The first five readings are 19 kΩ, 18 kΩ, 23 kΩ, 21 kΩ and 17 kΩ. If the mean of the measurements and the true value are equal, the last reading is _____ kΩ (rounded off to nearest integer).

Official answer: **22**

SOLUTION

1. The true value is $R_t = 20$ kΩ. The question says the mean of all six readings equals it: $\bar{R} = \frac{1}{6}\sum_{i=1}^{6} R_i = 20$ kΩ.
2. So the six readings must add up to $\sum_{i=1}^{6} R_i = 6 \times 20 = 120$ kΩ.
3. Add the first five readings: $19 + 18 + 23 + 21 + 17 = 98$ kΩ.
4. The sixth reading is what remains:
$$R_6 = 120 - 98 = 22 \text{ k}\Omega$$
This is already a whole number, so rounding does not change it.
5. Check: the deviations of the six readings from 20 kΩ are $-1, -2, +3, +1, -3, +2$. They add up to zero, so the mean of the six readings is exactly 20 kΩ, as the question requires.

KEY IDEA

If the arithmetic mean equals the true value, the deviations of the readings from the true value must add up to zero. Equivalently, the sum of all n readings must be n times the true value, and the missing reading is whatever is left over.

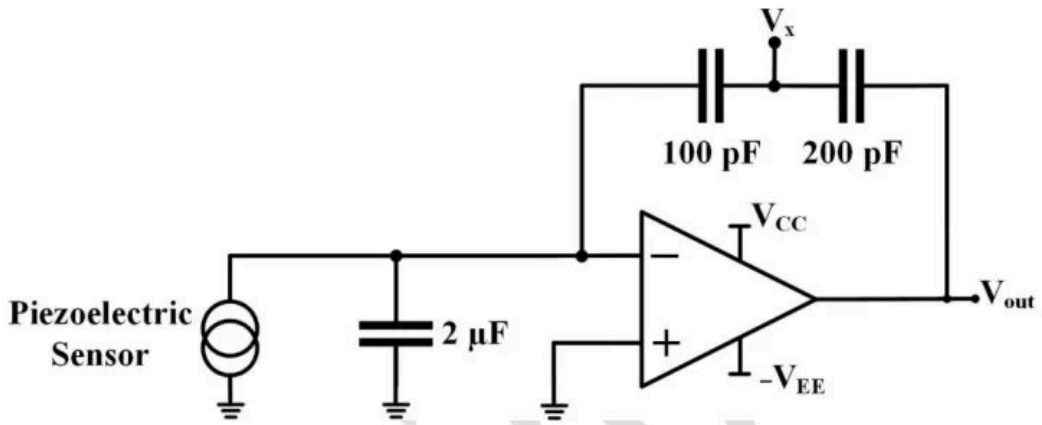
WATCH OUT FOR

Averaging only the five known readings (98/5 = 19.6 kΩ) and reporting that, or dividing by 5 instead of 6 when setting up the mean. The condition covers all six readings, so the total must be 6 × 20 = 120 kΩ.

Q.29 NAT · 1 MARK

Analog Electronics › Op-amps

Q.29 Consider the readout circuit of a piezoelectric sensor shown in the figure.



When the piezoelectric sensor generates a charge q_p, the resulting change in voltage V_x is -2 V. Then the corresponding change in the voltage V_{out} is _____V **(rounded off to nearest integer)**.

Note: Assume all components are ideal.

Official answer: **-3**

SOLUTION

1. **Assumptions:** the op-amp is ideal (infinite open-loop gain, zero input current), and nothing loads the node V_x, so no current leaves it. All voltages and charges below are *changes* caused by the sensor charge q_p.
2. **Virtual short at the op-amp inputs:** the $+$ input is grounded, so $V_+ = 0$ and negative feedback forces $V_- = 0$. The $2\ \mu$F capacitor from the inverting node to ground therefore has 0 V across it and takes no charge. It does not affect the result.
3. **Charge conservation at the inverting node:** no current enters the op-amp and none goes into the $2\ \mu$F capacitor, so all of q_p goes into the feedback branch. The 100 pF and 200 pF capacitors are in series and nothing is drawn from V_x, so each one carries the same charge q_p.
4. **Charge on the 100 pF capacitor (C_1):** the voltage across it is $V_- - V_x = 0 - (-2) = 2$ V, so
$$q_p = C_1(V_- - V_x) = (100\ \text{pF})(2\ \text{V}) = 200\ \text{pC}$$
5. **Same charge on the 200 pF capacitor (C_2):**
$$V_x - V_{out} = \frac{q_p}{C_2} = \frac{200\ \text{pC}}{200\ \text{pF}} = 1\ \text{V}$$
6. **Output:** $V_{out} = V_x - 1 = -2 - 1 = -3$ V.
7. **Check using the series capacitance:** $C_{eq} = \dfrac{100 \times 200}{100 + 200} = \dfrac{200}{3}$ pF, so
$$V_{out} = -\frac{q_p}{C_{eq}} = -\frac{200\ \text{pC}}{(200/3)\ \text{pF}} = -3\ \text{V}$$
The same result in one line: $V_{out} = V_x\left(1 + \dfrac{C_1}{C_2}\right) = -2 \times 1.5 = -3$ V.

KEY IDEA

This is a charge amplifier. The virtual ground holds the inverting node at 0 V, so the sensor's shunt capacitance (2 µF) takes no charge and all of q_p goes onto the feedback capacitors. Since nothing loads V_x, the two series capacitors carry the same charge. So V_x gives you q_p through the 100 pF capacitor, and q_p then gives you the extra drop across the 200 pF capacitor.

WATCH OUT FOR

Taking V_x as the output, or splitting the voltage the wrong way. With equal charge, the drop across each series capacitor goes as $1/C$, so the 200 pF capacitor drops half as much as the 100 pF one (1 V, not 4 V). Another mistake is letting the 2 µF sensor capacitance share the charge. It sits at virtual ground and takes none.

Q.30 NAT · 1 MARK

Networks (Electric Circuits) › AC, power & resonance

The voltage applied and the current drawn by a circuit are

$$v(t) = 95 + 200\cos(120\pi t) + 90\cos(360\pi t - 60^\circ)\ \text{V}$$
$$i(t) = 4\cos(120\pi t - 60^\circ) + 1.5\cos(240\pi t - 75^\circ)\ \text{A}$$

The average power absorbed by the circuit is _____ W (rounded off to nearest integer).

Official answer: **200**

SOLUTION

1. **Average power when the waveforms are non-sinusoidal.** Every component is a harmonic of 60 Hz (120π, 240π and 360π rad/s), so both waveforms repeat with period $T = \frac{1}{60}$ s. Over that period, the product of a voltage and a current at **different** frequencies averages to zero. Only a DC voltage with a DC current, or a voltage and a current **at the same frequency**, give average power:
$$P = V_0 I_0 + \sum_n \frac{V_{m,n} I_{m,n}}{2}\cos(\theta_{v,n} - \theta_{i,n})$$
Here $V_{m,n}$ and $I_{m,n}$ are peak amplitudes. The same thing in rms form is $V_n I_n \cos(\theta_{v,n} - \theta_{i,n})$.
2. **Sort the components by frequency.** DC: $V_0 = 95$ V, $I_0 = 0$. At 120π rad/s (60 Hz): voltage of peak 200 V at 0°, current of peak 4 A at -60°. As rms phasors these are $\frac{200}{\sqrt{2}}\angle 0^\circ = 141.42\angle 0^\circ$ V and $\frac{4}{\sqrt{2}}\angle -60^\circ = 2.828\angle -60^\circ$ A. At 240π rad/s (120 Hz): current only, with peak 1.5 A at -75°. At 360π rad/s (180 Hz): voltage only, with peak 90 V at -60°.
3. **DC term.** $P_0 = V_0 I_0 = 95 \times 0 = 0$ W, because the current has no DC component. The 95 V source term only multiplies sinusoidal currents, and a sinusoid averages to zero over a full period.
4. **60 Hz term.** Both voltage and current are present at this frequency:
$$P_1 = \frac{200 \times 4}{2}\cos\left(0^\circ - (-60^\circ)\right) = 400 \times 0.5 = 200\ \text{W}$$
The rms phasors give the same result: $141.42 \times 2.828 \times \cos 60^\circ = 400 \times 0.5 = 200$ W.
5. **120 Hz and 180 Hz terms.** Each of these frequencies has only a voltage or only a current, so there is nothing to pair it with. Products such as $\cos(120\pi t)\cos(240\pi t - 75^\circ)$ or $\cos(360\pi t - 60^\circ)\cos(240\pi t - 75^\circ)$ average to zero over T. So $P_2 = P_3 = 0$.
6. **Total.**
$$P = P_0 + P_1 + P_2 + P_3 = 0 + 200 + 0 + 0 = 200\ \text{W}$$

KEY IDEA

Voltage and current components at different frequencies (a DC voltage paired with an AC current counts as different) transfer no average power over a common period. Only same-frequency pairs contribute, each giving $\frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$. Here the only such pair is the 60 Hz one: $\frac{200\times 4}{2}\cos 60^\circ = 200$ W.

WATCH OUT FOR

Students often multiply the DC voltage by an AC current (writing 95×4 or similar), or pair the 90 V (180 Hz) voltage with the 1.5 A (120 Hz) current. Both give zero average power, because the frequencies differ. A second common slip is using $V_m I_m \cos\phi$ with peak values and forgetting the factor $\frac{1}{2}$, which gives 400 W. Using the wrong sign in the phase difference is harmless here, since $\cos(\pm 60^\circ) = 0.5$.

Q.31 NAT · 1 MARK

Signals and Systems › Signals basics

The current $i(t)$ drawn by a circuit is given as

$$i(t) = 4 + 30\cos(t) - 20\sin(t) + 15\cos(3t) - 10\sin(3t)\ \text{A}$$

The root-mean-square value of $i(t)$ is _____ A (rounded off to one decimal place).

Official answer: **27.0 to 30.0**

SOLUTION

1. **Orthogonality of sinusoids:** every term of $i(t)$ repeats with the common period $T = 2\pi$. Averaged over one period, every cross term averages to zero: dc × sinusoid, sinusoids at different frequencies, and $\cos$ × $\sin$ at the same frequency. So the mean-square value of $i(t)$ is the sum of the mean-square values of its separate components:
$$I_{\text{rms}}^2 = I_0^2 + \frac{I_{m1}^2}{2} + \frac{I_{m3}^2}{2}$$
2. **DC term:** the mean-square contribution is $4^2 = 16$.
3. **Fundamental ($\omega = 1$ rad/s):** $30\cos t - 20\sin t$ is a single sinusoid with peak $I_{m1} = \sqrt{30^2 + 20^2} = \sqrt{1300}$, so its mean-square value is $\dfrac{1300}{2} = 650$.
4. **Third harmonic ($\omega = 3$ rad/s):** $15\cos 3t - 10\sin 3t$ has peak $I_{m3} = \sqrt{15^2 + 10^2} = \sqrt{325}$, so its mean-square value is $\dfrac{325}{2} = 162.5$.
5. **Total mean square:**
$$I_{\text{rms}}^2 = 16 + 650 + 162.5 = 828.5\ \text{A}^2$$
6. **RMS value:**
$$I_{\text{rms}} = \sqrt{828.5} \approx 28.78\ \text{A} \approx 28.8\ \text{A}$$

KEY IDEA

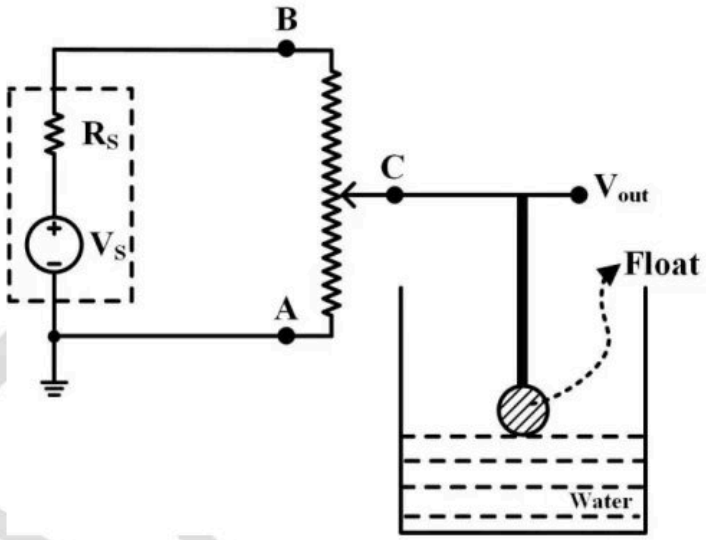
For a sum of a dc term and sinusoids at distinct frequencies, the mean squares add because the cross terms average to zero: $I_{\text{rms}} = \sqrt{I_0^2 + \sum_k I_{mk}^2/2}$. First merge the $\cos$ and $\sin$ terms at each frequency into one sinusoid of peak $\sqrt{a^2 + b^2}$. The dc term counts at full value, not halved.

WATCH OUT FOR

Common mistakes: (1) halving the dc term, i.e. using $4^2/2$ instead of 4^2; (2) adding peak or rms amplitudes directly instead of their squares, which gives $4 + 30/\sqrt{2} + \ldots$; (3) treating the $\cos$ and $\sin$ terms at the same frequency as if their peaks add linearly ($30 + 20$) instead of in quadrature ($\sqrt{30^2 + 20^2}$). The negative signs on the sine terms have no effect on the rms value.

Q.32 NAT · 1 MARK

Q.32 A linear potentiometer $(0 - 10\ \text{k}\Omega)$ is used to measure the water level as shown in the figure. The resistance between A and C varies linearly from 0 to 10 kΩ for a change in water level from 0 to 20 cm. The sensor is excited using a DC voltage source, $V_S = 10$ V with an internal resistance, $R_S = 200\ \Omega$. If $V_{out} = 5$ V, the water level is _____ cm **(rounded off to one decimal place)**.



Official answer: **10.1 to 10.3**

SOLUTION

1. The potentiometer is linear, so the resistance from A to the wiper C is proportional to the water level h: $R_{AC} = \dfrac{h}{20\ \text{cm}} \times 10\ \text{k}\Omega$.
2. V_{out} is taken between the wiper C and ground (A), and nothing is connected across it, so the output is unloaded. The whole potentiometer $R_{BA} = 10\ \text{k}\Omega$ is therefore in series with the source resistance R_S across V_S.
3. The current is the same for every wiper position: $I = \dfrac{V_S}{R_S + R_{BA}} = \dfrac{10\ \text{V}}{200\ \Omega + 10000\ \Omega} = \dfrac{10}{10200}\ \text{A}$.
4. V_{out} is the drop across the section A to C only: $V_{out} = I\, R_{AC}$.
5. For $V_{out} = 5$ V: $R_{AC} = \dfrac{5}{I} = \dfrac{5 \times 10200}{10}\ \Omega = 5100\ \Omega = 5.1\ \text{k}\Omega$.
6. Water level: $h = \dfrac{R_{AC}}{10\ \text{k}\Omega} \times 20\ \text{cm} = \dfrac{5.1}{10} \times 20\ \text{cm} = 10.2\ \text{cm}$.

KEY IDEA

A potentiometer with a source resistance forms a loaded divider. With an unloaded output the full track plus R_S carry the current, so $V_{out} = V_S \dfrac{R_{AC}}{R_S + R_{pot}}$. Because R_S is in series, the output cannot reach V_S and the wiper must sit above mid-travel to give half of V_S.

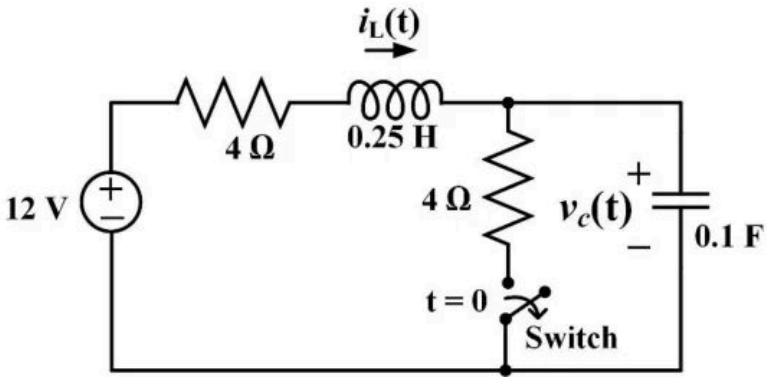
WATCH OUT FOR

Ignoring R_S and taking $V_{out} = V_S \cdot R_{AC}/R_{pot}$. This gives $R_{AC} = 5\ \text{k}\Omega$ and $h = 10.0\ \text{cm}$, which falls outside the key range of 10.1 to 10.3. Another slip is to treat the wiper as dividing the 10 k track into two parts that both see the load. The output is unloaded, so only the total series resistance of 10.2 k sets the current.

Q.33 NAT · 1 MARK

Networks (Electric Circuits) › Transients

Q.33 The switch in the following figure has been closed for a long time ($t < 0$). It is opened at $t = 0$ seconds. The value of dv_c/dt at $t = 0^+$ is _____ V/s **(rounded off to nearest integer)**.



Official answer: **15**

SOLUTION

1. **Steady state for $t < 0$ (switch closed).** In DC steady state the inductor acts as a short circuit and the capacitor as an open circuit. The source therefore drives one current through the series 4 Ω resistor, the inductor and the lower 4 Ω resistor:
$$i_L(0^-) = \frac{12}{4+4} = 1.5\ \text{A}$$
2. **Capacitor voltage before switching.** The capacitor is in parallel with the lower 4 Ω resistor, so $v_c(0^-) = 1.5 \times 4 = 6\ \text{V}$. The slope does not need this value, but it fixes the state of the circuit.
3. **Continuity at the switching instant.** Neither the inductor current nor the capacitor voltage can jump, so $i_L(0^+) = i_L(0^-) = 1.5\ \text{A}$ and $v_c(0^+) = v_c(0^-) = 6\ \text{V}$.
4. **KCL at the top node at $t = 0^+$.** With the switch open, the 4 Ω branch carries no current. All of the inductor current therefore flows into the capacitor: $i_C(0^+) = i_L(0^+) = 1.5\ \text{A}$, entering the + terminal of v_c.
5. **Capacitor law $i_C = C\,\dfrac{dv_c}{dt}$.**
$$\left.\frac{dv_c}{dt}\right|_{0^+} = \frac{i_C(0^+)}{C} = \frac{1.5}{0.1} = 15\ \text{V/s}$$

KEY IDEA

The inductor current cannot change instantly. Once the switch opens, the capacitor is the only path left for that current, so $i_C(0^+) = i_L(0^-)$ and $dv_c/dt\,|_{0^+} = i_L(0^-)/C$.

WATCH OUT FOR

Some students treat the capacitor current as zero at $t = 0^+$ because it was zero in steady state, and so answer 0. Only the capacitor voltage is continuous, not its current. Others keep the lower 4 Ω resistor in the circuit after the switch opens, splitting the current and getting a smaller slope. A third error is to take $i_L(0^-) = 12/4 = 3$ A by forgetting the second resistor, which gives 30.

Q.34 NAT · 1 MARK

Engineering Mathematics › Numerical methods

Consider a system given by the following first order differential equation:

$$\frac{dy}{dt} = y + 2t - t^2$$

where, $y(0) = 1$ and $0 \le t < \infty$. Using a step size $h = 0.1$ for the improved Euler method, the value of $y(t)$ at $t = 0.1$ is _____ (rounded off to two decimal places).

Official answer: **1.10 to 1.12**

SOLUTION

1. Write the right-hand side as $f(t, y) = y + 2t - t^2$, with $t_0 = 0$, $y_0 = 1$, $h = 0.1$, so $t_1 = 0.1$.
2. Improved Euler (Heun's method) is a predictor-corrector: predict $y_1^* = y_0 + h\,f(t_0, y_0)$, then correct with $y_1 = y_0 + \frac{h}{2}\,[f(t_0, y_0) + f(t_1, y_1^*)]$.
3. Slope at the start: $f(0, 1) = 1 + 2(0) - 0^2 = 1$.
4. Predictor (a plain Euler step): $y_1^* = 1 + 0.1(1) = 1.1$.
5. Slope at the predicted end point: $f(0.1,\ 1.1) = 1.1 + 2(0.1) - (0.1)^2 = 1.1 + 0.2 - 0.01 = 1.29$.
6. Corrector (average of the two slopes):
$$y_1 = 1 + \frac{0.1}{2}\,(1 + 1.29) = 1 + 0.05(2.29) = 1.1145$$
7. Rounded to two decimal places, $y(0.1) \approx 1.11$ (the third decimal is 4, so the value rounds down).
8. Check: the exact solution is $y = t^2 + e^t$. It satisfies $y(0) = 0 + 1 = 1$, and $y' = 2t + e^t = (t^2 + e^t) + 2t - t^2$. So $y(0.1) = 0.01 + e^{0.1} \approx 1.1152$. The improved Euler value 1.1145 is close to this, as expected from a second-order method with $h = 0.1$.

KEY IDEA

Improved Euler predicts with a plain Euler step, then corrects by averaging the slope at the start, $f(t_0, y_0)$, with the slope at the predicted end point, $f(t_1, y_1^*)$: $y_1 = y_0 + \frac{h}{2}\,[f(t_0, y_0) + f(t_1, y_1^*)]$. The predicted y_1^* must go into the second slope evaluation, not y_0.

WATCH OUT FOR

Stopping at the plain Euler step gives 1.10, which only just touches the edge of the key range and is the wrong method. Another slip is to evaluate the second slope at (t_1, y_0) instead of (t_1, y_1^*), which gives $f(0.1, 1) = 1.19$ and $y_1 = 1.1095 \approx 1.11$: the right number by luck, from the wrong working. Also, some textbooks use 'modified Euler' for the midpoint rule. Here that also gives $1.11475 \approx 1.11$, but Heun's form is the standard reading of 'improved Euler'.

Q.35 NAT · 1 MARK

Engineering Mathematics › Probability & statistics

Indian Premier League has divided the sixteen cricket teams into two equal pools: Pool-A and Pool-B. Four teams of Pool-A have blue logo jerseys while the rest four have red logo jerseys. Five teams of Pool-B have

blue logo jerseys while the rest three have red logo jerseys.

If one team from each pool reaches the final, the probability that one team has a blue logo jersey and another has a red logo jersey is _____ (rounded off to one decimal place).

Official answer: **0.5**

SOLUTION

1. Each pool has $16/2 = 8$ teams. Pool-A has 4 blue and 4 red; Pool-B has 5 blue and 3 red. Each team in a pool is taken to be equally likely to be that pool's finalist, and the two pools are independent.
2. Probabilities within each pool (subscript b = blue, r = red): $P(A_b) = \frac{4}{8}$, $P(A_r) = \frac{4}{8}$, $P(B_b) = \frac{5}{8}$, $P(B_r) = \frac{3}{8}$.
3. "One blue and one red" happens in exactly two ways that cannot both occur: (Pool-A blue and Pool-B red) or (Pool-A red and Pool-B blue).
4. Multiply for each independent pair, then add the two exclusive cases:
$$P = \frac{4}{8}\cdot\frac{3}{8} + \frac{4}{8}\cdot\frac{5}{8} = \frac{12}{64} + \frac{20}{64} = \frac{32}{64}$$
5. $P = \frac{1}{2} = 0.5$. Check with the complement: $P(\text{same colour}) = \frac{4\cdot5+4\cdot3}{64} = \frac{32}{64}$ (both blue plus both red), so $1 - \frac{32}{64} = 0.5$.

KEY IDEA

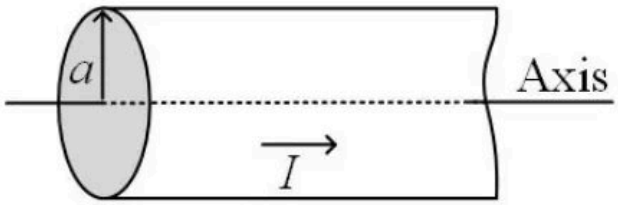
The two finalists come from different pools, so the events are independent. Split "one blue, one red" into its two exclusive cases (A blue with B red, and A red with B blue), multiply within each case and add the two. Checking with the complement (both the same colour) confirms the result.

WATCH OUT FOR

Counting only one case (for example A blue, B red $= \frac{12}{64} \approx 0.2$) and forgetting the reverse case. Another mistake is pooling all 16 teams and using $\frac{9}{16}\cdot\frac{7}{16}\cdot 2$ or $\binom{16}{2}$. That ignores the fact that exactly one finalist comes from each pool.

Q.36 MCQ · 2 MARKS

Electromagnetics › Magnetostatics

Q.36	A wire of circular cross section with radius a is shown in the figure. The current density is given by $\mathbf{J} = ks^2$, where k is a constant, s is the radial distance from the axis and $0 \le s \le a$. The total current I in the wire is
	
(A)	$\frac{\pi k a^4}{2}$
(B)	$\frac{2\pi k a^3}{3}$
(C)	$\frac{\pi k a^3}{2}$
(D)	$\frac{\pi k a^4}{4}$

Official answer: **A**

SOLUTION

1. The current flows along the axis, so $\mathbf{J} = ks^2\,\hat{a}_z$ is normal to the circular cross-section. The total current is the flux of $\mathbf{J}$ through that cross-section: $I = \int_S \mathbf{J} \cdot d\mathbf{S}$.
2. The integrand depends on s alone, so split the disc into thin rings of radius s and width ds. Each ring has area $dS = 2\pi s\,ds$ (in cylindrical coordinates, $dS = s\,ds\,d\phi$ integrated over ϕ from 0 to 2π).
3. $$I = \int_0^{2\pi}\int_0^a ks^2\, s\,ds\,d\phi = 2\pi k \int_0^a s^3\,ds$$
4. $$I = 2\pi k\,\frac{a^4}{4} = \frac{\pi k a^4}{2}$$
5. Check the powers: $J \propto s^2$ and the area element brings one more s, so s^3 integrates to a^4. That rules out options (B) and (C), which carry a^3. Option (D) is half the right value. The answer is (A).

KEY IDEA

When the current density varies across the cross-section, the total current is the surface integral $I = \int_S \mathbf{J} \cdot d\mathbf{S}$, not $J \times \text{area}$. For J that depends only on s, use rings of area $2\pi s\, ds$, so $I = \int_0^a J(s)\, 2\pi s\, ds$.

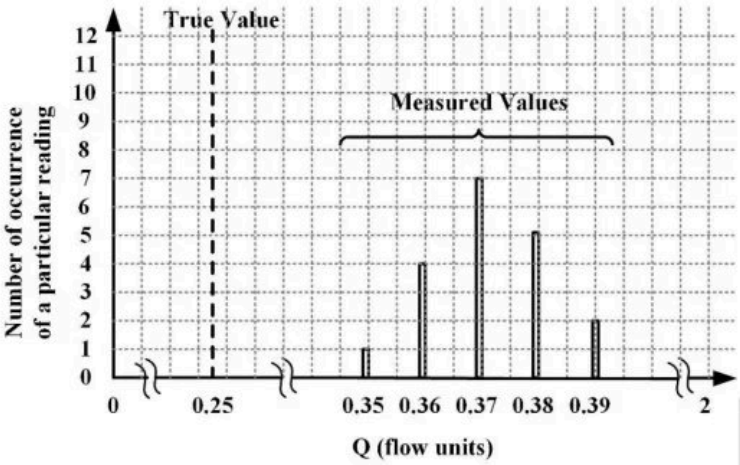
WATCH OUT FOR

Dropping the s in the area element $s\, ds\, d\phi$ gives $2\pi k a^3/3$, which is option (B). Taking the ring area as $\pi s\, ds$ in place of $2\pi s\, ds$ gives $\pi k a^4/4$, which is option (D). Multiplying $J(a) = ka^2$ by πa^2 gives $\pi k a^4$, which is double the right value.

Q.37 MCQ · 2 MARKS

Electrical and Electronic Measurements › Errors & standards

Q.37 The measured values from a flow instrument, whose range is between 0 and 2 flow units, are shown in the histogram. The systematic error (bias) and the maximum error (in flow units), respectively are



(A)	0.12 and 0.14
(B)	0.01 and 0.10
(C)	0.10 and 0.14
(D)	0.04 and 0.12

Official answer: **A**

SOLUTION

1. Read the histogram. The true value is $Q_t = 0.25$ flow units. The readings are 0.35 (1 time), 0.36 (4 times), 0.37 (7 times), 0.38 (5 times) and 0.39 (2 times), so the number of readings is $N = 1 + 4 + 7 + 5 + 2 = 19$.

2. The systematic error (bias) is the difference between the mean of the readings and the true value. Random scatter averages out over many readings, but a bias does not, so the mean is where the bias shows up.
3. Mean reading:
$$\bar{Q} = \frac{0.35(1) + 0.36(4) + 0.37(7) + 0.38(5) + 0.39(2)}{19} = \frac{0.35 + 1.44 + 2.59 + 1.90 + 0.78}{19} = \frac{7.06}{19} \approx 0.3716 \text{ flow units}$$
4. Bias: $\bar{Q} - Q_t = 0.3716 - 0.25 \approx 0.12$ flow units. As a check, the most frequent reading (the mode, 0.37) gives the same value: $0.37 - 0.25 = 0.12$.
5. The maximum error is the largest deviation of any single reading from the true value. The two extreme readings are 0.35 (error 0.10) and 0.39 (error 0.14), so the maximum error is $0.39 - 0.25 = 0.14$ flow units.
6. So the bias is 0.12 and the maximum error is 0.14 flow units, which is option (A).

KEY IDEA

Always measure errors from the true value. The bias is the mean of the readings minus the true value. The maximum error is the largest single-reading deviation from the true value. How far the readings spread around their own mean is precision, not accuracy.

WATCH OUT FOR

The usual mistake is measuring from the wrong reference. The spread of the readings, 0.39 - 0.35 = 0.04, describes precision, not maximum error, and taking it for the maximum error leads to option (D). Using the reading nearest the true value (0.35 - 0.25 = 0.10) as the bias leads to option (C). A third slip is dividing by the 0 to 2 full-scale range, which gives a percentage of full scale when the question asks for flow units.

Q.38 MCQ · 2 MARKS

Signals and Systems › z-transform

Consider a discrete-time sequence

$$x[n] = \begin{cases} (0.2)^n, & 0 \le n \le 7 \\ 0, & \text{otherwise} \end{cases}$$

The region of convergence of $X(z)$, the z-transform of $x[n]$, consists of

(A) all values of z except $z = 0.2$

(B) all values of z

(C) all values of z except $z = 0$

(D) all values of z except $z = \infty$

Official answer: **C**

SOLUTION

1. Write the z-transform directly from the definition. Since $x[n]$ is non-zero only for $0 \le n \le 7$:
$$X(z) = \sum_{n=0}^{7} (0.2)^n z^{-n} = 1 + 0.2z^{-1} + (0.2)^2 z^{-2} + \cdots + (0.2)^7 z^{-7}$$
This is a finite sum of 8 terms, so it converges wherever every term is finite. No condition like $|z| > 0.2$ comes in, because there is no infinite series to converge.
2. Check each term for finite, non-zero z: the $n = 0$ term is the constant 1, and every term with $n \ge 1$ contains $z^{-n} = 1/z^n$, which is finite for every $z \neq 0$. At $z = 0$ these terms are infinite, so $z = 0$ is excluded.
3. Check $z = \infty$: every z^{-n} with $n \ge 1$ goes to 0, so $X(z) \to 1$, which is finite. The sequence has no terms with $n < 0$, so $X(z)$ contains no positive powers of z, and $z = \infty$ belongs to the ROC.
4. Confirm with the closed form (finite geometric series, ratio $0.2z^{-1}$):

$$X(z) = \frac{1-(0.2)^8 z^{-8}}{1-0.2z^{-1}} = \frac{z^8-(0.2)^8}{z^7(z-0.2)}$$

5. The factor $z-0.2$ in the denominator is not a real pole. The numerator vanishes at $z = 0.2\,e^{j2\pi k/8}$, $k = 0, 1, \ldots, 7$, and the $k = 0$ zero is exactly $z = 0.2$, so it cancels. This must happen, because the sum in step 1 is plainly finite at $z = 0.2$ (it equals $1 + 1 + \cdots + 1 = 8$).
6. The only poles left are the 7 poles at $z = 0$ (from z^7). So the ROC is the whole z-plane except $z = 0$, which is option (C).

KEY IDEA

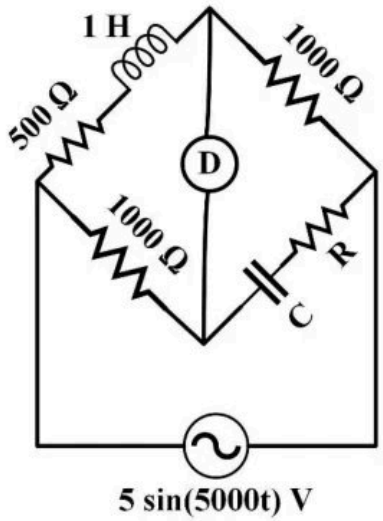
A finite-length sequence has a finite sum for its z-transform, so the ROC is the whole z-plane with at most two points taken out: $z = 0$ is excluded if the sequence has any non-zero sample at $n > 0$ (a z^{-n} term), and $z = \infty$ is excluded if it has any non-zero sample at $n < 0$ (a $z^{|n|}$ term). Here all samples are at $0 \le n \le 7$, so only $z = 0$ is excluded.

WATCH OUT FOR

Writing the geometric-series closed form $\frac{1-(0.2)^8 z^{-8}}{1-0.2z^{-1}}$, seeing the denominator vanish at $z = 0.2$, and choosing (A) or setting the ROC to $|z| > 0.2$ as for the infinite sequence $(0.2)^n u[n]$. That pole is cancelled by a zero of the numerator. The other common slip is choosing (B) because 'finite sequences converge everywhere', forgetting that the z^{-n} terms blow up at $z = 0$.

Q.39 MCQ · 2 MARKS

Electrical and Electronic Measurements › Bridges

Q.39	In the bridge circuit shown in the figure, under balanced condition, the values of R and C respectively, are
	
(A)	1.010 Ω and 19.802 μF
(B)	9.901 Ω and 0.505 μF
(C)	19.802 Ω and 1.01 μF
(D)	39.604 Ω and 2.02 μF

Official answer: **C**

SOLUTION

1. Identify the arms. The detector D is connected between the top and bottom nodes, and the source between the left and right nodes. So the opposite-arm pairs are (left-top, bottom-right) and (top-right, left-bottom). From $v = 5\sin(5000t)$ V, the angular frequency is $\omega = 5000\ \text{rad/s}$.
2. Write the arm impedances: $Z_1 = 500 + j\omega L = 500 + j(5000)(1) = 500 + j5000\ \Omega$ (left-top), $Z_2 = 1000\ \Omega$ (top-right), $Z_3 = 1000\ \Omega$ (left-bottom), and $Z_4 = R - \dfrac{j}{\omega C}$ (bottom-right, R and C in series).
3. At balance the products of opposite arms are equal: $Z_1 Z_4 = Z_2 Z_3$. So $Z_4 = \dfrac{Z_2 Z_3}{Z_1} = \dfrac{10^6}{500 + j5000}$.
4. Rationalise the denominator:
$$Z_4 = \frac{10^6\,(500 - j5000)}{500^2 + 5000^2} = \frac{10^6\,(500 - j5000)}{25.25 \times 10^6} = 19.802 - j198.02\ \Omega$$
5. Equate the real parts: $R = 19.802\ \Omega$.

6. Equate the imaginary parts: $\frac{1}{\omega C} = 198.02\ \Omega$, so $C = \frac{1}{5000 \times 198.02}\ \text{F} = 1.01 \times 10^{-6}\ \text{F} = 1.01\ \mu\text{F}$.
7. Check: Z_1 is inductive (positive angle), so the arm opposite it must be capacitive (negative angle) for $Z_1 Z_4$ to be a real number, 10^6. The series RC arm is capacitive, and R comes out positive, so the balance can actually be reached. $R = 19.802\ \Omega$ and $C = 1.01\ \mu\text{F}$ is option (C).

KEY IDEA

An AC bridge balances when the products of the complex impedances of opposite arms are equal, $Z_1 Z_4 = Z_2 Z_3$. The real parts and the imaginary parts must each match. The inductive arm does not balance on its own, so the answer depends on frequency. Take $\omega = 5000\ \text{rad/s}$ straight from $5\sin(5000t)$.

WATCH OUT FOR

One mistake is to read 5000 as the frequency in hertz and use $\omega = 2\pi \times 5000$. The argument of the sine is ωt, so $\omega = 5000\ \text{rad/s}$. Another is to pair adjacent arms instead of opposite ones, for example $Z_4 = Z_1 Z_2 / Z_3$. That makes Z_4 inductive, which a series RC arm cannot be.

Q.40 MCQ · 2 MARKS

Signals and Systems › Laplace

Laplace transform of a signal $x(t)$ is

$$X(s) = \frac{1}{s^2 + 13s + 42}$$

Let $u(t)$ be the unit step function. Choose the signal $x(t)$ from the following options if the region of convergence is $-7 < \text{Re}\{s\} < -6$.

(A) $-e^{-6t}u(t) - e^{-7t}u(-t)$

(B) $-e^{-6t}u(-t) - e^{-7t}u(t)$

(C) $e^{-6t}u(t) - e^{-7t}u(-t)$

(D) $-e^{-6t}u(-t) - e^{-7t}u(-t)$

Official answer: **B**

SOLUTION

1. Factor the denominator: $s^2 + 13s + 42 = (s+6)(s+7)$, so the poles are at $s = -6$ and $s = -7$.
2. Partial fractions:
$$X(s) = \frac{1}{(s+6)(s+7)} = \frac{1}{s+6} - \frac{1}{s+7}$$
Check: $\frac{(s+7)-(s+6)}{(s+6)(s+7)} = \frac{1}{(s+6)(s+7)}$.
3. The ROC $-7 < \text{Re}\{s\} < -6$ is a vertical strip between the two poles, so $x(t)$ is two-sided. Each pole's term is right-sided or left-sided depending on which side of the strip the pole lies.
4. The pole $s = -6$ lies to the RIGHT of the strip, so its term must have ROC $\text{Re}\{s\} < -6$ (left-sided). Using the pair $-e^{-at}u(-t) \leftrightarrow \frac{1}{s+a}$, $\text{Re}\{s\} < -a$:
$$\frac{1}{s+6} \ \rightarrow\ -e^{-6t}u(-t)$$
5. The pole $s = -7$ lies to the LEFT of the strip, so its term must have ROC $\text{Re}\{s\} > -7$ (right-sided). Using the pair $e^{-at}u(t) \leftrightarrow \frac{1}{s+a}$, $\text{Re}\{s\} > -a$:
$$-\frac{1}{s+7} \ \rightarrow\ -e^{-7t}u(t)$$
6. Add the two terms:
$$x(t) = -e^{-6t}u(-t) - e^{-7t}u(t)$$
The intersection of the two ROCs, $\text{Re}\{s\} < -6$ and $\text{Re}\{s\} > -7$, is exactly the given strip, which confirms it. This is option (B).

KEY IDEA

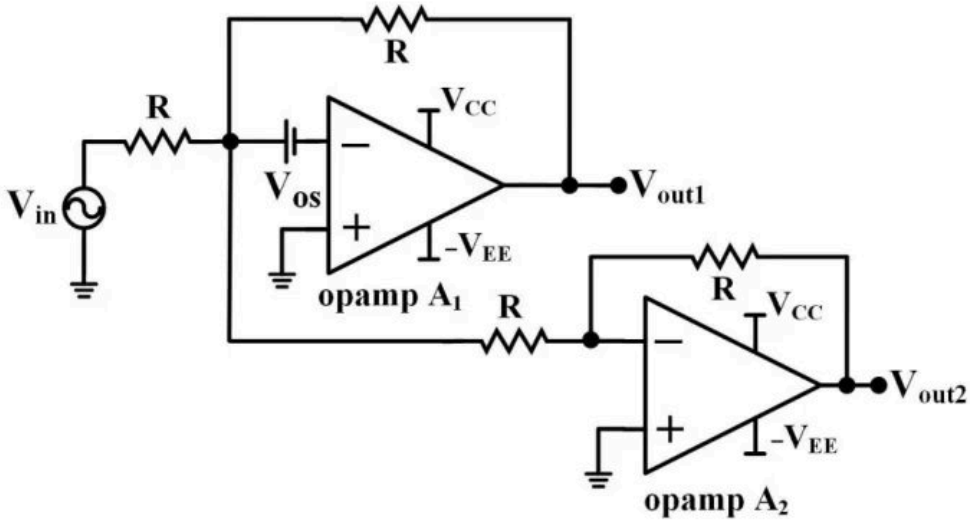
When the ROC is a strip between poles, a pole to the right of the strip gives a left-sided (anti-causal) term, $-e^{-at}u(-t)$, and a pole to the left gives a right-sided (causal) term, $e^{-at}u(t)$. The ROC of the whole signal is where the individual ROCs overlap.

WATCH OUT FOR

Treating every term as causal (the one-sided Laplace habit), or dropping the minus sign in the left-sided pair $-e^{-at}u(-t) \leftrightarrow \frac{1}{s+a}$. Quick check on the other options: in (A) the term $-e^{-6t}u(t)$ needs $\mathrm{Re}\{s\} > -6$ and $-e^{-7t}u(-t)$ needs $\mathrm{Re}\{s\} < -7$, and no s satisfies both. (C) has the same empty ROC. (D) is purely left-sided with ROC $\mathrm{Re}\{s\} < -7$, and its transform is $\frac{1}{s+6} + \frac{1}{s+7}$, which is not $X(s)$.

Q.41 MCQ · 2 MARKS

Analog Electronics › Op-amps

Q.41	In the figure shown, both the opamps A_1 and A_2 are ideal, except that the opamp A_1 has an offset voltage (V_{os}) of 1 mV. For $V_{in} = 0$ V, the values of the output voltages V_{out1} and V_{out2}, respectively, are
	
(A)	3 mV and -1 mV
(B)	1 mV and 0 mV
(C)	1 mV and -1 mV
(D)	2 mV and 0 mV

Official answer: **A**

SOLUTION

1. Assumptions: both op-amps are ideal (infinite gain, zero input current) and each has negative feedback, so each holds a virtual short between its inputs. The offset of A_1 is drawn as a battery $V_{os} = 1\ \text{mV}$ in series with its inverting input, with the long plate (positive terminal) on the side of the junction node. Call the junction of the three resistors node X.
2. Virtual short at A_1: its $+$ input is grounded, so its inverting input sits at 0 V. Going through the battery from its $-$ terminal to its $+$ terminal gives $V_X = 0 + V_{os} = +1$ mV. So node X is **not** a virtual ground.
3. Virtual short at A_2: its $+$ input is grounded, so its inverting input is at 0 V. The resistor R from node X therefore carries a current V_X/R from node X towards A_2.
4. KCL at node X with $V_{in} = 0$. The battery branch carries no current, because the input current of A_1 is zero. Summing the currents entering X:

$$\frac{0 - V_X}{R} + \frac{V_{out1} - V_X}{R} - \frac{V_X - 0}{R} = 0 \;\Rightarrow\; V_{out1} = 3V_X$$

5. So $V_{out1} = 3 \times 1\ \text{mV} = 3\ \text{mV}$. Another way to see it: node X has two resistors to 0 V (the source resistor, since $V_{in} = 0$, and the resistor to the virtual ground of A_2) and the feedback resistor to V_{out1}. This gives a gain of $1 + \dfrac{R_f}{R \parallel R} = 1 + \dfrac{R}{R/2} = 3$ from V_X to V_{out1}.
6. A_2 is an inverting amplifier with gain $-\dfrac{R}{R} = -1$, driven by V_X:
$$V_{out2} = -V_X = -1\ \text{mV}$$
7. Hence $V_{out1} = 3\ \text{mV}$ and $V_{out2} = -1\ \text{mV}$, which is option (A).

KEY IDEA

A series offset source moves the node it is connected to. The virtual short pins the op-amp's inverting input at 0 V, so the junction node sits at $+V_{os} = 1\ \text{mV}$ instead of at virtual ground. That node sees two resistors to 0 V and one feedback resistor, so it is amplified by the noise gain $1 + R/(R \parallel R) = 3$. The second op-amp simply inverts V_X with gain -1.

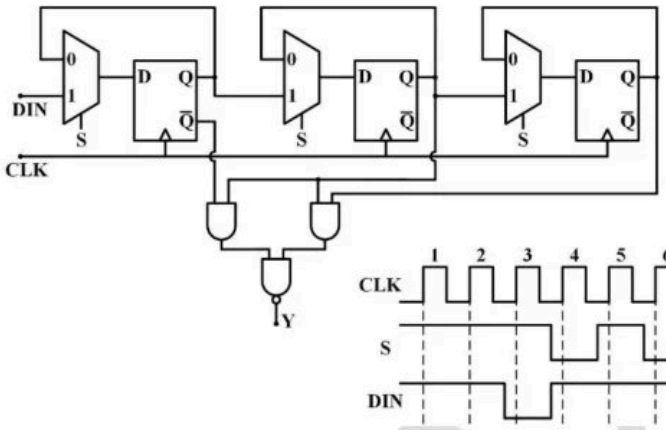
WATCH OUT FOR

A common mistake is to treat node X as a virtual ground, which gives $V_{out2} = 0$ (options B or D). Another is to count only the source resistor when working out the gain, giving $1 + R/R = 2$ and $V_{out1} = 2\ \text{mV}$ (option D). The resistor to the virtual ground of A_2 also loads node X, so the gain is 3. Reading the battery polarity backwards flips both signs.

Q.42 MCQ · 2 MARKS

Digital Electronics › Sequential

Q.42 In the figure shown, the positive edge triggered D flip-flops are initially reset to Q = 0. The logic gates and the multiplexers have no propagation delay. After reset, a train of clock pulses (CLK) are applied. The logic-states of the inputs DIN, S and the clock pulses are also shown in the figure. Assuming no timing violations, the sequence of output Y from the 3rd clock to the 5th clock, $Y_3Y_4Y_5$ is



(A) 001

(B) 010

(C) 000

(D) 011

Official answer: **A**

SOLUTION

1. **Read the MUX wiring.** In each stage the 2:1 MUX drives the D input, with input 0 on top and input 1 below: $D_i = \overline{S}\,Q_i + S\,(\text{input } 1)$, where input 1 is DIN, Q_1 and Q_2 for stages 1, 2 and 3. So $S = 1$ shifts right ($Q_1 \leftarrow \text{DIN},\ Q_2 \leftarrow Q_1,\ Q_3 \leftarrow Q_2$) and $S = 0$ holds the state, because each flip-flop reloads its own Q.
2. **Output logic** (overline = complement): the left AND gate takes $\overline{Q_1}$ and Q_2, the right AND gate takes Q_2 and Q_3, and both feed a NAND gate:

$$Y = \overline{(\overline{Q_1}\,Q_2)(Q_2\,Q_3)} = \overline{\overline{Q_1}\,Q_2\,Q_3}$$

So $Y = 0$ only in the state $Q_1Q_2Q_3 = 011$, and $Y = 1$ in every other state.

3. **Sample S and DIN at each rising edge.** A positive-edge D flip-flop stores whatever is on D at the instant of the edge. From the timing diagram, S falls just after edge 3 and DIN rises just after edge 3, so at edges 1 to 5: $S = 1, 1, 1, 0, 1$ and $\mathrm{DIN} = 1, 1, 0, 1, 1$.
4. **Step through the states**, starting from the reset state $Q_1Q_2Q_3 = 000$. Edge 1 ($S = 1$, DIN $= 1$): shift gives 100. Edge 2 ($S = 1$, DIN $= 1$): 110. Edge 3 ($S = 1$, DIN $= 0$): $Q_1 = 0,\ Q_2 = 1,\ Q_3 = 1$, so 011.
5. Edge 4 ($S = 0$): every flip-flop reloads its own output, so the state stays 011. Edge 5 ($S = 1$, DIN $= 1$): $Q_1 = 1$, $Q_2 =$ old $Q_1 = 0$, $Q_3 =$ old $Q_2 = 1$, so 101.
6. **Evaluate Y after each clock.** After clock 3 the state is 011, so $\overline{Q_1}Q_2Q_3 = 1 \cdot 1 \cdot 1 = 1$ and $Y_3 = 0$. After clock 4 the state is still 011, so $Y_4 = 0$. After clock 5 the state is 101, so $\overline{Q_1} = 0$, the product is 0 and $Y_5 = 1$.
7. Therefore $Y_3Y_4Y_5 = 001$, which is option (A).

KEY IDEA

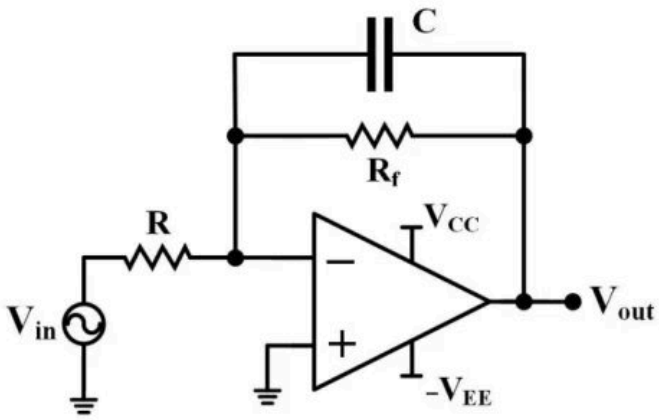
Each MUX in front of a D flip-flop picks between holding and shifting. With $S = 1$ each stage loads the stage to its left (DIN for stage 1), so the circuit is a right-shift register. With $S = 0$ each stage reloads its own Q and nothing changes. The NAND of the two AND terms reduces to $Y = \overline{\overline{Q_1}\,Q_2\,Q_3}$, so you only need to find when the register sits in state 011. Take S and DIN at the instant of each rising edge.

WATCH OUT FOR

The usual slip is to miss the hold on clock 4. S is 0 at that edge, but students often shift DIN = 1 in anyway, which gives states 011, 101, 110 and $Y_3Y_4Y_5 = 011$, the trap option (D). A second slip is reading S or DIN just after an edge instead of at it. Both change shortly after edge 3, so at edge 3 they are still S = 1 and DIN = 0. A third is misreading the left AND input as Q_1 when the wire actually comes from $\overline{Q_1}$.

Q.43 MCQ · 2 MARKS

Analog Electronics › Filters & waveforms

Q.43	In the figure shown, R = 1 kΩ and C = 0.1 μF. For a dc gain of −10, the 3 dB cut-off frequency (**rounded off to one decimal place**) is Assume the opamp is ideal. 
(A)	159.1 Hz
(B)	1591.5 Hz
(C)	1750.7 Hz
(D)	175.0 Hz

Official answer: **A**

SOLUTION

1. **Assumption: ideal op-amp.** The non-inverting input is grounded, so the virtual short holds the inverting input at $0\ \mathrm{V}$, and no current flows into the op-amp's input terminals.
2. **Feedback impedance.** The feedback network is R_f in parallel with C:
$$Z_f = R_f \parallel \frac{1}{j\omega C} = \frac{R_f}{1 + j\omega R_f C}.$$
3. **Transfer function.** For an inverting amplifier, $\dfrac{V_{out}}{V_{in}} = -\dfrac{Z_f}{R}$, so
$$\frac{V_{out}}{V_{in}} = -\frac{R_f}{R} \cdot \frac{1}{1 + j\omega R_f C}.$$
This is a first-order low-pass filter.

4. **DC gain fixes R_f.** At $\omega = 0$ the capacitor is an open circuit, so the gain is $-\dfrac{R_f}{R} = -10$. That gives $R_f = 10R = 10\ \text{k}\Omega$.
5. **3 dB frequency.** The gain magnitude falls to $\dfrac{1}{\sqrt{2}}$ of its dc value when $\omega R_f C = 1$, so $f_{3\text{dB}} = \dfrac{1}{2\pi R_f C}$. The pole is set by R_f and C only. The input resistor R changes the gain, not the cut-off.
6. $$f_{3\text{dB}} = \frac{1}{2\pi(10^4\ \Omega)(0.1 \times 10^{-6}\ \text{F})} = \frac{1}{2\pi \times 10^{-3}\ \text{s}} = 159.15\ \text{Hz}$$
7. **Matching the options.** Strict rounding of $159.155\ \text{Hz}$ to one decimal place gives $159.2\ \text{Hz}$, but no option prints that. Option (A), $159.1\ \text{Hz}$, is the same value cut off after one decimal place. All four options were produced the same way. (B) $1591.5\ \text{Hz}$ is $\dfrac{1}{2\pi RC}$, which uses the wrong resistor $R = 1\ \text{k}\Omega$ (exact value 1591.55). (C) $1750.7\ \text{Hz}$ is $\dfrac{1}{2\pi(R \parallel R_f)C}$ with $R \parallel R_f = 909.1\ \Omega$. (D) $175.0\ \text{Hz}$ is one-tenth of (C) (exact value 175.07). Only (A) comes from the correct expression $\dfrac{1}{2\pi R_f C}$, so the answer is **(A)**.

KEY IDEA

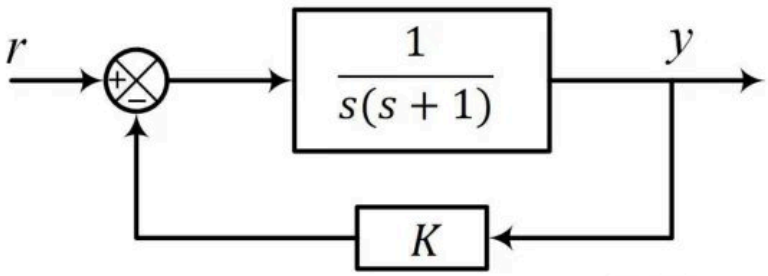
In an inverting amplifier with $R_f \parallel C$ in the feedback path, the gain is $-\dfrac{R_f}{R} \cdot \dfrac{1}{1 + j\omega R_f C}$. The dc gain $-\dfrac{R_f}{R}$ sets R_f, and the single pole sits at $f_{3\text{dB}} = \dfrac{1}{2\pi R_f C}$. The input resistor R sets the gain but plays no part in the cut-off.

WATCH OUT FOR

Using the input resistor in the cut-off formula. $\dfrac{1}{2\pi RC}$ gives $1591.5\ \text{Hz}$ (option B), and $\dfrac{1}{2\pi(R \parallel R_f)C}$ gives $1750.7\ \text{Hz}$ (option C). A second trap: strict rounding of $159.155\ \text{Hz}$ gives $159.2\ \text{Hz}$, which is not printed. Candidates who look only for 159.2 can wrongly reject (A), the only option that comes from $\dfrac{1}{2\pi R_f C}$.

Q.44 MCQ · 2 MARKS

Control Systems › Time response

Q.44	Consider the feedback control system shown in the figure. The steady-state error $e_{ss} = \lim_{t\to\infty} (r(t) - y(t))$ due to unit step reference $r(t)$ is
	
(A)	$\frac{K-1}{K}$
(B)	$\frac{1}{2}$
(C)	0
(D)	$\frac{1-K}{K}$

Official answer: **A**

SOLUTION

1. Closed-loop transfer function, negative feedback with $G(s) = \dfrac{1}{s(s+1)}$ and $H(s) = K$:

$$\frac{Y(s)}{R(s)} = \frac{G}{1+GH} = \frac{\frac{1}{s(s+1)}}{1+\frac{K}{s(s+1)}} = \frac{1}{s^2+s+K}$$

2. Stability check: the characteristic equation $s^2 + s + K = 0$ is second order, so it is stable exactly when all coefficients are positive, that is for $K > 0$. Under that condition the final value theorem can be used.
3. The question defines the error as $e = r - y$. In a non-unity feedback loop that is not the actuating signal $r - Ky$, so the usual $\dfrac{1}{1+K_p}$ formula does not apply. Work it out directly:

$$E(s) = R(s) - Y(s) = R(s)\left[1 - \frac{1}{s^2+s+K}\right] = R(s)\,\frac{s^2+s+K-1}{s^2+s+K}$$

4. Final value theorem with $R(s) = \dfrac{1}{s}$:

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{1}{s} \cdot \frac{s^2 + s + K - 1}{s^2 + s + K} = \frac{K - 1}{K}$$

5. Check: the closed-loop DC gain is $\frac{1}{K}$, so $y(\infty) = \frac{1}{K}$ and $e_{ss} = 1 - \frac{1}{K} = \frac{K-1}{K}$. The error is zero only when $K = 1$ (unity feedback). This is option (A).

KEY IDEA

With non-unity feedback, the error $r - y$ is not the signal leaving the summing junction. Find the closed-loop transfer function $T(s) = \frac{1}{s^2 + s + K}$. For a unit step, $e_{ss} = 1 - T(0) = 1 - \frac{1}{K}$, provided the loop is stable ($K > 0$).

WATCH OUT FOR

Picking option (C), 0, because the forward path $\frac{1}{s(s+1)}$ is type 1. The type-1 integrator drives the actuating signal $r - Ky$ to zero, not $r - y$. It forces $y(\infty) = 1/K$, so the output tracks the step only when $K = 1$. A sign slip in $1 - \frac{1}{K}$ gives option (D), $\frac{1-K}{K}$.

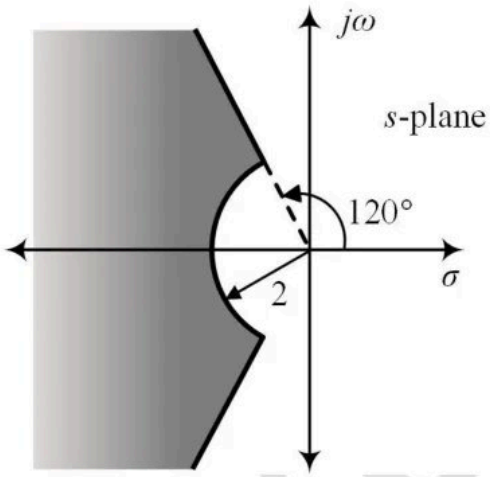
Q.45 MCQ · 2 MARKS

Control Systems › Time response

Q.45 The transfer function of a system is

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Choose the range of ξ and ω_n (in rad/s) from the following options such that the poles lie on the shaded region of the s-plane as shown in the figure.



(A) $\xi \geq \frac{1}{2}$ and $\omega_n \geq 2$

(B) $\xi \geq \frac{1}{4}$ and $\omega_n \geq 2$

(C) $\xi \geq \frac{1}{2}$ and $\omega_n \geq \sqrt{3}$

(D) $\xi \geq \frac{1}{4}$ and $\omega_n \geq \sqrt{3}$

Official answer: **A**

SOLUTION

1. **Pole geometry.** For $0 < \xi < 1$ the poles of $G(s)$ are

$$s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

Their distance from the origin is

$$|s| = \sqrt{\xi^2\omega_n^2 + \omega_n^2(1-\xi^2)} = \omega_n$$

If θ is the angle each pole makes with the negative real axis, then $\cos\theta = \dfrac{\xi\omega_n}{\omega_n} = \xi$. So on the s-plane ω_n sets the radius and ξ sets the angle. At $\xi = 1$ both poles sit at $-\omega_n$ with $\theta = 0$, which fits the same picture. The figure's rays and arc are drawn in exactly these radius and angle terms.

2. **Read the boundary rays.** The rays are at 120° from the positive σ axis. Measured from the negative real axis, that is $\theta = 180^\circ - 120^\circ = 60^\circ$, above and below the axis.
3. **Damping condition.** The shaded region lies between the two rays and contains the negative real axis, so every pole in it has $\theta \le 60^\circ$. Since $\cos\theta$ decreases on $0^\circ \le \theta \le 90^\circ$,
$$\xi = \cos\theta \ge \cos 60^\circ = \frac{1}{2}$$
4. **Natural-frequency condition.** The shaded region lies outside the arc of radius 2, so $|s| \ge 2$. Because $|s| = \omega_n$, this gives $\omega_n \ge 2\ \text{rad/s}$.
5. **Combine the conditions and check the other options.** Together the conditions are $\xi \ge \dfrac{1}{2}$ and $\omega_n \ge 2$, which is option (A). Option (B) allows $\xi = \dfrac{1}{4}$, which gives $\theta = \cos^{-1}\dfrac{1}{4} \approx 75.5^\circ > 60^\circ$, so the poles would fall outside the wedge. Options (C) and (D) allow values such as $\omega_n = 1.8$, between $\sqrt{3} \approx 1.73$ and 2, which puts the poles inside the arc, in the unshaded region.
6. **Where $\sqrt{3}$ comes from.** The corner where the upper ray meets the arc is $2\angle 120^\circ = 2\cos 120^\circ + j\,2\sin 120^\circ = -1 + j\sqrt{3}$. The number $\sqrt{3}$ is that point's height above the real axis, not its distance from the origin. The distance, which is what ω_n measures, is 2.

KEY IDEA

For a complex pole pair of the standard second-order system, $|s| = \omega_n$, so circles centred at the origin are lines of constant ω_n. The angle from the negative real axis is $\cos^{-1}\xi$, so rays from the origin are lines of constant ξ. A wedge of half-angle θ around the negative real axis therefore means $\xi \ge \cos\theta$, and the region outside a circle of radius r means $\omega_n \ge r$.

WATCH OUT FOR

One mistake is to use the marked 120° directly, which gives $\cos 120^\circ = -\frac{1}{2}$. The damping angle has to be measured from the NEGATIVE real axis, which makes it 60° and gives $\xi \ge \frac{1}{2}$. The other mistake is to take the corner point's imaginary part $\sqrt{3}$, from $-1 + j\sqrt{3}$, as the bound on ω_n (options C and D). The bound is the radius, 2, not the height. The value $\xi \ge \frac{1}{4}$ in option B matches no boundary in the figure: $\cos^{-1}\frac{1}{4} \approx 75.5^\circ$, which lies outside the wedge.

Q.46 MCQ · 2 MARKS

Engineering Mathematics › Vector calculus

Let C be the closed curve in the xy-plane, traversed in the counterclockwise direction along the boundary of the rectangle with vertices at $(0,0), (2,0), (2,1), (0,1)$. The value of the line integral

$$\oint_C (-e^y\,dx + e^x\,dy)$$

is

(A) $e^2 + 2e - 3$

(B) $e^2 - 2e - 3$

(C) $e^2 + e - 1$

(D) $e^2 + e + 1$

Official answer: **A**

SOLUTION

1. Write the integral as $\oint_C (P\,dx + Q\,dy)$ with $P = -e^y$ and $Q = e^x$. C is the boundary of the rectangle, a simple closed curve traversed counterclockwise, and P, Q have continuous partial derivatives everywhere, so Green's theorem applies:

$$\oint_C (P\,dx + Q\,dy) = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

2. Partial derivatives: $\frac{\partial Q}{\partial x} = e^x$ and $\frac{\partial P}{\partial y} = -e^y$, so the integrand is $e^x - (-e^y) = e^x + e^y$.
3. The region is the rectangle $R : 0 \le x \le 2,\ 0 \le y \le 1$. Split the double integral into two parts:
$$\iint_R (e^x + e^y)\,dA = \int_0^1 \int_0^2 e^x\,dx\,dy + \int_0^2 \int_0^1 e^y\,dy\,dx$$
4. First term: $\left(\int_0^2 e^x\,dx\right)(1) = e^2 - 1$.
5. Second term: $(2)\left(\int_0^1 e^y\,dy\right) = 2(e-1) = 2e - 2$.
6. Add them: $(e^2 - 1) + (2e - 2) = e^2 + 2e - 3$. This is option (A).
7. Check with the direct line integral. On the bottom edge ($y = 0$, x: $0 \to 2$, $dy = 0$) the integral is $\int_0^2 -1\,dx = -2$. On the right edge ($x = 2$, y: $0 \to 1$, $dx = 0$) it is $\int_0^1 e^2\,dy = e^2$. On the top edge ($y = 1$, x: $2 \to 0$, $dy = 0$) it is $\int_2^0 -e\,dx = 2e$. On the left edge ($x = 0$, y: $1 \to 0$, $dx = 0$) it is $\int_1^0 1\,dy = -1$. The total is $-2 + e^2 + 2e - 1 = e^2 + 2e - 3$, which agrees.

KEY IDEA

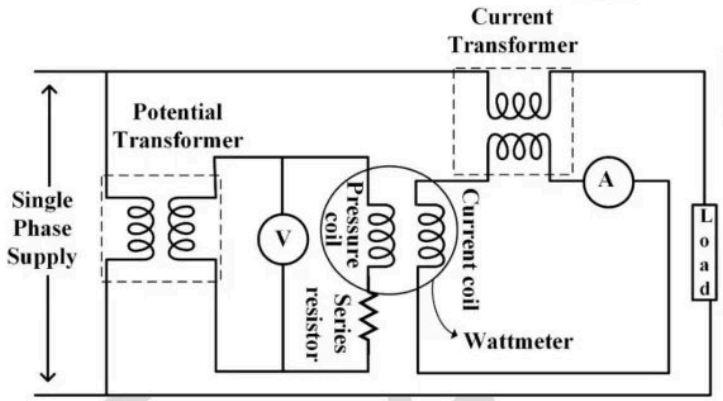
Green's theorem turns a counterclockwise line integral around a closed curve into the double integral of $\partial Q/\partial x - \partial P/\partial y$ over the region inside it. Here the integrand $e^x + e^y$ splits into two parts, and over a rectangle each part is a product of two one-variable integrals.

WATCH OUT FOR

Losing the minus sign on $P = -e^y$. Doing so gives the integrand $e^x - e^y$ and the value $e^2 - 2e + 1$, which is not among the options. A second mistake is using the wrong side lengths: the e^y term has to be multiplied by the x-width (2), and the e^x term by the y-height (1). Swapping them gives $2e^2 + e - 3$.

Q.47 MCQ · 2 MARKS

Electrical and Electronic Measurements › Instrument transformers

Q.47	In the figure shown, assume • α is the phase angle between the load current and the load voltage • β is the phase angle by which pressure coil current lags the pressure coil voltage of the wattmeter • γ is the phase angle between currents in the pressure coil and the current coil of the wattmeter • δ is the phase angle of the voltage transformer • θ is the phase angle of the current transformer When the load has a lagging phase angle of α, which one of the following options is correct? 
(A)	$\alpha = -\gamma \pm \delta \pm \theta - \beta$
(B)	$\alpha = -\gamma \pm \delta \pm \theta + \beta$
(C)	$\alpha = \gamma \pm \delta \pm \theta + \beta$
(D)	$\alpha = \gamma \pm \delta \pm \theta - \beta$

Official answer: **C**

SOLUTION

1. Take the load voltage V as the reference phasor, at angle 0. The load is lagging, so the load current I sits at angle $-\alpha$ (it lags V by α).
2. **Voltage (potential) transformer:** the pressure-coil circuit, and the voltmeter, are connected across the PT secondary. The reversed secondary voltage V_s differs in phase from V by the PT phase angle δ, and it can lead or lag. Let V_s lead V by δ', where $\delta' = \pm\delta$ (positive when it leads). Then V_s is at angle δ'.

3. **Current transformer:** the current coil and the ammeter carry the CT secondary current. The reversed secondary current I_s differs in phase from I by the CT phase angle θ, and it can also lead or lag. Let I_s lead I by θ', where $\theta' = \pm\theta$. Then I_s is at angle $-\alpha + \theta'$.
4. **Inside the wattmeter:** the pressure coil has some inductance, so its current I_p lags its applied voltage V_s by β. That puts I_p at angle $\delta' - \beta$.
5. With a lagging load, the current-coil current I_s lags the pressure-coil current I_p by γ, the angle between the two coil currents. So I_s is also at angle $\delta' - \beta - \gamma$.
6. Set the two expressions for the angle of I_s equal:
$$-\alpha + \theta' = \delta' - \beta - \gamma \;\Rightarrow\; \alpha = \gamma + \beta + \theta' - \delta'$$
7. The signs of $\theta' = \pm\theta$ and $-\delta' = \mp\delta$ depend only on whether each transformer's secondary quantity leads or lags, and the two are independent. So
$$\alpha = \gamma \pm \delta \pm \theta + \beta$$
This is option (C).
8. Check: with an ideal PT and CT ($\delta = \theta = 0$) and a purely resistive pressure coil ($\beta = 0$), I_p is in phase with V and I_s is in phase with I. The coil currents are then exactly α apart, and the formula correctly gives $\alpha = \gamma$.

KEY IDEA

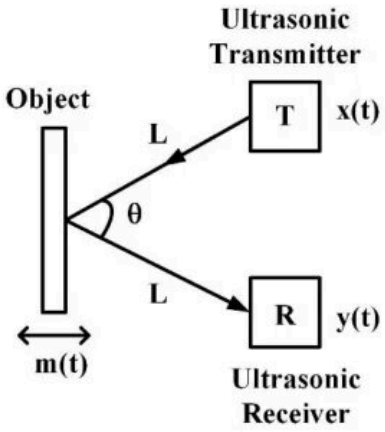
Draw every phasor relative to the load voltage. The CT and PT each rotate their secondary quantity by their own phase angle, which can go either way, hence the $\pm$. The pressure-coil inductance makes I_p lag its voltage by β. So the angle between the coil currents, γ, is the load angle α reduced by β and shifted by the two transformer angles: $\alpha = \gamma + \beta \pm \delta \pm \theta$.

WATCH OUT FOR

Getting the sign of β wrong, which leads to option (D). Because the pressure-coil current lags its voltage by β, it swings toward the lagging load current, so the gap between the coil currents is smaller than the load angle: $\gamma = \alpha - \beta$ (ignoring transformer errors), not $\alpha + \beta$. Options (A) and (B) also flip the sign of γ. That would make the coil currents close to $-\alpha$ apart for a lagging load, which is impossible.

Q.48 MCQ · 2 MARKS

Communications › Analog modulation

Q.48	Consider an ultrasonic measurement system shown in the figure. The ultrasonic transmitter (T) sends a continuous wave signal $x(t) = \cos(2\pi f_1 t)$ volts towards an object whose vibration is modeled as $m(t) = 0.5\sin(2\pi f_2 t)$ volts. Neglecting the phase shift due to any other effect, the received signal at the receiver (R) is $y(t) = \cos(2\pi f_1 t + \beta\cos(2\pi f_2 t))$ volts. Assuming the frequency sensitivity factor as 500 Hz/volt, $f_1 = 40$ kHz, $f_2 = 1$ kHz, the modulation index (β) and the frequency deviation in $y(t)$, respectively, are
	
(A)	0.25 and $\pm$ 250 Hz
(B)	0.5 and $\pm$ 500 Hz
(C)	1 and $\pm$ 1000 Hz
(D)	0.75 and $\pm$ 1000 Hz

Official answer: **A**

SOLUTION

1. The received signal $y(t) = \cos\left(2\pi f_1 t + \beta\cos(2\pi f_2 t)\right)$ is an angle-modulated carrier. Its instantaneous phase is $\theta_i(t) = 2\pi f_1 t + \beta\cos(2\pi f_2 t)$, so β is the peak phase deviation in radians.
2. Differentiate the phase to get the instantaneous frequency:

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i}{dt} = f_1 - \beta f_2 \sin(2\pi f_2 t)$$

The peak deviation from the carrier is therefore $\Delta f = \beta f_2$, which gives $\beta = \dfrac{\Delta f}{f_2}$.

3. For frequency modulation with sensitivity $k_f = 500\ \text{Hz/V}$, the frequency deviation is set by the peak message amplitude. Here $A_m = 0.5\ \text{V}$, since $m(t) = 0.5\sin(2\pi f_2 t)$.
4. Frequency deviation:
$$\Delta f = k_f A_m = 500 \times 0.5 = 250\ \text{Hz}$$
so the instantaneous frequency swings ± 250 Hz about $f_1 = 40$ kHz.
5. Modulation index:
$$\beta = \frac{\Delta f}{f_2} = \frac{250}{1000} = 0.25$$
6. Check: with $\beta = 0.25$, the peak of $\beta f_2 \sin(2\pi f_2 t)$ is $0.25 \times 1000 = 250$ Hz, which agrees with $k_f A_m$. The sign of the sinusoidal term depends on how $m(t)$ and the phase term are paired, but the peak deviation does not.
7. Result: $\beta = 0.25$ and $\Delta f = \pm 250\ \text{Hz}$, which is option (A).

KEY IDEA

In FM the peak frequency deviation is the frequency sensitivity times the peak message amplitude, $\Delta f = k_f A_m$. The modulation index is the deviation divided by the message frequency, $\beta = \Delta f / f_2$. Differentiating the given phase confirms that βf_2 is the peak deviation.

WATCH OUT FOR

Using $A_m = 1$ instead of the 0.5 V peak of $m(t)$ gives $\Delta f = 500$ Hz and $\beta = 0.5$, which is option (B). Another slip is to treat the carrier frequency of 40 kHz as the message frequency when computing β.

Q.49 MSQ · 2 MARKS

Engineering Mathematics › Complex analysis

The complex functions $f(z) = u(x,y) + i\,v(x,y)$ and $\overline{f(z)} = u(x,y) - i\,v(x,y)$ are both analytic in a given domain. Choose the correct option(s) from the following.

(A) $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y} = 0$

(B) $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x} \neq 0$

(C) $\dfrac{df(z)}{dz} = 0$

(D) $\dfrac{df(z)}{dz} \neq 0$

Official answer: **A, C**

SOLUTION

1. Since $f = u + i\,v$ is analytic, it satisfies the Cauchy-Riemann equations: $u_x = v_y$ and $u_y = -v_x$.
2. Write $\overline{f(z)} = u + i(-v)$: its real part is u and its imaginary part is $-v$. Since $\overline{f}$ is analytic too, its Cauchy-Riemann equations give $u_x = (-v)_y = -v_y$ and $u_y = -(-v)_x = v_x$.
3. Combine the first equations: $u_x = v_y$ and $u_x = -v_y$ give $2v_y = 0$, so
$$u_x = v_y = 0.$$
This is option (A), so (A) is true.
4. Combine the second equations: $u_y = -v_x$ and $u_y = v_x$ give $2v_x = 0$, so
$$u_y = -v_x = 0.$$
Both sides are zero. Option (B) says they are $\neq 0$, so (B) is false.
5. For an analytic function, $f'(z) = u_x + i\,v_x$. Here $f'(z) = 0 + i \cdot 0 = 0$ at every point of the domain, so (C) is true and (D) is false. (All four first partial derivatives of u and v vanish, so on a connected domain f is a constant.)

6. Correct options: (A) and (C).

KEY IDEA

Write the Cauchy-Riemann equations for $f = u + iv$ and for $\overline{f} = u + i(-v)$. Each pair of equations has the same left side and right sides of opposite sign, so every first partial derivative of u and v must be zero. That makes $f'(z) = u_x + i\,v_x = 0$, and on a connected domain f is constant.

WATCH OUT FOR

Using $+v$ instead of $-v$ as the imaginary part of $\overline{f}$ in its Cauchy-Riemann equations. Doing that just repeats the equations for f and gives no new information. Another trap is picking (B) because it looks like the standard CR equation $u_y = -v_x$. The equation does hold, but both sides equal zero, so the "$\neq 0$" part of (B) is false.

Q.50 NAT · 2 MARKS

Engineering Mathematics › Probability & statistics

Q.50 The readings recorded from a 20-psig pressure gauge are given in the Table. The regression line obtained for the data is $y = 0.04\,x + 10.32$. The regression coefficient of determination, $R^2 =$ _____ **(rounded off to three decimal places)**.

x	1	2	3	4	5
y (psig)	10.3	10.5	10.4	10.5	10.5

Official answer: **0.450 to 0.550**

SOLUTION

1. Means: $\bar{x} = \dfrac{15}{5} = 3$ and $\bar{y} = \dfrac{52.2}{5} = 10.44$.
2. Deviations from the mean: $x_i - \bar{x} = -2, -1, 0, 1, 2$ and $y_i - \bar{y} = -0.14, 0.06, -0.04, 0.06, 0.06$.
3. Sums of squares and cross-products: $S_{xx} = 4 + 1 + 0 + 1 + 4 = 10$, $S_{xy} = 0.28 - 0.06 + 0 + 0.06 + 0.12 = 0.40$, $S_{yy} = 0.0196 + 0.0036 + 0.0016 + 0.0036 + 0.0036 = 0.032$.
4. Check against the given line: slope $b = S_{xy}/S_{xx} = 0.40/10 = 0.04$ and intercept $\bar{y} - b\bar{x} = 10.44 - 0.12 = 10.32$. The given line is the least-squares fit, so the usual formula for R^2 applies.
5. Coefficient of determination for simple linear regression:

$$R^2 = \frac{S_{xy}^2}{S_{xx}\,S_{yy}} = \frac{(0.40)^2}{10 \times 0.032} = \frac{0.16}{0.32} = 0.5$$

6. Cross-check with residuals: the fitted values $\hat{y} = 10.36, 10.40, 10.44, 10.48, 10.52$ give residuals $-0.06, 0.10, -0.04, 0.02, -0.02$, so $SS_E = 0.0036 + 0.0100 + 0.0016 + 0.0004 + 0.0004 = 0.016$. With $SS_T = S_{yy} = 0.032$,

$$R^2 = 1 - \frac{SS_E}{SS_T} = 1 - \frac{0.016}{0.032} = 0.500$$

KEY IDEA

R^2 is the fraction of the spread in y that the line accounts for. For a least-squares line it equals $1 - SS_E/SS_T$, which for a straight line simplifies to $S_{xy}^2/(S_{xx}S_{yy})$ (the square of the correlation coefficient). First confirm that the given line really is the least-squares fit; here $b = S_{xy}/S_{xx} = 0.04$ and $a = \bar{y} - b\bar{x} = 10.32$.

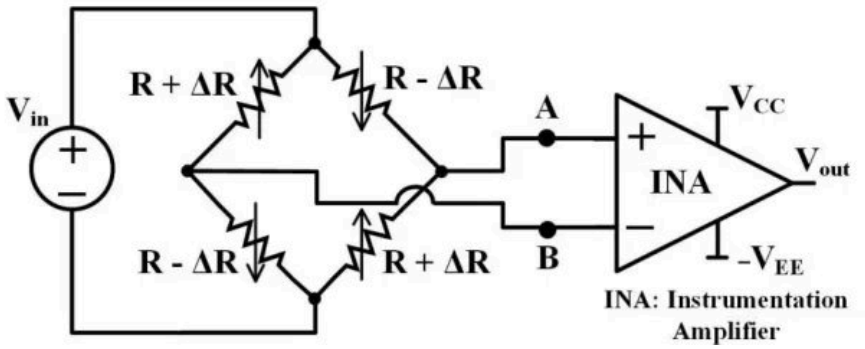
WATCH OUT FOR

Common mistakes: reporting the correlation coefficient $r = \sqrt{0.5} \approx 0.707$ in place of R^2; using raw sums of y^2 instead of deviations from the mean, which loses the small differences (of order 0.01) to rounding; and dividing by $n - 1$ in only one of S_{xy}, S_{xx} or S_{yy}. The ratio stays correct only when all three are treated the same way.

Q.51 NAT · 2 MARKS

Networks (Electric Circuits) › Laws & node/mesh

Q.51 In the figure shown, R = 4.5 kΩ, ΔR = 1.5 kΩ, and INA is assumed to be ideal. The equivalent resistance between A and B is ____ kΩ **(rounded off to nearest integer)**.



Official answer: **4**

SOLUTION

1. An ideal INA has infinite input resistance, so it draws no current and does not load terminals A-B. The equivalent resistance is therefore that of the bridge alone, seen between A (the right node) and B (the left node).
2. To find the equivalent resistance, deactivate the independent source. The ideal voltage source V_{in} is replaced by a short circuit, which joins the top node T and the bottom node Bot into a single node.
3. From B (left node) to the merged T/Bot node, the two left arms are in parallel:
$$(R + \Delta R) \parallel (R - \Delta R) = \frac{(R + \Delta R)(R - \Delta R)}{(R + \Delta R) + (R - \Delta R)} = \frac{R^2 - \Delta R^2}{2R}$$
Numerically this is $6\ \text{k}\Omega \parallel 3\ \text{k}\Omega = 2\ \text{k}\Omega$.
4. From A (right node) to the merged T/Bot node, the two right arms are also in parallel:
$$(R - \Delta R) \parallel (R + \Delta R) = \frac{R^2 - \Delta R^2}{2R}$$
Numerically this is $3\ \text{k}\Omega \parallel 6\ \text{k}\Omega = 2\ \text{k}\Omega$.
5. The two parallel pairs are in series through the merged node:
$$R_{AB} = 2 \cdot \frac{R^2 - \Delta R^2}{2R} = \frac{R^2 - \Delta R^2}{R}$$
6. Substitute $R = 4.5\ \text{k}\Omega$ and $\Delta R = 1.5\ \text{k}\Omega$:
$$R_{AB} = \frac{4.5^2 - 1.5^2}{4.5}\ \text{k}\Omega = \frac{20.25 - 2.25}{4.5}\ \text{k}\Omega = \frac{18}{4.5}\ \text{k}\Omega = 4\ \text{k}\Omega$$
This agrees with the numerical check $2 + 2 = 4\ \text{k}\Omega$.
7. The result is already an integer, so rounding changes nothing: $R_{AB} = 4\ \text{k}\Omega$.

KEY IDEA

Seen from the detector diagonal (A-B), with the excitation source shorted, a Wheatstone bridge is two parallel pairs in series: $R_{AB} = (R_1 \parallel R_2) + (R_3 \parallel R_4)$. Each pair is the two arms that meet at one detector node. An ideal INA draws no input current, so it does not appear in the result.

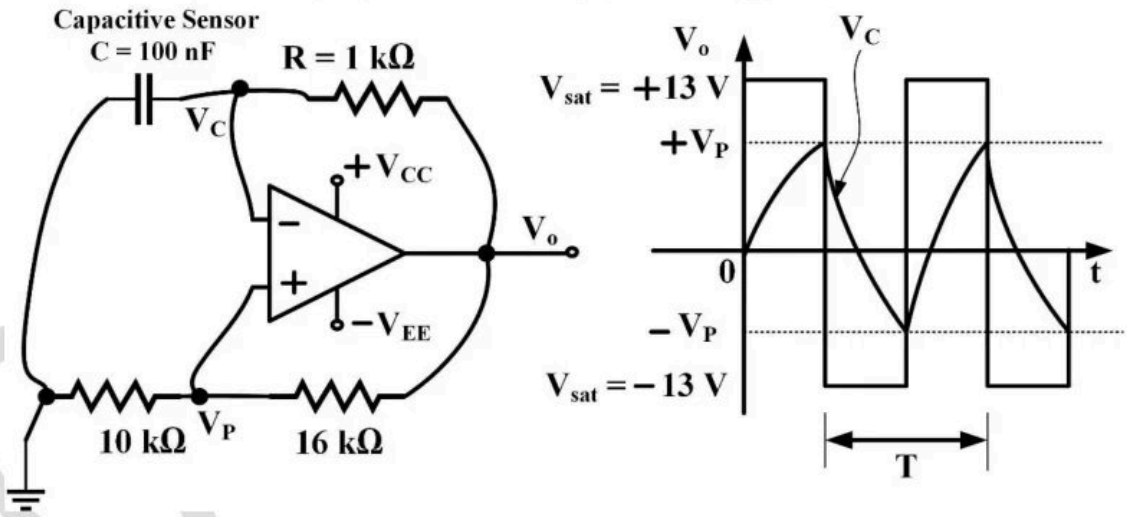
WATCH OUT FOR

Opening the voltage source instead of shorting it. The two paths from A to B are then each $(R - \Delta R) + (R + \Delta R) = 2R$, and in parallel they give $R = 4.5\ \text{k}\Omega$, which rounds to 5, not the key's 4. An ideal voltage source must be replaced by a short circuit. A second slip is to include a finite INA input resistance in parallel with the bridge. An ideal INA has infinite input resistance.

Q.52 NAT · 2 MARKS

Analog Electronics › Filters & waveforms

Q.52 Consider the capacitive sensor circuit and its output voltage shown in the figure. The circuit is switched ON at t = 0. Assuming the opamp to be ideal, the frequency of the output voltage V_o is _____ kHz (**rounded off to two decimal places**).



Official answer: **6.00 to 6.30**

SOLUTION

1. **Identify the circuit.** This is a symmetric op-amp astable (relaxation oscillator). R and C feed the inverting input; the $10\ \text{k}\Omega/16\ \text{k}\Omega$ divider feeds a fraction of the output back to the non-inverting input (positive feedback). Assumption: the op-amp is ideal (no input current, instant switching) and its output sits at $\pm V_{sat} = \pm 13\ \text{V}$, as the figure shows.
2. **Feedback fraction (voltage divider).** No current enters the $+$ input, so
$$\beta = \frac{10}{10+16} = \frac{10}{26}, \qquad V_P = \beta V_{sat} = \frac{10}{26}(13) = 5\ \text{V}$$
The comparator therefore switches when V_C reaches $+5\ \text{V}$ (while $V_o = +13\ \text{V}$) or $-5\ \text{V}$ (while $V_o = -13\ \text{V}$).
3. **Start-up (first half-cycle only).** At $t = 0$, $V_C = 0 < V_P$, so $V_o = +13\ \text{V}$ and C charges from 0 toward $+13\ \text{V}$. This first interval, from 0 to $+5\ \text{V}$, is shorter than a normal half-period ($RC \ln \frac{13}{8}$), which is why the figure marks T later on. It does not set the frequency.

4. **Steady-state half-period (RC charging law).** With $V_o = +13$ V, C charges through R from -5 V toward $+13$ V:

$$V_C(t) = V_{sat} + (-V_P - V_{sat})e^{-t/RC}$$

Setting $V_C(t_1) = +V_P$ gives

$$t_1 = RC \ln \frac{V_{sat} + V_P}{V_{sat} - V_P} = RC \ln \frac{13+5}{13-5} = RC \ln 2.25$$

The discharge from $+5$ V toward -13 V, ending at -5 V, is the mirror image, so it takes the same time.

5. **Period.**

$$T = 2RC \ln \frac{1+\beta}{1-\beta} = 2RC \ln \frac{36/26}{16/26} = 2RC \ln 2.25$$

With $RC = (1\ \text{k}\Omega)(100\ \text{nF}) = 10^{-4}$ s:

$$T = 2(10^{-4})(0.81093) = 1.6219 \times 10^{-4}\ \text{s}$$

6. **Frequency.**

$$f = \frac{1}{T} = \frac{1}{1.6219 \times 10^{-4}\ \text{s}} \approx 6165.8\ \text{Hz} \approx 6.17\ \text{kHz}$$

KEY IDEA

In a symmetric op-amp astable, the capacitor swings between $\pm\beta V_{sat}$, charging each time toward $\pm V_{sat}$. Each half-period is $RC \ln \frac{1+\beta}{1-\beta}$, so $T = 2RC \ln \frac{1+\beta}{1-\beta} = 2RC \ln\left(1 + \frac{2R_1}{R_2}\right)$. Here $\beta = \frac{R_1}{R_1+R_2}$, R_1 is the resistor from the $+$ input to ground (10 kΩ) and R_2 is the one from the $+$ input to the output (16 kΩ). The value of V_{sat} cancels, so the ±13 V only confirms that the thresholds are ±5 V.

WATCH OUT FOR

Swapping the divider resistors. $\beta = \frac{16}{26}$, or $T = 2RC \ln\left(1 + \frac{2\cdot16}{10}\right) = 2RC \ln 4.2$, gives about 3.48 kHz. Taking $\beta = \frac{10}{16}$ (the resistor ratio instead of the divider fraction) gives about 3.41 kHz. A third mistake is timing the start-up interval (0 to $+V_P$, which is $RC \ln \frac{13}{8}$) as if it were a half-period; the figure marks T in steady state, where V_C swings the full $-V_P$ to $+V_P$.

Q.53 NAT · 2 MARKS

Signals and Systems › DTFT & DFT

The 4-point DFTs of two sequences $x[n]$ and $y[n]$ are $X[k] = [1, -j, 1, j]$ and $Y[k] = [1, 3j, 1, -3j]$, respectively. Assuming $z[n]$ represents the 4-point circular convolution of $x[n]$ and $y[n]$, the value of $z[0]$ is _____ (rounded off to nearest integer).

Note: The DFT of a N-point sequence $x[n]$ is defined as

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{\frac{-j2\pi nk}{N}}$$

Official answer: **2**

SOLUTION

1. Circular convolution property of the DFT: a 4-point circular convolution in n becomes a point-by-point product in k, so $Z[k] = X[k]\,Y[k]$ for $k = 0, 1, 2, 3$.
2. Multiply term by term:

$$Z[0] = (1)(1) = 1$$
$$Z[1] = (-j)(3j) = -3j^2 = 3$$
$$Z[2] = (1)(1) = 1$$
$$Z[3] = (j)(-3j) = -3j^2 = 3$$

so $Z[k] = [1, 3, 1, 3]$.

3. Inverse DFT: the forward DFT given in the question has no scale factor, so the $\frac{1}{N}$ goes on the inverse: $z[n] = \frac{1}{4}\sum_{k=0}^{3} Z[k]\, e^{\frac{j2\pi nk}{4}}$.

4. At $n = 0$ every exponential equals 1, so $z[0]$ is the average of the DFT samples:
$$z[0] = \frac{1}{4}(1 + 3 + 1 + 3) = \frac{8}{4} = 2$$
5. Check: $Z[k]$ is real and $Z[1] = Z[3]$, which is what a real, circularly even $z[n]$ produces, so the result is consistent. $z[0] = 2$.

KEY IDEA

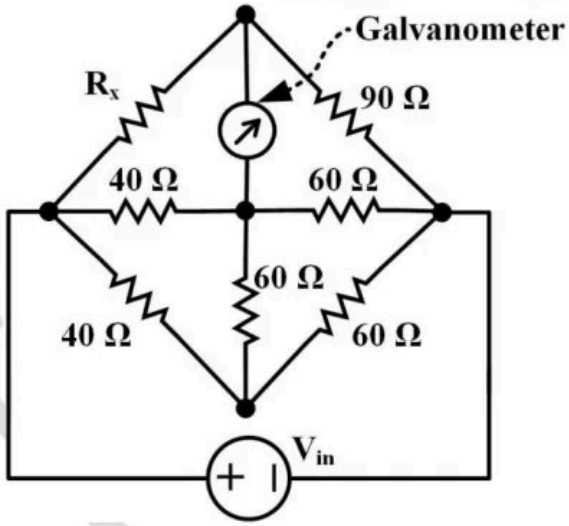
A circular convolution of length N becomes a product of the N-point DFTs, $Z[k] = X[k]Y[k]$. Setting $n = 0$ in the inverse DFT turns every exponential into 1, so $z[0] = \frac{1}{N}\sum_k Z[k]$. You never have to recover $x[n]$ or $y[n]$.

WATCH OUT FOR

Leaving out the $\frac{1}{N}$ in the inverse DFT gives $z[0] = 8$. Because the forward DFT given here has no scale factor, the whole $\frac{1}{4}$ belongs to the inverse. The other slip is a sign error in $j \cdot j$: $(-j)(3j) = -3j^2 = +3$, not -3. With that error $Z[1]$ and $Z[3]$ come out as -3, and the sum gives $z[0] = -1$.

Q.54 NAT · 2 MARKS

Networks (Electric Circuits) › Laws & node/mesh

Q.54	Consider the figure shown. For zero deflection in the galvanometer, the required value of resistor R_x is ____ Ω **(rounded off to nearest integer).**
	

Official answer: **58 to 62**

SOLUTION

1. Label the nodes: top T, left L, centre C, right R, bottom B. The source V_{in} is connected between L (+) and R (–). Take $V_R = 0$, so $V_L = V_{in}$.
2. Zero galvanometer deflection means no current in the galvanometer branch T–C, so $V_T = V_C$. Because that branch carries no current, treat it as open when you find the node voltages.
3. Look at the lower network. C and B each connect to L through $40\ \Omega$ and to R through $60\ \Omega$. That is a balanced bridge of its own, so $V_C = V_B$ and the $60\ \Omega$ resistor between C and B carries no current. (Check: with that resistor removed, both nodes sit at $0.6\,V_{in}$, so it really does carry no current, and the solution is unique.)
4. Voltage divider along L–C–R:

$$V_C = V_{in}\,\frac{60}{40+60} = 0.6\,V_{in}$$

5. Voltage divider along L–T–R (no current leaves T through the galvanometer):

$$V_T = V_{in}\,\frac{90}{R_x + 90}$$

6. Balance condition $V_T = V_C$:

$$\frac{90}{R_x+90} = 0.6 \;\Rightarrow\; R_x + 90 = 150 \;\Rightarrow\; R_x = 60\ \Omega$$

This is the usual Wheatstone ratio $\dfrac{R_x}{90} = \dfrac{40}{60}$.

KEY IDEA

With zero galvanometer current, nodes T and C are at the same potential. The lower network ($40\ \Omega$ and $60\ \Omega$ from L to R through both C and B) is itself balanced, so the $60\ \Omega$ resistor from C to B carries no current. V_C is then just the $40{:}60$ divider, and the balance reduces to the ratio $R_x/90 = 40/60$.

WATCH OUT FOR

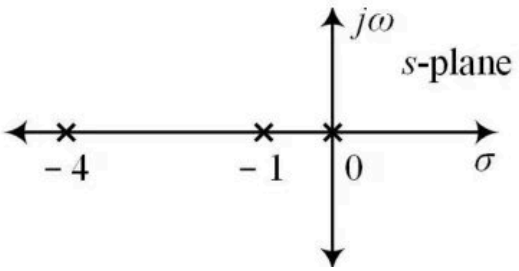
Setting up the ratio with the arms swapped, for example $R_x/90 = 60/40$, which gives $135\ \Omega$. Another mistake is assuming the bottom node B is wired to the source, or trying to solve the full five-node network when the C–B resistor carries no current. The arms that must be in the same ratio are the ones that meet at the source terminals: R_x and $40\ \Omega$ both touch L, and $90\ \Omega$ and $60\ \Omega$ both touch R.

Q.55 NAT · 2 MARKS

Control Systems › Root locus

Q.55 Consider a unity negative feedback system with its open-loop pole-zero map as shown in the figure. If the point $s = j\alpha$, $\alpha > 0$, lies on the root locus, the value of α is _____ **(rounded off to nearest integer).**

Note: The poles are marked with × in the figure.



Official answer: **2**

SOLUTION

1. Read the map: poles at 0, -1 and -4, no zeros. With unity negative feedback, $G(s)H(s) = \dfrac{K}{s(s+1)(s+4)}$.
2. Characteristic equation: $1 + G(s) = 0 \Rightarrow s(s+1)(s+4) + K = 0 \Rightarrow s^3 + 5s^2 + 4s + K = 0$.
3. Routh array: $s^3 : 1,\ 4$; $s^2 : 5,\ K$; $s^1 : \dfrac{5\cdot 4 - K}{5} = \dfrac{20-K}{5}$; $s^0 : K$.
4. The locus crosses the $j\omega$-axis when the s^1 row becomes zero: $20 - K = 0 \Rightarrow K = 20$.
5. Auxiliary equation from the s^2 row: $5s^2 + 20 = 0 \Rightarrow s^2 = -4 \Rightarrow s = \pm j2$.
6. Check by substituting $s = j\omega$:

$$-j\omega^3 - 5\omega^2 + j4\omega + K = 0$$

Imaginary part: $\omega(4-\omega^2)=0 \Rightarrow \omega = 2\ \text{rad/s}$ (taking $\omega > 0$). Real part: $K - 5\omega^2 = 0 \Rightarrow K = 20 > 0$, so the point lies on the positive-K root locus.

7. Angle check at $s = j2$: the pole angles are 90° (from 0), $\tan^{-1}(2) = 63.43^\circ$ (from -1) and $\tan^{-1}(0.5) = 26.57^\circ$ (from -4). They sum to 180°, so $\angle G(j2) = -180^\circ$ and the angle condition holds.
8. Hence $s = j\alpha = j2$, giving $\alpha = 2$.

KEY IDEA

A point on the $j\omega$-axis lies on the root locus exactly where the closed loop is marginally stable. Make the s^1 row of the Routh array zero to get the critical gain, then solve the auxiliary equation from the s^2 row to get the crossing frequency. As a quick cross-check, the pole angles at $s = j2$ ($90^\circ + 63.43^\circ + 26.57^\circ$) add up to exactly 180°.

WATCH OUT FOR

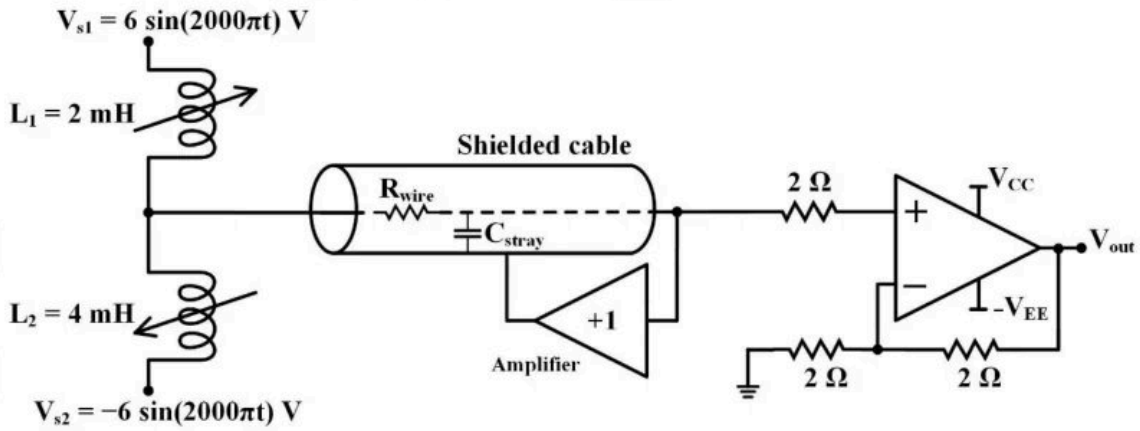
Stopping at the critical gain and entering $K = 20$ when the question asks for the crossing frequency $\alpha = 2$. Another slip is expanding $s(s+1)(s+4)$ as $s^3 + 5s^2 + 5s$ (the s coefficient is $1 \cdot 4 = 4$), which moves the crossing to $j\sqrt{5}$. Some students also treat the trivial root $\omega = 0$ as the answer, but it corresponds to $K = 0$ and is ruled out by $\alpha > 0$.

Q.56 NAT · 2 MARKS

Analog Electronics › Noise & shielding (IN)

Q.56 A shielded cable with $C_{stray} = 20$ pF and $R_{wire} = 10\ \Omega$ is used to connect the inductive sensors as shown in the figure. The RMS value of V_{out} is ____V **(rounded off to two decimal places).**

Note: Assume all components are ideal, and sensors are not magnetically coupled.



Official answer: **2.81 to 2.85**

SOLUTION

1. **What the guard does.** The unity-gain buffer copies the voltage of the cable's inner wire onto the shield, so both plates of C_{stray} sit at the same potential. With zero voltage across it, C_{stray} carries no current and has no effect on the signal. This is a driven shield. (Even without the guard its effect would be negligible here. At $f = 1$ kHz, $X_C = \dfrac{1}{2\pi(1000)(20\times10^{-12})} \approx 7.96\ \text{M}\Omega$, while the sensor divider's Thevenin impedance is only $\omega(L_1 \parallel L_2) \approx 8.4\ \Omega$.)
2. **The cable draws no current.** An ideal op-amp input and an ideal buffer input draw zero current, and C_{stray} carries none. So no current flows in R_{wire} or in the $2\ \Omega$ input resistor. Both drop 0 V, and the +

input sits at the sensor junction voltage V_m.

3. **The sensors form an unloaded inductive divider.** L_1 and L_2 carry the same current and are not magnetically coupled, so each has impedance $j\omega L$ with $\omega = 2000\pi$ rad/s ($f = 1$ kHz). The two sources are in phase (one is the negative of the other), so the $j\omega$ factors cancel and the divider acts on the amplitudes directly.
4. **Superposition (Millman's theorem) at the junction:**
$$V_m = \frac{V_{s1}\,(j\omega L_2) + V_{s2}\,(j\omega L_1)}{j\omega(L_1 + L_2)} = \frac{6(4) + (-6)(2)}{2 + 4}\,\sin(2000\pi t) = 2\sin(2000\pi t)\ \text{V}$$
5. **Check with KVL:** $I\,j\omega(L_1 + L_2) = V_{s1} - V_{s2} = 12\sin(2000\pi t)$. The drop across L_1 has amplitude $12 \cdot \dfrac{2}{6} = 4$, so $V_m = 6 - 4 = 2$ (amplitude). This agrees with the previous step.
6. **Non-inverting amplifier gain.** Assume an ideal op-amp (virtual short between the inputs, no input current) and an output that stays inside the supply rails:
$$A_v = 1 + \frac{R_f}{R_g} = 1 + \frac{2\ \Omega}{2\ \Omega} = 2$$
7. **Output:** $V_{\text{out}} = 2 \times 2\sin(2000\pi t) = 4\sin(2000\pi t)$ V
8. **RMS value:**
$$V_{\text{out,rms}} = \frac{4}{\sqrt{2}} = 2\sqrt{2} \approx 2.83\ \text{V}$$

KEY IDEA

The buffer holds the shield at the inner wire's voltage, so C_{stray} has no voltage across it and carries no current. The ideal amplifier inputs draw no current either, so R_{wire} and the $2\ \Omega$ input resistor drop nothing. The op-amp therefore sees the unloaded inductive-divider voltage $V_m = \dfrac{V_{s1}L_2 + V_{s2}L_1}{L_1 + L_2} = 2\sin(2000\pi t)$. A non-inverting gain of 2 makes the peak 4 V, and $4/\sqrt{2} \approx 2.83$ V RMS.

WATCH OUT FOR

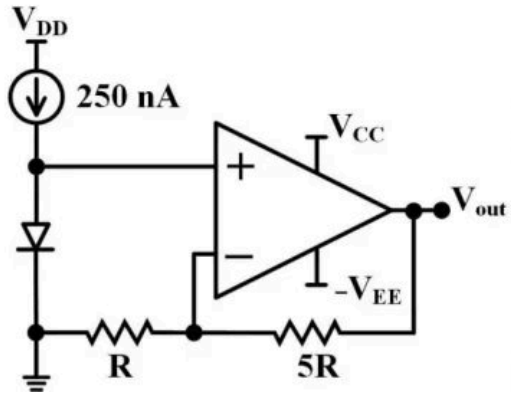
Swapping the divider weights. The junction voltage weights V_{s1} by the FAR inductor L_2, not by L_1. Writing $\dfrac{6(2) + (-6)(4)}{6}$ gives -2, which has the same magnitude only by coincidence, so check the sign with KVL. Other slips: using an inverting gain of $-R_f/R_g = -1$ instead of the non-inverting $1 + R_f/R_g = 2$ (answer 1.41); loading the divider with R_{wire}, the $2\ \Omega$ resistor or C_{stray} even though no current flows; and reporting the peak (4 V) instead of the RMS value.

Q.57 NAT · 2 MARKS

Analog Electronics › Diode circuits

Q.57

In the figure shown, the diode current is given by $I_D = I_S\, e^{\frac{\alpha V_D}{T}}$. V_D is the diode voltage in volts, T is the absolute temperature in Kelvin, $\alpha = 1.16 \times 10^4$ K/V, and $I_S = 10^{-15}$ A is the saturation current. The dc current source, opamp and the resistors are ideal, and are assumed to be temperature independent. The change in the output voltage (V_{out}) per Kelvin change in temperature is _____ mV **(rounded off to one decimal place).**



Official answer: **9.5 to 10.5**

SOLUTION

1. **Diode bias.** The ideal 250 nA source forces $I_D = 250\ \text{nA}$ through the diode. Its anode is at the top node and its cathode is at the grounded rail, so this current forward-biases it. The op-amp's $+$ input is tied to the anode, so it sits at V_D.
2. **Invert the diode law.** From $I_D = I_S\, e^{\alpha V_D/T}$:
$$V_D = \frac{T}{\alpha} \ln\left(\frac{I_D}{I_S}\right)$$
3. **Op-amp stage (ideal op-amp, negative feedback, virtual short).** The feedback resistor $5R$ runs from V_{out} to the $-$ input, and R runs from the $-$ input to ground. So $V_- = V_{out}\dfrac{R}{R+5R} = V_+ = V_D$. This is a non-inverting amplifier:
$$V_{out} = \left(1 + \frac{5R}{R}\right) V_D = 6\, V_D = \frac{6T}{\alpha} \ln\left(\frac{I_D}{I_S}\right)$$
4. **Temperature derivative.** I_D comes from an ideal source, and I_S, α and the resistor ratio are given as temperature independent. So only the explicit T changes:
$$\frac{dV_{out}}{dT} = \frac{6}{\alpha} \ln\left(\frac{I_D}{I_S}\right)$$
5. **Evaluate the logarithm:**
$$\ln\left(\frac{250 \times 10^{-9}}{10^{-15}}\right) = \ln(2.5 \times 10^8) = \ln 2.5 + 8 \ln 10 = 0.9163 + 18.4207 = 19.337$$
6. **Result:**
$$\frac{dV_{out}}{dT} = \frac{6 \times 19.337}{1.16 \times 10^4}\ \text{V/K} = 1.0002 \times 10^{-2}\ \text{V/K} \approx 10.0\ \text{mV/K}$$
7. **Sign and sanity check.** The result is positive. With I_S held constant and a fixed current, V_D is proportional to absolute temperature (the PTAT principle), so $\dfrac{dV_D}{dT} = \dfrac{V_D}{T}$. At $T = 300$ K this model

gives $V_D = \dfrac{300}{1.16 \times 10^4} \times 19.337 \approx 0.50\ \text{V}$, a realistic silicon diode drop, and $\dfrac{dV_D}{dT} \approx 1.67\ \text{mV/K}$. The gain of 6 turns that into $10.0\ \text{mV/K}$. The answer does not depend on which temperature is chosen.

KEY IDEA

A fixed current in a diode whose saturation current is held constant gives $V_D = \frac{T}{\alpha}\ln(I_D/I_S)$, which is proportional to absolute temperature. The non-inverting amplifier multiplies it by $1 + 5R/R = 6$, so $dV_{out}/dT = \frac{6}{\alpha}\ln(I_D/I_S)$.

WATCH OUT FOR

(1) Using the familiar $-2\ \text{mV}/^\circ\text{C}$ rule for a real diode. That rule comes from I_S rising steeply with temperature, and this question fixes I_S at a constant, so here the voltage rises with temperature. (2) Taking the gain as $5R/R = 5$, the inverting-amplifier gain, when the circuit is non-inverting with gain $1 + 5 = 6$; this gives 8.3 mV. (3) Dropping the $8 \ln 10$ term or using $\log_{10}$ in place of $\ln$.

Q.58 NAT · 2 MARKS

Digital Electronics › ADC & DAC

An ADC has a full scale voltage of 1.4 V, resolution of 200 mV, and produces binary output data. The input signal of the ADC has a bandwidth of 500 MHz, and it samples the data at the Nyquist rate. The parallel data output is converted to a serial bit stream using a parallel-to-serial converter. The data rate at the output of the parallel-to-serial converter is _____ Gbps (rounded off to nearest integer).

Official answer: **3**

SOLUTION

1. **Number of quantisation steps.** The full-scale range divided by the step size is $\dfrac{V_{FS}}{\Delta} = \dfrac{1.4\ \text{V}}{0.2\ \text{V}} = 7$ steps. The output levels are $0, 0.2, 0.4, \ldots, 1.4\ \text{V}$, which is $7 + 1 = 8$ distinct levels.
2. **Number of bits.** With the convention $\Delta = \dfrac{V_{FS}}{2^n - 1}$ we get $2^n - 1 = 7$, so $n = 3$ exactly. With the other convention, $\Delta = \dfrac{V_{FS}}{2^n}$, we need $\dfrac{1.4}{2^n} \le 0.2$, so $2^n \ge 7$. $n = 2$ gives $1.4/4 = 0.35\ \text{V}$, which is too coarse, and $n = 3$ gives $1.4/8 = 0.175\ \text{V}$, which meets the requirement. So the smallest whole number of bits is $n = 3$. Both conventions give $n = 3$ bits per sample.
3. **Nyquist sampling rate.** $f_s = 2f_{max} = 2 \times 500\ \text{MHz} = 1\ \text{GHz}$, which is 10^9 samples per second.
4. **Serial bit rate.** Each sample produces an n-bit parallel word. The parallel-to-serial converter has to send all n bits one after another within one sample period $T_s = 1/f_s$, so $R_b = n\, f_s$.
5. $$R_b = 3 \times 10^9\ \text{bits/s} = 3\ \text{Gbps}$$

KEY IDEA

The number of bits per sample comes from the full-scale voltage and the resolution: $1.4/0.2 = 7$ steps, which means 8 levels and so 3 bits. The Nyquist rate for a 500 MHz band is 10^9 samples/s. The serial bit rate is the bits per sample times the sampling rate: $3 \times 1\ \text{GHz} = 3\ \text{Gbps}$.

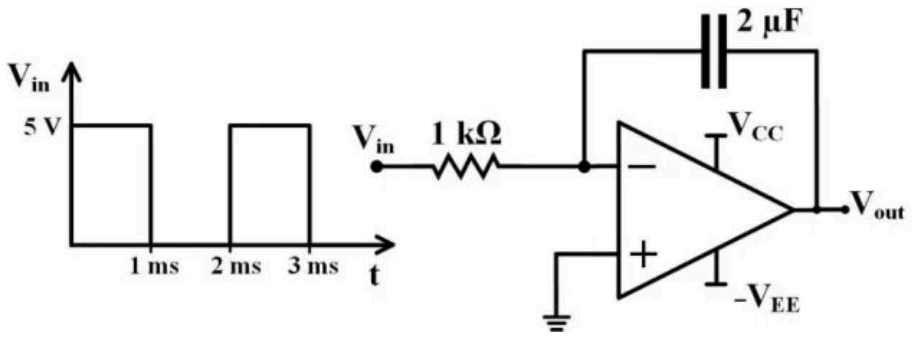
WATCH OUT FOR

Using the bandwidth (500 MHz) as the sampling rate instead of the Nyquist rate $2 \times 500\ \text{MHz}$, which gives 1.5 Gbps. Another mistake is taking $\log_2 7 \approx 2.81$ as the bit count and giving about 2.8 Gbps. The bit count has to be a whole number, and 7 steps means 8 levels, so it needs 3 bits.

Q.59 NAT · 2 MARKS

Analog Electronics › Op-amps

Q.59 In the circuit shown, assume the opamp is ideal and the initial charge on the capacitor is zero. The output voltage at time t = 2 ms is _____ V **(rounded off to one decimal place)**.



Official answer: **-2.6 to -2.4**

SOLUTION

1. Assumptions: the op-amp is ideal (infinite gain, zero input current, output inside the supply rails, so not saturated) and the capacitor starts uncharged, so $V_{out}(0) = 0$.
2. Virtual short: the + input is grounded, so the – input sits at 0 V (virtual ground). The resistor current is therefore $i = \dfrac{V_{in}}{R}$.
3. KCL at the inverting node: no current enters the op-amp, so all of i flows through the capacitor. The input side of C is at 0 and the output side at V_{out}, so $C\dfrac{d(0 - V_{out})}{dt} = \dfrac{V_{in}}{R}$, which gives
$$V_{out}(t) = V_{out}(0) - \frac{1}{RC}\int_0^t V_{in}\,dt = -\frac{1}{RC}\int_0^t V_{in}\,dt$$
4. Time constant: $RC = (1\ \text{k}\Omega)(2\ \mu\text{F}) = 2\ \text{ms}$.
5. For $0 \le t \le 1$ ms, $V_{in} = 5$ V is constant, so the output ramps down linearly:
$$V_{out}(1\ \text{ms}) = -\frac{5\ \text{V} \times 1\ \text{ms}}{2\ \text{ms}} = -2.5\ \text{V}$$
6. For $1\ \text{ms} < t < 2\ \text{ms}$, $V_{in} = 0$. No current flows through R, so none flows through C either. The capacitor keeps its charge and V_{out} stays at -2.5 V.
7. At $t = 2$ ms the second pulse is only just starting. The integral is a continuous function of t, so the step in V_{in} has not added any area yet:
$$V_{out}(2\ \text{ms}) = -2.5\ \text{V}$$

KEY IDEA

This is an inverting integrator. Its output equals minus the area under V_{in} divided by RC, starting from the initial capacitor voltage. While $V_{in} = 0$ there is no current, so the output holds its last value. Here the area up to 2 ms is $5\ \text{V} \times 1\ \text{ms}$ and $RC = 2$ ms, so $V_{out} = -2.5$ V.

WATCH OUT FOR

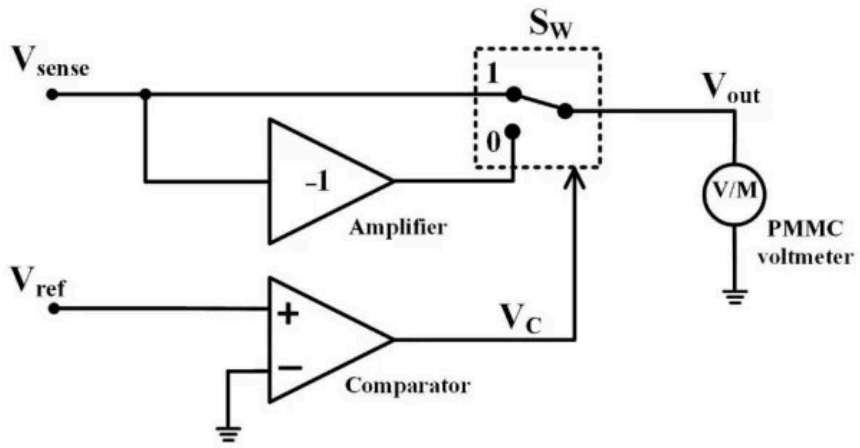
(1) Treating the circuit as a passive RC charging exponentially with $\tau = RC$, e.g. $-5(1 - e^{-0.5}) \approx -2.0$ V. The virtual ground keeps the resistor current constant, so the output ramp is linear. (2) Assuming the output decays back to 0 during the 1–2 ms gap. With zero input there is no current, so the capacitor holds its charge. (3) Dropping the minus sign of the inverting integrator and answering $+2.5$. (4) Unit slips in RC ($1\ \text{k}\Omega \times 2\ \mu\text{F} = 2$ ms, not 2 s).

Q.60 NAT · 2 MARKS

Electrical and Electronic Measurements › V, I, power & energy

Q.60 In the figure shown, S_W is a switch whose position changes from 1 to 0 when V_C changes from logic HIGH to LOW and vice versa. The bandwidth of the permanent magnet moving coil (PMMC) type voltmeter is 1 Hz. If $V_{sense} = 2\sin(4000\pi t)$ V and $V_{ref} = 4\sin(2000\pi t)$ V, the voltmeter reading is _____V **(rounded off to nearest integer)**.

Note: Assume all components are ideal.



Official answer: **0**

SOLUTION

1. **Comparator with its $-$ input grounded:** V_C is HIGH when $V_{\text{ref}} > 0$ and LOW when $V_{\text{ref}} < 0$. $V_{\text{ref}} = 4\sin(2000\pi t)$ has frequency $1\ \text{kHz}$, so its period is $T = 1\ \text{ms}$. For $0 < t < 0.5\ \text{ms}$, V_C is HIGH and S_W is at 1. For $0.5 < t < 1\ \text{ms}$, V_C is LOW and S_W is at 0.
2. **Switch output:** position 1 passes V_{sense} and position 0 passes $-V_{\text{sense}}$ (through the gain -1 amplifier). So $V_{\text{out}} = V_{\text{sense}} \cdot s(t)$, where $s(t) = \text{sgn}[\sin(2000\pi t)]$ is a ± 1 square wave at $1\ \text{kHz}$. The circuit is a phase-sensitive (synchronous) detector.
3. **PMMC with $1\ \text{Hz}$ bandwidth:** the meter responds only to the DC (average) value of V_{out}. Every AC component of V_{out} is at $1\ \text{kHz}$ or above, far outside its bandwidth.
4. **Average over one reference period:**
$$V_{\text{avg}} = \frac{1}{T}\left[\int_0^{T/2} 2\sin(4000\pi t)\,dt - \int_{T/2}^{T} 2\sin(4000\pi t)\,dt\right]$$
5. Across each half-period ($0.5\ \text{ms}$), the argument $4000\pi t$ advances by $4000\pi \times 0.5 \times 10^{-3} = 2\pi$. V_{sense} at $2\ \text{kHz}$ therefore completes exactly one full cycle in each half, so each integral is 0 and $V_{\text{avg}} = 0\ \text{V}$.
6. **Fourier check:** the square wave $s(t)$ has no DC term and only odd harmonics ($1, 3, 5, \ldots\ \text{kHz}$). Multiplying by a $2\ \text{kHz}$ sine gives components at $|2 \pm (2k+1)|\ \text{kHz}$. These are all odd multiples of $1\ \text{kHz}$ and are never $0\ \text{Hz}$, so V_{out} has no DC term.
7. The voltmeter reading is $0\ \text{V}$.

KEY IDEA

The comparator, the gain -1 amplifier and the switch together multiply V_{sense} by a ± 1 square wave locked to V_{ref}. This is a phase-sensitive detector. A PMMC meter with a $1\ \text{Hz}$ bandwidth reads only the DC part of the product. That DC part is non-zero only when V_{sense} has a component at the reference frequency or at one of its odd harmonics. Here $2\ \text{kHz}$ is an even harmonic of $1\ \text{kHz}$, so the reading is 0.

WATCH OUT FOR

One error is to treat the circuit as a full-wave rectifier, taking $V_{\text{out}} = |V_{\text{sense}}|$ and reading $\frac{2V_m}{\pi} = \frac{4}{\pi} \approx 1.27 \rightarrow 1\ \text{V}$. The switch follows V_{ref}, not V_{sense}, so the two signals are at different frequencies and the product is not rectified. Another error is to read an rms value, $\frac{2}{\sqrt{2}} \approx 1.41 \rightarrow 1\ \text{V}$. A PMMC meter responds to the average value, not the rms value.

Q.61 NAT · 2 MARKS

Electrical Machines › Transformers

A 50 kVA transformer has an efficiency of 95% at full load and unity power factor. Assume the core losses are negligible. The efficiency of the transformer at 75% of the full load and 0.8 power factor is _____% (rounded off to one decimal place).

Official answer: **95.2 to 95.4**

SOLUTION

1. Because core losses are negligible, the total loss at any load equals the copper loss alone.
2. At full load and unity power factor: rated output = 50 kW, and eta_fl = 50/(50+Pcu,fl) = 0.95, giving Pcu,fl = 50*(1/0.95 - 1) = 2.6316 kW.
3. Copper loss varies as the square of the per-unit load. At 75% load: Pcu = (0.75)^2 * 2.6316 = 1.4803 kW.
4. Core loss is negligible at every load, so total loss at 75% load is also 1.4803 kW.
5. Output at 75% load and 0.8 power factor: Pout = 0.75 *50* 0.8 = 30 kW.
6. Efficiency: eta = Pout/(Pout+Pcu) = 30/(30+1.4803) = 30/31.4803 = 0.9530.
7. So the efficiency is 95.3%.

KEY IDEA

With core loss negligible, all loss at any load is copper loss, which scales as the square of the per-unit load. Full-load unity-pf efficiency fixes the full-load copper loss; scale it to 75% load, recompute output at the new load fraction and power factor, and take the ratio.

WATCH OUT FOR

Forgetting that copper loss scales with the square of the load fraction (using 0.75x instead of 0.75 squared), or re-deriving core loss instead of taking it as negligible as stated.

Q.62 NAT · 2 MARKS

Electrical Machines › Induction machines

A three-phase squirrel-cage induction motor has a starting torque of 100% of the full load torque and a maximum torque of 300% of the full load torque. Neglecting the stator impedance, the slip at the maximum torque is _____% (rounded off to two decimal places).

Official answer: **17.00 to 17.30**

SOLUTION

1. Neglecting stator impedance, the torque-slip relation near the maximum-torque point is
$$\frac{T}{T_{max}} = \frac{2}{\frac{s}{s_{maxT}} + \frac{s_{maxT}}{s}}$$
2. At starting, $s = 1$, so
$$\frac{T_{st}}{T_{max}} = \frac{2\, s_{maxT}}{1 + s_{maxT}^2}$$

3. Given $T_{st} = 1.0\,T_{fl}$ and $T_{max} = 3.0\,T_{fl}$, so $\dfrac{T_{st}}{T_{max}} = \dfrac{1}{3}$.
4. Substituting: $\dfrac{2\,s_{maxT}}{1 + s_{maxT}^2} = \dfrac{1}{3} \Rightarrow 6\,s_{maxT} = 1 + s_{maxT}^2 \Rightarrow s_{maxT}^2 - 6\,s_{maxT} + 1 = 0$
5. Solving the quadratic:
$$s_{maxT} = \frac{6 \pm \sqrt{36 - 4}}{2} = 3 \pm 2\sqrt{2}$$
6. Since slip at maximum torque must satisfy $0 < s_{maxT} < 1$ (starting torque at $s = 1$ is only 100% of full load torque, well below the 300% peak, so the peak lies at a smaller slip than the starting slip), the valid root is
$$s_{maxT} = 3 - 2\sqrt{2} \approx 0.17157$$
7. Expressed as a percentage: $s_{maxT} \approx 17.16\%$.

KEY IDEA
With stator impedance neglected, the torque-slip relation about the maximum-torque point is T/Tmax = 2/(s/s_maxT + s_maxT/s). Evaluating at starting slip (s=1) with the given ratio Tst/Tmax = 1/3 gives a quadratic in s_maxT; the physically valid root (0 < s_maxT < 1) is the answer.

WATCH OUT FOR
Picking the wrong root of the quadratic (taking s = 3 + 2√2 ≈ 5.83, which is unphysical since slip cannot exceed 1 on this branch) instead of s = 3 - 2√2 ≈ 0.1716.

Q.63 NAT · 2 MARKS

Networks (Electric Circuits) › Coupled circuits

Two magnetically coupled coils, when connected in series-aiding configuration, have a total inductance of 500 mH. When connected in series-opposing configuration, the coils have a total inductance of 300 mH. If the self-inductance of both the coils are equal, then the coupling coefficient is _____ (rounded off to two decimal places).

Official answer: **0.25**

SOLUTION

1. Series-aiding connection (the two fluxes add): $L_{aid} = L_1 + L_2 + 2M = 500$ mH.
2. Series-opposing connection (the two fluxes oppose): $L_{opp} = L_1 + L_2 - 2M = 300$ mH.
3. Subtract the second equation from the first to get the mutual inductance: $4M = L_{aid} - L_{opp} = 500 - 300 = 200$ mH, so $M = 50$ mH.
4. Add the two equations to get the sum of the self-inductances: $2(L_1 + L_2) = 800$ mH, so $L_1 + L_2 = 400$ mH. The two self-inductances are equal, so $L_1 = L_2 = 200$ mH.
5. Coupling coefficient:
$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{50}{\sqrt{200 \times 200}} = \frac{50}{200} = 0.25$$

KEY IDEA
Aiding minus opposing inductance is $4M$, and aiding plus opposing is $2(L_1 + L_2)$. Once you have M and the two self-inductances, $k = M/\sqrt{L_1 L_2}$. You never need L_1 and L_2 separately unless they are needed for $\sqrt{L_1 L_2}$. Here they are equal, so $\sqrt{L_1 L_2} = L_1$.

WATCH OUT FOR
Writing $2M = L_{aid} - L_{opp}$ instead of $4M$ gives $M = 100$ mH and $k = 0.5$. A second slip is to divide M by $L_1 + L_2 = 400$ mH instead of by $\sqrt{L_1 L_2} = 200$ mH, which gives 0.125.

Q.64 NAT · 2 MARKS

Engineering Mathematics › Differential equations

The solution of an ordinary differential equation $y''' + 3y'' + 3y' + y = 30e^{-t}$ is

$$y(t) = (c_0 + c_1 t - c_2 t^2 + c_3 t^3) e^{-t}$$

Given $y(0) = 3$, $y'(0) = -3$ and $y''(0) = -47$, the value of $(c_0 + c_1 + c_2 + c_3)$ is _____ (rounded off to nearest integer).

Note: $y''' = d^3y/dt^3$, $y'' = d^2y/dt^2$, $y' = dy/dt$ and c_0, c_1, c_2, c_3 are constants.

Official answer: **33**

SOLUTION

1. **Characteristic equation:** $r^3 + 3r^2 + 3r + 1 = (r+1)^3 = 0$, so $r = -1$ is a triple root. The complementary function is $(A + Bt + Ct^2)e^{-t}$. This gives the c_0, c_1 and c_2 terms of the given form, with $C = -c_2$.
2. **Particular integral:** the forcing term e^{-t} corresponds to a root of multiplicity 3, so try $y_p = Kt^3e^{-t}$. The exponential shift rule $(D+1)^3\,[e^{-t}f(t)] = e^{-t}\,D^3 f(t)$ gives
$$(D+1)^3\left[Kt^3e^{-t}\right] = Ke^{-t}\,D^3(t^3) = 6Ke^{-t} = 30e^{-t} \;\Rightarrow\; K = 5$$
So the ODE alone fixes $c_3 = 5$. The initial conditions cannot change it, because D^3 of $c_0 + c_1 t - c_2 t^2$ is zero.
3. Write $y = u\,e^{-t}$ with $u = c_0 + c_1 t - c_2 t^2 + c_3 t^3$. Then $y' = (u' - u)e^{-t}$ and $y'' = (u'' - 2u' + u)e^{-t}$. At $t = 0$: $u(0) = c_0$, $u'(0) = c_1$, $u''(0) = -2c_2$.
4. $y(0) = c_0 = 3$.
5. $y'(0) = c_1 - c_0 = -3 \;\Rightarrow\; c_1 = 0$.
6. $y''(0) = -2c_2 - 2c_1 + c_0 = -2c_2 - 0 + 3 = -47 \;\Rightarrow\; c_2 = 25$. The question prints a minus sign in front of $c_2 t^2$.
7. **Check:** $u = 3 - 25t^2 + 5t^3$ gives $u''' = 30$, so the left side is $e^{-t} \cdot 30 = 30e^{-t}$. Also $y(0) = 3$, $y'(0) = 0 - 3 = -3$ and $y''(0) = -50 - 0 + 3 = -47$.
8. Sum: $c_0 + c_1 + c_2 + c_3 = 3 + 0 + 25 + 5 = 33$.

KEY IDEA

The operator is $(D+1)^3$, so substituting $y = u\,e^{-t}$ turns the ODE into $u''' = 30$. That fixes $c_3 = 5$ straight away. The three initial conditions, taken through $y' = (u' - u)e^{-t}$ and $y'' = (u'' - 2u' + u)e^{-t}$ at $t = 0$, then give c_0, c_1 and c_2 one after another.

WATCH OUT FOR

Missing the printed minus sign in front of $c_2 t^2$. With it, $u''(0) = -2c_2$ and $c_2 = 25$. If you drop the sign you get $c_2 = -25$ and a sum of -17. Another error is to try to find c_3 from the initial conditions: they only fix c_0, c_1 and c_2, while c_3 comes from the forcing term.

Q.65 NAT · 2 MARKS

Engineering Mathematics › Probability & statistics

A random variable X has a probability density function

$$f_X(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

The probability of $X > 2$ is _____ (rounded off to three decimal places).

Official answer: **0.130 to 0.140**

SOLUTION

1. Check the density: $\int_0^\infty e^{-x}\,dx = [-e^{-x}]_0^\infty = 0 - (-1) = 1$, and $f_X(x) \geq 0$ everywhere, so f_X is a valid pdf. It is the exponential distribution with rate $\lambda = 1$.
2. The probability is the area under the pdf to the right of 2:
$$P(X > 2) = \int_2^\infty f_X(x)\,dx = \int_2^\infty e^{-x}\,dx$$

3. Evaluate:

$$\int_2^{\infty} e^{-x}\,dx = \left[-e^{-x}\right]_2^{\infty} = 0 - \left(-e^{-2}\right) = e^{-2}$$

4. Cross-check with the exponential survival function $P(X > a) = e^{-\lambda a}$: with $\lambda = 1$ and $a = 2$ it gives e^{-2} again.
5. Numerically, $e^{-2} = 0.135335\ldots$, so $P(X > 2) \approx 0.135$ (rounded off to three decimal places).

KEY IDEA

For a continuous random variable, $P(X > a) = \int_a^{\infty} f_X(x)\,dx$. This pdf is the exponential distribution with $\lambda = 1$, whose survival function is $P(X > a) = e^{-\lambda a}$, so the answer is e^{-2} with no further working.

WATCH OUT FOR

Three common slips. (1) Integrating over $[0, 2]$ and reporting $P(X \le 2) = 1 - e^{-2} \approx 0.865$ instead of the right tail. (2) Evaluating the density at the point, $f_X(2) = e^{-2}$, and getting the right number for the wrong reason. For a continuous variable a probability is an area, and the two only agree here because $\lambda = 1$. (3) Rounding e^{-2} badly, for example writing 0.14 or 0.13 instead of 0.135 to three decimal places.

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