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GATE IN · 2023

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GATE IN 2023 Solved Paper

INSIDE

- All 65 questions, in the order of the paper
- The official answer to every question
- Step-by-step worked solutions
- The answer key on one page

Question paper and official answer key: IIT Kanpur

Online, with the questions linked to their topic notes: **iaeacademy.in/study/papers/in-2023**

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About this paper

65 questions, 100 marks. Question types: 31 MCQ · 1 MSQ · 33 NAT.

Official answer key: IIT Kanpur · https://gate.iitk.ac.in/GATE2023/doc/papers/2023/ANS/IN_ANS_GATE2023.pdf

We placed each question under a subject and topic ourselves; the exam paper does not label them.

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Answer key

Q	TYPE	MARKS	ANSWER
Q.1	MCQ	1	**D**
Q.2	MCQ	1	**C**
Q.3	MCQ	1	**D**
Q.4	MCQ	1	**C**
Q.5	MCQ	1	**B**
Q.6	MCQ	2	**Marks to all**
Q.7	MCQ	2	**A**
Q.8	MCQ	2	**D**
Q.9	MCQ	2	**A**
Q.10	MCQ	2	**D**
Q.11	MCQ	1	**A**
Q.12	MCQ	1	**A**
Q.13	MCQ	1	**D**
Q.14	MCQ	1	**C**
Q.15	MCQ	1	**A**
Q.16	MCQ	1	**A**
Q.17	MCQ	1	**D**
Q.18	MCQ	1	**B**
Q.19	MCQ	1	**B**
Q.20	MCQ	1	**C**
Q.21	MCQ	1	**C**
Q.22	MCQ	1	**B**
Q.23	MCQ	1	**A**
Q.24	NAT	1	**2**
Q.25	NAT	1	**2**
Q.26	NAT	1	**1110**
Q.27	NAT	1	**4.9 to 5.1**
Q.28	NAT	1	**-3**
Q.29	NAT	1	**25**
Q.30	NAT	1	**0.99 to 1.01**
Q.31	NAT	1	**1.0**
Q.32	NAT	1	**-0.5**
Q.33	NAT	1	**0.95 to 1.05**
Q.34	NAT	1	**0**
Q.35	NAT	1	**0**
Q.36	MCQ	2	**A**
Q.37	MCQ	2	**C**
Q.38	MCQ	2	**A**
Q.39	MCQ	2	**A**
Q.40	MCQ	2	**C**
Q.41	MCQ	2	**A**
Q.42	MCQ	2	**C**
Q.43	MCQ	2	**B**
Q.44	MSQ	2	**B, C**
Q.45	NAT	2	**0.620 to 0.640**
Q.46	NAT	2	**1.40 to 1.50**
Q.47	NAT	2	**0.43 to 0.45**
Q.48	NAT	2	**0.9 to 1.1**
Q.49	NAT	2	**240**
Q.50	NAT	2	**-2 to 2**
Q.51	NAT	2	**1.77 to 1.78**
Q.52	NAT	2	**34.5 to 35.5**
Q.53	NAT	2	**0.98 to 1.02**
Q.54	NAT	2	**4.00 to 4.10**
Q.55	NAT	2	**17.00 to 18.00**
Q.56	NAT	2	**0.64 to 0.70**
Q.57	NAT	2	**0.9 to 1.1**
Q.58	NAT	2	**0.95 to 1.05**
Q.59	NAT	2	**0.49 to 0.51**
Q.60	NAT	2	**99 to 101**
Q.61	NAT	2	**2.1 to 2.5**
Q.62	NAT	2	**1.9 to 2.1**
Q.63	NAT	2	**-8**
Q.64	NAT	2	**0.009 to 0.011**
Q.65	NAT	2	**99.0 to 101.0**

General Aptitude Q.1 to Q.10

Q.1 MCQ · 1 MARK

General Aptitude › Grammar & vocabulary

The village was nestled in a green spot, _____ the ocean and the hills.

(A) through

(B) in

(C) at

(D) between

Official answer: **D**

SOLUTION

1. The blank needs a preposition that places the green spot relative to two separate things, "the ocean" and "the hills", which are joined by "and".
2. Rule: *between* is the preposition for a position with one thing on each side. It fits the fixed pattern "between X and Y".
3. *through* (A) describes movement from one side of something to the other. A village that is "nestled" stays in one place, so it cannot be "through the ocean and the hills". Rejected.
4. *in* (B) would put the spot inside both the ocean and the hills at once. A green patch of land cannot be inside the ocean. Rejected.
5. *at* (C) marks a single point ("at the shore"). It cannot describe a position relative to two landmarks joined by "and". Rejected.
6. *between* (D) puts the ocean on one side of the spot and the hills on the other. That matches the sheltered picture "nestled" suggests. The sentence reads: "The village was nestled in a green spot, **between** the ocean and the hills."

KEY IDEA

When a sentence places something relative to two separate things joined by "and" ("_____ X and Y"), the preposition is *between*. *Among* is for a group of three or more, and *at*, *in* and *through* do not take a two-part "X and Y" object of place.

WATCH OUT FOR

Picking "in" because "nestled in" appears earlier in the sentence. That "in" belongs to "a green spot". The blank has to describe where the spot sits relative to the ocean and the hills, and "in the ocean and the hills" would put land inside the ocean.

Q.2 MCQ · 1 MARK

General Aptitude › Reasoning

Disagree : Protest : : Agree : _____

(By word meaning)

(A) Refuse

(B) Pretext

(C) Recommend

(D) Refute

Official answer: **C**

SOLUTION

1. **Find the relation in the first pair.** To *protest* is to say openly and actively that you **disagree**. The first word is an attitude, and the second is the outward action that expresses it (a stronger, public form of the same attitude).

2. **Turn the relation into a test sentence.** "When you disagree with something, you protest against it." The second pair has to fit the same frame with the opposite attitude: "When you agree with something, you _____ it." So we need an outward action that expresses **agreement or approval**.
3. **(A) Refuse** means to decline or say no. It expresses rejection, so it belongs with *disagree*, not with *agree*. Reject.
4. **(B) Pretext** is a noun meaning a false reason given to hide the real one. It is not an action that expresses any attitude. Reject.
5. **(D) Refute** means to prove that a statement or claim is wrong. It also expresses disagreement. Reject.
6. **(C) Recommend** means to speak in favour of something or endorse it publicly. That is an outward action expressing approval, which is agreement. It stands to *agree* in the same way that *protest* stands to *disagree*: both are active, outward expressions of the attitude in the first word.
7. Hence Disagree : Protest : : Agree : **Recommend**. The answer is **(C)**.

KEY IDEA

Put the first pair into a sentence ("When you disagree, you protest"), then keep the relation and swap the attitude ("When you agree, you _____"). Protest is the outward, active expression of disagreement, so the answer has to be the outward, active expression of agreement: recommend (to endorse or speak in favour of something).

WATCH OUT FOR

Choosing Refuse or Refute because they feel close to "Protest". This matches the answer to the second word of the first pair when it should match the relation. Both words express disagreement, so they have the wrong polarity for "Agree". Pretext is a noun that has nothing to do with either attitude.

Q.3 MCQ · 1 MARK

General Aptitude › Numerical

A 'frabjous' number is defined as a 3 digit number with all digits odd, and no two adjacent digits being the same. For example, 137 is a frabjous number, while 133 is not. How many such frabjous numbers exist?

(A) 125

(B) 720

(C) 60

(D) 80

Official answer: **D**

SOLUTION

1. The odd digits are $1, 3, 5, 7, 9$, so each position can take one of 5 digits. Zero is even and can never appear, so there is no leading-zero case to exclude.
2. Hundreds digit: any odd digit, so 5 choices.
3. Tens digit: odd and different from the hundreds digit (the two are adjacent), so $5 - 1 = 4$ choices.
4. Units digit: odd and different from the tens digit only. It is not adjacent to the hundreds digit, so it may repeat that digit (131 is allowed). That gives 4 choices.
5. The choices are made one after another and each count does not depend on which digit was picked earlier, so by the multiplication principle: $5 \times 4 \times 4 = 80$.
6. Check with the complement. All-odd 3-digit numbers: $5^3 = 125$. Let P = first two digits equal ($5 \times 5 = 25$ numbers) and Q = last two digits equal (25 numbers). $P \cap Q$ = all three digits equal (5 numbers). By inclusion-exclusion, numbers with at least one equal adjacent pair $= 25 + 25 - 5 = 45$. So frabjous numbers $= 125 - 45 = 80$.
7. The answer is 80, which is option (D).

KEY IDEA

Count position by position. The first digit has 5 odd choices. Each later digit must differ only from the digit right before it, so it has 4 choices. The multiplication principle then gives $5 \times 4 \times 4$.

WATCH OUT FOR

Reading "no two adjacent digits the same" as "all three digits different" gives $5 \times 4 \times 3 = 60$, option (C). That wrongly rules out numbers like 131, where only the non-adjacent first and last digits match. Ignoring the adjacency rule altogether gives $5^3 = 125$, option (A).

Q.4 MCQ · 1 MARK

General Aptitude › Statistics & probability

Which one among the following statements must be TRUE about the mean and the median of the scores of all candidates appearing for GATE 2023?

(A) The median is at least as large as the mean.

(B) The mean is at least as large as the median.

(C) At most half the candidates have a score that is larger than the median.

(D) At most half the candidates have a score that is larger than the mean.

Official answer: **C**

SOLUTION

1. "Must be TRUE" means the statement has to hold for **every** possible set of scores, so one counterexample is enough to rule an option out.
2. **Option (A), test with a right-skewed set:** scores $1, 2, 9$ give median $= 2$ and mean $= \dfrac{1+2+9}{3} = 4$. The median is smaller than the mean, so (A) is not always true.
3. **Option (B), test with a left-skewed set:** scores $1, 8, 9$ give median $= 8$ and mean $= \dfrac{1+8+9}{3} = 6$. The mean is smaller than the median, so (B) is not always true.
4. **Option (D), same left-skewed set:** mean $= 6$, and 2 of the 3 candidates (scores 8 and 9) are above it. $\dfrac{2}{3} > \dfrac{1}{2}$, so more than half the candidates score above the mean and (D) is not always true. The mean has no fixed position in the ordered list.
5. **Option (C), definition of the median:** arrange the n scores in increasing order $x_1 \le x_2 \le \cdots \le x_n$. The median is the middle value (n odd) or the average of the two middle values (n even).
6. For n odd, every score up to and including the middle one is $\le$ the median, so only the $\dfrac{n-1}{2}$ scores after it can be larger. For n even, the lower half $x_1, \ldots, x_{n/2}$ are all $\le x_{n/2} \le$ median, so only the $\dfrac{n}{2}$ scores in the upper half can be larger. Either way the count is $\le \dfrac{n}{2}$, and scores tied at the median only lower it.
7. So (C) holds for every distribution of scores. Only (C) must be true.

KEY IDEA

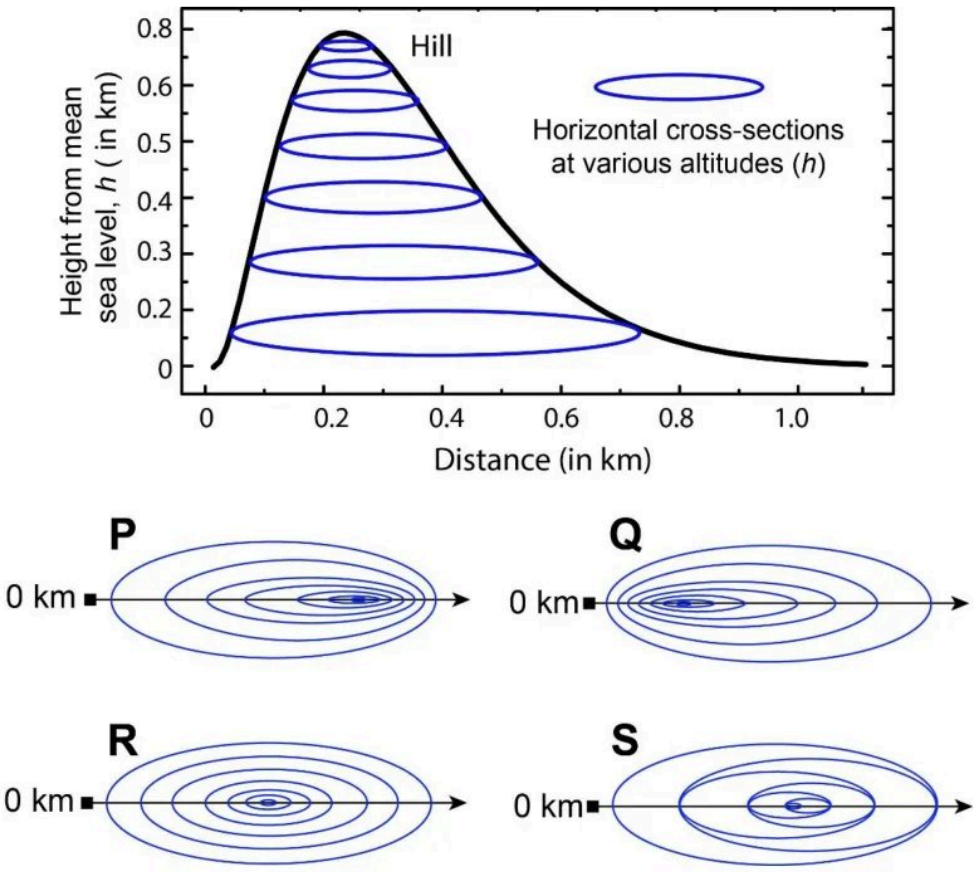
The median is defined by position: at least half of the ordered scores sit at or below it, so no more than half can be strictly above it. The mean depends on the size of the scores, not their position. Depending on skew it can lie above or below the median, and more than half the scores can exceed it.

WATCH OUT FOR

Choosing (A) or (B) from a remembered rule such as "mean > median for skewed data". That rule depends on the direction of skew, so neither direction holds for every set of scores. A second mistake is treating the mean as a "middle" value and picking (D). A single left-skewed set like $1, 8, 9$ puts two of three scores above the mean.

Q.5 MCQ · 1 MARK

General Aptitude › Spatial

Q.5	In the given diagram, ovals are marked at different heights (h) of a hill. Which one of the following options **P**, **Q**, **R**, and **S** depicts the top view of the hill?
	
(A)	**P**
(B)	**Q**
(C)	**R**
(D)	**S**

Official answer: **B**

SOLUTION

1. **Top view = the cross-sections seen from above.** Looking straight down, each horizontal cross-section at height h appears as a closed curve. It covers exactly the same stretch of the distance axis as its oval does in the side view. Higher sections are smaller and lie inside lower ones, so the top view is a set of nested closed curves (a contour map). In every option the arrow is the distance axis, with $0\ \text{km}$ at the left, the same direction as in the graph.

2. **Read the left ends of the ovals.** From the lowest oval to the highest they are at about $0.04, 0.07, 0.09, 0.12, 0.14, 0.17, 0.19$ km. Consecutive gaps are only 0.02 to 0.03 km, and all seven left ends fit within $0.19 - 0.04 = 0.15$ km. The left ends are **crowded**.
3. **Read the right ends.** From the lowest oval to the highest they are at about $0.74, 0.57, 0.47, 0.41, 0.36, 0.31, 0.28$ km. The gaps are $0.17, 0.10, 0.06, 0.05, 0.05, 0.03$ km, and together they spread over $0.74 - 0.28 = 0.46$ km, about three times the left-side spread. The right ends are **widely spaced**.
4. **Compare the two sides between the same pair of ovals.** Two consecutive ovals differ by the same height on the left side as on the right side. So for each pair, the side with the smaller horizontal gap is the steeper side. Every left gap is smaller than the matching right gap (for the lowest pair it is 0.03 against 0.17 km). So the hill is steeper on the 0 km side at every level, which matches the profile's steep rise and long gentle tail. The ovals are not at equal height steps, so compare left and right between the same two ovals, not one level against another. In the top view this means the contours are bunched on the left and spread out on the right.
5. **Where the summit sits.** The centre of the top oval is at $\frac{0.19 + 0.28}{2} \approx 0.24$ km and the centre of the lowest is at $\frac{0.04 + 0.74}{2} = 0.39$ km. The centres move right as h falls, so the innermost contour is off-centre towards the left. The peak is at about 0.23 km, which is $\frac{0.23 - 0.04}{0.74 - 0.04} \approx 0.27$ of the way across the outermost contour, counting from its left edge. That is roughly a quarter of the way across.
6. **Check each option.** R has ellipses with one common centre and equal gaps on the left and right. That is a symmetric hill, but here the centres shift from 0.24 to 0.39 km, so R is wrong. P bunches the contours on the right, with the summit about three-quarters of the way across. That is the mirror image of this hill, so P is wrong. In S each ellipse touches the next one, alternately at the left and the right. Two contours at different heights meeting at a point would mean a vertical cliff there. In the profile, every oval has its own left end and its own right end, and no two coincide, so S is wrong.
7. **Q** has its left edges bunched near the 0 km mark and its right edges widely spaced. Its innermost oval is about a quarter of the way across from the left. That matches the profile, so the answer is Q, option **(B)**.

KEY IDEA

The top view of a hill is its horizontal cross-sections seen from above, which is a contour map. Each oval in the side view becomes a closed curve covering the same distances. Where the hill is steep, one cross-section ends close to the next, so the contours bunch together. Where it is gentle, they spread out. This hill is steep on the 0 km side and has a long gentle tail to the right. So its contours must be bunched on the left and spread on the right, with the summit contour off-centre towards 0 km.

WATCH OUT FOR

Choosing P, the mirror image, by ignoring the 0 km mark that fixes left and right in every option. Other traps: picking R because a hill 'should' give concentric rings, which is true only for a symmetric hill. Reading crowded contours as a gentle slope instead of a steep one. Not noticing that the touching contours in S would mean vertical cliffs, which this hill does not have.

Q.6 MCQ · 2 MARKS

General Aptitude › Reasoning

Residency is a famous housing complex with many well-established individuals among its residents. A recent survey conducted among the residents of the complex revealed that all of those residents who are well established in their respective fields happen to be academicians. The survey also revealed that most of these academicians are authors of some best-selling books.

Based only on the information provided above, which one of the following statements can be logically inferred with *certainty*?

(A) Some residents of the complex who are well established in their fields are also authors of some best-selling books.

(B) All academicians residing in the complex are well established in their fields.

(C) Some authors of best-selling books are residents of the complex who are well established in their fields.

(D) Some academicians residing in the complex are well established in their fields.

Official answer: **Marks to all**

Q.7 MCQ · 2 MARKS

General Aptitude › Numerical

Ankita has to climb 5 stairs starting at the ground, while respecting the following rules:

1. At any stage, Ankita can move either one or two stairs up.
2. At any stage, Ankita cannot move to a lower step.

Let $F(N)$ denote the number of possible ways in which Ankita can reach the N^{th} stair. For example, $F(1) = 1,\ F(2) = 2,\ F(3) = 3.$

The value of $F(5)$ is _____.

(A) 8

(B) 7

(C) 6

(D) 5

Official answer: **A**

SOLUTION

1. Condition on the last move. Ankita can never move down, and each move goes up by 1 or 2 stairs, so every way of reaching stair N ends with exactly one of two moves: a 1-stair step from stair $N-1$ or a 2-stair step from stair $N-2$.
2. Every way ends in one of these two moves, so the two cases cover all ways. They also do not overlap, because a way ending in a 1-stair step cannot also end in a 2-stair step. The ways that end with a 1-stair step are exactly the ways of reaching stair $N-1$, and there are $F(N-1)$ of them. Likewise $F(N-2)$ ways end with a 2-stair step. The addition rule for disjoint cases gives
$$F(N) = F(N-1) + F(N-2)$$
This is the Fibonacci recurrence.
3. Check it against the given values: $F(3) = F(2) + F(1) = 2 + 1 = 3$. This matches the question.
4. $F(4) = F(3) + F(2) = 3 + 2 = 5$
5. $F(5) = F(4) + F(3) = 5 + 3 = 8$
6. Direct check by counting 2-stair steps. With k two-stair steps, Ankita needs $5 - 2k$ one-stair steps, which makes $5 - k$ moves in total. Choosing which k of those moves are 2-stair steps gives $\binom{5-k}{k}$ orderings, where $k = 0, 1, 2$. So
$$\binom{5}{0} + \binom{4}{1} + \binom{3}{2} = 1 + 4 + 3 = 8$$
7. $F(5) = 8$, which is option (A).

KEY IDEA

Look at the last move. The final move onto stair N is either a 1-stair step from $N-1$ or a 2-stair step from $N-2$. Those two cases are disjoint and cover every way, so $F(N) = F(N-1) + F(N-2)$, the Fibonacci recurrence. Starting from $F(1) = 1$ and $F(2) = 2$, the values run 1, 2, 3, 5, 8.

WATCH OUT FOR

One common mistake is to count only the combinations of step sizes (all 1s; three 1s and one 2; one 1 and two 2s), which gives 3, instead of counting orderings. For example, 1+2+2, 2+1+2 and 2+2+1 are three different ways. Another is to use the standard Fibonacci sequence 1, 1, 2, 3, 5 and pick 5, option (D). The question's own values $F(1) = 1$, $F(2) = 2$ shift the sequence by one place, so $F(5) = 8$.

Q.8 MCQ · 2 MARKS

General Aptitude › Reading

The information contained in DNA is used to synthesize proteins that are necessary for the functioning of life. DNA is composed of four nucleotides: Adenine (A), Thymine (T), Cytosine (C), and Guanine (G). The information contained in DNA can then be thought of as a sequence of these four nucleotides: A, T, C, and G. DNA has coding and non-coding regions. Coding regions—where the sequence of these nucleotides are read in groups of three to produce individual amino acids—constitute only about 2% of human DNA. For example, the triplet of nucleotides CCG codes for the amino acid glycine, while the triplet GGA codes for the amino acid proline. Multiple amino acids are then assembled to form a protein.

Based only on the information provided above, which of the following statements can be logically inferred with *certainty*?

(i) The majority of human DNA has no role in the synthesis of proteins.

(ii) The function of about 98% of human DNA is not understood.

(A) only (i)

(B) only (ii)

(C) both (i) and (ii)

(D) neither (i) nor (ii)

Official answer: **D**

SOLUTION

1. **What 'inferred with *certainty*' asks for.** A statement qualifies only if the passage forces it to be true. Outside knowledge does not count, and neither does a statement that is merely likely.
2. **What the passage actually says.** DNA has coding and non-coding regions. Coding regions, where the nucleotides are read in groups of three to produce individual amino acids, make up only about 2% of human DNA. So about 98% is non-coding, meaning it is not read in triplets to produce amino acids.
3. **Test (i): "The majority of human DNA has no role in the synthesis of proteins."** The passage defines only what the coding part does, which is to be read in triplets to produce amino acids. It never says what the non-coding part does. In particular, it never says the non-coding part has *no role* in protein synthesis. That part could still help in some other way (for example, by affecting when or how much protein is made), and nothing in the passage rules this out.
4. So 'not read into amino acids' does not force 'no role in synthesis'. (i) may be true, but the passage does not make it certain. (i) cannot be inferred.
5. **Test (ii): "The function of about 98% of human DNA is not understood."** The passage says nothing about how much is known or understood about any part of DNA. 'Non-coding' describes what a region does not do. It says nothing about the state of scientific knowledge.
6. (ii) brings in a new idea, understanding, that the passage never mentions. (ii) cannot be inferred either.
7. Neither statement follows with certainty from the passage, so the answer is **(D) neither (i) nor (ii)**.

KEY IDEA

An inference 'with certainty' must be forced by the passage alone. Being 'non-coding' (not read in triplets into amino acids) is a narrower claim than having 'no role in protein synthesis', and it says nothing at all about whether a region's function is 'understood'.

WATCH OUT FOR

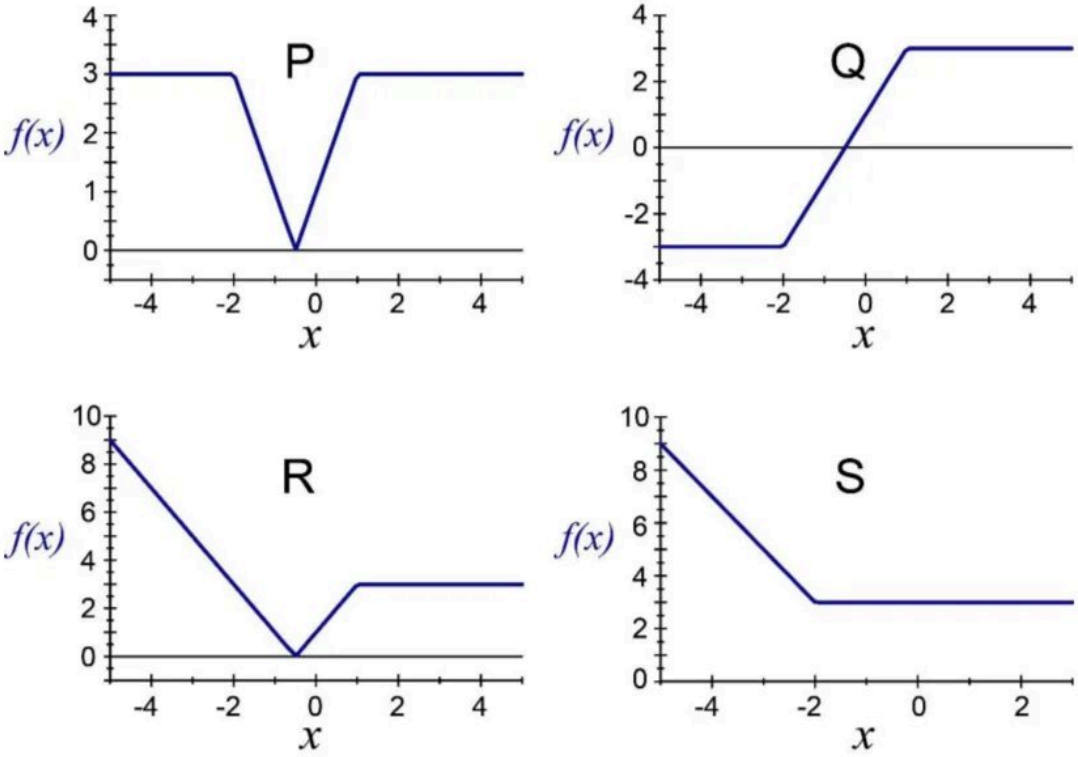
Picking (A): reading 'only about 2% is coding' as 'the other 98% has no role in making proteins'. The passage says only that the 98% is not read into amino acids, not that it plays no part. Picking (B) or (C) is also a mistake, because it brings in the familiar outside idea that non-coding DNA is poorly understood, which the passage never states. Answer from the passage alone, not from what you already know about biology.

Q.9 MCQ · 2 MARKS

General Aptitude › Numerical

Q.9 Which one of the given figures P, Q, R and S represents the graph of the following function?

$$f(x) = \big|\,|x+2| - |x-1|\,\big|$$



(A) P

(B) Q

(C) R

(D) S

Official answer: **A**

SOLUTION

1. The inner moduli change sign at $x = -2$ (from $|x+2|$) and at $x = 1$ (from $|x-1|$). Work out $g(x) = |x+2| - |x-1|$ on the three intervals these points create, then apply the outer modulus: $f(x) = |g(x)|$.
2. For $x \le -2$: $x + 2 \le 0$ and $x - 1 < 0$, so $|x+2| = -(x+2)$ and $|x-1| = 1 - x$. Then $g(x) = -(x+2) - (1-x) = -3$, so $f(x) = |-3| = 3$. On the left the graph is flat at height 3.

3. For $x \geq 1$: both $x+2$ and $x-1$ are non-negative, so $g(x) = (x+2) - (x-1) = 3$ and $f(x) = 3$. On the right the graph is also flat at height 3.
4. For $-2 \leq x \leq 1$: $x+2 \geq 0$ and $x-1 \leq 0$, so $g(x) = (x+2) - (1-x) = 2x+1$ and $f(x) = |2x+1|$. It equals $|-3| = 3$ at $x = -2$, drops to 0 at $x = -\frac{1}{2}$ and rises back to 3 at $x = 1$.
5. Putting it together:

$$f(x) = \begin{cases} 3, & x \leq -2 \\ |2x+1|, & -2 \leq x \leq 1 \\ 3, & x \geq 1 \end{cases}$$

   The graph never goes below 0, never goes above 3, and has a V touching 0 at $x = -\frac{1}{2}$ between two flat arms at height 3.
6. Now check the figures. Q goes down to -3 on the left. That is $g(x)$ before the outer modulus is applied, and an absolute value can never be negative. R and S both reach about 9 at $x \approx -5$, but $f(-5) = 3$. Those left arms come from continuing the middle-interval formula past $x = -2$, and S also never dips to 0 at $x = -\frac{1}{2}$. Only P has flat arms at 3 on both sides and a V touching 0 at $x = -0.5$. The answer is P, option (A).

KEY IDEA

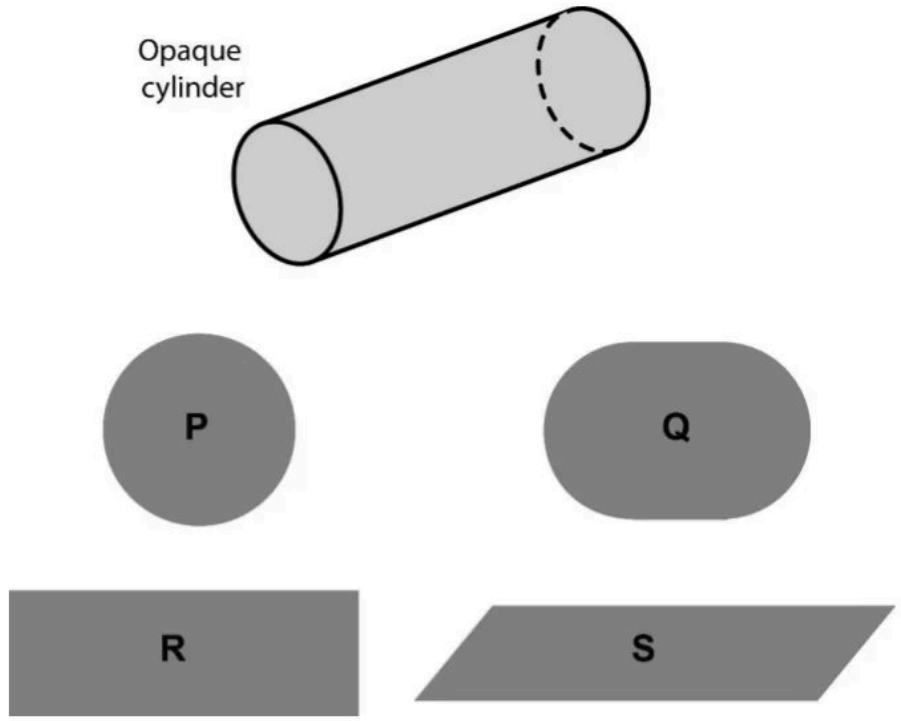
Break a sum of moduli at the points where each inner expression is zero ($x = -2$ and $x = 1$), write the function piece by piece, and apply the outer modulus last. A quick check: $|x+2| - |x-1|$ is (distance from x to -2) minus (distance from x to 1). The triangle inequality keeps that difference between -3 and 3, so $0 \leq f(x) \leq 3$ for every x. That alone rules out Q (it goes negative) and R and S (they reach about 9), leaving only P.

WATCH OUT FOR

Forgetting the outer modulus gives the ramp from -3 to 3, which is figure Q. Using the middle-interval formula $|2x+1|$ (or $-(2x+1)$) for all $x < -2$ instead of the constant 3 produces the rising left arm of R or S. Always evaluate each piece only on its own interval.

Q.10 MCQ · 2 MARKS

General Aptitude › Spatial

Q.10	An opaque cylinder (shown below) is suspended in the path of a parallel beam of light, such that its shadow is cast on a screen oriented perpendicular to the direction of the light beam. The cylinder can be reoriented in any direction within the light beam. Under these conditions, which one of the shadows **P**, **Q**, **R**, and **S** is NOT possible?
	
(A)	**P**
(B)	**Q**
(C)	**R**
(D)	**S**

Official answer: **D**

SOLUTION

1. A parallel beam gives an orthographic projection onto a screen perpendicular to the beam: every point of the cylinder moves straight along the beam onto the screen. Let d be the cylinder's diameter, L its length and θ the angle between its axis and the beam.
2. Take one end face first. It is a circle of diameter d, and its normal is the cylinder's axis, which makes angle θ with the beam. Orthographically, this circle projects to an ellipse. The major axis is always d and lies perpendicular to the projected axis. The minor axis is $d\cos\theta$ and lies along the projected axis.

3. The cylinder is the end disc slid a distance L along its axis. On the screen, that slide becomes a slide of $L\sin\theta$ along the projected axis. So the shadow is one end-ellipse swept $L\sin\theta$ along the projected axis, which is its minor-axis direction. The sweep is bounded by two straight lines parallel to the projected axis, each $\frac{d}{2}$ from it and touching the ellipses at the ends of their major axes, and it is capped by the outer halves of the two end-ellipses.
4. $\theta = 0^\circ$ (axis along the beam): $L\sin\theta = 0$ and $d\cos\theta = d$, so the two end faces project to the same circle of diameter d. The shadow is a circle, so **P** is possible.
5. $0^\circ < \theta < 90^\circ$ (axis tilted): two parallel straight edges are capped by two rounded half-ellipse ends, giving a capsule shape. **Q** is possible.
6. $\theta = 90^\circ$ (axis across the beam): $d\cos\theta = 0$, so each end face flattens into a straight segment of length d perpendicular to the projected axis, and the sweep length is $L\sin 90^\circ = L$. The shadow is a $d \times L$ rectangle, so **R** is possible.
7. For every θ, the shadow is symmetric about the projected axis, which is parallel to its long edges. Its ends are straight only when $\theta = 90^\circ$, and then they meet the long edges at 90°. A slanted parallelogram has straight ends that are not perpendicular to its long edges, and it has no line of symmetry parallel to them. No orientation produces that, so **S** is NOT possible.

KEY IDEA

Under a parallel beam, the shadow is an orthographic projection. For a cylinder, that is always one end-ellipse (minor axis $d\cos\theta$ along the projected axis, major axis d across it) swept a distance $L\sin\theta$ along the projected axis. The result is a circle ($\theta = 0^\circ$), a capsule ($0^\circ < \theta < 90^\circ$) or a rectangle ($\theta = 90^\circ$). Every one of them is symmetric about a line parallel to its long edges, and any straight end meets those edges at a right angle.

WATCH OUT FOR

Picturing a tilted cylinder the way it looks in a perspective or oblique drawing, and assuming the tilt skews the rectangle into a parallelogram. Under parallel light, tilting the axis only curves the ends into half-ellipses; it never slants them. The opposite slip is to reject Q because the ends of a cylinder are flat. Those flat ends are circles seen at an angle, so they cast rounded shadows.

Instrumentation Engineering Q.11 to Q.65

Q.11 MCQ · 1 MARK

Engineering Mathematics › Linear algebra

Choose solution set S corresponding to the systems of two equations

$$x - 2y + z = 0$$
$$x - z = 0$$

Note: $\Re$ denotes the set of real numbers

(A) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \middle| \alpha \in \Re \right\}$

(B) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \middle| \alpha, \beta \in \Re \right\}$

(C) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \middle| \alpha, \beta \in \Re \right\}$

(D) $S = \left\{ \alpha \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \middle| \alpha \in \Re \right\}$

Official answer: **A**

SOLUTION

1. Write the system as $A\mathbf{x} = \mathbf{0}$ with $A = \begin{bmatrix} 1 & -2 & 1 \\ 1 & 0 & -1 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. The two rows are not multiples of each other, so $\operatorname{rank}(A) = 2$.
2. By the rank–nullity theorem, the solution space has dimension $n - \operatorname{rank}(A) = 3 - 2 = 1$. So S is a line through the origin, spanned by **one** vector. Options (B) and (C) each use two linearly independent vectors, so each describes a plane, which is 2-dimensional. Both are ruled out.
3. Now solve directly. The second equation gives $x = z$.
4. Put $z = x$ into the first equation: $x - 2y + x = 0 \Rightarrow 2x = 2y \Rightarrow y = x$.
5. Let $x = \alpha$. Then $(x, y, z) = (\alpha, \alpha, \alpha) = \alpha \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ for any $\alpha \in \Re$. Check: $1 - 2(1) + 1 = 0$ and $1 - 1 = 0$.
6. Check (D): $(1, 0, 1)$ gives $1 - 0 + 1 = 2 \neq 0$ in the first equation, so it is not a solution. The same vector lies in the set in (B) (take $\alpha = 0,\ \beta = 1$). In (C), $(2, 1, 2)$ gives $2 - 2 + 2 = 2 \neq 0$. This confirms that (B), (C) and (D) are all wrong.
7. So $S = \left\{ \alpha \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \ : \ \alpha \in \Re \right\}$, which is option (A).

KEY IDEA

For a homogeneous system $A\mathbf{x} = \mathbf{0}$ in n unknowns, the solution set is a subspace of dimension $n - \text{rank}(A)$. Here that is $3 - 2 = 1$, so S is a line spanned by one vector. Two independent equations in three unknowns leave one free parameter. Any proposed spanning vector must satisfy every equation.

WATCH OUT FOR

Choosing (D) because $(1, 0, 1)$ satisfies the second equation $x - z = 0$, without checking the first equation, where it gives $2 \neq 0$. A second mistake is choosing (B) or (C) because they contain the correct vector $(1, 1, 1)$. Adding a second independent vector turns the set into a plane, and that plane includes non-solutions such as $(1, 0, 1)$ or $(2, 1, 2)$.

Q.12 MCQ · 1 MARK

Electromagnetics › Maxwell's equations

Inductance of a coil is measured as 10 mH, using an LCR meter, when no other objects are present near the coil. The LCR meter uses a sinusoidal excitation at 10 kHz. If a pure copper sheet is brought near the coil, the same LCR meter will read _____.

(A) less than 10 mH

(B) 10 mH

(C) more than 10 mH

(D) less than 10 mH initially and then stabilizes to more than 10 mH

Official answer: **A**

SOLUTION

1. **Faraday's law:** the coil's alternating field at $f = 10\ \text{kHz}$ passes through the copper sheet and induces an emf in it. Copper is a very good conductor ($\sigma \approx 5.8 \times 10^7\ \text{S/m}$), so large eddy currents circulate in the sheet.
2. **Lenz's law:** these eddy currents set up a flux that opposes the coil's own flux. For the same coil current, less net flux links the coil, so $L = \lambda / I$ goes down.
3. **Copper is not magnetic:** $\mu_r \approx 1$ (it is very slightly diamagnetic), so the sheet gives the flux no easier path and nothing pushes L up. A ferromagnetic sheet such as iron would have the opposite effect.
4. **Circuit model (coil = primary, sheet = shorted one-turn secondary):** the coil's input impedance is
$$Z_{in} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2}$$
Rationalising the last term gives
$$L_{eff} = L_1 - \frac{\omega^2 M^2 L_2}{R_2^2 + \omega^2 L_2^2} < L_1, \qquad R_{eff} = R_1 + \frac{\omega^2 M^2 R_2}{R_2^2 + \omega^2 L_2^2} > R_1$$
So the sheet lowers the effective inductance and raises the effective resistance, because it absorbs eddy-current loss.
5. **Check that 10 kHz is high enough:** the skin depth is $\delta = \dfrac{1}{\sqrt{\pi f \mu_0 \sigma}}$ with $\mu_0 = 4\pi \times 10^{-7}\ \text{H/m}$, giving $\delta = \dfrac{1}{\sqrt{\pi (10^4)(4\pi \times 10^{-7})(5.8 \times 10^7)}} \approx 0.66\ \text{mm}$. A typical sheet is thicker than this, so it blocks most of the flux ($\omega L_2 \gg R_2$, and L_{eff} approaches $L_1 - M^2/L_2$). The drop in L is therefore large enough to see.
6. The LCR meter reports the reactive part of the impedance as an inductance, so it reads **less than 10 mH**. The reading drops as soon as the sheet is in place and stays there, because nothing makes it recover later. That rules out option (D). Answer: **(A)**.

KEY IDEA

A non-magnetic conductor near an AC coil behaves like a shorted secondary winding. Its eddy currents oppose the coil's flux (Lenz's law) and reflect a negative reactance into the coil, $-\omega^2 M^2 L_2/(R_2^2 + \omega^2 L_2^2)$. The measured inductance falls and the measured resistance rises. Only a magnetic material ($\mu_r \gg 1$) would raise L.

WATCH OUT FOR

Many students pick (C) because they assume any metal near a coil increases its inductance, as an iron core does. Copper has $\mu_r \approx 1$, so it adds no magnetic path, and its eddy currents cancel part of the flux. Option (D) is also wrong: the reduction is a steady-state AC effect with no later rise above 10 mH.

Q.13 MCQ · 1 MARK

Sensors and Industrial Instrumentation › Pressure & flow

Which of the following flow meters offers the lowest resistance to the flow?

(A) Turbine flow meter

(B) Orifice flow meter*

(C) Venturi meter

(D) Electromagnetic flow meter

Official answer: **D**

SOLUTION

1. Resistance to flow means the permanent pressure loss that a meter introduces, which comes from any obstruction it places in the flow path.
2. Orifice meter: a thin plate with a small hole narrows the flow abruptly. The jet separates and mixes downstream, so a large part of the differential pressure Δp is lost for good. This is the highest loss of the four.
3. Venturi meter: the gradual cone and diffuser recover most of the pressure, so the permanent loss is much smaller than for an orifice. It is still a constriction in the pipe.
4. Turbine meter: the rotor, blades and hub sit in the stream and obstruct it, and bearing friction adds to the pressure drop.
5. Electromagnetic meter: the measuring section is an open, unobstructed pipe with electrodes flush to the wall. By Faraday's law, $e = B\,D\,v$ is generated in the moving conductive liquid, so there is no moving or intruding part.
6. So its pressure drop is essentially that of a plain straight pipe, the lowest resistance of the four. The answer is (D).

KEY IDEA

A meter with no obstruction in the pipe (electromagnetic) causes essentially no permanent pressure loss. Head-type meters (orifice, Venturi) and the turbine meter all put something in the stream.

WATCH OUT FOR

Choosing the Venturi meter because it has low loss compared with the orifice. Low is not zero: it still narrows the pipe, whereas the electromagnetic meter does not.

Q.14 MCQ · 1 MARK

Sensors and Industrial Instrumentation › Sensing principles

Q.14 Pair the quantities (p) to (s) with the measuring devices (i) to (iv).

(i) Linear Variable Differential Transformer (LVDT)	(p) Torque
(ii) Thermistor	(q) Pressure
(iii) Strain gauge	(r) Linear position
(iv) Diaphragm	(s) Temperature

(A)	(i) - (r), (ii) - (s), (iii) - (q), (iv) - (p)
(B)	(i) - (p), (ii) - (s), (iii) - (r), (iv) - (q)
(C)	(i) - (r), (ii) - (s), (iii) - (p), (iv) - (q)
(D)	(i) - (q), (ii) - (s), (iii) - (p), (iv) - (r)

Official answer: **C**

SOLUTION

1. **LVDT.** Moving the core changes the difference between the two secondary voltages, and that difference is proportional to the core displacement. So (i) pairs with (r), linear position.
2. **Thermistor.** Its resistance depends strongly on temperature, $R = R_0 e^{\beta\left(\frac{1}{T}-\frac{1}{T_0}\right)}$. So (ii) pairs with (s), temperature.
3. **Diaphragm.** A pressure difference deflects the diaphragm, and the deflection is sensed by a secondary element. So (iv) pairs with (q), pressure.
4. **Strain gauge.** The only quantity left is (p), torque. Gauges bonded on a twisted shaft sense the surface strain, with gauge factor $G = \dfrac{\Delta R/R}{\varepsilon}$, and so measure torque. So (iii) pairs with (p). The pressure and position quantities are already taken by the diaphragm and the LVDT.
5. **Check the options.** The pairing is (i)-(r), (ii)-(s), (iii)-(p), (iv)-(q). (A) puts the diaphragm with torque and the strain gauge with pressure. (B) puts the LVDT with torque. (D) puts the LVDT with pressure and the diaphragm with linear position. Only (C) matches.

KEY IDEA

Match each device to the quantity it is designed to sense, starting with the unambiguous ones (LVDT for displacement, thermistor for temperature, diaphragm for pressure). The strain gauge then takes the one left, torque, through the strain it senses on a twisted shaft.

WATCH OUT FOR

Pairing the strain gauge with pressure because pressure transducers often use strain gauges. Here the diaphragm is the pressure-sensing element, and the strain gauge fits torque. The other slip is pairing the LVDT with torque or pressure instead of linear displacement.

Q.15 MCQ · 1 MARK

Electromagnetics › Electrostatics

Capacitance 'C' of a parallel plate structure is calculated as 20 pF using $C = \dfrac{\varepsilon_o \varepsilon_r A}{d}$, where ε_o is the permittivity of free space, ε_r is the relative permittivity of the dielectric, A is the overlapping area of the electrodes and d is the distance between them. The value of C is then measured using an LCR meter. If the meter is assumed to be ideal and it introduces no error due to cable capacitance, which one of the following readings is likely to be correct?

(A) 20.5 pF

(B) 20 pF

(C) 19.5 pF

(D) 10 pF

Official answer: **A**

SOLUTION

1. **What the formula assumes.** $C = \dfrac{\varepsilon_0 \varepsilon_r A}{d}$ comes from Gauss's law with a *uniform* field $E = \dfrac{\sigma}{\varepsilon_0 \varepsilon_r}$ that exists only in the region between the plates and nowhere else. It ignores what happens at the edges of the plates. This is exact only for infinitely large plates.
2. **What a real structure does.** Real plates have a finite size, so they have **fringing fields**: flux lines bulge outward past the edges, and some flux also links the outer (back) faces of the plates. Each of these lines ends on charge held by the electrodes, so at the same applied voltage V the plates hold more charge than the ideal model gives.
3. **Direction of the error.** Since $C = \dfrac{Q}{V}$ and the fringing flux adds charge $Q_{\text{fringe}} > 0$ at the same V,
$$C_{\text{actual}} = \frac{Q_{\text{ideal}} + Q_{\text{fringe}}}{V} > \frac{\varepsilon_0 \varepsilon_r A}{d} = 20 \text{ pF}.$$
So the true capacitance of a finite parallel-plate structure is always slightly **greater** than the value from the formula.
4. **Size of the error.** In a practical parallel-plate structure the gap d is small compared with the plate dimensions, so fringing adds only a small correction, typically a few percent. The true value is therefore just above 20 pF, not far above it.
5. **Eliminate options.** The question says the meter is ideal and that cable capacitance adds no error, so the reading equals the true capacitance of the structure. (B) 20 pF would mean no fringing at all, which happens only for infinite plates. (C) 19.5 pF is off in the wrong direction, because fringing adds charge and cannot lower C. (D) 10 pF is a 50% drop, which no edge effect can produce. That leaves **20.5 pF**, option (A).

KEY IDEA

$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$ assumes a uniform field confined between infinitely large plates. Real plates have fringing flux at the edges and on the outer faces, and that flux stores extra charge at the same voltage. So the real capacitance is always a little above the formula value. With meter and cable errors excluded, a correct reading is slightly more than 20 pF.

WATCH OUT FOR

Choosing 20 pF because the meter is called ideal. An ideal meter shows the true capacitance of the structure, and the formula itself is the approximation because it leaves out fringing. The second trap is 19.5 pF, which treats the difference as random scatter. Fringing always adds capacitance, so the error has a known direction.

Q.16 MCQ · 1 MARK

Digital Electronics › Sequential

Q.16 The table shows the present state Q(t), next state Q(t+1), and the control input in a flip-flop. Identify the flip-flop.

Q (t)	Q (t+1)	Input
0	0	0
0	1	1
1	0	1
1	1	0

(A) T flip-flop

(B) D flip-flop

(C) SR flip-flop

(D) JK flip-flop

Official answer: **A**

SOLUTION

1. The table has only **one** control-input column, so the device is a single-input flip-flop: T or D. SR and JK flip-flops each have two control inputs (S, R or J, K), so a one-column table cannot describe them. This rules out (C) and (D).
2. Read the rows where the input is 0: the state holds, $0 \to 0$ and $1 \to 1$.
3. Read the rows where the input is 1: the state flips, $0 \to 1$ and $1 \to 0$.

4. Holding on 0 and toggling on 1 is the defining behaviour of the T flip-flop. Its characteristic equation is $Q(t+1) = T \oplus Q(t)$.
5. Check every row against $Q(t+1) = \text{Input} \oplus Q(t)$: row 1, $0 \oplus 0 = 0$; row 2, $1 \oplus 0 = 1$; row 3, $1 \oplus 1 = 0$; row 4, $0 \oplus 1 = 1$. All four rows match.
6. Rule out D, whose characteristic equation is $Q(t+1) = D$. In row 3 the input is 1 but $Q(t+1) = 0$, and in row 4 the input is 0 but $Q(t+1) = 1$. So the next state does not simply copy the input, and the device cannot be a D flip-flop.
7. The flip-flop is a **T flip-flop**, option (A).

KEY IDEA

Look for the pattern that links the input to the change of state. A T flip-flop holds when $T = 0$ and toggles when $T = 1$, so $Q(t+1) = T \oplus Q(t)$. A D flip-flop copies its input, so $Q(t+1) = D$.

WATCH OUT FOR

Picking D because the input equals $Q(t+1)$ in the first two rows. Rows 3 and 4 break that pattern. The input actually equals $Q(t) \oplus Q(t+1)$, which means it tells the flip-flop whether to change state. That is the T flip-flop's excitation rule.

Q.17 MCQ · 1 MARK

Digital Electronics › Boolean logic

Match the Exclusive-OR (XOR) operations (i) to (iv) with the results (p) to (s), where X is a Boolean input.

OPERATION	RESULT
(i) $X \oplus X$	(p) 1
(ii) $X \oplus \overline{X}$	(q) 0
(iii) $X \oplus 0$	(r) $\overline{X}$
(iv) $X \oplus 1$	(s) X

(A) (i) - (q), (ii) - (r), (iii) - (s), (iv) - (p)

(B) (i) - (q), (ii) - (r), (iii) - (p), (iv) - (s)

(C) (i) - (p), (ii) - (s), (iii) - (q), (iv) - (r)

(D) (i) - (q), (ii) - (p), (iii) - (s), (iv) - (r)

Official answer: **D**

SOLUTION

1. Definition of XOR (complement written as -): $A \oplus B = A\overline{B} + \overline{A}B$, which is 1 exactly when the two inputs differ.
2. (i) $X \oplus X = X\overline{X} + \overline{X}X = 0 + 0 = 0$, since $X\overline{X} = 0$. The two inputs are always equal, so the result is (q).
3. (ii) $X \oplus \overline{X} = X \cdot \overline{\overline{X}} + \overline{X} \cdot \overline{X} = X \cdot X + \overline{X} \cdot \overline{X} = X + \overline{X} = 1$. The two inputs always differ, so the result is (p).
4. (iii) $X \oplus 0 = X \cdot \overline{0} + \overline{X} \cdot 0 = X \cdot 1 + 0 = X$. XOR with 0 passes the input through unchanged, so the result is (s).
5. (iv) $X \oplus 1 = X \cdot \overline{1} + \overline{X} \cdot 1 = X \cdot 0 + \overline{X} = \overline{X}$. XOR with 1 inverts the input (a controlled inverter), so the result is (r).
6. The matching is (i)-(q), (ii)-(p), (iii)-(s), (iv)-(r), which is option (D).

KEY IDEA
XOR outputs 1 exactly when its inputs differ. So a signal XORed with itself gives 0, and XORed with its complement gives 1. XOR with a constant makes a controlled inverter: a control of 0 passes X through unchanged, and a control of 1 inverts it to $\overline{X}$.

WATCH OUT FOR
Mixing up the two constant cases: taking $X \oplus 1 = 1$ or $X \oplus 0 = 0$ by treating XOR like OR or AND. Another mistake is swapping (i) and (ii), writing $X \oplus X = 1$ because "a signal matches itself". Options (A), (B) and (C) each contain one of these slips.

Q.18 MCQ · 1 MARK

Sensors and Industrial Instrumentation › Optical

A light emitting diode (LED) emits light when it is _____ biased. A photodiode provides maximum sensitivity to light when it is _____ biased.

(A) forward, forward

(B) forward, reverse

(C) reverse, reverse

(D) reverse, forward

Official answer: **B**

SOLUTION

1. An LED emits light by radiative recombination of electrons and holes in the junction. Forward bias injects carriers across the junction, so they recombine and emit photons. The LED must be **forward** biased.
2. In a photodiode, incident photons create electron-hole pairs in and near the depletion region. The photocurrent is the flow of these collected carriers.
3. Reverse bias widens the depletion region and raises the electric field. This gives a larger collection volume and faster sweep-out of carriers. It also lowers junction capacitance and keeps the dark current small, so the photocurrent is a clean, linear measure of the light.
4. So the photodiode is most sensitive when it is **reverse** biased (photoconductive mode).
5. The pair is (forward, reverse), which is option (B).

KEY IDEA
An LED needs forward bias so carriers recombine and emit light. A photodiode needs reverse bias so its wide depletion region and strong field collect photo-generated carriers efficiently.

WATCH OUT FOR
Assuming both devices work the same way, so that a photodiode is also forward biased, or mixing up the two. Forward bias on a photodiode makes it behave like an ordinary diode, and the photocurrent is then swamped by the large forward current.

Q.19 MCQ · 1 MARK

Engineering Mathematics › Complex analysis

$F(z) = \dfrac{1}{1-z}$, when expanded as a power series around $z = 2$, would result in $F(z) = \sum_{k=0}^{\infty} a_k (z-2)^k$, with the region of convergence (ROC) $|z-2| < 1$. The coefficients a_k, $k \geq 0$, are given by the expression _____.

(A) $(-1)^k$

(B) $(-1)^{k+1}$

(C) $\left(\dfrac{1}{2}\right)^k$

(D) $\left(\frac{-1}{2}\right)^{k+1}$

Official answer: **B**

SOLUTION

1. Substitute $w = z - 2$, so $z = w + 2$ and the ROC becomes $|w| < 1$. Then $1 - z = 1 - (w + 2) = -(1 + w)$.
2. So
$$F(z) = \frac{1}{1 - z} = -\frac{1}{1 + w}.$$
3. Use the geometric series $\frac{1}{1 + w} = \sum_{k=0}^{\infty} (-1)^k w^k$. It holds for $|w| < 1$, which is exactly the given ROC $|z - 2| < 1$.
4. Hence
$$F(z) = -\sum_{k=0}^{\infty} (-1)^k (z - 2)^k = \sum_{k=0}^{\infty} (-1)^{k+1} (z - 2)^k,$$
so $a_k = (-1)^{k+1}$.
5. Check with Taylor's formula $a_k = \frac{F^{(k)}(2)}{k!}$. Since $F^{(k)}(z) = \frac{k!}{(1 - z)^{k+1}}$, we get $a_k = \frac{1}{(1 - 2)^{k+1}} = (-1)^{k+1}$. As a quick test, $a_0 = F(2) = \frac{1}{1 - 2} = -1$. Option (A) gives $a_0 = 1$, (C) gives 1 and (D) gives $-\frac{1}{2}$, so only (B) fits. Answer: option (B).

KEY IDEA

Shift to the new centre with $w = z - 2$ and rewrite $1 - z$ as $-(1 + w)$. That turns F into $-\frac{1}{1 + w}$, a geometric series that converges for $|w| < 1$. You can confirm the result in one line with $a_0 = F(2) = -1$.

WATCH OUT FOR

The usual slip is dropping the minus sign in $1 - z = -(1 + w)$. That gives $a_k = (-1)^k$, which is option (A). A second trap is mixing up the centre: the familiar $\frac{1}{1 - z} = \sum z^k$ is centred at $z = 0$, and forcing a factor of $\frac{1}{2}$ into the expansion leads to options (C) or (D), which belong to a different expansion with a different ROC. Checking $a_0 = F(2) = -1$ catches all three.

Q.20 MCQ · 1 MARK

Engineering Mathematics › Differential equations

The solution $x(t), t \geq 0$, to the differential equation $\ddot{x} = -k\dot{x}, k > 0$ with initial conditions $x(0) = 1$ and $\dot{x}(0) = 0$ is

(A) $x(t) = 2e^{-kt} + 2kt - 1$

(B) $x(t) = 2e^{-kt} - 1$

(C) $x(t) = 1$

(D) $x(t) = 2e^{-kt} - kt - 1$

Official answer: **C**

SOLUTION

1. Write the equation as $\ddot{x} + k\dot{x} = 0$. It is linear with constant coefficients, so its auxiliary equation is $m^2 + km = 0$, i.e. $m(m + k) = 0$, giving $m = 0$ and $m = -k$.
2. General solution: $x(t) = A + Be^{-kt}$, so $\dot{x}(t) = -kBe^{-kt}$.
3. Apply $\dot{x}(0) = 0$: $-kB = 0$. Since $k > 0$, $B = 0$.

4. Apply $x(0) = 1$: $A + B = 1$, so $A = 1$. Hence $x(t) = 1$ for all $t \geq 0$. Check: $\dot{x} = 0$ and $\ddot{x} = 0$, so $\ddot{x} = -k\dot{x}$ holds, and both initial conditions hold.
5. Quicker route: put $v = \dot{x}$. Then $\dot{v} = -kv$ with $v(0) = 0$, so $v(t) = v(0)e^{-kt} = 0$. The velocity is zero at every instant, so x stays at its starting value $x(0) = 1$.
6. Check the other options. (B): $\dot{x} = -2ke^{-kt}$, so $\dot{x}(0) = -2k \neq 0$. (D): $\dot{x} = -2ke^{-kt} - k$, so $\dot{x}(0) = -3k \neq 0$. (A) satisfies both initial conditions, since $x(0) = 2 - 1 = 1$ and $\dot{x} = -2ke^{-kt} + 2k$ gives $\dot{x}(0) = 0$, but it fails the equation itself: $\ddot{x} = 2k^2e^{-kt}$, while $-k\dot{x} = 2k^2e^{-kt} - 2k^2$, and these differ by $2k^2 \neq 0$.
7. So the answer is (C), $x(t) = 1$.

KEY IDEA

The equation involves only $\dot{x}$ and $\ddot{x}$, so substitute $v = \dot{x}$ to get the first-order equation $\dot{v} = -kv$. Its solution is $v(t) = v(0)e^{-kt}$. With $v(0) = 0$ the velocity is identically zero, so $x(t)$ is the constant $x(0)$. Equivalently, the roots $m = 0, -k$ give $x = A + Be^{-kt}$, and $\dot{x}(0) = 0$ forces $B = 0$.

WATCH OUT FOR

Checking only the initial conditions. Option (A) satisfies both $x(0) = 1$ and $\dot{x}(0) = 0$ and is picked for that reason alone, but it does not satisfy $\ddot{x} = -k\dot{x}$. Every option also has to be substituted into the differential equation. A second slip is to read $\ddot{x} = -k\dot{x}$ as $\ddot{x} = -kx$ (a harmonic oscillator), which gives oscillating answers.

Q.21 MCQ · 1 MARK

Control Systems › Bode & margins

A system has the transfer-function

$$\frac{Y(s)}{X(s)} = \frac{s - \pi}{s + \pi}$$

Let $u(t)$ be the unit-step function. The input $x(t)$ that results in a steady-state output $y(t) = \sin \pi t$ is _____.

(A) $x(t) = \sin(\pi t)\, u(t)$

(B) $x(t) = \sin\left(\pi t + \frac{\pi}{2}\right) u(t)$

(C) $x(t) = \sin\left(\pi t - \frac{\pi}{2}\right) u(t)$

(D) $x(t) = \cos\left(\pi t + \frac{\pi}{4}\right) u(t)$

Official answer: **C**

SOLUTION

1. The only pole is at $s = -\pi$, in the left half-plane, so the system is stable and a sinusoidal steady state exists. The transient from the step switch-on decays as $e^{-\pi t}$. Sinusoidal steady-state rule: an input $\sin(\omega t + \theta)\, u(t)$ gives the steady-state output $|H(j\omega)| \sin(\omega t + \theta + \angle H(j\omega))$.
2. The output frequency is $\omega = \pi$ rad/s. An LTI system does not change frequency, so the input must also be at $\omega = \pi$. Every option is, so the answer comes down to the phase (and gain).
3. Put $s = j\pi$:
$$H(j\pi) = \frac{j\pi - \pi}{j\pi + \pi} = \frac{-1 + j}{1 + j} = \frac{(-1 + j)(1 - j)}{(1 + j)(1 - j)} = \frac{-1 + 2j + 1}{2} = \frac{2j}{2} = j$$
4. So $|H(j\pi)| = 1$, as expected for an all-pass network (the zero at $+\pi$ mirrors the pole at $-\pi$), and $\angle H(j\pi) = +\frac{\pi}{2}$. Checking with angles: $\angle(-\pi + j\pi) - \angle(\pi + j\pi) = 135^\circ - 45^\circ = 90^\circ$.
5. At this frequency the system adds $+\pi/2$ of phase and leaves the amplitude unchanged. For the output to be $\sin(\pi t)$, the input must therefore have phase $-\pi/2$:
$$x(t) = \sin\left(\pi t - \frac{\pi}{2}\right) u(t) \;\Rightarrow\; y_{ss}(t) = 1 \cdot \sin\left(\pi t - \frac{\pi}{2} + \frac{\pi}{2}\right) = \sin \pi t$$

6. Ruling out the others: (A) gives $\sin(\pi t + \pi/2)$. (B) gives $\sin(\pi t + \pi) = -\sin \pi t$. (D) is $\cos(\pi t + \pi/4) = \sin(\pi t + 3\pi/4)$, which gives $\sin(\pi t + 5\pi/4)$. None of these is $\sin \pi t$, so the answer is (C).

KEY IDEA

In sinusoidal steady state a stable LTI system scales the amplitude by $|H(j\omega)|$ and shifts the phase by $\angle H(j\omega)$, and it never changes the frequency. To find the input from the output, you undo that shift. Here $(s - \pi)/(s + \pi)$ is all-pass: the gain is 1 at every frequency, and at $\omega = \pi$ the phase is exactly $+90^\circ$. So the input has to lag the output by $\pi/2$.

WATCH OUT FOR

The usual slip is getting the sign of the phase wrong. The numerator $j\pi - \pi$ sits in the second quadrant, at 135° and not 45°. People who treat $(s - \pi)/(s + \pi)$ as a lag, or read the numerator's angle as -45°, conclude that the system shifts the phase by $-\pi/2$. That leads them to pick (B), which actually produces $-\sin \pi t$. A second mistake is to pick (A) on the grounds that $|H| = 1$, so the input must equal the output. That forgets that an all-pass network still shifts the phase.

Q.22 MCQ · 1 MARK

Digital Electronics › Logic families

Choose the fastest logic family among the following:

(A) Transistor-Transistor Logic

(B) Emitter-Coupled Logic

(C) CMOS Logic

(D) Resistor-Transistor Logic

Official answer: **B**

SOLUTION

1. **What sets gate speed.** A gate's propagation delay t_{pd} is the time between an input change and the resulting output change. Two effects dominate it: (i) in bipolar logic whose transistors saturate, the storage time t_s needed to remove excess base charge before a transistor can turn off; (ii) the time needed to charge or discharge the load capacitance C_L through the output resistance, which grows with the voltage swing the output has to cover.
2. **RTL (D).** Resistor-Transistor Logic uses a saturating BJT switch with a passive resistor pull-up. It pays the full storage time at turn-off, and on the rising edge the resistor charges C_L slowly. Typical t_{pd} is around $25\ \text{ns}$, which makes it one of the slowest families.
3. **TTL (A).** TTL transistors still saturate, so storage time remains. The active totem-pole output drives the edges harder than RTL's resistor does, so standard TTL has $t_{pd} \approx 10\ \text{ns}$. Its swing is still about $3\ \text{V}$. Schottky versions clamp the base-collector junction to keep the transistors out of deep saturation, which shortens the delay but does not remove the large swing.
4. **CMOS (C).** CMOS has no storage delay, but its output swings rail to rail (about $5\ \text{V}$ on a $5\ \text{V}$ supply), and its MOSFETs charge C_L through a fairly high on-resistance, so delay rises with load. Standard CMOS is typically $25\ \text{ns}$ or slower (the 4000 series is slower still at $5\ \text{V}$). High-speed HC parts reach about $8\ \text{ns}$ to $10\ \text{ns}$, which puts them level with standard TTL, not ahead of ECL.
5. **ECL (B).** Emitter-Coupled Logic is built on a differential current-switch pair fed by a constant current source. The current is steered from one transistor to the other, so each transistor is either cut off or in the active region. Neither ever saturates, so there is no storage time. The logic swing is small ($V_{OH} \approx -0.9\ \text{V}$, $V_{OL} \approx -1.75\ \text{V}$, about $0.85\ \text{V}$ in total), and the emitter-follower outputs have low output resistance, so C_L charges quickly through a small voltage change. Typical t_{pd} is about $1\ \text{ns}$ to $2\ \text{ns}$ for the 10K series and under $1\ \text{ns}$ for the 100K series.
6. **Conclusion.** Of the four families, only ECL both avoids saturation and uses a sub-volt swing with low-resistance outputs. That gives it the smallest propagation delay: about $1\ \text{ns}$ to $2\ \text{ns}$, against about $10\ \text{ns}$ for standard TTL and roughly $25\ \text{ns}$ for RTL and standard CMOS. The cost is high static power, because

the current source conducts all the time. The fastest logic family is Emitter-Coupled Logic, so the answer is option (B).

KEY IDEA

ECL is the fastest family because its current-switch transistors never saturate, so there is no storage delay. It also uses a small swing (about 0.85 V) and low-resistance emitter-follower outputs, so the load capacitance charges quickly. The price is the highest static power dissipation of the common families.

WATCH OUT FOR

Picking CMOS because it is the dominant modern technology. CMOS's strength is very low static power, not speed: standard CMOS gates are slower than TTL, and high-speed CMOS only matches standard TTL. A second mistake is confusing 'fastest' with 'best speed-power product'. ECL wins on raw delay but not on power.

Q.23 MCQ · 1 MARK

Engineering Mathematics › Calculus

What is $\lim_{x\to 0} f(x)$, where $f(x) = x \sin \frac{1}{x}$?

(A) 0

(B) 1

(C) ∞

(D) Limit does not exist

Official answer: **A**

SOLUTION

1. As $x \to 0$, $\sin \frac{1}{x}$ keeps swinging between -1 and 1, so it has no limit of its own. It is still **bounded**: $\left|\sin \frac{1}{x}\right| \le 1$ for every $x \neq 0$.
2. Multiply by $|x|$ to get a bound on f:
$$0 \le \left| x \sin \frac{1}{x} \right| = |x| \left| \sin \frac{1}{x} \right| \le |x| \quad (x \neq 0)$$
3. **Squeeze (sandwich) theorem:** both bounds tend to 0, because $\lim_{x\to 0} 0 = 0$ and $\lim_{x\to 0} |x| = 0$. So $\lim_{x\to 0} \left| x \sin \frac{1}{x} \right| = 0$.
4. Since $-|f(x)| \le f(x) \le |f(x)|$ and both outer terms tend to 0, the squeeze theorem gives $f(x) \to 0$ as well:
$$\lim_{x\to 0} x \sin \frac{1}{x} = 0$$
5. Put another way, a function that tends to 0 (here x) multiplied by a bounded function (here $\sin \frac{1}{x}$) tends to 0. The answer is (A).

KEY IDEA

A function that tends to zero, multiplied by a bounded function, tends to zero. Here $|x \sin(1/x)| \le |x|$, and the squeeze theorem finishes it. The oscillation of $\sin(1/x)$ does not matter, because its amplitude is being shrunk to zero by the factor x.

WATCH OUT FOR

Picking (D) because $\sin \frac{1}{x}$ oscillates and has no limit. That is true of $\sin \frac{1}{x}$ by itself, but not of $x \sin \frac{1}{x}$, whose swings are held inside $\pm|x|$. Picking (B) by rewriting the function as $\frac{\sin(1/x)}{1/x}$ and applying the standard limit $\frac{\sin t}{t} \to 1$. That standard limit needs $t \to 0$, but here $t = \frac{1}{x} \to \infty$, and $\frac{\sin t}{t} \to 0$ as $t \to \infty$. ($x \sin \frac{1}{x} \to 1$ is correct only as $x \to \infty$.)

Q.24 NAT · 1 MARK

Control Systems › Routh–Hurwitz

The number of zeros of the polynomial $P(s) = s^3 + 2s^2 + 5s + 80$ in the right-half plane is _____.

Official answer: **2**

SOLUTION

1. Every coefficient is positive. For a cubic that is necessary for stability but not sufficient, so we build the Routh array to count the right-half-plane (RHP) roots.
2. The first two rows come straight from the coefficients:
$$\begin{array}{ll} s^3: & 1 \quad 5 \\ s^2: & 2 \quad 80 \end{array}$$
3. Row s^1: $\dfrac{(2)(5) - (1)(80)}{2} = \dfrac{10 - 80}{2} = -35$
4. Row s^0: 80, carried down from the s^2 row.
5. First column: $1,\ 2,\ -35,\ 80$. The sign changes twice, once from 2 to -35 and once from -35 to 80.
6. By the Routh criterion, the number of sign changes in the first column equals the number of roots in the right-half plane. So $P(s)$ has **2** RHP zeros.
7. Cross-check with the cubic rule. For $s^3 + a_2 s^2 + a_1 s + a_0$ with all coefficients positive, the roots all lie in the left-half plane only if $a_2 a_1 > a_0$. Here $a_2 a_1 = 2 \times 5 = 10 < a_0 = 80$, so the polynomial is unstable. Also $P(-5) = -20$ and $P(-4) = 28$, so there is one real root between them, near $s \approx -4.64$. The roots sum to $-a_2 = -2$, so the complex pair has real part $\approx \dfrac{-2 + 4.64}{2} \approx +1.32$. Both of those roots are in the RHP, which confirms the count of 2.

KEY IDEA

In the Routh array, the number of sign changes down the first column equals the number of roots in the right-half plane. For a cubic $s^3 + a_2 s^2 + a_1 s + a_0$ with positive coefficients, the one test that matters is whether $a_2 a_1 > a_0$. When it fails, as it does here, the s^1 entry turns negative and two roots lie in the RHP.

WATCH OUT FOR

Seeing that every coefficient is positive and answering 0. Positive coefficients are necessary but not sufficient for stability once the order is 3 or more. A second mistake is to spot that the system is unstable and answer 1. The number of sign changes is the number of RHP roots, and here there are two, $2 \to -35$ and $-35 \to 80$.

Q.25 NAT · 1 MARK

Control Systems › Nyquist

The number of times the Nyquist plot of $G(s)H(s) = \dfrac{1}{2}\dfrac{(s-1)(s-2)}{(s+1)(s+2)}$ encircles the origin is ____.

Official answer: **2**

SOLUTION

1. Argument principle about the origin: as s goes clockwise around the Nyquist contour (up the $j\omega$ axis from $-j\infty$ to $+j\infty$, then around the infinite semicircle enclosing the right half-plane), the plot of $G(s)H(s)$ circles the origin $N = Z - P$ times clockwise. Here Z counts the right-half-plane zeros of $G(s)H(s)$ itself (not of $1 + G(s)H(s)$), and P counts its right-half-plane poles. No pole lies on the $j\omega$ axis, so the contour needs no indentation.
2. Zeros are at $s = 1, 2$, both in the RHP, so $Z = 2$. Poles are at $s = -1, -2$, both in the LHP, so $P = 0$. That gives $N = 2 - 0 = 2$ clockwise encirclements.
3. Check with the magnitude. Because $|j\omega - 1| = |j\omega + 1|$ and $|j\omega - 2| = |j\omega + 2|$,
$$|G(j\omega)H(j\omega)| = \frac{1}{2} \quad \text{for all } \omega,$$

so the plot lies on a circle of radius $\frac{1}{2}$ centred at the origin. It is an all-pass function.

4. Check with the phase. For $\omega > 0$, the vector $j\omega - 1$ lies in the second quadrant, so its angle is $180^\circ - \tan^{-1}\omega$; similarly for $j\omega - 2$. Hence
$$\angle G(j\omega)H(j\omega) = \left(180^\circ - \tan^{-1}\omega\right) + \left(180^\circ - \tan^{-1}\tfrac{\omega}{2}\right) - \tan^{-1}\omega - \tan^{-1}\tfrac{\omega}{2} = 360^\circ - 2\tan^{-1}\omega - 2\tan^{-1}\tfrac{\omega}{2}$$
5. As ω goes from 0 to ∞, the phase falls steadily from 360° to $360^\circ - 180^\circ - 180^\circ = 0^\circ$, starting and ending at the point $G(0)H(0) = \frac{1}{2}$. That is one full clockwise turn around the origin.
6. The branch for ω from $-\infty$ to 0 is the complex conjugate (mirror image in the real axis) of the $\omega > 0$ branch, traversed in the opposite order of ω. Reflection and reversal together keep the sense of rotation, so it adds a second clockwise turn. The infinite semicircle maps to the single point $G(\infty)H(\infty) = \frac{1}{2}$ and adds nothing. Total: 2 encirclements (clockwise), which agrees with $N = Z - P$.

KEY IDEA

When a question asks about encircling the origin, not the point -1, apply the argument principle to $G(s)H(s)$ itself. Then $N = Z - P$ uses the open-loop right-half-plane zeros and poles. Here the two RHP zeros at $s = 1, 2$ give $N = 2$ clockwise. The all-pass magnitude of $\frac{1}{2}$ and the 720° total phase swing confirm it directly.

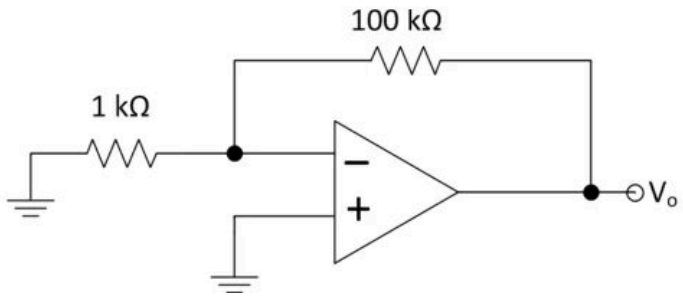
WATCH OUT FOR

Answering 1 by tracing only the $\omega = 0$ to ∞ half of the plot, which is one circle. The mirror half from $-\infty$ to 0 adds a second turn. Other traps: counting zeros of $1 + G(s)H(s)$ (the closed-loop rule for the -1 point) in place of the zeros of $G(s)H(s)$, and deciding there is no encirclement because the magnitude is constant.

Q.26 NAT · 1 MARK

Analog Electronics › Op-amps

Q.26 The opamp in the circuit shown is ideal, except that it has an input bias current of 1 nA and an input offset voltage of 10 μV. The resulting worst-case output voltage will be ±_____ μV (**rounded off to the nearest integer**).



Official answer: **1110**

SOLUTION

1. **Model.** Treat the non-ideal op-amp as an ideal op-amp plus (i) an offset source $V_{OS} = 10\ \mu\text{V}$ in series with the non-inverting input and (ii) a bias current $I_B = 1\ \text{nA}$ flowing into each input. No signal is applied, so everything at V_o is error. The circuit is linear, so find each error term by superposition. Here $R_1 = 1\ \text{k}\Omega$ (inverting node to ground) and $R_f = 100\ \text{k}\Omega$ (inverting node to output).
2. **Offset-voltage term** (bias currents set to zero). V_{OS} sits at the + input, and R_1 to ground with R_f as feedback make a non-inverting amplifier. The noise gain applies:
$$V_{o1} = V_{OS}\left(1 + \frac{R_f}{R_1}\right) = 10\ \mu\text{V} \times \left(1 + \frac{100\ \text{k}\Omega}{1\ \text{k}\Omega}\right) = 10 \times 101 = 1010\ \mu\text{V}$$
3. **Bias current into the + input.** The + input is tied straight to ground. Its bias current flows through zero resistance, produces no voltage, and adds nothing to V_o.
4. **Bias current into the – input** (V_{OS} set to zero). The virtual short holds $v_- = v_+ = 0$, so no current flows in the 1 kΩ resistor, because both its ends are at 0 V. All of I_B must therefore reach the – input through the 100 kΩ feedback resistor, from the output:

$$V_{o2} = I_B R_f = 1\ \text{nA} \times 100\ \text{k}\Omega = 100\ \mu\text{V}$$

5. **Worst case.** The polarity of V_{OS} is not specified, so the worst case is when the two terms add:
$$|V_o|_{\max} = 1010\ \mu\text{V} + 100\ \mu\text{V} = 1110\ \mu\text{V}$$
6. So $V_o = \pm 1110\ \mu\text{V}$.

KEY IDEA
The input offset voltage is amplified by the noise gain $1 + R_f/R_1$, whatever configuration the circuit is drawn in. When the + input is grounded directly, the bias current into the – input is forced through R_f alone, because the virtual short leaves no voltage across R_1. That gives $I_B R_f$. Superpose the two and add magnitudes for the worst case.

WATCH OUT FOR
Multiplying V_{OS} by the inverting gain $R_f/R_1 = 100$ gives 1000 µV instead of 1010 µV, which would be the correct figure only for a signal applied through R_1. Other errors are dropping the bias-current term (answer 1010), or letting part of I_B flow through the 1 kΩ resistor, when that resistor has 0 V across it.

Q.27 NAT · 1 MARK

Electromagnetics › Magnetostatics

The force per unit length between two infinitely long parallel conductors, with a gap of 2 cm between them is 10 μN/m. When the gap is doubled, the force per unit length will be _____ μN/m (**rounded off to one decimal place**).

Official answer: **4.9 to 5.1**

SOLUTION

1. Field of one infinitely long conductor carrying current I_1, at distance d from it (Ampère's circuital law over a circle of radius d):
$$B_1 = \frac{\mu_0 I_1}{2\pi d}, \qquad \mu_0 = 4\pi \times 10^{-7}\ \text{H/m}$$
Its direction is circumferential, so it is perpendicular to the second conductor.
2. Force on a length L of the second conductor, carrying I_2 (Lorentz force $d\mathbf{F} = I_2\, d\mathbf{l} \times \mathbf{B}_1$, with $d\mathbf{l} \perp \mathbf{B}_1$):
$$\frac{F}{L} = I_2 B_1 = \frac{\mu_0 I_1 I_2}{2\pi d}$$
3. Only the gap changes; the currents I_1, I_2 and μ_0 stay the same. Therefore $\dfrac{F}{L} \propto \dfrac{1}{d}$.
4. Take the ratio of the two cases:
$$\frac{(F/L)_2}{(F/L)_1} = \frac{d_1}{d_2} = \frac{2\ \text{cm}}{4\ \text{cm}} = \frac{1}{2}$$
5. Hence
$$(F/L)_2 = \frac{10\ \mu\text{N/m}}{2} = 5.0\ \mu\text{N/m}$$

KEY IDEA
The force per unit length between two long parallel currents is $\dfrac{F}{L} = \dfrac{\mu_0 I_1 I_2}{2\pi d}$. The field of a long straight wire falls off as $1/d$, not $1/d^2$, so with the currents unchanged the force is inversely proportional to the separation.

WATCH OUT FOR
Treating the force like Coulomb's law between point charges, as $1/d^2$, which gives 2.5 µN/m. The field of an infinitely long line current falls off as $1/d$, so doubling the gap only halves the force. A second error is trying to find the actual currents: they are not needed, because only the ratio matters.

Q.28 NAT · 1 MARK

Signals and Systems › Signals basics

Consider the discrete-time signal $x[n] = u[-n+5] - u[n+3]$,

where $u[n] = \begin{cases} 1; & n \geq 0 \\ 0; & n < 0 \end{cases}$.

The smallest n for which $x[n] = 0$ is _____.

Official answer: **-3**

SOLUTION

1. **Time-reversed, shifted step.** $u[-n+5] = 1$ when $-n+5 \geq 0$, that is for $n \leq 5$, and $u[-n+5] = 0$ for $n > 5$ (i.e. $n \geq 6$).
2. **Shifted step.** $u[n+3] = 1$ when $n+3 \geq 0$, that is for $n \geq -3$, and $u[n+3] = 0$ for $n < -3$ (i.e. $n \leq -4$).
3. **Region $n \leq -4$:** $u[-n+5] = 1$ and $u[n+3] = 0$, so $x[n] = 1 - 0 = 1$.
4. **Region $-3 \leq n \leq 5$:** both steps equal 1, so $x[n] = 1 - 1 = 0$.
5. **Region $n \geq 6$:** $u[-n+5] = 0$ and $u[n+3] = 1$, so $x[n] = 0 - 1 = -1$.
6. Putting the three regions together,

$$x[n] = \begin{cases} 1, & n \leq -4 \\ 0, & -3 \leq n \leq 5 \\ -1, & n \geq 6 \end{cases}$$

$x[n]$ is non-zero for every $n \leq -4$ and first becomes zero at $n = -3$, so the smallest n with $x[n] = 0$ is -3.

KEY IDEA

Find the switching point of each step. $u[-n+5]$ is 1 for $n \leq 5$ (a reflected step, so it is on to the left), and $u[n+3]$ is 1 for $n \geq -3$ (on to the right). These two breakpoints cut the time axis into three regions. The difference is zero only where the two steps overlap, $-3 \leq n \leq 5$, and the left end of that overlap is the answer.

WATCH OUT FOR

Getting the direction of the reflected step wrong. Some students read $u[-n+5]$ as 1 for $n \geq 5$ or $n \geq -5$, when it is actually 1 for $n \leq 5$. Two other slips: answering -4, the last point where $x[n] = 1$, instead of -3, the first point where $x[n] = 0$; and assuming that $x[n] = 0$ again for $n \geq 6$, when it is -1 there.

Q.29 NAT · 1 MARK

Signals and Systems › Signals basics

Let $y(t) = x(4t)$, where $x(t)$ is a continuous-time periodic signal with fundamental period of 100 s. The fundamental period of $y(t)$ is _____ s (**rounded off to the nearest integer**).

Official answer: **25**

SOLUTION

1. Periodicity of x: $x(t+100) = x(t)$ for all t, and 100 s is the smallest such positive shift. Every period of x is therefore a positive integer multiple of 100 s.
2. Test a shift T on y: $y(t+T) = x\big(4(t+T)\big) = x(4t+4T)$.
3. So $y(t+T) = y(t)$ for all t exactly when $4T$ is a period of x, i.e. $4T = 100k$ for a positive integer k.
4. Smallest positive choice ($k = 1$):

$$T_y = \frac{T_x}{4} = \frac{100}{4} = 25 \text{ s}$$

5. Time compression by a factor of 4 makes $y(t)$ complete four cycles in every 100 s, so the fundamental period of $y(t)$ is 25 s.

KEY IDEA

Time scaling $y(t) = x(at)$ with $a > 0$ divides the fundamental period by a: $T_y = T_x/a$. Compression ($a > 1$) shortens the period and expansion ($0 < a < 1$) lengthens it.

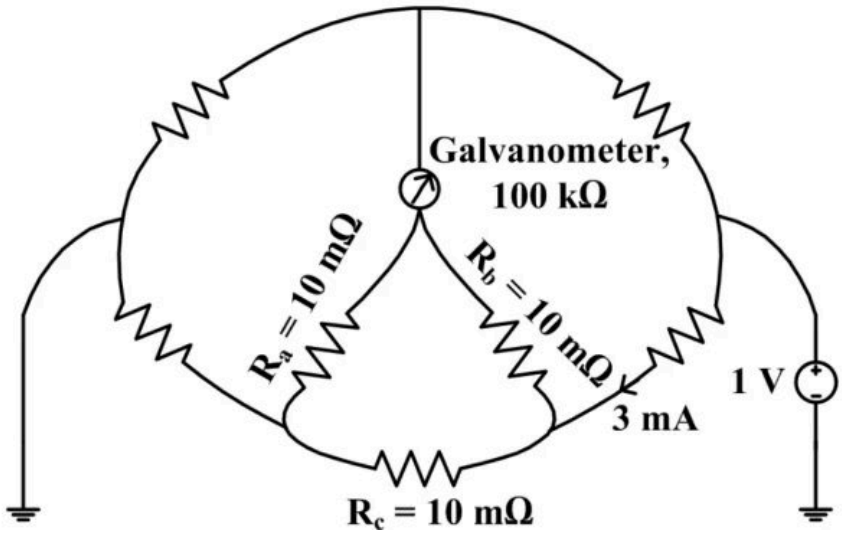
WATCH OUT FOR

Multiplying instead of dividing, which gives $4 \times 100 = 400$ s. Replacing t by $4t$ compresses the signal in time, so each cycle takes a quarter as long and the period is $100/4 = 25$ s.

Q.30 NAT · 1 MARK

Electrical and Electronic Measurements › Bridges

Q.30 When the bridge given below is balanced, the current through the resistor R_a is _____mA (**rounded off to two decimal places**).



Official answer: **0.99 to 1.01**

SOLUTION

1. At balance the galvanometer carries no current: $I_g = 0$. The inner node G connects only to the galvanometer, R_a and R_b, so by KCL at G the current in R_a equals the current in R_b.
2. Because $I_g = 0$, R_a and R_b form a single series path between the bottom junctions P and Q: $R_a + R_b = 10 + 10 = 20\ \text{m}\Omega$. This path is in parallel with the link $R_c = 10\ \text{m}\Omega$, which also joins P and Q.
3. The $3\ \text{mA}$ in the lower-right arm arrives at Q. Nothing else is attached to Q except R_c and R_b, so the whole $3\ \text{mA}$ divides between these two parallel paths and recombines at P, then continues through the lower-left arm towards ground.
4. Current divider: each path takes a share proportional to the resistance of the other path.

$$I_{R_a} = I\,\frac{R_c}{R_c + (R_a + R_b)} = 3\ \text{mA} \times \frac{10}{10 + 20}$$

5. $$I_{R_a} = 3 \times \frac{1}{3} = 1.00\ \text{mA}$$

   The other $2\ \text{mA}$ flows through R_c (check: $2 + 1 = 3\ \text{mA}$). Once the bridge is balanced, the $1\ \text{V}$ source, the unlabelled outer arms and the $100\ \text{k}\Omega$ galvanometer resistance do not affect the answer.

KEY IDEA

In a balanced Kelvin double bridge the galvanometer current is zero. The inner arms R_a and R_b therefore carry the same current and act as one series path of $R_a + R_b$, in parallel with the link R_c. The $3\ \text{mA}$ in the arm feeding the link splits by a simple current divider: $I_{R_a} = 3 \times \frac{10}{10+20} = 1\ \text{mA}$.

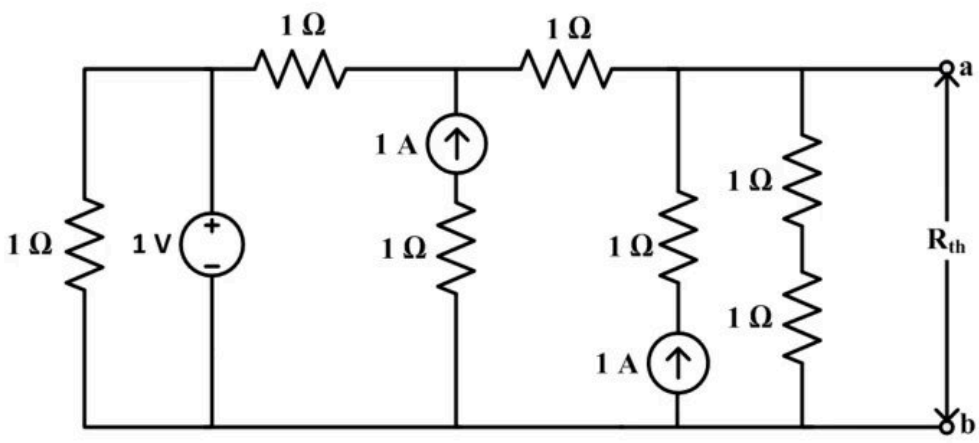
WATCH OUT FOR

One mistake is to put R_a in parallel with R_c alone and forget that R_b is in series with it, which gives $1.5\ \text{mA}$. Another is to flip the divider and use the path's own resistance in the numerator, $3 \times \frac{20}{30} = 2\ \text{mA}$, which is actually the current through R_c. A third is trying to solve the whole network with the $1\ \text{V}$ source and the $100\ \text{k}\Omega$ galvanometer. At balance neither affects the answer, and the outer arms have no values printed anyway.

Q.31 NAT · 1 MARK

Networks (Electric Circuits) › Theorems

Q.31 In the circuit given, the Thevenin equivalent resistance R_{th} across the terminals 'a' and 'b' is _____ Ω **(rounded off to one decimal place).**



Official answer: **1.0**

SOLUTION

1. **Deactivate the independent sources.** The circuit has no dependent sources, so R_{th} is simply the resistance seen between a and b once the sources are switched off. Replace the $1\ \text{V}$ source with a short circuit and each $1\ \text{A}$ source with an open circuit.
2. **Left end:** the shorted $1\ \text{V}$ source joins the top-left node to the bottom rail (terminal b). The left $1\ \Omega$ now sits across a short, so it carries no current and drops out. The top-left node is at the same potential as b.
3. **Middle branch:** the $1\ \text{A}$ source is now an open, so its series $1\ \Omega$ carries no current and that branch drops out. The two top $1\ \Omega$ resistors are therefore in series, running from a to the top-left node, which is b: $1 + 1 = 2\ \Omega$.
4. **Right side:** the branch with $1\ \Omega$ in series with the second $1\ \text{A}$ source is also an open, so it drops out too. The last branch is the two $1\ \Omega$ resistors in series, straight from a to b: $1 + 1 = 2\ \Omega$.
5. **Combine the two paths in parallel across a-b:**

$$R_{th} = 2\ \Omega \parallel 2\ \Omega = \frac{2 \times 2}{2 + 2} = 1.0\ \Omega$$

KEY IDEA

With only independent sources present, R_{th} is found by deactivating them: short every voltage source and open every current source. Any resistor in series with a current source then carries no current and drops out, and any resistor directly across a voltage source is shorted out.

WATCH OUT FOR

Keeping the $1\ \Omega$ resistors that sit in series with the $1\ \text{A}$ sources in the network. Once a current source is opened, its whole branch carries no current, so those resistors do not count (keeping them gives about $0.46\ \Omega$). A second mistake is opening the $1\ \text{V}$ source instead of shorting it. That leaves the left $1\ \Omega$ in the path and gives $3 \parallel 2 = 1.2\ \Omega$.

Q.32 NAT · 1 MARK

Engineering Mathematics › Probability & statistics

X is a discrete random variable which takes values 0, 1 and 2. The probabilities are $P(X = 0) = 0.25$ and $P(X = 1) = 0.5$. With $E[\,.\,]$ denoting the expectation operator, the value of $E[X] - E[X^2]$ is _____ (rounded off to one decimal place).

Official answer: **-0.5**

SOLUTION

1. The probabilities must add up to 1, so $P(X = 2) = 1 - 0.25 - 0.5 = 0.25$.
2. Mean, from $E[X] = \sum x\, P(X = x)$: $E[X] = 0(0.25) + 1(0.5) + 2(0.25) = 0 + 0.5 + 0.5 = 1$.
3. Second moment, from $E[g(X)] = \sum g(x)\, P(X = x)$ with $g(x) = x^2$: $E[X^2] = 0^2(0.25) + 1^2(0.5) + 2^2(0.25) = 0 + 0.5 + 1 = 1.5$.
4. Therefore
$$E[X] - E[X^2] = 1 - 1.5 = -0.5$$
5. Check: by linearity, $E[X] - E[X^2] = E[X(1 - X)]$. The product $x(1 - x)$ is 0 at $x = 0$ and at $x = 1$, and $2(1 - 2) = -2$ at $x = 2$. So the value is $-2 \times 0.25 = -0.5$, which agrees.

KEY IDEA

First find the missing probability from $\sum P = 1$. Then use $E[g(X)] = \sum g(x)\, P(X = x)$: take $g(x) = x$ for the mean and $g(x) = x^2$ for the second moment. Linearity also gives the answer in one line as $E[X(1 - X)]$, and only $x = 2$ adds anything to that sum.

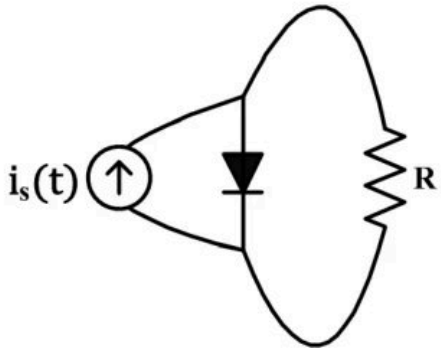
WATCH OUT FOR

There are two common slips. One is leaving out the $x = 2$ term because its probability is not printed, when it must be found as $1 - 0.25 - 0.5 = 0.25$. The other is computing $E[X^2]$ as $(E[X])^2 = 1$, which gives 0 instead of -0.5. $E[X^2]$ is the second moment, not the square of the mean. The two differ by the variance, which here is 0.5.

Q.33 NAT · 1 MARK

Analog Electronics › Diode circuits

Q.33 The diode in the circuit is ideal. The current source $i_s(t) = \pi \sin(3000\pi t)$ mA. The magnitude of the average current flowing through the resistor R is _____mA (**rounded off to two decimal places**).



Official answer: **0.95 to 1.05**

SOLUTION

1. **Circuit.** The source, the ideal diode and R are all in parallel between a top node and a bottom node. The diode's anode is on the top node and its cathode is on the bottom node. The source arrow points up, so a positive i_s pushes current into the top node. Take i_R as the current flowing down through R, from the top node to the bottom node.
2. **Positive half-cycle** ($i_s > 0$): the current entering the top node can flow through the diode from anode to cathode, which is its forward direction, so the diode is ON. An ideal ON diode is a short circuit, so the voltage across R is 0 and $i_R = 0$. All of i_s flows through the diode. This is self-consistent because the diode current i_s is positive in its forward direction.
3. **Negative half-cycle** ($i_s < 0$): the source now pulls current out of the top node. If the diode were ON it would have to carry this current from cathode to anode, which an ideal diode cannot do, so the diode is OFF (open). The source and R then form a single loop, so $i_R = i_s = \pi \sin(3000\pi t)\ \text{mA} < 0$. Check: the voltage of the top node relative to the bottom node is $i_R R < 0$, so the diode is reverse-biased, which confirms it is OFF.
4. **Waveform through R.** i_R is a half-wave-rectified sinusoid of peak magnitude $I_m = \pi\ \text{mA}$. It is nonzero only in the negative half-cycles of i_s, where it flows upward through R. Because a current source drives the circuit, i_R does not depend on the value of R.
5. **Average of a half-wave-rectified sine** over one period T, with $\omega T = 2\pi$:
$$\frac{1}{T}\int_0^{T/2} I_m \sin(\omega t)\, dt = \frac{I_m}{\omega T}\Big[-\cos(\omega t)\Big]_0^{T/2} = \frac{I_m}{2\pi}\cdot 2 = \frac{I_m}{\pi}$$
The same result holds for the negative lobes, with a minus sign: the average of i_R is $-\dfrac{I_m}{\pi}$.
6. **Substitute:**
$$|I_{avg}| = \frac{\pi\ \text{mA}}{\pi} = 1.00\ \text{mA}$$
The minus sign only records that the resistor current flows upward, which is why the question asks for the magnitude. The frequency (1500 Hz) does not affect the average.

KEY IDEA

The ideal diode shorts R whenever the source current flows in the diode's forward direction, so R carries only the other half-cycles: a half-wave-rectified sine of peak π mA. Its average over a full period is $I_m/\pi = 1$ mA, whatever the value of R, because a current source drives the circuit.

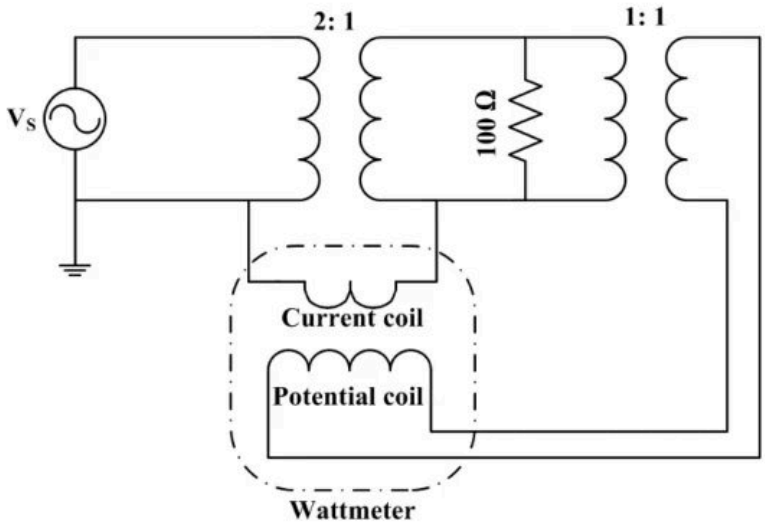
WATCH OUT FOR

Averaging over only the half-cycle in which R conducts gives $2I_m/\pi = 2$ mA. The average must be taken over the full period, which gives I_m/π. A second trap is to think the answer needs the value of R: with a current source and the diode open, all of i_s goes through R, whatever its resistance.

Q.34 NAT · 1 MARK

Electrical and Electronic Measurements › V, I, power & energy

Q.34 The full-scale range of the wattmeter shown in the circuit is 100 W. The turns ratio of the individual transformers are indicated in the figure. The RMS value of the ac source voltage V_s is 200 V. The wattmeter reading will be _____W (**rounded off to the nearest integer**).



Official answer: **0**

SOLUTION

1. Potential coil: the T1 secondary voltage is $200 \times \frac{1}{2} = 100$ V. This voltage sits across both the $100\ \Omega$ resistor and T2's primary. T2 is $1:1$, so the potential coil, which is connected across T2's secondary, sees $V_{PC} = 100$ V.
2. Current coil: one end is on the grounded return wire of the source/T1-primary loop and the other end is on the lower wire of the T1-secondary loop. So the coil is not in series with any loop that carries load current.
3. An ideal transformer couples its windings only magnetically, so the primary loop and the secondary loop share no conductor. The current coil is the only wire that joins the two circuits.
4. Apply KCL to the isolated secondary network. Any current that enters it through one conductor has to leave through another, and there is no second conductor. Therefore $I_{CC} = 0$. All the coil does is fix the secondary circuit's potential relative to ground.
5. Wattmeter deflection is proportional to $V_{PC}\, I_{CC} \cos\phi = 100 \times 0 \times \cos\phi = 0$, so the meter reads 0 W.

6. For contrast, the resistor really does dissipate $\frac{100^2}{100} = 100\ \mathrm{W}$, which equals the meter's full scale. The current coil never carries that current, though, so the meter does not register it.

KEY IDEA

A wattmeter shows a reading only when current flows through its current coil. Here the current coil is the only conductor between the primary circuit and the secondary circuit, which the transformer isolates from each other. KCL gives no return path, so $I_{CC} = 0$ and the reading is $0\ \mathrm{W}$ whatever the potential-coil voltage.

WATCH OUT FOR

Working out the resistor's power, $\frac{100^2}{100} = 100\ \mathrm{W}$, and answering 100 W, the meter's full scale. That assumes the current coil carries the secondary load current, but it does not: it only bridges the grounded primary return to the secondary loop and forms no closed path.

Q.35 NAT · 1 MARK

Electrical Machines › DC machines

The no-load steady-state output voltage of a DC shunt generator is 200 V when it is driven in the clockwise direction at its rated speed. If the same machine is driven at the rated speed but in the opposite direction, the steady-state output voltage will be _____V (rounded off to the nearest integer).

Official answer: **0**

SOLUTION

1. A DC shunt generator has no independent excitation source; it relies on a small residual flux in the poles. As the armature rotates (clockwise case, given), this residual flux induces a small EMF, which drives a small current through the shunt field winding.
2. Self-excitation ('voltage build-up') occurs only if the mmf produced by this field current reinforces the residual flux: flux rises, EMF rises, field current rises further, a cumulative process that builds the no-load terminal voltage up to 200 V at rated speed (as given for the clockwise direction).
3. The induced EMF's polarity depends on the direction of rotation, since $e \propto \frac{d\phi}{dt}$ (equivalently $e \propto v \times B$ for the moving conductors) changes sign when the direction of rotation reverses.
4. The physical connection of the field winding to the armature terminals/brushes is fixed hardware and does not change merely because the shaft is driven in the opposite direction.
5. Therefore, running the machine in the opposite (counter-clockwise) direction reverses the polarity of the small residual-flux-generated EMF, and hence reverses the direction of current flowing through the same, unchanged shunt field winding.
6. This reversed field current now produces an mmf that opposes (demagnetizes) the residual flux instead of reinforcing it, because the sense of reinforcement that existed in the clockwise case is tied to the specific combination of rotation direction and fixed winding connection.
7. With an opposing/demagnetizing mmf, cumulative voltage build-up cannot occur: there is no regenerative loop, and the residual flux is driven toward cancellation rather than being reinforced.
8. Hence the machine fails to self-excite when driven in the reverse direction, and the steady-state no-load output voltage is $0\ \mathrm{V}$ (rounded off to the nearest integer).

KEY IDEA

A DC shunt generator self-excites only if the field-current mmf produced by the small residual-flux EMF reinforces (adds to) the residual flux. This "sense of reinforcement" depends on both the direction of rotation and the fixed physical connection of the field winding to the armature; reversing only the rotation direction reverses the EMF polarity and hence the field-current direction, which flips the field mmf from reinforcing to opposing the residual flux, killing self-excitation.

WATCH OUT FOR

Assuming the output voltage stays at 200 V (or drops to some other nonzero value) because "the machine is unchanged", forgetting that self-excitation depends on the *relative direction* between the residual-flux EMF and the fixed field-winding connection, so reversing rotation reverses this relative sense and can convert a cumulatively-connected (self-exciting) machine into a differentially-connected (de-exciting) one.

Q.36 MCQ · 2 MARKS

Signals and Systems › LTI & convolution

The impulse response of an LTI system is $h(t) = \delta(t) + 0.5\,\delta(t-4)$, where $\delta(t)$ is the continuous-time unit impulse signal. If the input signal $x(t) = \cos\left(\frac{7\pi}{4}t\right)$, the output is _____.

(A) $0.5\cos\left(\frac{7\pi}{4}t\right)$

(B) $1.5\cos\left(\frac{7\pi}{4}t\right)$

(C) $0.5\sin\left(\frac{7\pi}{4}t\right)$

(D) $1.5\sin\left(\frac{7\pi}{4}t\right)$

Official answer: **A**

SOLUTION

1. Convolving a signal with $\delta(t-t_0)$ delays it by t_0: $x(t) * \delta(t-t_0) = x(t-t_0)$. Convolution is linear, so
$$y(t) = x(t) * h(t) = x(t) + 0.5\,x(t-4).$$
2. Substitute the input:
$$y(t) = \cos\left(\frac{7\pi}{4}t\right) + 0.5\cos\left(\frac{7\pi}{4}(t-4)\right) = \cos\left(\frac{7\pi}{4}t\right) + 0.5\cos\left(\frac{7\pi}{4}t - 7\pi\right)$$
3. 7π is an odd multiple of π, so $\cos(\theta - 7\pi) = \cos(\theta - \pi) = -\cos\theta$.
4. So
$$y(t) = \cos\left(\frac{7\pi}{4}t\right) - 0.5\cos\left(\frac{7\pi}{4}t\right) = 0.5\cos\left(\frac{7\pi}{4}t\right).$$
5. Check with the frequency response (a sinusoid is an eigenfunction of an LTI system): $H(j\omega) = 1 + 0.5\,e^{-j4\omega}$. At $\omega = \frac{7\pi}{4}$ we get $4\omega = 7\pi$ and $e^{-j7\pi} = \cos 7\pi - j\sin 7\pi = -1$, so $H = 1 - 0.5 = 0.5\angle 0°$. The output is the input scaled by 0.5 with no phase shift, $0.5\cos\left(\frac{7\pi}{4}t\right)$: option (A).

KEY IDEA

An impulse at t_0 in $h(t)$ adds a copy of the input delayed by t_0. Here the delay of 4 s is a phase lag of $\frac{7\pi}{4} \times 4 = 7\pi$ rad, an odd multiple of π. That turns the delayed copy into $-0.5\cos\left(\frac{7\pi}{4}t\right)$, which cancels half of the direct term. Put another way, $|H(j\omega)| = |1 + 0.5e^{-j4\omega}|$ is at its minimum of 0.5, with zero phase, at this frequency.

WATCH OUT FOR

Adding the two impulse strengths without accounting for the delay ($1 + 0.5 = 1.5$) gives option (B). That is only right if the phase lag 4ω is an even multiple of π. A second trap is thinking the delay turns cosine into sine (options C and D). A phase shift of 7π only flips the sign; a sine would need an odd multiple of $\pi/2$.

Q.37 MCQ · 2 MARKS

Signals and Systems › Laplace

The Laplace transform of the continuous-time signal $x(t) = e^{-3t}u(t-5)$ is _____, where $u(t)$ denotes the continuous-time unit step signal.

(A) $\dfrac{e^{-5s}}{s+3}$, $Real\{s\} > -3$

(B) $\dfrac{e^{-5(s-3)}}{s-3}$, $Real\{s\} > 3$

(C) $\dfrac{e^{-5(s+3)}}{s+3}$, $Real\{s\} > -3$

(D) $\dfrac{e^{-5(s-3)}}{s+3}$, $Real\{s\} > -3$

Official answer: **C**

SOLUTION

1. The step switches on at $t = 5$, but the exponential is written in terms of t, not $(t-5)$. To use the time-shift property, rewrite it in terms of $(t-5)$:
$$x(t) = e^{-3t}u(t-5) = e^{-3(t-5)-15}\,u(t-5) = e^{-15}\,e^{-3(t-5)}\,u(t-5)$$
2. Base pair: $e^{-3t}u(t) \longleftrightarrow \dfrac{1}{s+3}$, with ROC: $\mathrm{Re}\{s\} > -3$.
3. Time-shift property: $g(t-t_0)u(t-t_0) \longleftrightarrow e^{-st_0}G(s)$. It leaves the ROC unchanged because e^{-st_0} is finite for every s. With $t_0 = 5$: $e^{-3(t-5)}u(t-5) \longleftrightarrow \dfrac{e^{-5s}}{s+3}$, $\mathrm{Re}\{s\} > -3$.
4. Multiply by the constant e^{-15}:
$$X(s) = e^{-15}\cdot\frac{e^{-5s}}{s+3} = \frac{e^{-5s-15}}{s+3} = \frac{e^{-5(s+3)}}{s+3}, \quad \mathrm{Re}\{s\} > -3$$
5. Check by direct integration. $x(t)$ is zero for $t < 5$, so
$$X(s) = \int_5^\infty e^{-3t}e^{-st}\,dt = \int_5^\infty e^{-(s+3)t}\,dt = \left[\frac{e^{-(s+3)t}}{-(s+3)}\right]_5^\infty = \frac{e^{-5(s+3)}}{s+3}$$
The upper limit goes to zero only when $\mathrm{Re}\{s+3\} > 0$, which gives $\mathrm{Re}\{s\} > -3$. The signal is right-sided, so this right half-plane ROC is what we expect.
6. Rule out the others. (A) leaves out the factor e^{-15}, so it treats the signal as $e^{-3(t-5)}u(t-5)$. (B) is the transform of $e^{+3t}u(t-5)$. (D) has the wrong sign inside the exponent of the numerator. The answer is option (C).

KEY IDEA

The time-shift property $g(t-t_0)u(t-t_0) \longleftrightarrow e^{-st_0}G(s)$ only works when the whole function is shifted, not just the step. So first write $e^{-3t} = e^{-15}e^{-3(t-5)}$. The constant e^{-15} multiplies the transform, and the ROC stays $\mathrm{Re}\{s\} > -3$. The same result follows from shifting in s: $e^{-at}f(t) \longleftrightarrow F(s+a)$, where $u(t-5) \longleftrightarrow \dfrac{e^{-5s}}{s}$ gives $\dfrac{e^{-5(s+3)}}{s+3}$.

WATCH OUT FOR

The usual mistake is to apply the time shift to the step alone and write $\dfrac{e^{-5s}}{s+3}$ (option A). That drops the factor e^{-15}, because $e^{-3t}u(t-5)$ is not the same as $e^{-3(t-5)}u(t-5)$. Getting the sign of the exponential wrong leads to (B) or (D).

Q.38 MCQ · 2 MARKS

Sensors and Industrial Instrumentation › Optical

In a p-i-n photodiode, a pulse of light containing 8×10^{12} incident photons at wavelength $\lambda_0 = 1.55\ \mu\mathrm{m}$ gives rise to an average 4×10^{12} electrons collected at the terminals of the device. The quantum efficiency of the photodiode at this wavelength is _____ %.

(A) 50

(B) 54.2

(C) 62.5

(D) 80

Official answer: **A**

SOLUTION

1. Quantum efficiency is the number of collected electron-hole pairs per incident photon: $\eta = \dfrac{N_e}{N_p}$.
2. Incident photons: $N_p = 8 \times 10^{12}$. Collected electrons: $N_e = 4 \times 10^{12}$.
3. $$\eta = \frac{4 \times 10^{12}}{8 \times 10^{12}} = 0.5$$
4. As a percentage, $\eta = 0.5 \times 100 = 50\,\%$.
5. The wavelength $1.55\ \mu\text{m}$ is not needed here. It would matter only for the responsivity $R = \dfrac{\eta q \lambda}{hc}$, which the question does not ask for. The answer is option (A).

KEY IDEA
Quantum efficiency is the ratio of collected electrons to incident photons. It is a pure count ratio and does not depend on wavelength.

WATCH OUT FOR
Bringing the wavelength into the calculation, for example by computing responsivity or photon energy. The question gives photon and electron counts directly, so the wavelength is a distractor.

Q.39 MCQ · 2 MARKS

Engineering Mathematics › Complex analysis

Let $f(z) = j\dfrac{1-z}{1+z}$, where z denotes a complex number and j denotes $\sqrt{-1}$. The inverse function $f^{-1}(z)$ maps the real axis to the _____.

(A) unit circle with centre at the origin

(B) unit circle with centre not at the origin

(C) imaginary axis

(D) real axis

Official answer: **A**

SOLUTION

1. Write $w = f(z) = j\dfrac{1-z}{1+z}$. This is a bilinear (Mobius) map. Solve it for z to get the inverse: $w(1+z) = j(1-z) \Rightarrow w + wz = j - jz \Rightarrow z(w+j) = j - w$.
2. So the inverse map is
$$f^{-1}(w) = \frac{j-w}{j+w}.$$
3. Substitute a point of the real axis, $w = u$ with u real: $z = \dfrac{j-u}{j+u}$.
4. Take the modulus. The numerator is $-u + j$ and the denominator is $u + j$, and both have modulus $\sqrt{u^2+1}$:
$$|z| = \frac{|j-u|}{|j+u|} = \frac{\sqrt{u^2+1}}{\sqrt{u^2+1}} = 1 \quad \text{for every real } u.$$
5. So every image point lies on $|z| = 1$. Check a few points: $u = 0 \to z = \dfrac{j}{j} = 1$; $u = 1 \to z = \dfrac{-1+j}{1+j} = \dfrac{(-1+j)(1-j)}{(1+j)(1-j)} = \dfrac{2j}{2} = j$; $u \to \pm\infty \to z \to -1$. All three lie on the unit circle. A Mobius map is one-

to-one, so as u runs over the whole real axis (with the point at infinity) the image covers the whole circle.

6. A bilinear map sends a line to either a line or a circle. The inverse map's only pole is at $w = -j$, which is not on the real axis, so the image is bounded: it is a circle, not a line. That circle is $|z| = 1$, whose centre is the origin. Answer: (A).

KEY IDEA

Invert the bilinear map and substitute a real value. For real u, the numbers $j - u$ and $j + u$ have the same modulus $\sqrt{u^2 + 1}$, so their ratio always has modulus 1. Put the other way round: f is a Cayley-type transform that takes the unit circle onto the real axis, so f^{-1} takes the real axis back onto the unit circle.

WATCH OUT FOR

Using the forward map f instead of f^{-1}. For real x, $f(x) = j\dfrac{1-x}{1+x}$ is purely imaginary, so f maps the real axis to the imaginary axis, which leads to the wrong option (C). A second slip is getting the image circle right but not checking its centre. The points 1, j and -1 are all at distance 1 from the origin, which fixes the centre at 0 and rules out (B).

Q.40 MCQ · 2 MARKS

Digital Electronics › Boolean logic

The simplified form of the Boolean function $F(W, X, Y, Z) = \Sigma(4, 5, 10, 11, 12, 13, 14, 15)$ with the minimum number of terms and smallest number of literals in each term is _____.

(A) $WX + \overline{W}X\overline{Y} + W\overline{X}Y$

(B) $WX + WY + X\overline{Y}$

(C) $X\overline{Y} + WY$

(D) $\overline{X}Y + \overline{W}\,\overline{Y}$

Official answer: **C**

SOLUTION

1. Write the minterms in binary, with W as the MSB: $4 = (0100)_2$, $5 = (0101)_2$, $10 = (1010)_2$, $11 = (1011)_2$, $12 = (1100)_2$, $13 = (1101)_2$, $14 = (1110)_2$, $15 = (1111)_2$. Complement is written with an overline throughout, e.g. $\overline{Y}$.
2. First quad: $\{4, 5, 12, 13\}$. All four have $X = 1$ and $Y = 0$, while W and Z take every combination. This quad gives $X\overline{Y}$.
3. Second quad: $\{10, 11, 14, 15\}$. All four have $W = 1$ and $Y = 1$, while X and Z take every combination. This quad gives WY.
4. Coverage and minimality. $\{4, 5, 12, 13\} \cup \{10, 11, 14, 15\}$ is exactly the eight given minterms, so $X\overline{Y} + WY$ covers F with nothing extra. Both terms are essential prime implicants: minterm 4 has $W = 0$ and $Y = 0$, so only $X\overline{Y}$ covers it, and minterm 10 has $X = 0$, so only WY covers it. No single literal can equal F. $X = 1$ would also include 6 and 7, and $W = 1$ would include 8 and 9, none of which are in F. So no implicant has fewer than 2 literals, and fewer than 2 terms is impossible.
5. Therefore
$$F = X\overline{Y} + WY$$
This is 2 terms of 2 literals each.
6. Eliminate the other options. (A) $WX + \overline{W}X\overline{Y} + W\overline{X}Y$ covers $\{12, 13, 14, 15\} \cup \{4, 5\} \cup \{10, 11\}$, which equals F, but it uses 3 terms, two of them with 3 literals, so it is not minimal. (B) adds WX, which is redundant: $\{12, 13, 14, 15\}$ is already covered by the two quads. This is the consensus theorem, $X\overline{Y} + WY + WX = X\overline{Y} + WY$. (D) $\overline{X}Y + \overline{W}\,\overline{Y}$ is true at minterm $0 = (0000)_2$ through $\overline{W}\,\overline{Y}$, and 0 is not in F, so (D) is a different function.
7. Answer: (C) $X\overline{Y} + WY$.

KEY IDEA

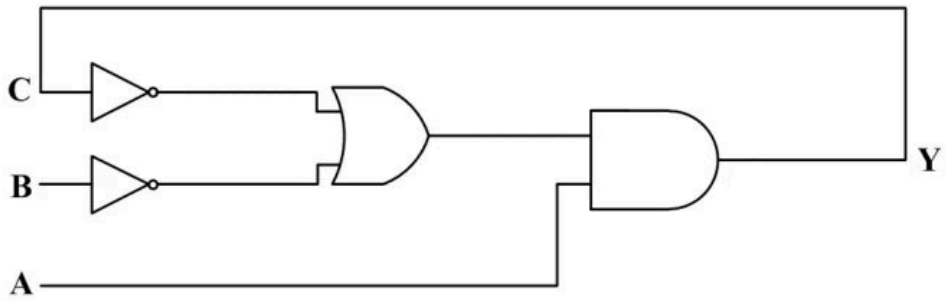
Plot the 8 minterms on a 4-variable K-map. They form exactly two non-overlapping quads: $\{4, 5, 12, 13\}$ gives $X\overline{Y}$ and $\{10, 11, 14, 15\}$ gives WY. Both are essential prime implicants, and together they cover every minterm. The third prime implicant, WX, is the consensus of these two and is redundant.

WATCH OUT FOR

Choosing (B) because the quad $\{12, 13, 14, 15\}$ (WX) is a valid prime implicant. It is the consensus term of $X\overline{Y}$ and WY, so it adds no coverage. Choosing (A) is the same kind of error: it describes the correct function but is not the minimal form the question asks for.

Q.41 MCQ · 2 MARKS

Digital Electronics › Sequential

Q.41	For the given digital circuit, A = B = 1. Assume that AND, OR, and NOT gates have propagation delays of 10 ns, 10 ns, and 5 ns respectively. All lines have zero propagation delay. Given that C = 1 when the circuit is turned on, the frequency of steady-state oscillation of the output Y is _____.
	
(A)	20 MHz
(B)	15 MHz
(C)	40 MHz
(D)	50 MHz

Official answer: **A**

SOLUTION

1. Write the logic. With $\overline{X}$ meaning complement: $Y = A \cdot (\overline{C} + \overline{B})$. The feedback wire from the output back to the C input makes $C = Y$.
2. Put in the constants $A = B = 1$. Then $\overline{B} = 0$, so the OR gate passes $\overline{C}$ straight through ($\overline{C} + 0 = \overline{C}$). Because $A = 1$, the AND gate passes the OR output straight through ($1 \cdot \overline{C} = \overline{C}$). So $Y = \overline{C}$, but only after a delay.
3. Since $C = Y$, the closed path contains exactly one inversion. That makes it a ring oscillator: no stable state is possible, because $Y = \overline{Y}$ has no solution. So Y keeps toggling.

4. Delay once around the loop, through NOT, then OR, then AND: $t_{loop} = 5 + 10 + 10 = 25\ \text{ns}$. The NOT gate on B is not in the loop, because B is a constant and that gate's output never changes.
5. Trace from $C = Y = 1$ at $t = 0$. The NOT output falls to 0 at $5\ \text{ns}$, the OR output at $15\ \text{ns}$, and $Y = C$ at $25\ \text{ns}$. That new 0 goes round again: the NOT output rises to 1 at $30\ \text{ns}$, the OR output at $40\ \text{ns}$, and Y at $50\ \text{ns}$. So Y stays at each level for $25\ \text{ns}$. How the gates power up can affect the first few nanoseconds, but not the steady-state period.
6. In an inverting loop, the output needs one trip round the loop to go from 1 to 0 and a second trip to go back to 1. So the period is twice the loop delay: $T = 2\,t_{loop} = 2 \times 25\ \text{ns} = 50\ \text{ns}$.
7. $$f = \frac{1}{T} = \frac{1}{50\ \text{ns}} = 20\ \text{MHz}$$

   This is option (A).

KEY IDEA

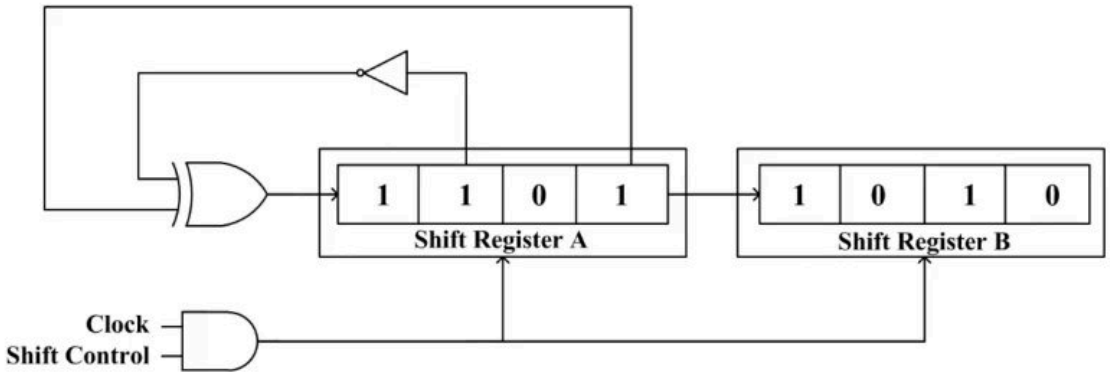
A closed loop with an odd number of inversions is a ring oscillator. Its period is twice the total propagation delay round the loop, and only gates whose inputs actually change count towards that delay. Here the constant inputs A and B make the OR and AND gates act as plain delays, so the loop is effectively one inverter plus 20 ns of extra delay.

WATCH OUT FOR

Taking the period as a single trip round the loop gives $25\ \text{ns}$ and 40 MHz (option C). You need two trips, one for each edge. Counting only the OR and AND delays and missing the NOT gate in the loop gives $20\ \text{ns}$, which students often misread as the period and so pick 50 MHz (option D). Adding the NOT gate on B is also wrong: B is constant, so that gate is not in the loop.

Q.42 MCQ · 2 MARKS

Digital Electronics › Sequential

Q.42	In the circuit shown, the initial binary content of shift register A is 1101 and that of shift register B is 1010. The shift registers are positive-edge triggered, and the gates have no delay. When the shift control is high, what will be the binary content of the shift registers A and B after four clock pulses?
	
(A)	A = 1101, B = 1101
(B)	A = 1110, B = 1001
(C)	A = 0101, B = 1101
(D)	A = 1010, B = 1111

Official answer: **C**

SOLUTION

1. **Clock gating.** The AND gate passes the clock only when Shift Control is 1. Shift Control is high here, so both registers shift on every rising edge of the clock.
2. **Label the cells and read the connections.** Call A's cells $A_1A_2A_3A_4$ and B's cells $B_1B_2B_3B_4$, left to right. Both registers shift right, and A_4 feeds B_1. The inverter complements A_2, and the long top wire brings back A_4, so the serial input of A is
$$D = A_4 \oplus \overline{A_2}$$
where $\overline{A_2}$ means the complement of A_2.
3. **Next-state rule on each rising edge.**
$$A^+ = (D,\ A_1,\ A_2,\ A_3), \qquad B^+ = (A_4,\ B_1,\ B_2,\ B_3)$$

The gates have no delay, so D is always worked out from the contents present *just before* the edge.

4. **Clock 1.** $A = 1101$, so $A_2 = 1$, $A_4 = 1$ and $D = 1 \oplus \overline{1} = 1 \oplus 0 = 1$. After the edge, $A = 1110$ and $B = 1101$.
5. **Clock 2.** $A = 1110$, so $A_2 = 1$, $A_4 = 0$ and $D = 0 \oplus \overline{1} = 0 \oplus 0 = 0$. After the edge, $A = 0111$ and $B = 0110$.
6. **Clock 3.** $A = 0111$, so $A_2 = 1$, $A_4 = 1$ and $D = 1 \oplus \overline{1} = 1 \oplus 0 = 1$. After the edge, $A = 1011$ and $B = 1011$.
7. **Clock 4.** $A = 1011$, so $A_2 = 0$, $A_4 = 1$ and $D = 1 \oplus \overline{0} = 1 \oplus 1 = 0$. After the edge, $A = 0101$ and $B = 1101$.
8. **Check.** After four shifts, every one of A's original four bits has moved into B, in the same order. So B must be 1101, which rules out (B) and (D). Only the XOR feedback separates (A) from (C), and the clock-by-clock working gives $A = 0101$. The answer is option (C): $A = 0101$, $B = 1101$.

KEY IDEA

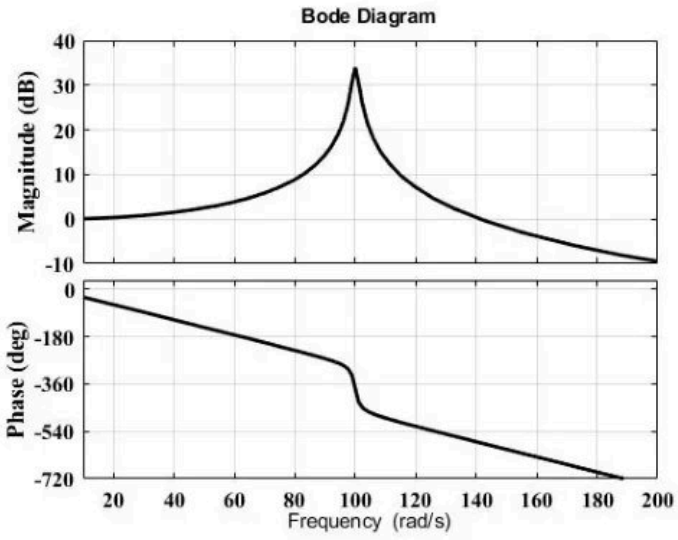
A 4-bit shift register fully empties in four shifts. So B ends up holding A's original contents (1101), and A ends up holding the four feedback bits $D = A_4 \oplus \overline{A_2}$ it produced, the newest on the left. Each feedback bit is worked out from the register contents just before its clock edge.

WATCH OUT FOR

Treating A as a plain ring counter, with A_4 fed straight back to A_1. That brings A back to 1101 after four shifts and gives option (A), but it ignores the XOR and the inverter. Other slips are dropping the inverter on A_2, taking the tap from the wrong cell, and working out D from contents that have already shifted instead of the contents just before the edge.

Q.43 MCQ · 2 MARKS

Control Systems › Bode & margins

Q.43	The magnitude and phase plots shown in the figure match with the transfer-function_____.
	
(A)	$\dfrac{10000}{s^2+2s+10000}$
(B)	$\dfrac{10000}{s^2+2s+10000}e^{-0.05s}$
(C)	$\dfrac{10000}{s^2+2s+10000}e^{-0.5\times10^{-12}s}$
(D)	$\dfrac{100}{s^2+2s+100}$

Official answer: **B**

SOLUTION

1. **Read the magnitude plot.** The gain is 0 dB at low frequency and there is one sharp resonant peak at $\omega \approx 100$ rad/s. Every option has $G(0)=1$, which is 0 dB. For $\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$ with small ζ, the peak is at $\omega_r=\omega_n\sqrt{1-2\zeta^2}\approx\omega_n$. So $\omega_n=100$ rad/s and $\omega_n^2=10000$. Option (D) has $\omega_n^2=100$, so $\omega_n=10$ rad/s and its peak would be at 10 rad/s. (D) is out.
2. **Check the peak height and the high-frequency end.** For $s^2+2s+10000$: $2\zeta\omega_n=2$, so $\zeta=0.01$.

$$M_r=\frac{1}{2\zeta\sqrt{1-\zeta^2}}\approx 50 \;\Rightarrow\; 20\log_{10}50\approx 34\text{ dB},$$

which matches the plotted peak of about 33 to 34 dB. At $\omega=200$:

$$|G(j200)| = \frac{10000}{|10000 - 40000 + j400|} \approx \frac{10000}{30003} \approx 0.333 \;\Rightarrow \approx -9.5\ \text{dB},$$

which matches the curve ending just above -10 dB. Because $|e^{-j\omega T}| = 1$ for every real ω, options (A), (B) and (C) have exactly the same magnitude plot. The magnitude cannot tell them apart, so the phase has to.

3. **Phase of the quadratic alone.**

$$\angle \frac{10000}{10000 - \omega^2 + j2\omega} = -\tan^{-1}\frac{2\omega}{10000 - \omega^2},$$

taken between 0° and -180°. It starts at 0°, equals -90° at $\omega = 100\ \text{rad/s}$ and approaches -180°, but it never goes past -180°. With $\zeta = 0.01$, almost all of that change happens close to $100\ \text{rad/s}$: about -11° at $95\ \text{rad/s}$ and about -168° at $105\ \text{rad/s}$. This accounts for the steep drop at $100\ \text{rad/s}$. The plotted phase reaches -720°, though, so option (A) cannot produce it.

4. **The rest of the phase is a transport delay.** Away from $100\ \text{rad/s}$ the phase falls along a straight line on this *linear* frequency axis. A pure delay does exactly that: $\angle e^{-j\omega T} = -\omega T$ rad is linear in ω, has no lower limit and leaves the magnitude unchanged. So $G(s)$ is the quadratic multiplied by e^{-sT}, and its total phase is

$$\phi(\omega) = -\tan^{-1}\frac{2\omega}{10000 - \omega^2} - \frac{180}{\pi}\omega T \ \text{degrees}.$$

5. **Estimate T at a point far from resonance.** The curve leaves the bottom of the axes, at -720°, at about $\omega \approx 189\ \text{rad/s}$. At that frequency the quadratic has almost finished its swing:

$$-\left(180^\circ - \tan^{-1}\frac{378}{35721 - 10000}\right) \approx -(180^\circ - 0.8^\circ) \approx -179^\circ.$$

That leaves about $-720^\circ + 179^\circ \approx -541^\circ \approx -9.44$ rad for the delay, so

$$T \approx \frac{9.44}{189} \approx 0.05\ \text{s}.$$

6. **Check $T = 0.05$ s at the low-frequency end, and rule out (C).** At the left edge of the plot, $\omega = 10\ \text{rad/s}$, the delay gives $-0.5\ \text{rad} \approx -28.6^\circ$ and the quadratic about -0.1°. The total is about -29°, which matches the curve starting near -30°. Near $\omega = 62\ \text{rad/s}$ the delay gives $-3.1\ \text{rad} \approx -178^\circ$ and the quadratic about -1°. That puts the -180° crossing a little past $60\ \text{rad/s}$, which is also where the plot crosses it. Option (C) has $T = 0.5 \times 10^{-12}$ s. Even at $200\ \text{rad/s}$ its delay adds only $200 \times 0.5 \times 10^{-12} = 10^{-10}\ \text{rad} \approx 6 \times 10^{-9}$ degrees. Its phase would look the same as (A)'s and stay above -180°, so (C) is out.

7. **Conclusion.**

$$G(s) = \frac{10000}{s^2 + 2s + 10000}\, e^{-0.05s},$$

which is option (B).

KEY IDEA

A pure delay e^{-sT} does not change $|G(j\omega)|$. It adds phase $-\omega T$, which is a straight line on a linear frequency axis and has no lower limit. A second-order rational transfer function on its own can never go below -180°. So the magnitude plot gives the quadratic (ω_n from where the peak sits, ζ from how high it is), and the extra linear phase gives the delay T.

WATCH OUT FOR

Choosing (A) because its magnitude plot matches exactly, without noticing that its phase stops at -180° while the plot goes to -720°. Another trap is choosing (C) because it 'has a delay' without checking the size: $T = 0.5 \times 10^{-12}$ s adds only about 10^{-10} rad of phase here. Some students also take $\omega_n = 10000$ instead of $\omega_n^2 = 10000$, or pick (D) by mistaking 100 for ω_n when it is ω_n^2 in that option. Finally, a straight phase line on a *linear* frequency axis points to a delay, not to extra poles; pole and zero asymptotes are straight on a *log* axis.

Q.44 MSQ · 2 MARKS

Signals and Systems › Fourier transform

A continuous real-valued signal $x(t)$ has finite positive energy and $x(t) = 0, \forall\, t < 0$. From the list given below, select **ALL** the signals whose continuous-time Fourier transform is purely imaginary.

(A) $x(t) + x(-t)$

(B) $x(t) - x(-t)$

(C) $j\,(x(t) + x(-t))$

(D) $j\,(x(t) - x(-t))$

Official answer: **B, C**

SOLUTION

1. Let $x(t) \leftrightarrow X(j\omega)$. The transform exists because $x(t)$ has finite energy. Time reversal gives $x(-t) \leftrightarrow X(-j\omega)$. Because $x(t)$ is real, conjugate symmetry gives $X(-j\omega) = X^*(j\omega)$, so $x(-t) \leftrightarrow X^*(j\omega)$.
2. **(A)** $x(t) + x(-t) \leftrightarrow X(j\omega) + X^*(j\omega) = 2\,\mathrm{Re}\{X(j\omega)\}$. This is **real**, not imaginary. (A real, even signal has a real, even transform.)
3. **(B)** $x(t) - x(-t) \leftrightarrow X(j\omega) - X^*(j\omega) = j\,2\,\mathrm{Im}\{X(j\omega)\}$. This is **purely imaginary**. (A real, odd signal has an imaginary, odd transform.)
4. **(C)** By linearity, $j\,(x(t) + x(-t)) \leftrightarrow j\,2\,\mathrm{Re}\{X(j\omega)\}$. This is j times a real function, so it is **purely imaginary**.
5. **(D)** By linearity, $j\,(x(t) - x(-t)) \leftrightarrow j \cdot j\,2\,\mathrm{Im}\{X(j\omega)\} = -2\,\mathrm{Im}\{X(j\omega)\}$. This is **real**.
6. Why the causality and positive-energy conditions are given: since $x(t) = 0$ for $t < 0$, for every $t > 0$ we have $x(-t) = 0$, so the even part $x_e(t) = \frac{1}{2}\,(x(t) + x(-t))$ and the odd part $x_o(t) = \frac{1}{2}\,(x(t) - x(-t))$ both equal $x(t)/2$ there. All the energy of $x(t)$ sits in $t \geq 0$ and is positive, so neither part is the zero signal, and by uniqueness neither $2\,\mathrm{Re}\{X(j\omega)\}$ nor $2\,\mathrm{Im}\{X(j\omega)\}$ is identically zero. So the transforms in (A) and (D) are genuinely nonzero real functions, not trivially zero (a zero function would count as both real and imaginary). Without causality an even $x(t)$ could make (B) and (D) vanish.
7. Hence the transforms of (B) and (C) are purely imaginary. **Answer: B, C.**

KEY IDEA

For a real signal, $x(-t) \leftrightarrow X^*(j\omega)$. So the even part $x(t) + x(-t)$ has transform $2\,\mathrm{Re}\{X(j\omega)\}$, which is real, and the odd part $x(t) - x(-t)$ has transform $j\,2\,\mathrm{Im}\{X(j\omega)\}$, which is imaginary. Multiplying a signal by j multiplies its transform by j, which turns real into imaginary and imaginary into real. Causality with positive energy makes sure the even and odd parts are both nonzero, so none of the four transforms is trivially zero.

WATCH OUT FOR

Marking only (B) because you remember "real and odd gives imaginary" and forgetting that the factor j in (C) turns the real transform of the even part into an imaginary one. The opposite mistake is marking (D) because it "has a j in it": there $j \cdot j = -1$, so its transform comes out real.

Q.45 NAT · 2 MARKS

Electromagnetics › Waveguides & fibres

A silica-glass fiber has a core refractive index of 1.47 and a cladding refractive index of 1.44. If the cladding is completely stripped out and the core is dipped in water having a refractive index of 1.33, the numerical aperture of the modified fiber is _____ (rounded off to three decimal places).

Official answer: **0.620 to 0.640**

SOLUTION

1. **Identify the new cladding.** Once the glass cladding is removed and the core is dipped in water, the water surrounds the core and does the cladding's job. So $n_1 = 1.47$ (core) and $n_2 = 1.33$ (water). The

original cladding index 1.44 no longer plays any part.

2. **Numerical aperture of a step-index guide.** Light is launched from air, so $n_0 = 1$:
$$\mathrm{NA} = n_0 \sin\theta_a = \sqrt{n_1^2 - n_2^2}$$
3. **Substitute:**
$$n_1^2 = (1.47)^2 = 2.1609, \qquad n_2^2 = (1.33)^2 = 1.7689$$
4. $$n_1^2 - n_2^2 = 2.1609 - 1.7689 = 0.3920$$
5. $$\mathrm{NA} = \sqrt{0.3920} \approx 0.62610 \;\Rightarrow\; \mathrm{NA} \approx 0.626$$
This is less than 1, so it is a valid acceptance-angle result: $\theta_a = \sin^{-1}(0.626) \approx 38.8^\circ$.
6. **Sanity check:** the original fiber had $\mathrm{NA} = \sqrt{1.47^2 - 1.44^2} = \sqrt{0.0873} \approx 0.295$. Water has a much lower index than the glass cladding, so the core-to-surround index contrast is larger and the NA should rise. It does: $0.295 \to 0.626$.

KEY IDEA

The NA is set by whatever medium directly surrounds the core. Remove the glass cladding and put the core in water, and the water becomes the cladding: $\mathrm{NA} = \sqrt{n_{\text{core}}^2 - n_{\text{surround}}^2}$ with $n_{\text{surround}} = 1.33$.

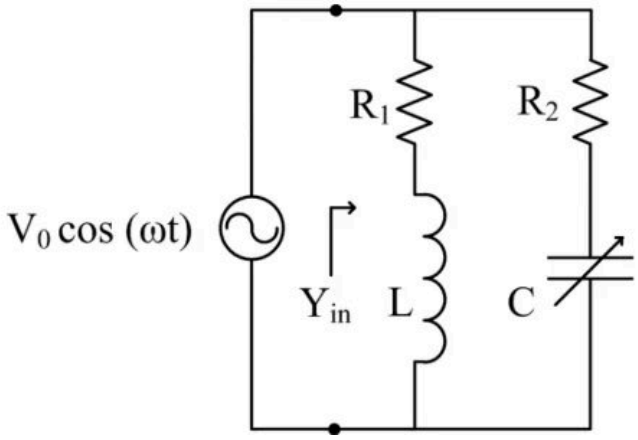
WATCH OUT FOR

Using the original cladding index 1.44, which gives about 0.295. Another slip is taking the square root of the index difference, $\sqrt{1.47 - 1.33} \approx 0.374$, when the formula needs the difference of the squares. Some students also divide by n_{water} as if light were launched from water. The NA is defined for launch from air ($n_0 = 1$); water here only forms the cladding.

Q.46 NAT · 2 MARKS

Networks (Electric Circuits) › AC, power & resonance

Q.46 In the circuit shown, $\omega = 100\pi$ rad/s, $R_1 = R_2 = 2.2\ \Omega$ and $L = 7$ mH. The capacitance C for which Y_{in} is purely real is _____ mF (**rounded off to two decimal places**).



Official answer: **1.40 to 1.50**

SOLUTION

1. Reactances: $X_L = \omega L = 100\pi \times 7 \times 10^{-3} = 2.199\ \Omega$ and $X_C = \dfrac{1}{\omega C}$. Write $R = R_1 = R_2 = 2.2\ \Omega$.
2. Admittances of the two parallel branches:
$$Y_1 = \frac{1}{R + jX_L} = \frac{R - jX_L}{R^2 + X_L^2}, \qquad Y_2 = \frac{1}{R - jX_C} = \frac{R + jX_C}{R^2 + X_C^2}$$
3. Parallel admittances add, so $Y_{in} = Y_1 + Y_2$. For Y_{in} to be purely real its imaginary part must be zero:

$$\frac{X_L}{R^2 + X_L^2} = \frac{X_C}{R^2 + X_C^2}$$

4. Cross-multiply, $X_L(R^2 + X_C^2) = X_C(R^2 + X_L^2)$, and rearrange into a quadratic in X_C:
$$X_L X_C^2 - (R^2 + X_L^2)X_C + R^2 X_L = 0 \;\Rightarrow\; (X_C - X_L)(X_L X_C - R^2) = 0$$
5. Root 1, $X_C = X_L$: $C = \dfrac{1}{\omega^2 L} = \dfrac{1}{(100\pi)^2 \times 7 \times 10^{-3}} = \dfrac{1}{690.87} = 1.4474 \times 10^{-3}\ \text{F}$.
6. Root 2, $X_C = R^2/X_L$: $C = \dfrac{X_L}{\omega R^2} = \dfrac{L}{R^2} = \dfrac{7 \times 10^{-3}}{4.84} = 1.4463 \times 10^{-3}\ \text{F}$.
7. Because $X_L = 2.199\ \Omega$ is almost exactly equal to $R = 2.2\ \Omega$, the two roots are nearly equal. Substituting either one back gives a real Y_{in} (about $0.455\ \text{S}$), and both round to the same value: $C \approx 1.45\ \text{mF}$.

KEY IDEA

Work with admittances, because parallel admittances simply add. Set the imaginary part of $Y_1 + Y_2$ to zero. The condition factorises into $X_C = X_L$ or $X_C X_L = R^2$ (that is, $C = L/R^2$). Here $\omega L \approx R$, so the network is almost the classic constant-resistance circuit ($R_1 = R_2 = \sqrt{L/C}$). That is why the two roots nearly coincide at about 1.45 mF.

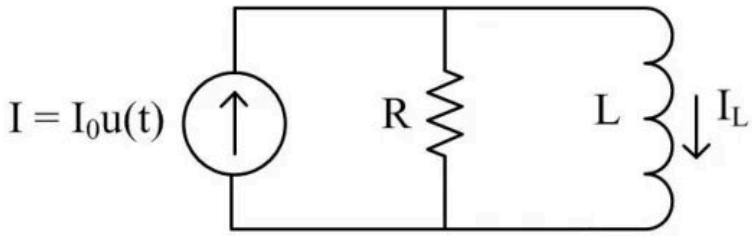
WATCH OUT FOR

Setting $\omega L = 1/(\omega C)$ straight away, as if this were a series or pure-LC resonance, without deriving the condition. That happens to land in the range here, but only because of the numbers. A second common slip is writing $Y_2 = 1/(R + jX_C)$ with the wrong sign on the capacitive reactance. That makes the two imaginary parts add instead of cancel, so no real solution exists.

Q.47 NAT · 2 MARKS

Networks (Electric Circuits) › Transients

Q.47 The R-L circuit with R = 10 kΩ and L = 1 mH is excited by a step current $I_0u(t)$. At $t = 0^-$, there is a current $I_L = I_0/5$ flowing through the inductor. The minimum time taken for the current through the inductor to reach 99% of its final value is _____μs (**rounded off to two decimal places**).



Official answer: **0.43 to 0.45**

SOLUTION

1. **Time constant.** With the current source open-circuited, the inductor sees only R, so
$$\tau = \frac{L}{R} = \frac{1 \times 10^{-3}}{10 \times 10^3} = 10^{-7}\ \text{s} = 0.1\ \mu\text{s}$$

2. **Initial condition.** Inductor current cannot change instantaneously, so $i_L(0^+) = i_L(0^-) = \dfrac{I_0}{5}$.
3. **Final value.** For $t > 0$ the source delivers I_0. In steady state the inductor behaves as a short circuit, so it carries all of the source current and none flows in R: $i_L(\infty) = I_0$.
4. **First-order response.** $i_L(t) = i_L(\infty) + [i_L(0^+) - i_L(\infty)]\, e^{-t/\tau} = I_0 - \dfrac{4I_0}{5}\, e^{-t/\tau}$ for $t \geq 0$.
5. **Condition for 99% of the final value.** $I_0 - \dfrac{4I_0}{5}e^{-t/\tau} = 0.99\, I_0 \;\Rightarrow\; \dfrac{4}{5}e^{-t/\tau} = 0.01 \;\Rightarrow\; e^{-t/\tau} = \dfrac{1}{80}$.
6. **Solve for t.**

$$t = \tau \ln 80 = 0.1\ \mu\text{s} \times 4.382 = 0.4382\ \mu\text{s} \approx 0.44\ \mu\text{s}$$

The current rises monotonically from $I_0/5$ towards I_0, so this is the first (minimum) time it reaches 99% of its final value.

KEY IDEA

This is a first-order RL circuit, so $i_L(t) = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{-t/\tau}$ with $\tau = L/R$, where R is the resistance the inductor sees with the current source open-circuited. The non-zero initial current matters: the gap still to close is only $\frac{4}{5}I_0$. The 99% point is where that gap has shrunk to $0.01I_0$, so $e^{-t/\tau} = \frac{0.01}{0.8} = \frac{1}{80}$.

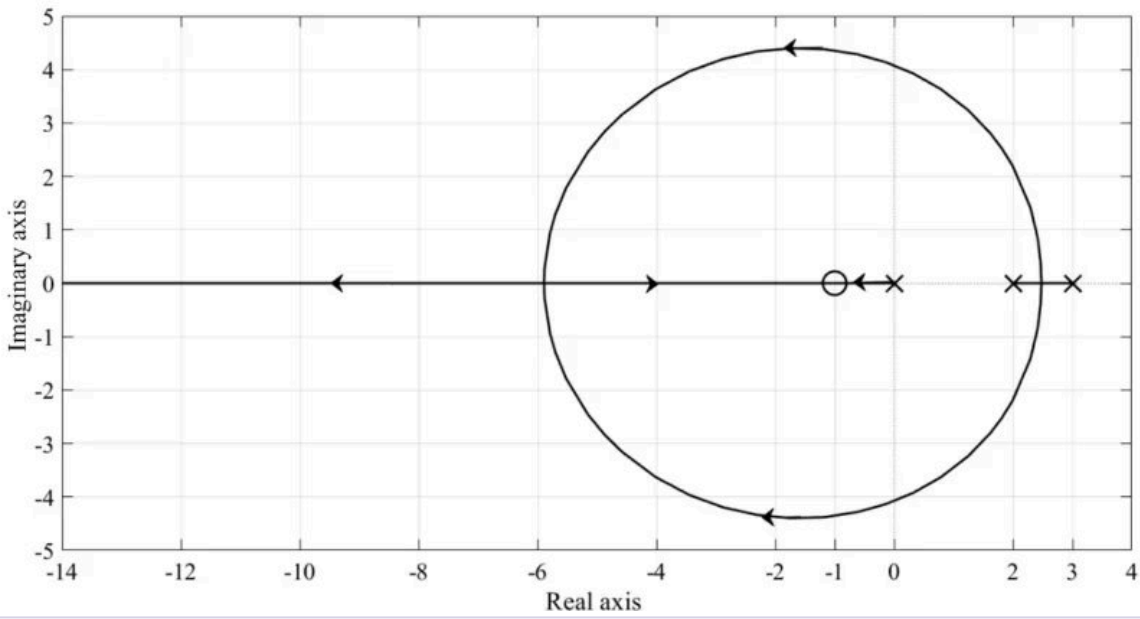
WATCH OUT FOR

Ignoring the initial current and using the zero-state rule $t = \tau \ln 100 = 0.46\ \mu\text{s}$, which falls outside the key. A second mistake is shorting the current source when finding τ (that would give $\tau = 0$). An ideal current source is replaced by an open circuit, so the inductor sees R alone.

Q.48 NAT · 2 MARKS

Control Systems › Compensators & PID

Q.48 Consider a standard negative feedback configuration with $G(s) = \frac{1}{(s-2)(s-3)}$ and the controller $C(s) = K_P + \frac{K_I}{s} + K_D s$. The root-locus of $G(s)C(s)$ is presented in the figure below. The gain $C(j\omega) = 2$ at $\omega = 1$ rad/s. The value of K_D is _____ (**rounded off to one decimal place**).



Official answer: **0.9 to 1.1**

SOLUTION

1. Write the PID as one fraction: $C(s) = \frac{K_D s^2 + K_P s + K_I}{s}$. So $G(s)C(s) = \frac{K_D s^2 + K_P s + K_I}{s(s-2)(s-3)}$ has poles at $0, 2, 3$ and at most two zeros, the roots of the PID numerator.
2. Real-axis rule: a point on the real axis is on the locus if the total number of real poles and zeros to its right is odd. For a point in $(-1, 0)$ there are 3 poles to the right, which is odd, so the segment from 0 to -1 is on the locus. The plot also shows the whole real axis left of -1 on the locus. If -1 were a single zero (either $K_D = 0$, or a second zero further left), a point just left of -1 would see $3 + 1 = 4$ poles and zeros to its right. That is even, so the point would not be on the locus, which contradicts the figure. Only one zero marker is drawn, so the second zero must also sit at -1: the PID has a double zero at $s = -1$, and left of -1 the count is $3 + 2 = 5$ (odd).
3. Branch check: 3 poles and 2 finite zeros give $3 - 2 = 1$ asymptote, at $180°$. This matches the single branch running to $-\infty$ on the plot. The pole at 0 ends on one of the zeros at -1, and the branch coming right from the break-in point ends on the other.
4. Breakaway check with $K = -\frac{s(s-2)(s-3)}{(s+1)^2}$. Setting $\frac{dK}{ds} = 0$ and cancelling the factor $(s+1)$ gives
$$s^3 + 3s^2 - 16s + 6 = 0,$$
with roots $s \approx 2.48$, $s \approx -5.89$ and $s \approx 0.41$. The root at 0.41 is not on the locus, since only 2 poles lie to its right. The other two are the breakaway point near 2.5 and the break-in point near -5.9 seen on the plot, which confirms the double zero at -1.
5. So $K_D s^2 + K_P s + K_I = K_D(s+1)^2 = K_D s^2 + 2K_D s + K_D$. That gives $K_P = 2K_D$ and $K_I = K_D$, and

$$C(s) = \frac{K_D(s+1)^2}{s}.$$

The drawn locus is the positive-gain locus, and its gain is K_D, so $K_D > 0$.

6. Evaluate at $\omega = 1$:

$$C(j1) = \frac{K_D(1+j)^2}{j} = \frac{K_D \cdot 2j}{j} = 2K_D.$$

Checking term by term: $C(j1) = K_P + j(K_D - K_I) = 2K_D$. $C(j1)$ is real and positive, so reading 'gain $= 2$' either literally or as a magnitude gives the same equation.

7. Set $2K_D = 2$, so $K_D = 1.0$ (and hence $K_P = 2$, $K_I = 1$).

KEY IDEA

Read the controller's zeros off the root locus. A PID adds a pole at the origin and two zeros. The whole real axis left of -1 is on the locus, which by the odd-count rule is only possible if both zeros sit at -1. So $C(s) = K_D(s+1)^2/s$, and at $\omega = 1$ the proportional, integral and derivative terms combine into the real number $2K_D$.

WATCH OUT FOR

Taking the single 'o' marker as a single zero, $C(s) = K_P(s+1)/s$. That leaves K_D undetermined (or wrongly zero), and it contradicts the plot, because a single zero at -1 would take the real axis left of -1 off the locus. Overlapping zeros are drawn as one marker, so count them with the real-axis rule, not by eye. A second slip is using $|j\omega + 1|$ instead of $|j\omega + 1|^2$ in the magnitude, which gives $K_D = \sqrt{2}$.

Q.49 NAT · 2 MARKS

Engineering Mathematics › Probability & statistics

How many five-digit numbers can be formed using the integers 3, 4, 5 and 6 with exactly one digit appearing twice?

Official answer: **240**

SOLUTION

1. **What the number looks like.** Read the condition the standard way: one digit is repeated (it appears twice) and no other digit repeats. A five-digit number has 5 places. The repeated digit fills 2 of them, and the other 3 places hold three different digits, none equal to the repeated one. Only 3 other digits are left in $\{3, 4, 5, 6\}$, so all three are used once each ($2 + 1 + 1 + 1 = 5$). Every such number therefore uses all four digits, and exactly one of them appears twice.
2. **Choose the repeated digit.** Any of the 4 digits can be the one that appears twice: $\binom{4}{1} = 4$ ways.
3. **Arrange the multiset.** Once the repeated digit is fixed, the five digits form a multiset with one item appearing twice. Its number of distinct orders is $\frac{5!}{2!} = \frac{120}{2} = 60$ (divide by $2!$ because swapping the two equal digits gives the same number).
4. **Leading digit.** None of the digits is 0, so every order is a genuine five-digit number and nothing has to be subtracted.
5. **Multiplication principle.**

$$N = 4 \times \frac{5!}{2!} = 4 \times 60 = 240$$

6. **Check by placing the digits.** Choose the repeated digit (4 ways). Choose its two places: $\binom{5}{2} = 10$. Put the other three digits in the remaining 3 places: $3! = 6$. That gives $4 \times 10 \times 6 = 240$, the same count.

KEY IDEA

There are 5 places and only 4 digits, and just one digit may repeat, so the digit pattern has to be $2+1+1+1$: all four digits appear and one of them appears twice. Choose the repeated digit (4 ways), then count the orders of a multiset, $\frac{5!}{2!} = 60$. That gives $4 \times 60 = 240$.

WATCH OUT FOR

One slip is to forget to choose which digit repeats, which gives $\frac{5!}{2!} = 60$. Another is to count $4 \times 5! = 480$ without dividing by 2! for the two identical digits. A third is to pick the repeated digit and then choose 3 of the remaining digits as $\binom{3}{3}$ while also multiplying by $\binom{4}{3}$ or a similar factor, which counts the same number more than once. Subtracting orders with a leading zero is also wrong here, because 0 is not one of the digits.

Q.50 NAT · 2 MARKS

Control Systems › Bode & margins

The phase margin of the transfer function $G(s) = \dfrac{2(1-s)}{(1+s)^2}$ is _____ degrees. (rounded off to the nearest integer).

Official answer: **-2 to 2**

SOLUTION

1. Write the frequency response: $G(j\omega) = \dfrac{2(1-j\omega)}{(1+j\omega)^2}$. The zero is at $s = +1$, in the right half-plane, so the system is non-minimum phase.
2. Magnitude. The RHP zero has the same magnitude as an LHP zero: $|1-j\omega| = \sqrt{1+\omega^2}$. So
$$|G(j\omega)| = \frac{2\sqrt{1+\omega^2}}{1+\omega^2} = \frac{2}{\sqrt{1+\omega^2}}$$
3. Gain crossover frequency, from $|G(j\omega_{gc})| = 1$: $\sqrt{1+\omega_{gc}^2} = 2$, so $\omega_{gc}^2 = 3$ and $\omega_{gc} = \sqrt{3}\ \text{rad/s}$.
4. Phase. The RHP zero $(1-j\omega)$ contributes $-\tan^{-1}\omega$, a lag like a pole's. The double pole adds $-2\tan^{-1}\omega$. The gain 2 is positive, so it adds no phase. Therefore $\angle G(j\omega) = -3\tan^{-1}\omega$.
5. At gain crossover: $\angle G(j\sqrt{3}) = -3\tan^{-1}\sqrt{3} = -3(60^\circ) = -180^\circ$. Direct check: $1-j\sqrt{3} = 2\angle -60^\circ$ and $(1+j\sqrt{3})^2 = 4\angle 120^\circ$, so $G(j\sqrt{3}) = \dfrac{2\cdot 2\angle -60^\circ}{4\angle 120^\circ} = 1\angle -180^\circ = -1$.
6. Phase margin:
$$\text{PM} = 180^\circ + \angle G(j\omega_{gc}) = 180^\circ - 180^\circ = 0^\circ$$
The Nyquist plot passes through -1 at $\omega = \sqrt{3}\ \text{rad/s}$, so the gain and phase crossover frequencies coincide and the closed loop is marginally stable.

KEY IDEA

A right-half-plane zero $(1-s)$ has the same magnitude as the left-half-plane zero $(1+s)$, but its phase is the opposite: it contributes $-\tan^{-1}\omega$, a lag. Get the gain crossover from the magnitude, then add up the phase with the correct sign for each factor.

WATCH OUT FOR

Treating the zero $(1-s)$ as if it were $(1+s)$ and giving it a phase lead of $+\tan^{-1}\omega$. That makes $\angle G(j\sqrt{3}) = 60^\circ - 120^\circ = -60^\circ$ and a wrong PM of 120°. Another slip is to read the leading minus in $-(s-1)$ as an extra 180°. Written as $2(1-s)$ the gain is $+2$ and adds no phase.

Q.51 NAT · 2 MARKS

Sensors and Industrial Instrumentation › Sensing principles

A wire-wound 'resistive potentiometer type' angle sensor with 72 turns is used in an application. The first turn of the potentiometer is connected to ground while its last turn is connected to 3.6 V. The width of the

wiper covers two turns ensuring make-before-break. The output (wiper) voltage when the wiper is on top of both the turns 35 and 36 is _____ V (rounded off to two decimal places).

Official answer: **1.77 to 1.78**

SOLUTION

1. Treat the winding as 72 equal turns in series across the 3.6 V supply, with the ground lead at the start of turn 1 and the 3.6 V lead at the end of turn 72. Each turn then drops
$$\Delta V = \frac{3.6\ \text{V}}{72} = 0.05\ \text{V}.$$
2. When the wiper touches a single turn n, there are n turns between it and ground, so $V_n = n\,\Delta V$. This scale reads $V_{72} = 3.6$ V on the last turn. So $V_{35} = 35 \times 0.05 = 1.75$ V and $V_{36} = 36 \times 0.05 = 1.80$ V.
3. Make-before-break means the wiper is two turns wide. On turns 35 and 36 together it touches both contact points at once, so it shorts the one turn of wire between them (turn 36). With no load on the wiper, that turn carries no current and drops out of the divider.
4. The divider now has 71 turns in circuit: 35 turns (1 to 35) between the wiper and ground, and 36 turns (37 to 72) between the wiper and the 3.6 V end.
$$V_o = 3.6 \times \frac{35}{71} = 1.7746\ \text{V}.$$
5. Check: 1.7746 V sits almost exactly halfway between the single-turn readings 1.75 V and 1.80 V (their mean is 1.775 V). That is the point of a two-turn wiper: it adds a half-turn step between neighbouring turns and so doubles the resolution.
6. $V_o \approx 1.77$ V (rounded to two decimal places).

KEY IDEA
With N turns across V, each turn drops V/N, and the wiper on turn n alone reads nV/N. A make-before-break wiper that bridges two neighbouring turns shorts the turn between them, so it reads almost exactly halfway between the two single-turn values. This doubles the resolution.

WATCH OUT FOR
Reading the bridged position as turn 35 alone (35 × 0.05 = 1.75 V) ignores the half-turn step the two-turn wiper adds. Switching to a 71-interval scale (turn 1 at 0 V) is just as wrong. On this question's scale each of the 72 turns drops 3.6/72 = 0.05 V, and the bridged position reads about 1.775 V.

Q.52 NAT · 2 MARKS

Sensors and Industrial Instrumentation › Sensing principles

The two secondaries of a linear variable differential transformer (LVDT) showed a magnitude of 2 V (RMS) for zero displacement position of the core. It is noted that the phase of one of the secondaries has a deviation of one degree from the expected phase. Other than this deviation, the LVDT is ideal.

If the differential output sensitivity of the LVDT is 1 mV (RMS)/1 µm, the output for zero displacement is _____ µm (rounded off to one decimal place).

Official answer: **34.5 to 35.5**

SOLUTION

1. At the null position of an ideal LVDT the two secondary voltages have equal magnitude and equal phase, so the series-opposition (differential) output $V_1 - V_2$ is zero. Here the only defect is that one secondary is 1° out of phase.
2. Take each secondary magnitude as $V = 2$ V (RMS), with $V_1 = 2\angle 0^\circ$ and $V_2 = 2\angle 1^\circ$, so $\theta = 1^\circ$.
3. The differential output is the phasor difference: $|V_1 - V_2| = V\,|1 - e^{j\theta}| = 2V \sin\left(\frac{\theta}{2}\right) = 4\sin\left(\frac{\theta}{2}\right)$ V.
4. Substituting: $|V_1 - V_2| = 4\sin(0.5^\circ) = 4 \times 0.0087265 \approx 0.03491\ \text{V} = 34.91\ \text{mV}$ (RMS).

5. This residual (quadrature) voltage is nearly 90° out of phase with the in-phase signal, so moving the core cannot cancel it. It remains at the null position as the output for zero displacement.
6. Using the sensitivity of $1\ \text{mV}/\mu\text{m}$, the equivalent displacement is $x = \dfrac{34.91\ \text{mV}}{1\ \text{mV}/\mu\text{m}} \approx 34.9\ \mu\text{m}$.

KEY IDEA

A phase error between the two secondaries cannot be nulled by core position. The differential output is the phasor difference, $2V\sin(\theta/2)$, which leaves a residual quadrature voltage at zero displacement. Dividing by the sensitivity gives the equivalent displacement.

WATCH OUT FOR

Treating the zero-displacement output as exactly zero because the magnitudes are equal. Subtracting magnitudes instead of phasors gives 0. Another slip is using $V\sin\theta$ (one secondary against the other as reference) or converting the angle in degrees directly instead of radians.

Q.53 NAT · 2 MARKS

Engineering Mathematics › Probability & statistics

Five measurements are made using a weighing machine, and the readings are 80 kg, 79 kg, 81 kg, 79 kg and 81 kg. The sample standard deviation of the measurement is _____ kg (rounded off to two decimal places).

Official answer: **0.98 to 1.02**

SOLUTION

1. Sample mean: $\bar{x} = \dfrac{80+79+81+79+81}{5} = \dfrac{400}{5} = 80\ \text{kg}$.
2. Deviations from the mean, $x_i - \bar{x}$: $0,\ -1,\ +1,\ -1,\ +1$.
3. Sum of squared deviations: $\sum(x_i-\bar{x})^2 = 0+1+1+1+1 = 4\ \text{kg}^2$.
4. The question asks for the *sample* standard deviation, so divide by $n-1$ (Bessel's correction), not by n:

$$s = \sqrt{\frac{\sum(x_i-\bar{x})^2}{n-1}} = \sqrt{\frac{4}{5-1}} = \sqrt{1} = 1\ \text{kg}$$

5. Rounded off to two decimal places, $s = 1.00\ \text{kg}$.

KEY IDEA

The sample standard deviation divides the sum of squared deviations by $n-1$ (Bessel's correction), not by n: $s = \sqrt{\sum(x_i-\bar{x})^2/(n-1)}$. Here the mean is 80 kg, the squared deviations add up to 4, and $s = \sqrt{4/4} = 1$.

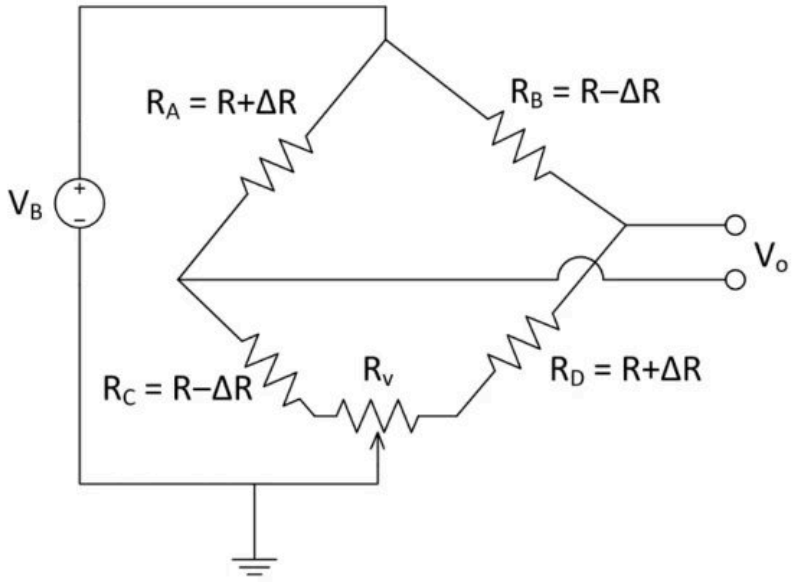
WATCH OUT FOR

Dividing by $n = 5$ gives the population standard deviation, $\sqrt{4/5} \approx 0.89\ \text{kg}$. That falls outside the key's range of 0.98 to 1.02. The word "sample" in the question means you divide by $n-1 = 4$.

Q.54 NAT · 2 MARKS

Sensors and Industrial Instrumentation › Sensing principles

Q.54 Four strain gauges R_A, R_B, R_C and R_D, each with nominal resistance R, are connected in a bridge configuration. When a force is applied, R_A and R_D increase by ΔR and R_B and R_C decrease by ΔR as shown. A potentiometer with total resistance R_v is connected as shown. If $R = 100\ \Omega$, and $\Delta R = 1\ \Omega$, the minimum value of resistance R_v required to balance the bridge is _____ Ω **(rounded off to two decimal places).**



Official answer: **4.00 to 4.10**

SOLUTION

1. Circuit: the sensors are in all four arms of a full bridge. The supply V_B drives the top node. Left arm: R_A then R_C then the left part R_1 of the potentiometer, down to the grounded wiper. Right arm: R_B then R_D then the right part R_2 of the potentiometer, down to the same ground. The output V_o is taken between the left node (between R_A and R_C) and the right node (between R_B and R_D). With $R = 100\,\Omega$ and $\Delta R = 1\,\Omega$: $R_A = R_D = 101\,\Omega$ and $R_B = R_C = 99\,\Omega$. The wiper splits the pot as $R_1 + R_2 = R_v$, with $R_1, R_2 \geq 0$.
2. Left node voltage (potential divider):
$$V_L = V_B \frac{99 + R_1}{101 + 99 + R_1} = V_B \frac{99 + R_1}{200 + R_1}$$
3. Right node voltage:
$$V_R = V_B \frac{101 + R_2}{99 + 101 + R_2} = V_B \frac{101 + R_2}{200 + R_2}$$
4. Balance requires $V_o = V_L - V_R = 0$, so
$$(99 + R_1)(200 + R_2) = (101 + R_2)(200 + R_1)$$
5. Expand. The $R_1 R_2$ terms cancel:
$$19800 + 99R_2 + 200R_1 = 20200 + 101R_1 + 200R_2$$
which gives

$$99R_1 - 101R_2 = 400$$

6. Put $R_1 = R_v - R_2$: $99R_v - 200R_2 = 400$, so
$$R_v = \frac{400 + 200R_2}{99}$$
7. Since $R_2 \geq 0$ (the wiper cannot go past the end of the pot), R_v is smallest when the wiper sits at the end stop, $R_2 = 0$.
8. Then $99R_v = 400$, so
$$R_v = \frac{400}{99} \approx 4.04\,\Omega$$

KEY IDEA

Write the two arm voltages as potential dividers to the grounded wiper. Balance gives the linear condition $99R_1 - 101R_2 = 400$. The pot's total resistance is smallest when the wiper is at one end stop, so $R_2 = 0$.

WATCH OUT FOR

Treating the grounded wiper as a simple series trim in one arm, or ignoring that the pot is shared between both arms. Another slip is balancing with $R_1 = R_2$ (wiper at the centre) and getting a larger R_v, instead of finding the smallest pot that can reach balance, which puts the wiper at the end stop.

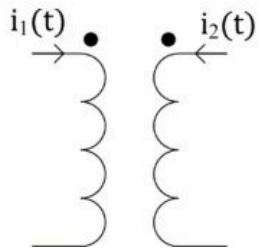
Q.55 NAT · 2 MARKS

Networks (Electric Circuits) › Coupled circuits

Q.55 A sinusoidal current of $i_1(t) = 1\sin(200\pi t)$ mA is flowing through a 4 H inductor which is mutually coupled to another 5 H inductor carrying

$$i_2(t) = 2\sin(200\pi t)\text{ mA}$$

as shown in the figure. The coupling coefficient between the inductors is 0.6. The peak energy stored in the circuit is _____μJ (**rounded off to two decimal places**).



Official answer: **17.00 to 18.00**

SOLUTION

1. Dot convention: in the figure, i_1 enters the dotted (top) end of the 4 H coil and i_2 enters the dotted (top) end of the 5 H coil. Both currents enter dots, so the mutual fluxes aid each other and the mutual energy term carries a plus sign, $+Mi_1i_2$.
2. Mutual inductance from the coupling coefficient:
$$M = k\sqrt{L_1L_2} = 0.6\sqrt{4\times 5} = 0.6\sqrt{20} = 2.683\text{ H}$$
3. Instantaneous energy stored in a coupled pair (both currents entering dots):
$$w(t) = \tfrac{1}{2}L_1i_1^2 + \tfrac{1}{2}L_2i_2^2 + Mi_1i_2$$
4. The two currents are in phase: $i_1 = 1\sin(200\pi t)$ mA and $i_2 = 2\sin(200\pi t)$ mA. So every term is proportional to $\sin^2(200\pi t)$, and the mutual term Mi_1i_2 is never negative. The energy is therefore largest when $\sin^2(200\pi t) = 1$, the instant when $i_1 = 1$ mA and $i_2 = 2$ mA at the same time.
5. Put in the peak values:

$$w_{\max} = \tfrac{1}{2}(4)(10^{-3})^2 + \tfrac{1}{2}(5)(2\times10^{-3})^2 + (2.683)(10^{-3})(2\times10^{-3})$$
$$= 2\ \mu\text{J} + 10\ \mu\text{J} + 5.367\ \mu\text{J}$$

6. Add the three terms: $w_{\max} = 17.366\ \mu\text{J} \approx 17.37\ \mu\text{J}$.

KEY IDEA

The energy stored in coupled inductors is $w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2$, with $M = k\sqrt{L_1 L_2}$. Use the plus sign when both currents enter (or both leave) their dotted ends. Here the two currents are in phase, so all three terms peak together, when both currents are at their peak values.

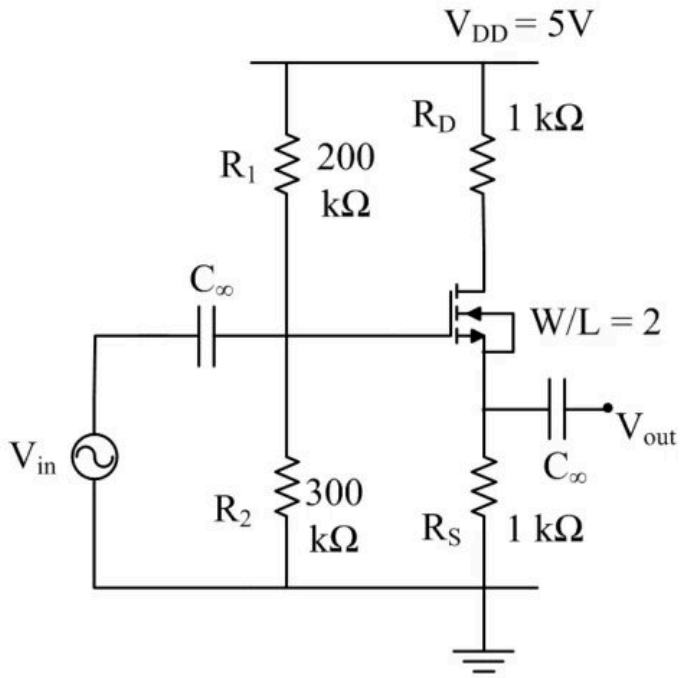
WATCH OUT FOR

Using the minus sign for the mutual term gives $12 - 5.37 = 6.63\ \mu\text{J}$. The right sign comes from reading the dots correctly: both currents enter dotted ends here. Other slips: leaving out the mutual term entirely (12 μJ), or taking $M = k\,L_1 L_2$ or $k(L_1 + L_2)/2$ instead of $k\sqrt{L_1 L_2}$. Some also use rms currents, but the question asks for the peak energy, which needs the peak currents.

Q.56 NAT · 2 MARKS

Analog Electronics › Amplifiers

Q.56 The figure below shows a feedback amplifier constructed using an nMOS transistor. Assume that $\mu_n C_{ox}$ = 1 mA/V^2, threshold voltage V_T = 1V and W/L = 2. The bias voltage at the drain terminal is 4 V. The capacitors C_∞ offer zero impedance at the signal frequency. The ratio V_{out}/V_{in} is _____ **(rounded off to two decimal places)**.



Official answer: **0.64 to 0.70**

SOLUTION

1. **Gate bias (voltage divider, no gate current):** $V_G = V_{DD}\dfrac{R_2}{R_1 + R_2} = 5 \times \dfrac{300}{200 + 300} = 3\ \text{V}$.

2. **Drain current from the stated drain bias (Ohm's law across R_D):** $I_D = \dfrac{V_{DD} - V_D}{R_D} = \dfrac{5-4}{1\ \text{k}\Omega} = 1\ \text{mA}$.
3. **Source voltage and V_{GS}:** $V_S = I_D R_S = 1\ \text{mA} \times 1\ \text{k}\Omega = 1\ \text{V}$, so $V_{GS} = 3 - 1 = 2\ \text{V}$ and the overdrive is $V_{GS} - V_T = 1\ \text{V}$.
4. **Consistency check (square law):** $I_D = \frac{1}{2}\mu_n C_{ox}\dfrac{W}{L}(V_{GS} - V_T)^2 = \frac{1}{2}(1\ \text{mA/V}^2)(2)(1\ \text{V})^2 = 1\ \text{mA}$, which matches the current found from R_D. Saturation holds because $V_{DS} = 4 - 1 = 3\ \text{V} > V_{GS} - V_T = 1\ \text{V}$.
5. **Transconductance:** $g_m = \mu_n C_{ox}\dfrac{W}{L}(V_{GS} - V_T) = 1 \times 2 \times 1 = 2\ \text{mA/V}$ (equivalently $g_m = \dfrac{2I_D}{V_{GS} - V_T} = \dfrac{2 \times 1}{1} = 2\ \text{mA/V}$).
6. **Small-signal model:** both C_∞ are short circuits and V_{DD} is AC ground. V_{in} drives the gate directly (the divider $R_1 \parallel R_2$ only loads the ideal source), and the output is taken at the source with no load, so this is a common-drain (source follower) stage with series-series feedback through R_S. Assumptions: $r_o \to \infty$ (no channel-length modulation is given), and no body effect because the body is tied to the source. R_D sits in the drain and does not affect the source-node gain.
7. **Source-follower gain:** with $i_d = g_m(v_g - v_s)$ and $v_s = i_d R_S$,

$$v_s = g_m R_S(v_{in} - v_s) \;\Rightarrow\; \frac{V_{out}}{V_{in}} = \frac{g_m R_S}{1 + g_m R_S} = \frac{2 \times 1}{1 + 2 \times 1} = \frac{2}{3} \approx 0.67$$

KEY IDEA

The drain bias of 4 V fixes $I_D = (5-4)/1\ \text{k}\Omega = 1\ \text{mA}$ straight away. That gives $V_S = 1\ \text{V}$, $V_{GS} = 2\ \text{V}$ and $g_m = 2\ \text{mA/V}$. Because the output is taken at the source, the stage is a source follower with gain $g_m R_S/(1 + g_m R_S)$. R_D only sets the bias and plays no part in the signal gain.

WATCH OUT FOR

Taking the output as if it came from the drain and using the degenerated common-source gain $-g_m R_D/(1 + g_m R_S) = -0.67$, which has the right magnitude but the wrong sign and the wrong physical meaning. Other slips: dropping the factor $\frac{1}{2}$ in the square law so that g_m comes out wrong, or adding a body-effect term g_{mb} even though the body is tied to the source.

Q.57 NAT · 2 MARKS

Engineering Mathematics › Calculus

Consider the real-valued function $g(x) = \max\{(x-2)^2, -2x + 7\}$, where $x \in (-\infty, \infty)$. The minimum value attained by $g(x)$ is _____ (rounded off to one decimal place).

Official answer: **0.9 to 1.1**

SOLUTION

1. **Where the two pieces cross.** Set them equal: $(x-2)^2 = -2x + 7 \Rightarrow x^2 - 4x + 4 = -2x + 7 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x-3)(x+1) = 0$. So they cross at $x = -1$ and $x = 3$.
2. **Which piece is larger where.** Take the difference $d(x) = (x-2)^2 - (-2x+7) = x^2 - 2x - 3 = (x-3)(x+1)$. It is ≤ 0 on $[-1, 3]$ and > 0 outside that interval. So the line $-2x + 7$ is the larger piece on $[-1, 3]$, and the parabola is larger everywhere else. (Check at $x = 2$: $(x-2)^2 = 0$ and $-2x + 7 = 3$.)
3. **Write g piecewise:**

$$g(x) = \begin{cases} (x-2)^2, & x \le -1 \\ -2x + 7, & -1 \le x \le 3 \\ (x-2)^2, & x \ge 3 \end{cases}$$

4. **Find the minimum of each piece on its own interval.** On $[-1, 3]$, $-2x + 7$ is strictly decreasing, so its smallest value is at $x = 3$: $-2(3) + 7 = 1$. For $x \ge 3$, $(x-2)^2$ is increasing because its vertex $x = 2$ is to the left, so $g(x) \ge (3-2)^2 = 1$. For $x \le -1$, $(x-2)^2$ is decreasing on that interval, so $g(x) \ge (-1-2)^2 = 9$.

5. **Conclusion.** The smallest of these values is 1. It occurs at the corner $x = 3$, where both pieces equal 1. So $g_{\min} = 1.0$.

KEY IDEA

The graph of $\max\{f, h\}$ is the upper boundary of the two curves. Its minimum is either at the lowest point of a piece inside that piece's own interval, or at a crossing point where the upper boundary switches from one curve to the other. Here the line is on top on $[-1, 3]$ and keeps falling up to $x = 3$. After $x = 3$ the parabola is on top and keeps rising. So the minimum is at the crossing $x = 3$.

WATCH OUT FOR

Taking the vertex of the parabola, $x = 2$, $g = 0$, as the answer. At $x = 2$ the parabola is not the larger piece: the line gives 3, so $g(2) = 3$. The opposite slip is to minimise $\min\{f, h\}$ instead of $\max\{f, h\}$. The vertex value 0 is only the minimum of the parabola alone, and g never gets that low.

Q.58 NAT · 2 MARKS

Electrical Machines › Transformers

A short-circuit test is conducted on a single-phase transformer by shorting its secondary. The frequency of input voltage is 1 kHz. The corresponding wattmeter reading, primary current and primary voltage are 8 W, 2 A and 6 V respectively. Assume that the no-load losses and the no-load currents are negligible, and the core has linear magnetic characteristics. Keeping the secondary shorted, the primary is connected to a 2 V (RMS), 1 kHz sinusoidal source in series with a $\dfrac{1}{2\pi\sqrt{5}}$ mF capacitor. The primary current (RMS) will be ____ A (rounded off to two decimal places).

Official answer: **0.95 to 1.05**

SOLUTION

1. Short-circuit test data (secondary shorted): $V_{sc} = 6\ \text{V}$, $I_{sc} = 2\ \text{A}$, $P_{sc} = 8\ \text{W}$.
2. $Z_{sc} = \dfrac{V_{sc}}{I_{sc}} = \dfrac{6}{2} = 3\ \Omega$.
3. $R_{sc} = \dfrac{P_{sc}}{I_{sc}^2} = \dfrac{8}{2^2} = 2\ \Omega$.
4. $X_{sc} = \sqrt{Z_{sc}^2 - R_{sc}^2} = \sqrt{9 - 4} = \sqrt{5}\ \Omega$. With the secondary shorted, the transformer's impedance as seen from the primary is $R_{sc} + jX_{sc} = 2 + j\sqrt{5}\ \Omega$.
5. New circuit: a $2\ \text{V}$ (RMS), $1\ \text{kHz}$ source in series with a capacitor $C = \dfrac{1}{2\pi\sqrt{5}}\ \text{mF}$, feeding the primary while the secondary stays shorted.
6. $\omega = 2\pi \times 1000 = 2000\pi\ \text{rad/s}$.
7. $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2000\pi \times \dfrac{1}{2\pi\sqrt{5}} \times 10^{-3}} = \sqrt{5}\ \Omega$.
8. Total series impedance = capacitor + shorted transformer impedance: $Z = -j\sqrt{5} + (2 + j\sqrt{5}) = 2\ \Omega$: the capacitive and leakage reactances cancel exactly (series resonance).
9. Primary current: $I = \dfrac{V}{Z} = \dfrac{2}{2} = 1.00\ \text{A}$.

KEY IDEA

A short-circuit test gives the transformer's series (leakage) impedance as seen from the primary; when a capacitor is placed in series with that impedance and its reactance is chosen to exactly cancel the leakage reactance, the circuit becomes purely resistive (series resonance), so the current is simply the source voltage divided by the short-circuit resistance.

WATCH OUT FOR

Forgetting that with the secondary shorted, the transformer itself still contributes its short-circuit impedance $R_{sc} + jX_{sc}$ to the new circuit (treating the capacitor as connected to an ideal short instead of to the transformer's leakage impedance), or making a sign/algebra slip when combining $-jX_C$ with $+jX_{sc}$ and not noticing they are equal in magnitude (resonance), leading to a wrong non-real or mismatched impedance.

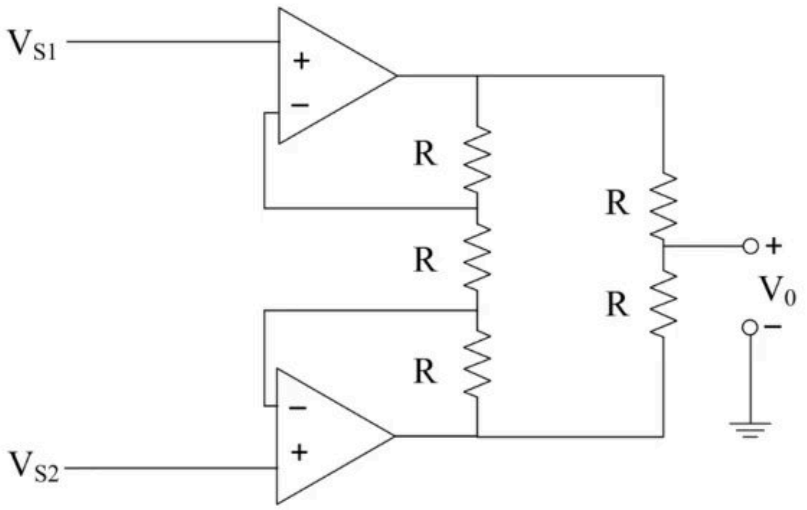
Q.59 NAT · 2 MARKS

Analog Electronics › Op-amps

Q.59 The opamps in the circuit are ideal. The input signals are

$V_{S1} = 3 + 0.10\sin(300t)$ V and $V_{S2} = -2 + 0.11\sin(300t)$ V.

The average value of the voltage V_0 is _____V (**rounded off to two decimal places**).



Official answer: **0.49 to 0.51**

SOLUTION

1. Assumptions: the op-amps are ideal, so no current flows into either input terminal, each op-amp's two inputs sit at the same voltage (virtual short), and each output behaves as an ideal voltage source (zero output resistance). Each op-amp's inverting input connects back to its own output through a resistor of the left chain, so both have negative feedback and the virtual short holds. The V_O terminal is open-circuited (no load).
2. Virtual short at each op-amp: node A (the top op-amp's inverting input, the junction of the upper and middle resistors of the left chain) is at $V_A = V_{S1}$. Node B (the bottom op-amp's inverting input, the junction of the middle and lower resistors) is at $V_B = V_{S2}$.

3. Ohm's law on the middle resistor: a current $I = \dfrac{V_{S1} - V_{S2}}{R}$ flows from A to B. No current enters the op-amp inputs, so the same I also flows down through the upper and lower resistors of the left chain.
4. Outputs of the two op-amps:
$$V_{o1} = V_A + IR = V_{S1} + (V_{S1} - V_{S2}) = 2V_{S1} - V_{S2}$$
$$V_{o2} = V_B - IR = V_{S2} - (V_{S1} - V_{S2}) = 2V_{S2} - V_{S1}$$
5. Right chain: two equal resistors R in series between V_{o1} and V_{o2}. The op-amp outputs are ideal voltage sources, so this divider does not change V_{o1} or V_{o2}, and with no load on the output V_O sits at their midpoint:
$$V_O = \frac{V_{o1} + V_{o2}}{2} = \frac{(2V_{S1} - V_{S2}) + (2V_{S2} - V_{S1})}{2} = \frac{V_{S1} + V_{S2}}{2}$$
6. Substitute the inputs:
$$V_O = \frac{(3 - 2) + (0.10 + 0.11)\sin(300t)}{2} = 0.5 + 0.105\sin(300t)\ \text{V}$$
7. Average value: over one period $T = \dfrac{2\pi}{300}$ s the sinusoid averages to zero, so the average of V_O is 0.50 V. Check with the DC levels alone: $V_{o1} = 2(3) - (-2) = 8$ V and $V_{o2} = 2(-2) - 3 = -7$ V, and their midpoint is $\dfrac{8 + (-7)}{2} = 0.5$ V.

KEY IDEA

This is the input stage of a three-op-amp instrumentation amplifier. The virtual shorts put V_{S1} and V_{S2} across the middle resistor, and the same current flows through all three resistors of the left chain, so $V_{o1} = 2V_{S1} - V_{S2}$ and $V_{o2} = 2V_{S2} - V_{S1}$. The difference between the outputs is amplified, but their average stays equal to the input common-mode voltage. The equal-resistor divider on the right picks out that average: $V_O = (V_{S1} + V_{S2})/2$. Only the DC parts of the inputs survive averaging, so the answer is $(3 - 2)/2 = 0.5$ V.

WATCH OUT FOR

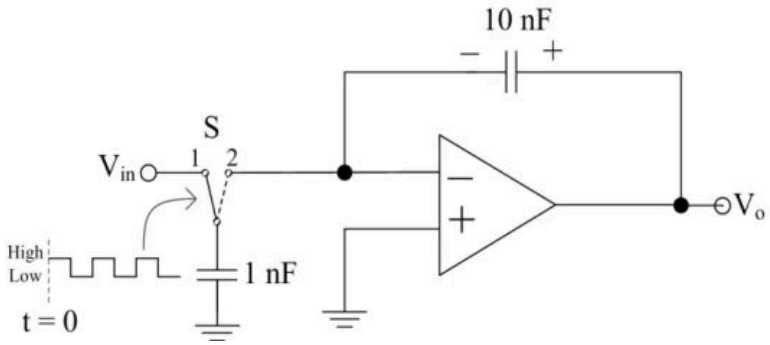
A common error is to treat the circuit as a differential amplifier and compute $V_{o1} - V_{o2} = 3(V_{S1} - V_{S2})$, which averages to 15 V. But V_O is measured from the midpoint of the right divider to ground, not across the two outputs, so it is the common-mode average $(V_{o1} + V_{o2})/2$. Other slips: getting the sign of IR wrong for the bottom op-amp ($V_{o2} = V_{S2} - IR$, because the current flows from B towards the bottom output), and averaging the sine amplitudes into the answer when a sinusoid averages to zero.

Q.60 NAT · 2 MARKS

Analog Electronics › Op-amps

Q.60 In the circuit shown, the input voltage $V_{in} = 100$ mV. The switch and the opamp are ideal. At time t = 0, the initial charge stored in the 10 nF capacitor is 1 nC, with the polarity as indicated in the figure. The switch S is controlled using a 1 kHz square-wave voltage signal V_s as shown. Whenever V_s is 'High', S is in position '1' and when V_s is 'Low', S is in position '2'.

At t = 20 ms, the magnitude of the voltage V_o will be _____ mV (**rounded off to the nearest integer**).



Official answer: **99 to 101**

SOLUTION

1. **Assumptions.** Ideal op-amp: infinite gain, no input current, and the output stays in its linear range (it only swings about ± 100 mV here). Ideal switch and ideal source: the 1 nF capacitor charges or discharges instantly when S changes position. Write $C_1 = 1$ nF and $C_f = 10$ nF.
2. **Initial output (virtual ground).** The feedback capacitor holds $Q_0 = 1$ nC, so $V_{C_f} = \dfrac{1\text{ nC}}{10\text{ nF}} = 100$ mV. Its '+' plate is on the output side and its '−' plate on the inverting-input side. The non-inverting input is grounded, so the inverting input is a virtual ground at 0 V, which gives $V_o(0) = +100$ mV.
3. **Timing.** $f = 1$ kHz $\Rightarrow T = 1$ ms. The waveform starts High at $t = 0$, so in every period S is at '1' for the first 0.5 ms and at '2' for the next 0.5 ms.
4. **Phase 1 (S at '1').** C_1 is connected straight across V_{in} and charges to $q = C_1 V_{in} = 1\text{ nF} \times 100\text{ mV} = 0.1$ nC, with the top plate positive. Any charge C_1 held before $t = 0$ does not matter, because the first thing that happens is this connection to V_{in}. During this phase C_1 is not connected to the op-amp, so V_o holds its value.
5. **Phase 2 (S at '2').** The top plate of C_1 is now tied to the virtual ground at 0 V, so C_1 discharges completely. Its 0.1 nC flows into the inverting node. The op-amp draws no input current, so all of this charge lands on the inverting-input ('−') plate of C_f.
6. **Change per period (charge conservation at the inverting node).** Adding $+0.1$ nC to the '−' plate, which carried -1 nC, leaves -0.9 nC on it. So every transfer lowers the output by
$$\Delta V_o = -\frac{C_1 V_{in}}{C_f} = -\frac{0.1\text{ nC}}{10\text{ nF}} = -10\text{ mV per clock period}$$
7. **Count the transfers.** With an ideal switch, each transfer happens the moment S moves to '2', at $t = 0.5, 1.5, 2.5, \ldots, 19.5$ ms. That is $N = 20$ transfers by $t = 20$ ms. At $t = 20$ ms S moves back to '1', which disconnects C_1 from the op-amp and leaves V_o unchanged.
8. **Result.**
$$V_o(20\text{ ms}) = 100\text{ mV} - 20 \times 10\text{ mV} = -100\text{ mV}$$

The magnitude is $|V_o| = 100\ \text{mV}$.

9. **Check with the equivalent resistor.** A switched capacitor behaves like $R_{eq} = \dfrac{1}{fC_1} = \dfrac{1}{(10^3)(10^{-9})} = 1\ \text{M}\Omega$. The average input current is $\dfrac{100\ \text{mV}}{1\ \text{M}\Omega} = 0.1\ \mu\text{A}$, which delivers $2\ \text{nC}$ in $20\ \text{ms}$ (exactly 20 whole periods). So $\Delta V_o = -\dfrac{2\ \text{nC}}{10\ \text{nF}} = -200\ \text{mV}$ and $V_o = 100 - 200 = -100\ \text{mV}$, which agrees.

KEY IDEA

This circuit is a switched-capacitor inverting integrator. In every clock period the 1 nF capacitor picks up $C_1 V_{in}$ from the input and dumps it at the virtual ground, onto the feedback capacitor, so the output steps by $-\dfrac{C_1}{C_f} V_{in} = -10\ \text{mV}$ each period. The 1 nC starting charge fixes the starting point: because the inverting input is a virtual ground, $V_o(0) = +Q_0/C_f = +100\ \text{mV}$. After 20 periods, $V_o = 100 - 200 = -100\ \text{mV}$.

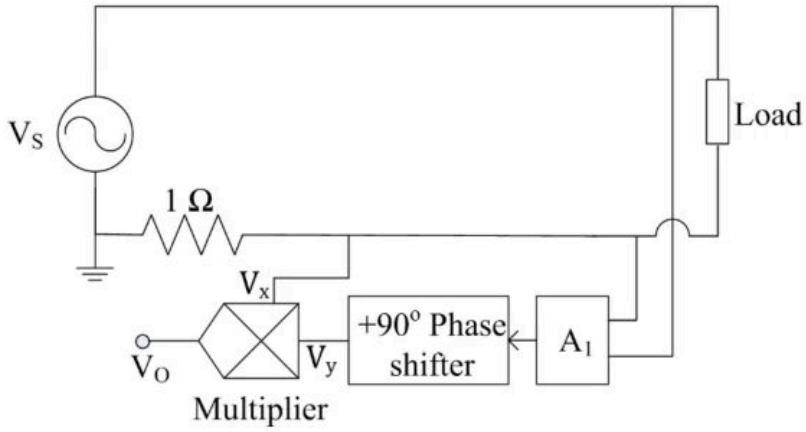
WATCH OUT FOR

1. Ignoring the 1 nC initial charge. That gives $|V_o| = 200\ \text{mV}$.
2. Reading the polarity backwards, so $V_o(0) = -100\ \text{mV}$. That gives $-300\ \text{mV}$, a magnitude of $300\ \text{mV}$.
3. Treating the stage as non-inverting and adding the steps: $100 + 200 = 300\ \text{mV}$.
4. Counting 19 or 40 transfers instead of 20. There is exactly one charge transfer per 1 ms period, at each High-to-Low edge.

Q.61 NAT · 2 MARKS

Electrical and Electronic Measurements › V, I, power & energy

Q.61 In the diagram shown, the frequency of the sinusoidal source voltage V_S is 50 Hz. The load voltage is 230 V (RMS), and the load impedance is $\frac{230}{\sqrt{2}} + j\frac{230}{\sqrt{2}}\ \Omega$. The value of attenuator $A_1 = \frac{1}{50\sqrt{2}}$. The multiplier output voltage $V_o = \frac{V_x V_y}{1\ V}$, where V_x and V_y are the inputs. The magnitude of the average value of the multiplier output V_o is _____ V (**rounded off to one decimal place**).



Official answer: **2.1 to 2.5**

SOLUTION

1. Load current, from Ohm's law with the given load voltage: $|Z| = \sqrt{\left(\frac{230}{\sqrt{2}}\right)^2 + \left(\frac{230}{\sqrt{2}}\right)^2} = 230\ \Omega$ and $\angle Z = \tan^{-1}(1) = 45^\circ$. Taking the load voltage as reference, $I = \frac{230\angle 0^\circ}{230\angle 45^\circ} = 1\angle -45^\circ\ \text{A (rms)}$, so the current lags the load voltage by 45°. The 50 Hz figure is not needed, because the impedance is given directly.
2. Current-sensing input: the load current returns to the grounded source terminal through the series $1\ \Omega$ resistor, and V_x is taken from the resistor's ungrounded end. So $V_x = I \times 1\ \Omega = 1\angle -45^\circ\ \text{V (rms)}$, in phase with the current.
3. Voltage-sensing input: A_1 scales the load voltage and the phase shifter adds $+90^\circ$, so $V_y = \frac{230}{50\sqrt{2}}\angle 90^\circ = 3.2527\angle 90^\circ\ \text{V (rms)}$. The polarity of A_1's input leads is not marked, and reversing them would only flip the sign of V_y.
4. The average of the product of two sinusoids at the same frequency is $\overline{v_x v_y} = V_{x,\text{rms}}\, V_{y,\text{rms}} \cos\theta$, where θ is the phase angle between them. Here $\theta = 90^\circ - (-45^\circ) = 135^\circ$.
5. $$\overline{V_o} = \frac{1 \times 3.2527 \times \cos 135^\circ}{1\ \text{V}} = \frac{4.6}{\sqrt{2}} \times \left(-\frac{1}{\sqrt{2}}\right)\ \text{V} = -2.30\ \text{V}$$
6. Check through reactive power: the 90° shift turns the multiplier into a varmeter, so $\overline{V_o} = \frac{Q \times 1\ \Omega \times A_1}{1\ \text{V}}$. With $Q = VI\sin\phi = 230 \times 1 \times \sin 45^\circ = 162.63\ \text{VAR}$, this gives $\frac{162.63}{50\sqrt{2}} = 2.30\ \text{V}$ in magnitude.

7. The sign depends only on the unmarked polarity of A_1's input and on the phase-shift convention. The question asks for the magnitude, so $|\overline{V_o}| = 2.3\ \text{V}$.

KEY IDEA

The multiplier's V_x input is proportional to the load current and its V_y input is proportional to the load voltage shifted by 90°. That makes the circuit a varmeter: the average output is $V_{x,\text{rms}} V_{y,\text{rms}} \cos(90^\circ + \phi)$, whose magnitude is $VI \sin\phi \times R_{\text{shunt}} \times A_1$ for a load with power-factor angle ϕ. Here that is $230 \times 1 \times \sin 45^\circ \times 1 \times \dfrac{1}{50\sqrt{2}} = 2.3\ \text{V}$.

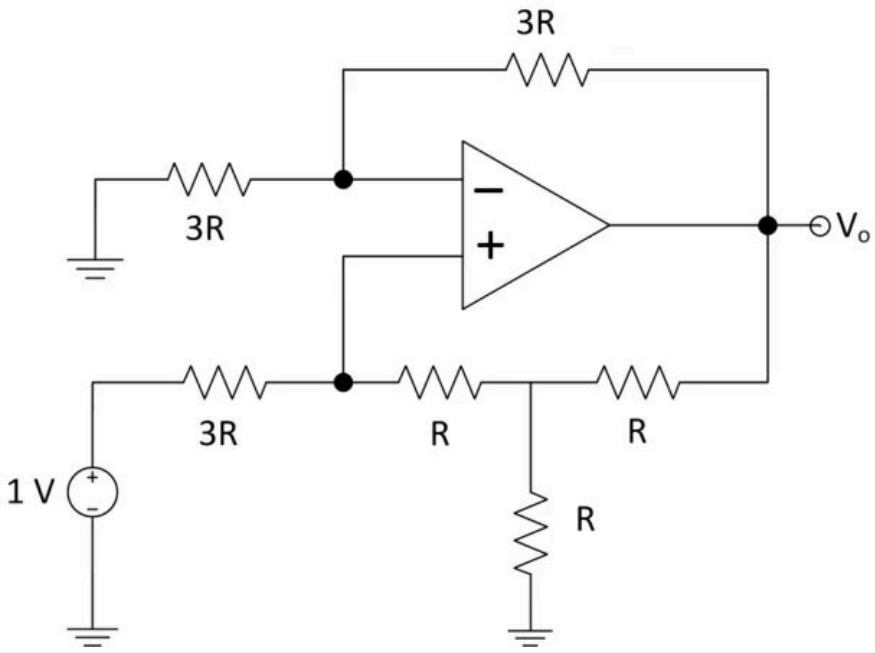
WATCH OUT FOR

Ignoring the $+90^\circ$ phase shifter and computing active power, $P \times R \times A_1 = 162.63/70.71 = 2.3\ \text{V}$, reaches the right number only by accident, because $\cos 45^\circ = \sin 45^\circ$. The method would fail for any other power factor. Two other slips: using peak values instead of rms (which gives $4.6\ \text{V}$), and taking $|I| = 230/(230/\sqrt{2}) = \sqrt{2}\ \text{A}$ by dividing by one component of Z instead of $|Z| = 230\ \Omega$.

Q.62 NAT · 2 MARKS

Analog Electronics › Op-amps

Q.62 In the circuit shown, assuming an ideal opamp, the value of the output voltage V_o = _____ V (**rounded off to one decimal place**).



Official answer: **1.9 to 2.1**

SOLUTION

1. Assume an ideal op-amp: infinite open-loop gain and zero input currents. Step 7 shows that the negative feedback outweighs the positive feedback, so the two inputs sit at the same voltage (virtual short): $V_- = V_+$.
2. Inverting side. No current enters the input, so the two $3R$ resistors form a voltage divider from V_o to ground: $V_- = V_o \dfrac{3R}{3R + 3R} = \dfrac{V_o}{2}$. The virtual short then gives $V_P = V_+ = \dfrac{V_o}{2}$, where P is the non-inverting-input node.
3. KCL at node P (no current into the input). The current in from the 1 V source through $3R$ equals the current out through R to node X:
$$\frac{1 - V_P}{3R} = \frac{V_P - V_X}{R} \Rightarrow 1 = 4V_P - 3V_X$$
4. KCL at node X. The currents in from P and from the output equal the current out to ground:

$$\frac{V_P - V_X}{R} + \frac{V_o - V_X}{R} = \frac{V_X}{R} \;\Rightarrow\; 3V_X = V_P + V_o$$

5. Substitute $V_P = V_o/2$ into the node-X equation: $3V_X = \frac{V_o}{2} + V_o = \frac{3V_o}{2} \Rightarrow V_X = \frac{V_o}{2}$. So $V_X = V_P$, and no current flows in the R between P and X.
6. Substitute into the node-P equation:
$$1 = 4 \cdot \frac{V_o}{2} - 3 \cdot \frac{V_o}{2} = \frac{V_o}{2} \;\Rightarrow\; V_o = 2\ \text{V}$$
7. Check. $V_P = V_X = 1$ V, so no current flows in the $3R$ from the 1 V source or in the R between P and X. At X, $\frac{2-1}{R} = \frac{1}{R}$ flows in from the output and $\frac{1}{R}$ flows out to ground, so KCL balances. Feedback sign: with the source set to zero, node X sees R to ground in parallel with $R + 3R = 4R$ to ground, which is $0.8R$. So $V_X = V_o \dfrac{0.8R}{R + 0.8R} = \dfrac{4}{9}V_o$ and $V_+ = \dfrac{3R}{4R} V_X = \dfrac{1}{3}V_o$. The positive-feedback fraction $\frac{1}{3}$ is smaller than the negative-feedback fraction $\frac{1}{2}$. Net feedback is negative, so the virtual short is valid.
8. Answer: $V_o = 2.0$ V.

KEY IDEA

The output feeds back to both inputs: negative feedback through the $3R$–$3R$ divider (fraction $\frac{1}{2}$) and positive feedback through the R-network (fraction $\frac{1}{3}$). Net feedback is negative, so the virtual short holds. It gives $V_+ = V_o/2$, and nodal analysis of the non-inverting network then fixes V_o.

WATCH OUT FOR

One mistake is to ignore the R from node X to the output and treat the non-inverting side as a plain divider of the 1 V source. That gives $V_+ = 1 \cdot \frac{2R}{5R} = 0.4$ V and $V_o = 0.8$ V. Another is to see positive feedback and assume the op-amp saturates, without checking that the negative feedback ($\frac{1}{2}$) is stronger than the positive feedback ($\frac{1}{3}$).

Q.63 NAT · 2 MARKS

Engineering Mathematics › Linear algebra

The rank of the matrix A given below is one. The ratio $\dfrac{\alpha}{\beta}$ is _____ (rounded off to the nearest integer).

$$A = \begin{bmatrix} 1 & 4 \\ -3 & \alpha \\ \beta & 6 \end{bmatrix}$$

Official answer: **-8**

SOLUTION

1. Rank one means the row space is one-dimensional, so every row is a scalar multiple of any one nonzero row. (Equivalently, every 2×2 minor is zero.) Row 1, $\begin{bmatrix} 1 & 4 \end{bmatrix}$, is nonzero, so rows 2 and 3 must both be multiples of it.
2. Row 2: its first entry is $-3 = -3 \times 1$, so row 2 $= -3\times$ row 1. The second entries must agree too: $\alpha = -3 \times 4 = -12$. (The minor check says the same: $\begin{vmatrix} 1 & 4 \\ -3 & \alpha \end{vmatrix} = \alpha + 12 = 0$.)
3. Row 3: its second entry is $6 = \frac{3}{2} \times 4$, so row 3 $= \frac{3}{2}\times$ row 1. The first entries must agree too: $\beta = \frac{3}{2} \times 1 = \frac{3}{2}$. (Minor check: $\begin{vmatrix} 1 & 4 \\ \beta & 6 \end{vmatrix} = 6 - 4\beta = 0$.)
4. Check the remaining minor with these values: $\begin{vmatrix} -3 & -12 \\ \frac{3}{2} & 6 \end{vmatrix} = (-3)(6) - (-12)\left(\frac{3}{2}\right) = -18 + 18 = 0$. So the rank really is 1 (it cannot be 0, since A has nonzero entries).
5. $$\frac{\alpha}{\beta} = \frac{-12}{3/2} = -8$$
The value is already an integer, so rounding changes nothing: the answer is -8.

KEY IDEA

A rank-one matrix has every row proportional to one nonzero row. That is the same as saying every 2×2 minor vanishes. Each unknown sits in a row with one known entry, and that known entry fixes the scale factor for its row.

WATCH OUT FOR

Using the wrong entry to fix a row's multiplier, for example taking row 3 as $6\times$ row 1 instead of $\frac{6}{4}\times$ row 1, which gives $\beta = 6$ and a ratio of -2. Another slip is computing $\beta/\alpha = -0.125$ and rounding it to 0. Also, setting only one minor to zero is not enough: you have to use both the row-2 and row-3 conditions.

Q.64 NAT · 2 MARKS

Electrical and Electronic Measurements › Digital instruments

A 1.999 V True RMS 3-1/2 digit multimeter has an accuracy of

$\pm 0.1\ \%$ of reading ± 2 digits.

It is used to measure 100 A (RMS) current flowing through a line using a 100:5 ratio, Class-1 current transformer with a burden of $0.1\ \Omega \pm 0.5\ \%$.

The worst-case absolute error in the multimeter output is _____ V (**rounded off to three decimal places**).

Official answer: **0.009 to 0.011**

SOLUTION

1. CT secondary current (ratio 100:5): $I_s = 100 \times \dfrac{5}{100} = 5\ \text{A}$. The line current of $100\ \text{A}$ is the CT's rated primary current, so the Class-1 limit applies in full.
2. The multimeter reads the voltage across the burden: $V = I_s R = 5 \times 0.1 = 0.5\ \text{V}$ (nominal reading). This lies on the $1.999\ \text{V}$ range.
3. A Class-1 CT has a ratio (current) error of up to $\pm 1\ \%$ at rated current, and the burden resistance is $\pm 0.5\ \%$. Since $V = I_s R$ is a product, the limiting relative errors add: $\pm(1 + 0.5) = \pm 1.5\ \%$.
4. Error in the voltage presented to the meter: $1.5\ \% \times 0.5\ \text{V} = \pm 0.0075\ \text{V}$.
5. Meter reading error: $0.1\ \%$ of reading $= 0.001 \times 0.5\ \text{V} = \pm 0.0005\ \text{V}$.
6. Digit error: on the $1.999\ \text{V}$ range of a 3½-digit meter one count (the least significant digit) is $0.001\ \text{V}$, so ± 2 digits $= \pm 0.002\ \text{V}$.
7. Worst case, all errors add in the same direction:

$$\Delta V = 0.0075 + 0.0005 + 0.002 = 0.0100\ \text{V}$$

   If you take $0.1\ \%$ of the worst-case reading $0.5075\ \text{V}$ instead, you get $0.01001\ \text{V}$, which still rounds to $0.010\ \text{V}$.

KEY IDEA

The meter measures $V = I_s R$ across the CT burden. For the worst case, add the limiting relative errors of the CT ratio (Class 1, so $\pm 1\ \%$) and the burden ($\pm 0.5\ \%$), turn that into volts at the $0.5\ \text{V}$ reading, then add the meter's own error: $0.1\ \%$ of reading plus 2 counts of $0.001\ \text{V}$ each on the $1.999\ \text{V}$ range.

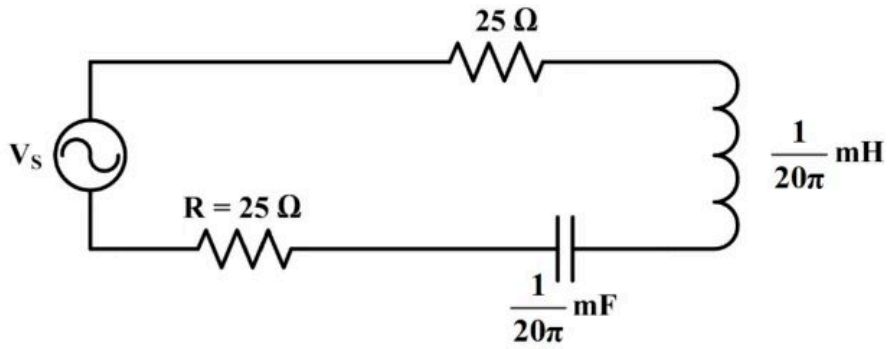
WATCH OUT FOR

The usual mistakes are leaving out the digit error or giving it the wrong size. On a 3½-digit meter, ± 2 digits on the $1.999\ \text{V}$ range is $\pm 0.002\ \text{V}$, and that alone is 20 % of the answer. Other slips: forgetting the burden tolerance, or combining the errors as a root-sum-square when the question asks for the worst case, which is a straight sum.

Q.65 NAT · 2 MARKS

Networks (Electric Circuits) › AC, power & resonance

Q.65 The voltage source Vs = $10\sqrt{2}\ sin(20000\pi t)$ V has an internal resistance of 50 Ω. The RMS value of the current through R is_____mA (**rounded off to one decimal place**).



*This option did not appear in GATE 2023 Examination. It appeared as "Turbine flow meter", that is, "Turbine flow meter" option was repeated.

Official answer: **99.0 to 101.0**

SOLUTION

1. Read the source. $V_s = 10\sqrt{2}\sin(20000\pi t)$ V has peak value $10\sqrt{2}$ V, so $V_{rms} = \dfrac{10\sqrt{2}}{\sqrt{2}} = 10$ V and $\omega = 20000\pi$ rad/s. Taking the source as the reference, the rms phasor is $V_s = 10\angle 0^\circ$ V.
2. Inductive reactance, with $L = \dfrac{1}{20\pi}$ mH $= \dfrac{1}{20\pi} \times 10^{-3}$ H:
$$X_L = \omega L = 20000\pi \times \frac{1}{20\pi} \times 10^{-3} = 1\ \Omega$$
3. Capacitive reactance, with $C = \dfrac{1}{20\pi}$ mF $= \dfrac{1}{20\pi} \times 10^{-3}$ F: $\omega C = 20000\pi \times \dfrac{1}{20\pi} \times 10^{-3} = 1$ S, so
$$X_C = \frac{1}{\omega C} = 1\ \Omega$$
4. Because $X_L = X_C$, the series loop is at resonance. The $+j1\ \Omega$ of the inductor and the $-j1\ \Omega$ of the capacitor cancel, so the loop impedance is purely resistive.
5. The circuit is a single series loop, so the source's 50 Ω internal resistance is in series with the drawn 25 Ω resistor and $R = 25\ \Omega$:
$$Z = 50 + 25 + 25 + j(1 - 1) = 100\ \Omega$$
6. The same current flows through every element of the loop, including R:
$$I = \frac{V_s}{Z} = \frac{10\angle 0^\circ}{100} = 0.1\angle 0^\circ \text{ A (rms)}$$
So the rms current through R is 0.1 A $= 100.0$ mA.

KEY IDEA

The element values are chosen so that $\omega = \dfrac{1}{\sqrt{LC}}$: here $\sqrt{LC} = \dfrac{1}{20\pi} \times 10^{-3}\ \text{s}$, so $\dfrac{1}{\sqrt{LC}} = 20000\pi\ \text{rad/s}$, which is exactly the source frequency. At series resonance the reactances cancel and the current is set only by the total loop resistance, and that total includes the source's internal resistance.

WATCH OUT FOR

Leaving out the 50 Ω internal resistance because it is not drawn, which gives $10/50 = 200\ \text{mA}$. Using the peak voltage $10\sqrt{2}\ \text{V}$ in place of the rms value, which gives $141.4\ \text{mA}$. Reading mF as μF, which makes $X_C = 1000\ \Omega$ and takes the circuit far from resonance.

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