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GATE IN · 2020

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GATE IN 2020 Solved Paper

INSIDE

- All 65 questions, in the order of the paper
- The official answer to every question
- Step-by-step worked solutions
- The answer key on one page

Question paper and official answer key: IIT Delhi

Online, with the questions linked to their topic notes: **iaeacademy.in/study/papers/in-2020**

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About this paper

65 questions, 100 marks. Question types: 30 MCQ · 35 NAT.

Official answer key: IIT Delhi

This paper numbers General Aptitude GA Q.1 to GA Q.10 and the subject questions Q.1 to Q.55: 65 questions in all.

We placed each question under a subject and topic ourselves; the exam paper does not label them.

This paper online, with the questions linked to their topic notes:
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Answer key

Q	TYPE	MARKS	ANSWER
GA Q.1	MCQ	1	**C**
GA Q.2	MCQ	1	**A**
GA Q.3	MCQ	1	**C**
GA Q.4	MCQ	1	**B**
GA Q.5	MCQ	1	**D**
GA Q.6	MCQ	2	**D**
GA Q.7	MCQ	2	**D**
GA Q.8	MCQ	2	**D**
GA Q.9	MCQ	2	**A**
GA Q.10	MCQ	2	**C**
Q.1	MCQ	1	**A**
Q.2	MCQ	1	**B**
Q.3	MCQ	1	**B**
Q.4	MCQ	1	**C**
Q.5	MCQ	1	**C**
Q.6	MCQ	1	**A**
Q.7	MCQ	1	**A**
Q.8	MCQ	1	**B**
Q.9	MCQ	1	**B**
Q.10	MCQ	1	**B**
Q.11	MCQ	1	**A**
Q.12	MCQ	1	**A**
Q.13	MCQ	1	**D**
Q.14	MCQ	1	**D**
Q.15	MCQ	1	**B**
Q.16	NAT	1	**0.09**
Q.17	NAT	1	**7**
Q.18	NAT	1	**1.7 to 1.8**
Q.19	NAT	1	**2**
Q.20	NAT	1	**0.78 to 0.79**
Q.21	NAT	1	**-4**
Q.22	NAT	1	**25**
Q.23	NAT	1	**223.3 to 223.5**
Q.24	NAT	1	**5**
Q.25	NAT	1	**2.5, or 3.5**
Q.26	MCQ	2	**D**
Q.27	MCQ	2	**A**
Q.28	MCQ	2	**D**
Q.29	MCQ	2	**A**
Q.30	MCQ	2	**A**
Q.31	NAT	2	**0.5**
Q.32	NAT	2	**0.7**
Q.33	NAT	2	**1**
Q.34	NAT	2	**115 to 116**
Q.35	NAT	2	**0**
Q.36	NAT	2	**5.0 to 5.1**
Q.37	NAT	2	**4**
Q.38	NAT	2	**1**
Q.39	NAT	2	**0.45 to 0.55**
Q.40	NAT	2	**7**
Q.41	NAT	2	**3.5 to 4.5**
Q.42	NAT	2	**199 to 201**
Q.43	NAT	2	**999 to 1001**
Q.44	NAT	2	**3.4 to 3.6**
Q.45	NAT	2	**0.95 to 1.05**
Q.46	NAT	2	**27.5 to 28.5**
Q.47	NAT	2	**200**
Q.48	NAT	2	**0**
Q.49	NAT	2	**14.5 to 15.0**
Q.50	NAT	2	**-0.13 to -0.11**
Q.51	NAT	2	**1.8**
Q.52	NAT	2	**0.95 to 1.05**
Q.53	NAT	2	**4.3 to 4.6**
Q.54	NAT	2	**10**
Q.55	NAT	2	**65.4 to 65.6**

General Aptitude GA Q.1 to Q.10

GA Q.1 MCQ · 1 MARK

General Aptitude › Grammar & vocabulary

He is known for his unscrupulous ways. He always sheds_____tears to deceive people.

(A) fox's

(B) crocodile's

(C) crocodile

(D) fox

Official answer: **C**

SOLUTION

1. The first sentence sets the context. *Unscrupulous* means having no moral principles, so the blank needs a phrase for a false show of emotion that is used to deceive people.
2. Idiom: **crocodile tears** means a false or insincere show of sorrow. It comes from the old belief that a crocodile weeps while it eats its prey. To *shed crocodile tears* is to pretend to be sad, which fits "to deceive people" exactly.
3. English has no idiom that pairs *fox* with *tears*. The fox stands for cunning, but it never appears with *tears*, so (A) and (D) are ruled out.
4. The idiom has a fixed form. *Crocodile* is a noun used like an adjective in front of *tears*, just as in *crocodile skin*. The possessive *crocodile's* is not the idiom, so (B) is ruled out.
5. Completed sentence: "He always sheds crocodile tears to deceive people." Answer: (C).

KEY IDEA

"Crocodile tears" is a fixed idiom for fake sorrow. The noun *crocodile* describes *tears* the way an adjective would, and the idiom never uses the possessive form.

WATCH OUT FOR

Picking (B) *crocodile's* because the tears seem to "belong" to the crocodile. The idiom is always written *crocodile tears*, with no possessive. The other trap is picking a *fox* option because "unscrupulous" suggests cunning, but no idiom pairs *fox* with *tears*.

GA Q.2 MCQ · 1 MARK

General Aptitude › Grammar & vocabulary

Jofra Archer, the England fast bowler, is_____than accurate.

(A) more fast

(B) faster

(C) less fast

(D) more faster

Official answer: **A**

SOLUTION

1. The sentence compares two qualities of the same person: how fast Archer is and how accurate he is. It does not compare Archer's speed with anyone else's.
2. Rule: when two qualities of one person or thing are compared, English uses "more + positive form of the adjective" ... than ..., however short the adjective is. For example, "He is more brave than wise." The "-er" comparative is not used in this construction.
3. (B) "faster than accurate" breaks this rule. The "-er" form compares one quality across two people or things ("Archer is faster than Broad"), not two qualities of one person.
4. (D) "more faster" is a double comparative, "more" together with "-er", and is always wrong.

5. (C) "less fast than accurate" has the correct form, but it says pace is Archer's weaker quality. That goes against the appositive "the England fast bowler", which introduces him by his pace. The sentence is built to set his defining trait, pace, against his accuracy.
6. So the blank is "more fast": "Jofra Archer, the England fast bowler, is more fast than accurate." That is, his pace is greater than his accuracy.

KEY IDEA
When you compare two qualities of the same subject, use "more + positive adjective ... than ...", even for a short adjective like "fast". The "-er" form (faster) is for comparing one quality across two different subjects.

WATCH OUT FOR
Choosing (B) "faster", because "fast" normally takes "-er". That form is right for "Archer is faster than Broad" but wrong when one person's speed is compared with his own accuracy. (D) "more faster" is a double comparative and is never correct.

GA Q.3 MCQ · 1 MARK

General Aptitude › Reasoning

Select the word that fits the analogy:

Build : Building :: Grow : _____

(A) Grown
(B) Grew
(C) Growth
(D) Growed

Official answer: **C**

SOLUTION

1. Find the relation in the first pair. "Build" is a verb, and "building" is the **noun** formed from it (the thing that results from building). So the relation is *verb : its noun form*.
2. Apply the same relation to "grow". We need the noun formed from the verb "grow".
3. Check (A) Grown. This is the past participle of "grow" (as in "has grown"). It can also work as an adjective ("a grown man"), but it is never a noun. Reject.
4. Check (B) Grew. This is the simple past tense of "grow", so it is a verb form. Reject.
5. Check (D) Growed. This is not a standard English word, because "grow" is an irregular verb (grow, grew, grown). Reject.
6. Check (C) Growth. This is the noun formed from "grow" (the process or result of growing), so it matches the relation build → building. Accept.
7. So Build : Building :: Grow : Growth. The answer is (C).

KEY IDEA
Find the grammatical relation in the first pair (verb → the noun formed from that verb), then pick the option with the same relation to "grow". The distractors are all verb forms of "grow" (past tense, past participle) or a non-word.

WATCH OUT FOR
Reading "Building" only as the "-ing" form of "build" and then looking for another inflected verb form of "grow" (Grown or Grew). Other students pick "Growed" because it copies the regular "-ed" pattern. In this analogy "building" works as a noun, so the answer must be the noun "growth".

GA Q.4 MCQ · 1 MARK

General Aptitude › Grammar & vocabulary

I do not think you know the case well enough to have opinions. Having said that, I agree with your other point.
What does the phrase “having said that” mean in the given text?

(A) as opposed to what I have said

(B) despite what I have said

(C) in addition to what I have said

(D) contrary to what I have said

Official answer: **B**

SOLUTION

1. **What the phrase means:** "having said that" is a discourse marker of *concession*. It means "even though I have just said that", or "nevertheless". The speaker keeps the earlier statement and then adds a point that seems to pull the other way.
2. **First sentence:** the speaker criticises the listener: "I do not think you know the case well enough to have opinions."
3. **Second sentence:** the speaker still agrees with the listener's other point. After that criticism, agreeing is the unexpected turn. So the two sentences are linked by concession ("in spite of that"), not by addition.
4. **Why not (A) or (D):** "as opposed to" and "contrary to" would mean the speaker contradicts or takes back the first sentence. The speaker does not withdraw it. The criticism still stands, and both statements are meant to be true together.
5. **Why not (C):** "in addition to" continues in the same direction, like "moreover". Agreeing with someone is not a further criticism, so this option loses the contrast between the two sentences.
6. **Why (B) works:** put the option in place of the phrase and read the sentence again: "*Despite what I have said*, I agree with your other point." The meaning does not change. The answer is (B).

KEY IDEA

"Having said that" is a concessive connector, like "nevertheless" or "even so". The speaker stands by the earlier statement and then adds a point that runs against it. Check an option by putting it in the phrase's place: the right one leaves the meaning unchanged.

WATCH OUT FOR

Picking (D) "contrary to" or (A) "as opposed to" because the two sentences contrast. Both of those mean the speaker is contradicting or withdrawing the first statement, but the speaker keeps it. Picking (C) "in addition to" misses the contrast altogether.

GA Q.5 MCQ · 1 MARK

General Aptitude › Geometry

Define $[x]$ as the greatest integer less than or equal to x, for each $x \in (-\infty, \infty)$. If $y = [x]$, then area under y for $x \in [1, 4]$ is _____.

(A) 1

(B) 3

(C) 4

(D) 6

Official answer: **D**

SOLUTION

1. The greatest integer (floor) function $[x]$ equals the integer n for $n \le x < n + 1$ and jumps up by 1 at each integer. On $[1, 4]$ its graph is three horizontal steps. All of them lie above the x-axis ($y \ge 1 > 0$), so the area under y is simply $\int_1^4 [x]\, dx$.
2. Split the interval at the integers: $[1, 2)$, $[2, 3)$, $[3, 4]$. On these pieces $y = 1$, $y = 2$ and $y = 3$. At the single point $x = 4$, $y = 4$, but a point has zero width and adds no area.
3. Each step is a rectangle 1 unit wide, so its area equals its height: $1 \times 1 = 1$, $1 \times 2 = 2$ and $1 \times 3 = 3$.
4. Add the three rectangles:

$$\int_1^4 [x]\,dx = 1 + 2 + 3 = 6$$

5. Quick check: the interval is 3 units wide. On $[2, 4]$ we have $y \geq 2 > 1$, so the area is strictly more than $1 \times 3 = 3$. Apart from the single point $x = 4$, $y \leq 3 < 4$, so the area is strictly less than $4 \times 3 = 12$. That leaves only 4 and 6 among the options, and the exact sum is 6. The answer is option (D).

KEY IDEA

$[x]$ is a step function, constant between consecutive integers. Split $[1, 4]$ at the integers. The area is then the sum of rectangles 1 unit wide whose heights are the step values 1, 2 and 3.

WATCH OUT FOR

Counting the value at $x = 4$ as a fourth step and getting $1 + 2 + 3 + 4 = 10$, when that point has zero width. Another slip is treating $[x]$ as x and using the trapezoid $\int_1^4 x\,dx = 7.5$. A third is taking just one step, or the width 3 alone, and picking 3.

GA Q.6 MCQ · 2 MARKS

General Aptitude › Reading

Crowd funding deals with mobilisation of funds for a project from a large number of people, who would be willing to invest smaller amounts through web-based platforms in the project.

Based on the above paragraph, which of the following is correct about crowd funding?

(A) Funds raised through unwilling contributions on web-based platforms.

(B) Funds raised through large contributions on web-based platforms.

(C) Funds raised through coerced contributions on web-based platforms.

(D) Funds raised through voluntary contributions on web-based platforms.

Official answer: **D**

SOLUTION

1. Pick out the key phrases in the passage. The funds come from "a large number of people", who "would be willing to invest", in "smaller amounts", "through web-based platforms".
2. Rule for reading comprehension: the correct option has to follow from the passage. Any option the passage contradicts is out.
3. (A) "unwilling contributions": the passage says the people "would be willing to invest", which is the opposite. Eliminate.
4. (B) "large contributions": the passage says "smaller amounts". The large thing is the number of people, not the size of each contribution. Eliminate.
5. (C) "coerced contributions": to coerce someone is to force them, and that contradicts "willing". Eliminate.
6. (D) "voluntary contributions": giving willingly is giving voluntarily, and "on web-based platforms" matches "through web-based platforms". This follows directly from the passage.
7. Answer: (D).

KEY IDEA

In the passage, "willing to invest" means the same as "voluntary". All four options repeat "on web-based platforms", so the only thing that tells them apart is the word describing the contributions. Test that word against the passage: willing means voluntary, not unwilling or coerced, and "smaller amounts" rules out large.

WATCH OUT FOR

Choosing (B) because the passage says "large number of people". "Large" describes how many people contribute, not how much each one gives. The passage says each person gives "smaller amounts".

GA Q.7 MCQ · 2 MARKS

General Aptitude › Reasoning

P, Q, R and S are to be uniquely coded using α and β. If P is coded as $\alpha\alpha$ and Q as $\alpha\beta$, then R and S, respectively, can be coded as _____.

(A) $\beta\alpha$ and $\alpha\beta$

(B) $\beta\beta$ and $\alpha\alpha$

(C) $\alpha\beta$ and $\beta\beta$

(D) $\beta\alpha$ and $\beta\beta$

Official answer: **D**

SOLUTION

1. The codes for P and Q have two symbols each, and every option also uses two-symbol codes. So we work with two-symbol strings made from α and β.
2. By the multiplication principle there are 2 choices for each of the 2 positions, so $2 \times 2 = 4$ such codes: $\alpha\alpha,\ \alpha\beta,\ \beta\alpha,\ \beta\beta$. That is exactly one code for each of the four letters.
3. "Uniquely coded" means no two letters can share a code. P already has $\alpha\alpha$ and Q already has $\alpha\beta$, so only $\beta\alpha$ and $\beta\beta$ are left for R and S.
4. Check each option. (A) gives S the code $\alpha\beta$, which Q already has. (B) gives S the code $\alpha\alpha$, which P already has. (C) gives R the code $\alpha\beta$, which Q already has. All three break uniqueness, so they are rejected.
5. (D) gives R $= \beta\alpha$ and S $= \beta\beta$. Neither code is used by P or Q, and the two are different from each other, so all four letters get different codes. The answer is (D).

KEY IDEA

Unique coding means one code per letter with no repeats. Two symbols over two positions give only $2 \times 2 = 4$ codes. Once P takes $\alpha\alpha$ and Q takes $\alpha\beta$, R and S must share out the two codes that are left, $\beta\alpha$ and $\beta\beta$.

WATCH OUT FOR

Checking only that R and S get different codes from each other, and forgetting to check them against P and Q. Option (A) looks fine on that test because $\beta\alpha \neq \alpha\beta$, but it gives S the code $\alpha\beta$, which Q already has.

GA Q.8 MCQ · 2 MARKS

General Aptitude › Numerical

The sum of the first n terms in the sequence 8, 88, 888, 8888, ... is_____.

(A) $\frac{81}{80}(10^n - 1) + \frac{9}{8}n$

(B) $\frac{81}{80}(10^n - 1) - \frac{9}{8}n$

(C) $\frac{80}{81}(10^n - 1) + \frac{8}{9}n$

(D) $\frac{80}{81}(10^n - 1) - \frac{8}{9}n$

Official answer: **D**

SOLUTION

1. Take out the common factor 8, then write each repunit as a string of 9s divided by 9: $8 + 88 + 888 + \cdots = 8(1 + 11 + 111 + \dots) = \frac{8}{9}(9 + 99 + 999 + \dots)$.
2. A string of k nines is one less than a power of 10: $\underbrace{99\dots9}_{k} = 10^k - 1$.
3. So the bracket is $\sum_{k=1}^{n}(10^k - 1) = (10 + 10^2 + \cdots + 10^n) - n$.

4. Sum the GP with first term $a = 10$ and ratio $r = 10$: $\dfrac{a(r^n - 1)}{r - 1} = \dfrac{10(10^n - 1)}{9}$.

5. Put the pieces together:

$$S_n = \frac{8}{9}\left[\frac{10(10^n - 1)}{9} - n\right] = \frac{80}{81}(10^n - 1) - \frac{8}{9}n$$

6. Check with $n = 1$: $\dfrac{80}{81} \cdot 9 - \dfrac{8}{9} = \dfrac{80 - 8}{9} = 8$. Check with $n = 2$: $\dfrac{80}{81} \cdot 99 - \dfrac{16}{9} = \dfrac{7920 - 144}{81} = 96 = 8 + 88$. Both are correct.

7. This matches option (D).

KEY IDEA

Write each term as $\dfrac{8}{9}(10^k - 1)$, since a string of k eights is $\dfrac{8}{9}$ of a string of k nines. The sum then splits into a geometric series in powers of 10 minus n copies of 1. The fastest way to decide between options like these is to put in $n = 1$ and $n = 2$ and check against 8 and 96.

WATCH OUT FOR

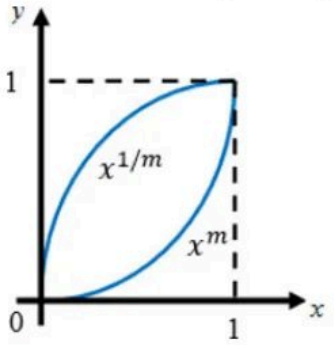
Forgetting to subtract the n (the "-1" inside each $10^k - 1$) gives a plus sign and points to (C). Starting the GP at $10^0 = 1$ instead of $10^1 = 10$ loses the factor 10 and gives a coefficient of $\frac{8}{81}$ instead of $\frac{80}{81}$. Mixing up $\frac{8}{9}$ and $\frac{9}{8}$ leads to (A) or (B). Putting in $n = 1$ (the sum must be 8) rules out every wrong option at once.

GA Q.9 MCQ · 2 MARKS

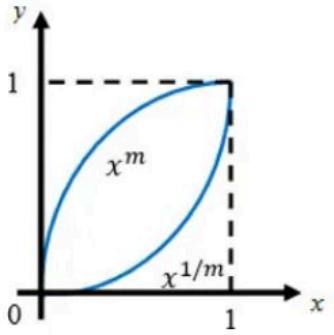
General Aptitude › Numerical

Q.No. 9 Select the graph that schematically represents BOTH $y = x^m$ and $y = x^{1/m}$ properly in the interval $0 \le x \le 1$, for integer values of m, where $m > 1$.

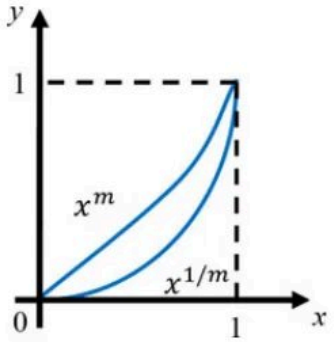
(A)



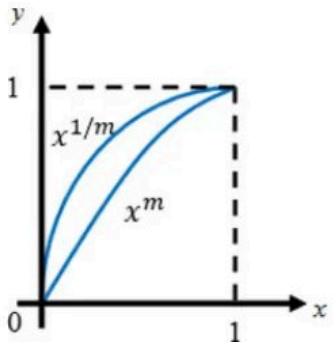
(B)



(C)



(D)



Official answer: **A**

SOLUTION

1. Endpoints: $0^m = 0^{1/m} = 0$ and $1^m = 1^{1/m} = 1$, so both curves join $(0, 0)$ to $(1, 1)$. All four options do this, so the endpoints alone cannot decide the answer.
2. Order between the endpoints: take $0 < x < 1$. Since $m > 1$, $x^{m-1} < 1$, so $x^m = x \cdot x^{m-1} < x$. Now put $t = x^{1/m}$, so $0 < t < 1$. By the same argument $x = t^m < t$, which means $x < x^{1/m}$. Together,

$$x^m < x < x^{1/m} \quad \text{for } 0 < x < 1.$$

Check with $x = \frac{1}{4}$ and $m = 2$: $x^2 = \frac{1}{16} < \frac{1}{4} < \frac{1}{2} = x^{1/2}$.

3. So the curve labelled $x^{1/m}$ must lie above the line $y = x$ and the curve labelled x^m must lie below it. That rules out (B), where the labels are reversed, and (C), where $x^{1/m}$ is drawn below x^m.
4. Shape of $y = x^m$: $y' = mx^{m-1}$ and $y'' = m(m-1)x^{m-2} > 0$ for $0 < x < 1$, so the curve is convex (it bends upward). Because $m - 1 \geq 1$, the slope mx^{m-1} is 0 at $x = 0$ and m at $x = 1$: flat at the start, steep at the end.
5. Shape of $y = x^{1/m}$: $y' = \frac{1}{m}x^{\frac{1}{m}-1}$ and $y'' = \frac{1}{m}\left(\frac{1}{m} - 1\right)x^{\frac{1}{m}-2} < 0$ for $0 < x < 1$, so the curve is concave. Since $\frac{1}{m} - 1 < 0$, the slope grows without bound as $x \to 0^+$, and it equals $\frac{1}{m}$ at $x = 1$: steep at the start, levelling off at the end.
6. The two functions are inverses of each other on $[0, 1]$ ($y = x^m \iff x = y^{1/m}$), so their graphs are mirror images in the line $y = x$. In (D) the curve labelled x^m is drawn concave and above the diagonal, which contradicts steps 2 and 4. So (D) is wrong.
7. (A) draws $x^{1/m}$ concave and above the diagonal, steep at the origin, and x^m convex and below it, flat at the origin, as mirror images of each other. This matches every property found above. The answer is (A).

KEY IDEA

On $0 < x < 1$, raising a fraction to a power greater than 1 makes it smaller and raising it to a power less than 1 makes it larger, so $x^m < x < x^{1/m}$. x^m is convex with slope 0 at the origin, $x^{1/m}$ is concave with a vertical tangent there, and as inverse functions their graphs are mirror images in $y = x$.

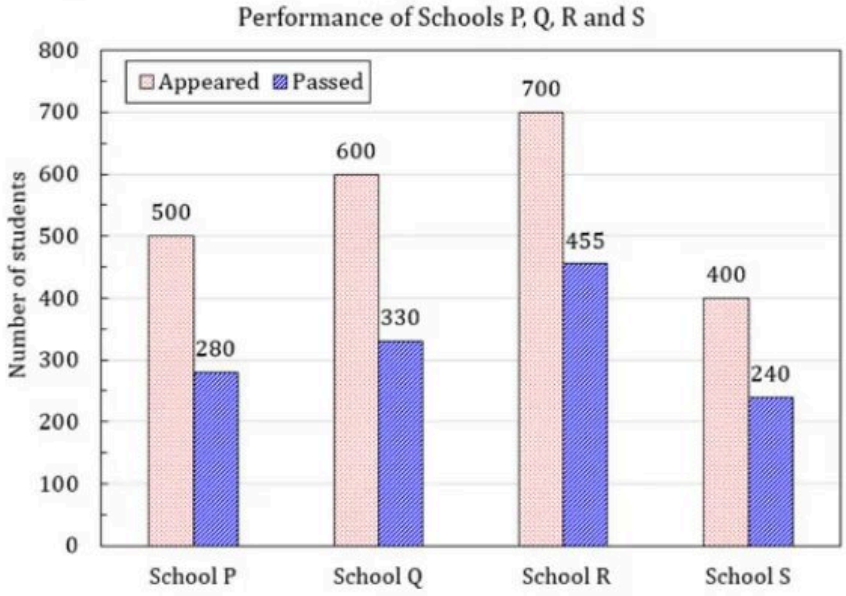
WATCH OUT FOR

Carrying over the intuition from $x > 1$, where x^m is the larger of the two, and so choosing (B), which has the right shapes but swapped labels. Another mistake is to check only the endpoints $(0, 0)$ and $(1, 1)$, which every option satisfies, and never test a point in between such as $x = \frac{1}{4}$.

GA Q.10 MCQ · 2 MARKS

General Aptitude › Data interpretation

Q.No. 10 The bar graph shows the data of the students who appeared and passed in an examination for four schools P, Q, R and S. The average of success rates (in percentage) of these four schools is _____.



(A) 58.5 %

(B) 58.8 %

(C) 59.0 %

(D) 59.3 %

Official answer: **C**

SOLUTION

1. Read the chart: School P, 500 appeared and 280 passed. School Q, 600 and 330. School R, 700 and 455. School S, 400 and 240.
2. A school's success rate is $\frac{\text{passed}}{\text{appeared}} \times 100$. For P: $\frac{280}{500} \times 100 = 56\%$.
3. For Q: $\frac{330}{600} \times 100 = 55\%$.
4. For R: $\frac{455}{700} \times 100 = 65\%$.
5. For S: $\frac{240}{400} \times 100 = 60\%$.
6. The question asks for the average of the four success rates, so take the arithmetic mean of the four percentages, with each school counted once:
$$\frac{56 + 55 + 65 + 60}{4} = \frac{236}{4} = 59.0\%$$
7. This is option (C).

KEY IDEA

"Average of success rates" means the simple mean of each school's own rate (passed / appeared x 100), with every school weighted equally. It is not the success rate of all four schools pooled together.

WATCH OUT FOR

Pooling the totals: total passed / total appeared = $\frac{1305}{2200} \times 100 \approx 59.3\%$. That gives option (D), a distractor. Pooling weights each school by how many students it has, so the large School R (65 %) pulls the figure up. The question asks for the average of the four rates.

Instrumentation Engineering Q.1 to Q.55

Q.1 MCQ · 1 MARK

Engineering Mathematics › Vector calculus

The unit vectors along the mutually perpendicular x, y and z axes are $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ respectively. Consider the plane $z = 0$ and two vectors $\vec{a}$ and $\vec{b}$ on that plane such that $\vec{a} \neq \alpha \vec{b}$ for any scalar α. A vector perpendicular to both $\vec{a}$ and $\vec{b}$ is ____.

(A) $\hat{k}$

(B) $\hat{\imath} - \hat{\jmath}$

(C) $-\hat{\jmath}$

(D) $\hat{\imath}$

Official answer: **A**

SOLUTION

1. Any vector in the plane $z = 0$ has zero z-component, so write $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath}$.
2. The condition $\vec{a} \neq \alpha\vec{b}$ for every scalar α means the two vectors are not parallel. So $a_1b_2 - a_2b_1 \neq 0$, and $\vec{a}$ and $\vec{b}$ together span the whole plane $z = 0$.
3. The cross product $\vec{a} \times \vec{b}$ is perpendicular to both vectors:
$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & 0 \\ b_1 & b_2 & 0 \end{vmatrix} = (a_1b_2 - a_2b_1)\,\hat{k}$$
Because $a_1b_2 - a_2b_1 \neq 0$, this is a nonzero multiple of $\hat{k}$.
4. Direct check of option (A): $\hat{k} \cdot \vec{a} = 0$ and $\hat{k} \cdot \vec{b} = 0$, since neither vector has a z-component. So $\hat{k}$, the normal to the plane $z = 0$, is perpendicular to both.
5. Options (B), (C) and (D) ($\hat{\imath} - \hat{\jmath}$, $-\hat{\jmath}$ and $\hat{\imath}$) are nonzero vectors lying in the plane $z = 0$. Suppose an in-plane vector $\vec{v}$ were perpendicular to both $\vec{a}$ and $\vec{b}$. Since $\vec{a}$ and $\vec{b}$ span the plane, $\vec{v}$ would be perpendicular to every vector in the plane, including itself, so $\vec{v} \cdot \vec{v} = 0$ and $\vec{v} = \vec{0}$. So none of (B), (C), (D) can be perpendicular to both. For example, $\hat{\imath} \cdot \vec{a} = a_1$ and $\hat{\imath} \cdot \vec{b} = b_1$, and these cannot both be zero, because then $a_1b_2 - a_2b_1 = 0$.
6. So the answer is $\hat{k}$, option (A).

KEY IDEA

Two non-parallel vectors in the plane $z = 0$ span that plane. The only direction perpendicular to both is the plane's normal, which is $\hat{k}$. The cross product shows this: $\vec{a} \times \vec{b} = (a_1b_2 - a_2b_1)\,\hat{k}$.

WATCH OUT FOR

Picking an in-plane option such as $\hat{\imath}$ or $-\hat{\jmath}$ because it is perpendicular to one particular vector you have in mind. The question asks for a vector perpendicular to BOTH of two non-parallel vectors in the plane, and no nonzero vector in the plane can be. Another slip is to overlook the condition $\vec{a} \neq \alpha\vec{b}$. It is what makes $\vec{a} \times \vec{b}$ nonzero, so the perpendicular direction is exactly $\hat{k}$.

Q.2 MCQ · 1 MARK

Engineering Mathematics › Numerical methods

Consider the recursive equation $X_{n+1} = X_n - h(F(X_n) - X_n)$, with initial condition $X_0 = 1$ and $h > 0$ being a very small valued scalar. This recursion numerically solves the ordinary differential equation ____

(A) $\dot{X} = -F(X),\ X(0) = 1$

(B) $\dot{X} = -F(X) + X,\ X(0) = 1$

(C) $\dot{X} = F(X),\ X(0) = 1$

(D) $\dot{X} = F(X) + X,\ X(0) = 1$

Official answer: **B**

SOLUTION

1. **Forward (explicit) Euler method:** for $\dot{X} = f(X)$ with step size h, the update rule is
$$X_{n+1} = X_n + h\, f(X_n).$$
2. Put the given recursion in that form by taking the minus sign inside the bracket:
$$X_{n+1} = X_n - h\big(F(X_n) - X_n\big) = X_n + h\big(-F(X_n) + X_n\big).$$
3. Matching the two forms term by term gives $f(X) = -F(X) + X$.
4. Check by taking the limit: $\dfrac{X_{n+1} - X_n}{h} = -F(X_n) + X_n$. As $h \to 0$ the left side tends to $\dot{X}$, so $\dot{X} = -F(X) + X$.
5. The recursion starts from $X_0 = 1$, so the initial condition is $X(0) = 1$.
6. The ODE is therefore $\dot{X} = -F(X) + X,\ X(0) = 1$, which is option (B).

KEY IDEA

A recursion of the form $X_{n+1} = X_n + h\, f(X_n)$ is forward Euler for $\dot{X} = f(X)$. Rearrange the given update until it has exactly that shape, then read off f. The starting value X_0 gives the initial condition.

WATCH OUT FOR

Getting the sign wrong on the X_n term inside the bracket. $-h(F - X_n)$ expands to $-hF + hX_n$, so $f = -F + X$, not $-F - X$. Students who read only the $-hF(X_n)$ part pick (A); students who drop the minus sign in front of h pick (C) or (D).

Q.3 MCQ · 1 MARK

Engineering Mathematics › Linear algebra

A set of linear equations is given in the form $Ax = b$, where A is a 2×4 matrix with real number entries and $b \neq 0$. Will it be possible to solve for x and obtain a **unique solution** by multiplying both left and right sides of the equation by A^T (the super script T denotes the transpose) and inverting the matrix $A^T A$? Answer is

____

(A) Yes, it is always possible to get a unique solution for any 2×4 matrix A.

(B) No, it is not possible to get a unique solution for any 2×4 matrix A.

(C) Yes, can obtain a unique solution provided the matrix $A^T A$ is well conditioned

(D) Yes, can obtain a unique solution provided the matrix A is well conditioned

Official answer: **B**

SOLUTION

1. Multiplying both sides of $Ax = b$ on the left by A^T gives the normal equations $A^T A\, x = A^T b$. Since A^T is 4×2 and A is 2×4, the matrix $A^T A$ is 4×4.
2. The rank of a matrix cannot exceed its smaller dimension. A is 2×4, so $\rho(A) \leq \min(2, 4) = 2$.
3. The rank of a product is at most the rank of either factor: $\rho(A^T A) \leq \min\big(\rho(A^T), \rho(A)\big) \leq 2$. (For real A, in fact $\rho(A^T A) = \rho(A)$.)
4. A 4×4 matrix is invertible only if its rank is 4. Since $\rho(A^T A) \leq 2 < 4$, $\det(A^T A) = 0$ for **every** 2×4 matrix A, so $(A^T A)^{-1}$ never exists and the proposed method cannot be carried out.
5. The same conclusion follows from the original system directly: $Ax = b$ has 4 unknowns and a coefficient matrix of rank at most 2, so there are at least $4 - 2 = 2$ free variables. If the system is consistent it has infinitely many solutions; it can never have a unique one.
6. Conditioning cannot rescue this. A singular matrix has an infinite condition number, so $A^T A$ can never be "well conditioned", which rules out (C). (D) fails for the same rank reason: however well behaved the entries of A are, $A^T A$ is still singular and the system is still underdetermined. Hence the answer is (B).

KEY IDEA

For an $m \times n$ matrix A with $m < n$, the $n \times n$ matrix $A^T A$ has rank $\rho(A) \leq m < n$, so it is always singular. An underdetermined system (fewer equations than unknowns) can never have a unique solution, and the normal equations cannot change that.

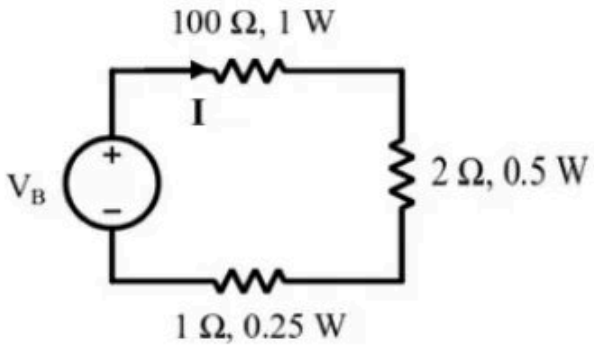
WATCH OUT FOR

Carrying over the least-squares recipe $x = (A^T A)^{-1} A^T b$ without checking when it applies. It gives a unique answer only when A has full column rank, which needs at least as many rows as columns ($m \geq n$). A second trap is picking (C) or (D) because they sound like numerical-analysis caveats: conditioning describes how sensitive an invertible matrix is to small errors, but here $A^T A$ is exactly singular, with infinite condition number, for every 2×4 matrix A.

Q.4 MCQ · 1 MARK

Networks (Electric Circuits) › Laws & node/mesh

Q.No. 4 In the circuit shown below, the safe maximum value for the current **I** is ___



(A) 1.0 A
(B) 0.5 A
(C) 0.1 A
(D) 0.05 A

Official answer: **C**

SOLUTION

1. **One loop, one current.** The circuit has a single loop, so the $100\ \Omega$, $2\ \Omega$ and $1\ \Omega$ resistors are in series and all carry the same current I (KCL at every node).
2. **Current limit of each resistor.** A resistor dissipates $P = I^2 R$, so the largest current it can carry without exceeding its rated power is $I_{\max} = \sqrt{P_{\text{rated}}/R}$.
3. $100\ \Omega,\ 1\ \text{W}$: $I_{\max} = \sqrt{\dfrac{1}{100}} = 0.1\ \text{A}$
4. $2\ \Omega,\ 0.5\ \text{W}$: $I_{\max} = \sqrt{\dfrac{0.5}{2}} = \sqrt{0.25} = 0.5\ \text{A}$
5. $1\ \Omega,\ 0.25\ \text{W}$: $I_{\max} = \sqrt{\dfrac{0.25}{1}} = 0.5\ \text{A}$
6. **The weakest element sets the limit.** Every resistor must stay within its rating, so the common series current can be no larger than the smallest of these limits: $I_{\text{safe}} = \min(0.1,\ 0.5,\ 0.5) = 0.1\ \text{A}$.
7. **Check.** At $I = 0.1\ \text{A}$: $P_{100} = (0.1)^2 \times 100 = 1\ \text{W}$ (exactly at its rating), $P_2 = (0.1)^2 \times 2 = 0.02\ \text{W}$ and $P_1 = (0.1)^2 \times 1 = 0.01\ \text{W}$, both well below their ratings. The answer is (C).

KEY IDEA

In a series circuit every element carries the same current. Work out each resistor's own current limit, $I_{\max} = \sqrt{P_{\text{rated}}/R}$. The safe current is the smallest of these limits, because the most restrictive element decides it.

WATCH OUT FOR

Adding the power ratings (1.75 W) and the resistances (103 Ω) and computing $\sqrt{1.75/103} \approx 0.13$ A. That current would overload the 100 Ω resistor, since each rating applies only to its own resistor. Another mistake is taking the largest limit (0.5 A, option B) when the smallest one governs.

Q.5 MCQ · 1 MARK

Signals and Systems › Fourier transform

A differentiator has a transfer function whose

(A) phase increases linearly with frequency

(B) magnitude remains constant

(C) magnitude increases linearly with frequency

(D) magnitude decreases linearly with frequency

Official answer: **C**

SOLUTION

1. **Time-domain definition.** An ideal differentiator gives $y(t) = \dfrac{dx(t)}{dt}$.
2. **Differentiation property of the Laplace/Fourier transform.** $\dfrac{dx}{dt} \leftrightarrow sX(s)$ (with zero initial condition), so $H(s) = \dfrac{Y(s)}{X(s)} = s$. On the $j\omega$-axis this is $H(j\omega) = j\omega$.
3. **Magnitude.** $|H(j\omega)| = |j\omega| = \omega$ for $\omega \geq 0$. It is directly proportional to frequency, so it increases linearly with ω: a straight line through the origin on a linear scale, or a slope of $+20\ \text{dB/decade}$ on a Bode plot.
4. **Phase.** $\angle H(j\omega) = \angle j\omega = +90^\circ$ for every $\omega > 0$. The phase is constant, not linear in ω, so (A) is false.
5. **Check the other options.** (B) needs $|H(j\omega)|$ independent of ω (a pure gain or an all-pass system), which $|H| = \omega$ is not. (D) needs $|H| \propto 1/\omega$, which is the integrator $H(s) = 1/s$, not the differentiator.
6. Only option (C) holds.

KEY IDEA

Differentiating in time multiplies the transform by s (by $j\omega$ on the frequency axis). So an ideal differentiator has $H(j\omega) = j\omega$. Its magnitude ω grows in direct proportion to frequency, and its phase stays fixed at $+90^\circ$.

WATCH OUT FOR

Mixing up magnitude and phase and picking (A). The phase of $j\omega$ is a constant $+90^\circ$, not linear in ω; linear phase belongs to a pure time delay $e^{-j\omega t_0}$. The other common slip is swapping differentiator and integrator: $|H| \propto 1/\omega$ (option D) is the integrator $1/s$.

Q.6 MCQ · 1 MARK

Control Systems › Compensators & PID

A phase lead network has the transfer function $G(s) = \dfrac{1 + 0.2s}{1 + 0.05s}$. The angular frequency at which the maximum phase shift for the network occurs is ___

(A) 10 rad/s

(B) 20 rad/s

(C) 100 rad/s

(D) 200 rad/s

Official answer: **A**

SOLUTION

1. Read off the corner frequencies from the time constants. The zero corner is at $\omega_z = \dfrac{1}{0.2} = 5\ \text{rad/s}$ and the pole corner is at $\omega_p = \dfrac{1}{0.05} = 20\ \text{rad/s}$. Because $\omega_z < \omega_p$, the network is a lead network.
2. The phase of the network is $\phi(\omega) = \tan^{-1}(0.2\omega) - \tan^{-1}(0.05\omega)$. Set $\dfrac{d\phi}{d\omega} = \dfrac{0.2}{1+0.04\omega^2} - \dfrac{0.05}{1+0.0025\omega^2} = 0$. Cross-multiplying gives $0.2 + 0.0005\omega^2 = 0.05 + 0.002\omega^2$, so $0.0015\,\omega^2 = 0.15$ and $\omega^2 = 100 = \dfrac{1}{0.2 \times 0.05}$.
3. So the maximum phase occurs at the geometric mean of the two corner frequencies:
$$\omega_m = \sqrt{\omega_z\,\omega_p} = \sqrt{5 \times 20} = \sqrt{100} = 10\ \text{rad/s}$$
4. Cross-check with the standard form $\dfrac{1+Ts}{1+\alpha Ts}$. Here $T = 0.2$ and $\alpha T = 0.05$, so $\alpha = 0.25$. Then $\omega_m = \dfrac{1}{T\sqrt{\alpha}} = \dfrac{1}{0.2 \times 0.5} = 10\ \text{rad/s}$, which is option (A).
5. This was not asked, but it is a useful consistency check: $\phi_m = \tan^{-1}(0.2 \times 10) - \tan^{-1}(0.05 \times 10) = \tan^{-1}(2) - \tan^{-1}(0.5) \approx 63.4^\circ - 26.6^\circ = 36.9^\circ$. This matches $\sin^{-1}\dfrac{1-\alpha}{1+\alpha} = \sin^{-1}\dfrac{0.75}{1.25} = \sin^{-1}(0.6) \approx 36.9^\circ$.

KEY IDEA

For a lead network $\dfrac{1+Ts}{1+\alpha Ts}$ with $\alpha < 1$, the phase peaks at the geometric mean of the zero and pole corner frequencies: $\omega_m = \sqrt{\omega_z\,\omega_p} = \dfrac{1}{T\sqrt{\alpha}}$. On a log-frequency axis this is the midpoint between the two corners.

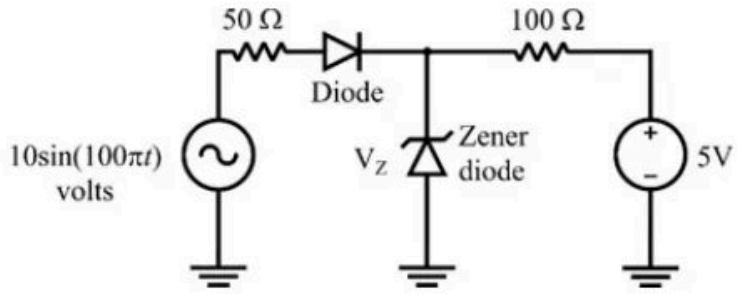
WATCH OUT FOR

Picking the pole corner, $1/0.05 = 20\ \text{rad/s}$ (option B), or taking $1/(0.2 \times 0.05) = 100$ (option C), which is ω_m^2 and needs a square root. The other slip is using the arithmetic mean $(5+20)/2 = 12.5$ in place of the geometric mean.

Q.7 MCQ · 1 MARK

Analog Electronics › Diode circuits

Q.No. 7 If the diodes in the circuit shown are ideal and the breakdown voltage V_Z of the Zener diode is 5 V, the power dissipated in the 100 Ω resistor (**in watts**) is ___



(A) 0
(B) 1
(C) 25/100
(D) 225/100

Official answer: **A**

SOLUTION

1. **Read the orientations.** Call the junction of the diode, the Zener and the $100\ \Omega$ resistor the node, at voltage V_N. The series diode has its anode towards the $50\ \Omega$ resistor, so it conducts only when $v_s = 10\sin(100\pi t)$ is above V_N. The Zener has its cathode at the node and its anode at ground. Taking it as ideal: it carries no current for $0 < V_N < 5$ V, it breaks down and holds $V_N = 5$ V when reverse current is forced into it, and it would conduct forward only if $V_N < 0$.
2. **Diode off ($v_s < V_N$).** The diode cuts the source off from the node. Only the 5 V battery and the $100\ \Omega$ resistor drive the node. The Zener cannot conduct below 5 V, so there is no path for current, and $V_N = 5$ V. That is exactly the breakdown voltage, with zero Zener current, so the assumption is consistent.
3. **Diode on ($v_s > 5$ V).** Suppose the Zener stayed off. KCL at the node would give $\frac{v_s - V_N}{50} = \frac{V_N - 5}{100}$. The left side is positive because the diode conducts, so $V_N > 5$ V. The Zener cannot block more than 5 V, so this is a contradiction. The Zener is therefore in breakdown and holds $V_N = 5$ V. All of the source current $\frac{v_s - 5}{50}$ flows down through the Zener.
4. **Negative half-cycle, or $0 \le v_s < 5$ V.** The diode's anode is below its cathode at 5 V, so the diode is reverse-biased and step 2 applies: $V_N = 5$ V. Forward-biasing the Zener would need $V_N < 0$. With the diode off, nothing can pull the node below the battery's 5 V.
5. **So $V_N = 5$ V at every instant.** Both ends of the $100\ \Omega$ resistor are at 5 V. By Ohm's law,
$$I_{100} = \frac{5 - V_N}{100} = \frac{5 - 5}{100} = 0$$
6. **Power:** $P_{100} = I_{100}^2 \times 100 = 0$ W at every instant, so the average power is also 0 W. **Answer: (A)**.

KEY IDEA

The Zener's breakdown voltage (5 V) equals the battery voltage (5 V). When the series diode is off, the battery sets the node at 5 V through the 100 Ω resistor with no current flowing. When the diode is on, the ideal Zener breaks down and clamps the node at 5 V, taking all the source current. Either way the voltage across the 100 Ω resistor is zero, so it carries no current.

WATCH OUT FOR

Ignoring the Zener clamp and putting the source peak minus the battery, $(10-5)=5$ V, across the $100\ \Omega$ resistor gives $25/100$ W (option C). Reading the battery polarity backwards, $(10+5)=15$ V, gives $225/100$ W (option D). The Zener pins the node at exactly the battery voltage, so the resistor never sees any voltage across it.

Q.8 MCQ · 1 MARK

Digital Electronics › Boolean logic

Given $f(A,B,C,D)=\sum m(0,1,2,6,8,9,10,11)+\sum d(3,7,14,15)$ is a Boolean function, where m represents min-terms and d represents don't-cares. The minimal sum of products expression for f is ___

(A) $f=A\overline{B}+CB$

(B) $f=\overline{B}+C$

(C) $f=\overline{D}+A$

(D) $f=\overline{A}B+\overline{C}D$

Official answer: **B**

SOLUTION

1. Complements are written with –. Draw the 4-variable K-map with rows $AB=00,01,11,10$ and columns $CD=00,01,11,10$. Put 1s at $0,1,2,6,8,9,10,11$ and X (don't-care) at $3,7,14,15$. The remaining cells, $4,5,12,13$, are 0s.
2. Octet 1: rows $AB=00$ and $AB=10$ are adjacent because the map wraps around. Together they hold cells $0,1,3,2,8,9,11,10$. Every one of these is a 1, except cell 3, which is an X and is taken as 1. The only variable that stays constant over the octet is $B=0$, so the term is $\overline{B}$.
3. Octet 2: columns $CD=11$ and $CD=10$ hold cells $3,2,7,6,15,14,11,10$, and each of these is a 1 or an X. The only constant variable is $C=1$, so the term is C.
4. Coverage check. $\overline{B}$ covers the minterms $0,1,2,8,9,10,11$, and C covers minterm 6, so every minterm is covered. Neither group contains a 0-cell, because each of $4,5,12,13$ has $B=1$ and $C=0$.
5. Minimality. No single literal covers every 1: $\overline{B}$ misses minterm 6, and C misses minterm 0. So the minimal SOP is two one-literal prime implicants:
$$f=\overline{B}+C$$
6. Check the other options against the map. (A) $A\overline{B}+BC$ misses minterms $0,1,2$, where $A=0$ and $B=0$. (C) $\overline{D}+A$ is 1 at cells 4 and 12, which are 0s. (D) $\overline{A}B+\overline{C}D$ misses minterm 0, and its $\overline{C}D$ term is 1 at cells 5 and 13, which are 0s. Only (B) is correct.

KEY IDEA

Put the don't-cares into the K-map as 1s wherever that makes a group bigger. Here the only 0s are $4,5,12,13$, and all four have $B=1$ and $C=0$. So $\overline{f}=B\overline{C}$, and $f=\overline{B}+C$.

WATCH OUT FOR

Forgetting the don't-cares, especially 3, 7, 14 and 15. Without them the octets break into quads and pairs, which gives a longer expression that is not in the options. Another mistake is picking (A) $A\overline{B}+CB$ because it looks close. It misses minterms 0, 1 and 2.

Q.9 MCQ · 1 MARK

Electrical and Electronic Measurements › Bridges

A Q meter is best suited for the measurement of the ___

(A) Quality factor of a capacitance.

(B) Distributed capacitance of a coil.

(C) Quality factor of piezoelectric sensor.

(D) Turns-ratio of a transformer

Official answer: **B**

SOLUTION

1. **How a Q meter works (series resonance).** The coil under test is connected in series with a calibrated variable capacitor C, and a small, fixed RF voltage E is injected into the loop. At resonance the voltage across the capacitor is $V_C = QE$. With E held constant, a voltmeter across C that is calibrated in Q reads the coil's quality factor directly. The instrument is designed to measure **coils** (inductors) at radio frequencies.
2. **Distributed (self) capacitance is a standard Q-meter measurement.** Resonate the coil at frequency f_1 with the tuning capacitor set to C_1. Then resonate it at $f_2 = 2f_1$ with the capacitor set to C_2. Each time, the coil's distributed capacitance C_d is in parallel with the tuning capacitor.
3. Apply the resonance condition $f = \dfrac{1}{2\pi\sqrt{L(C + C_d)}}$ at both frequencies. L is the same in both cases, so it cancels:
$$\frac{f_2}{f_1} = 2 = \sqrt{\frac{C_1 + C_d}{C_2 + C_d}} \;\Rightarrow\; C_1 + C_d = 4(C_2 + C_d)$$
4. Solving gives
$$C_d = \frac{C_1 - 4C_2}{3}$$
This needs only two readings of the capacitor dial, so the Q meter measures C_d directly and simply. Option (B) fits.
5. **Why not (A).** A good capacitor has very low loss, so its Q is much higher than a coil's, and the Q meter cannot read it directly. It has to be found indirectly: resonate a standard coil first without the capacitor and then with it, and calculate the capacitor's Q from the small difference between two large readings. That makes the result much less accurate, so the Q meter is not *best suited* to this measurement.
6. **Why not (C) and (D).** A piezoelectric sensor is a capacitive, charge-generating transducer. Its quality factor describes an electromechanical resonance, not the resonance of a coil with a tuning capacitor, which is what the Q meter's circuit is built to measure. A transformer's turns ratio is found from a voltage-ratio test (a ratio meter or a ratio bridge), not from a resonance or a Q reading.
7. So, of the four options, a Q meter is best suited to measuring the distributed capacitance of a coil. **Answer: (B).**

KEY IDEA

A Q meter is a series-resonant circuit made of the coil under test and a calibrated tuning capacitor. At resonance $V_C = QE$, so it reads a coil's Q directly. Resonating the same coil at f_1 and at $2f_1$ gives its distributed capacitance from two dial readings: $C_d = (C_1 - 4C_2)/3$.

WATCH OUT FOR

Picking (A) because the instrument is named after the quality factor. A Q meter reads a coil's Q directly. A low-loss capacitor has a much higher Q, which the meter can only find indirectly from the small difference between two readings, and that result is much less accurate. The standard "direct" uses of a Q meter are a coil's Q, its inductance and its distributed capacitance.

Q.10 MCQ · 1 MARK

Sensors and Industrial Instrumentation › Sensing principles

If I is the current flowing through a Hall effect sensor and B is the magnetic flux density perpendicular to the direction of the current (in the plane of the Hall effect sensor), the Hall voltage generated is ___

(A) Directly proportional to I and inversely proportional to B

(B) Directly proportional to both I and B

(C) Inversely proportional to both I and B

(D) Inversely proportional to I and directly proportional to B

Official answer: **B**

SOLUTION

1. A Hall element carries current I along its length. A field B perpendicular to the current pushes the moving carriers sideways with the Lorentz force $F = qvB$.
2. Charge collects on the side faces until the Hall electric field balances the magnetic force: $qE_H = qvB$, so $E_H = vB$.
3. The drift velocity follows from the current: $v = \dfrac{I}{nqA} = \dfrac{I}{nq\,w\,t}$, where w is the width, t is the thickness and n is the carrier density.
4. The Hall voltage across the width is $V_H = E_H\,w = vBw = \dfrac{IB}{nqt}$.
5. This is $V_H = \dfrac{R_H\,I\,B}{t}$ with Hall coefficient $R_H = \dfrac{1}{nq}$.
6. For a given sensor n, q and t are fixed, so $V_H \propto I\,B$. The Hall voltage is directly proportional to both I and B, which is option (B).

KEY IDEA

The Hall voltage is $V_H = \dfrac{IB}{nqt}$. For a fixed sensor it is linear in the bias current and in the perpendicular flux density, which is why Hall sensors measure magnetic field or current.

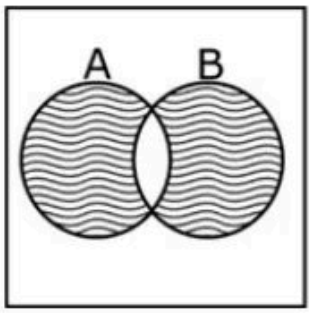
WATCH OUT FOR

Treating the Hall voltage as inversely proportional to B or I. The carrier density n and thickness t sit in the denominator, but I and B both sit in the numerator.

Q.11 MCQ · 1 MARK

Digital Electronics › Boolean logic

Q.No. 11 The Boolean expression for the shaded regions as shown in the figure is ___



(A) $(A+B)\bullet(\overline{A}+\overline{B})$

(B) $(A+\overline{B})\bullet(\overline{A}+B)$

(C) $(\overline{A}+B)\bullet(\overline{A}+\overline{B})$

(D) $(\overline{A}+\overline{B})\bullet(A+\overline{B})$

Official answer: **A**

SOLUTION

1. Read the Venn diagram. The shaded area is "in A but not in B" together with "in B but not in A". The overlap AB and the area outside both circles are not shaded. Complement is written with $\overline{\ }$ (an overbar), and the printed dot between brackets means AND.
2. Write it in sum-of-products form: $F = A\overline{B} + \overline{A}B = A \oplus B$, the exclusive-OR.
3. Turn it into product-of-sums form. F is 0 only in the two unshaded regions: $\overline{A}\,\overline{B}$ (outside both circles, $A = 0, B = 0$) and AB (the overlap, $A = 1, B = 1$). The maxterms for these are $M_0 = (A + B)$ and $M_3 = (\overline{A} + \overline{B})$, so $F = (A + B)(\overline{A} + \overline{B})$.
4. Check by multiplying out, using $A\overline{A} = 0$ and $B\overline{B} = 0$:
$$(A + B)(\overline{A} + \overline{B}) = A\overline{A} + A\overline{B} + \overline{A}B + B\overline{B} = A\overline{B} + \overline{A}B$$
This is exactly the shaded area, so option (A) is correct.
5. Rule out the other options. (B) $(A + \overline{B})(\overline{A} + B) = A\overline{A} + AB + \overline{A}\,\overline{B} + \overline{B}B = AB + \overline{A}\,\overline{B}$, the XNOR, which is exactly the unshaded area. (C) $(\overline{A} + B)(\overline{A} + \overline{B}) = \overline{A} + B\overline{B} = \overline{A}$, by the distributive law $(X + Y)(X + Z) = X + YZ$; that is everything outside circle A. (D) $(\overline{A} + \overline{B})(A + \overline{B}) = \overline{B} + \overline{A}A = \overline{B}$, by the same law; that is everything outside circle B.

KEY IDEA

Shading "A only" plus "B only" is the exclusive-OR, $A\overline{B} + \overline{A}B$. The options are in product-of-sums form, so build the POS from the regions that are NOT shaded: outside both circles gives the maxterm $(A + B)$ and the overlap gives $(\overline{A} + \overline{B})$. Their product is the answer.

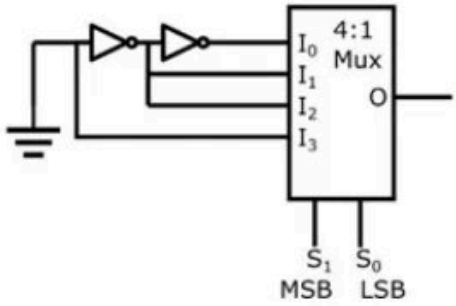
WATCH OUT FOR

Picking option (B), $(A + \overline{B})(\overline{A} + B)$, because it looks like a "mixed" XOR pattern. Multiplied out it is $AB + \overline{A}\,\overline{B}$, the XNOR, which is exactly the unshaded area. The other trap is putting the maxterms for the shaded regions into the POS instead of the maxterms for the unshaded ones.

Q.12 MCQ · 1 MARK

Digital Electronics › Combinational

Q.No. 12 The Boolean operation performed by the following circuit at the output O is ___



(A) $O = S_1 \oplus S_0$

(B) $O = S_1 \bullet \overline{S}_0$

(C) $O = S_1 + S_0$

(D) $O = S_0 \bullet \overline{S}_1$

Official answer: **A**

SOLUTION

1. Complement is written as - throughout. Find each data input by following the wires from ground (logic 0). I_3 is tapped from the grounded wire before either NOT gate, so $I_3 = 0$.
2. I_1 and I_2 are both taken from the output of the first NOT gate: $I_1 = I_2 = \overline{0} = 1$.
3. I_0 comes after both NOT gates: $I_0 = \overline{\overline{0}} = 0$.
4. With S_1 as the MSB, a 4:1 MUX passes I_k when $(S_1 S_0)_2 = k$, so its output is
$$O = \overline{S_1}\,\overline{S_0}\,I_0 + \overline{S_1}\,S_0\,I_1 + S_1\,\overline{S_0}\,I_2 + S_1\,S_0\,I_3$$
5. Substitute $(I_0, I_1, I_2, I_3) = (0, 1, 1, 0)$:
$$O = \overline{S_1}\,S_0 + S_1\,\overline{S_0}$$
6. This is the sum-of-products form of XOR, so $O = S_1 \oplus S_0$. Truth-table check: $S_1 S_0 = 00, 01, 10, 11$ gives $O = 0, 1, 1, 0$, which is XOR. The other options fail this check: (B) gives $0, 0, 1, 0$, (C) gives $0, 1, 1, 1$ and (D) gives $0, 1, 0, 0$. Answer: option (A).

KEY IDEA

A MUX with fixed data inputs is just a truth table. Input I_k is the output for select combination k (with S_1 as MSB), so once you know the four input values you can read the function straight off. Here the inputs are 0, 1, 1, 0 for select values 00, 01, 10, 11, which is XOR.

WATCH OUT FOR

Losing track of the inverter count on each wire. I_0 goes through two inverters, so it is back to 0, not 1. I_1 and I_2 come off the middle node after one inverter, so they are 1. If you treat I_0 as 1, you get $0, 1, 1, 1$ and pick OR, option (C). Swapping the MSB and LSB would not change this answer, because XOR is symmetric and $I_1 = I_2$, but it does matter in general.

Q.13 MCQ · 1 MARK

Signals and Systems › Signals basics

Consider the Signal $x[n] = \sin(2\pi n)\,u[n]$, where $u[n] = 1$ for $n = 0, 1, 2, 3, \ldots$ and $u[n] = 0$ otherwise.

The period of this signal $x[n]$ is ___

(A) 4

(B) 3

(C) 2

(D) 1

Official answer: **D**

Q.14 MCQ · 1 MARK

Control Systems › Bode & margins

The closed loop transfer function of a control system is given by $\frac{C(s)}{R(s)} = \frac{1}{s+1}$. For the input $r(t) = \sin t$, the steady state response $c(t)$ is ___

(A) 1

(B) $\frac{1}{\sqrt{2}} \cos t$

(C) $\frac{1}{\sqrt{2}} \sin\left(t + \frac{\pi}{4}\right)$

(D) $\frac{1}{\sqrt{2}} \sin\left(t - \frac{\pi}{4}\right)$

Official answer: **D**

SOLUTION

1. The only pole of $G(s) = \frac{1}{s+1}$ is at $s = -1$, in the left half-plane, so the system is stable and its transient term e^{-t} dies out. For a stable system with input $r(t) = A \sin \omega t$, the steady-state output is a sinusoid at the same frequency:
$$c_{ss}(t) = A\,|G(j\omega)|\, \sin\left(\omega t + \angle G(j\omega)\right)$$
2. The input $\sin t$ has $A = 1$ and $\omega = 1$ rad/s. Substituting $s = j\omega = j1$ gives $G(j1) = \frac{1}{1+j}$.
3. Magnitude: $|G(j1)| = \frac{1}{\sqrt{1^2 + 1^2}} = \frac{1}{\sqrt{2}}$.
4. Phase: $\angle G(j1) = 0 - \tan^{-1}\left(\frac{1}{1}\right) = -45^\circ = -\frac{\pi}{4}$. The pole introduces a phase lag, so the output lags the input.
5. Therefore
$$c_{ss}(t) = \frac{1}{\sqrt{2}} \sin\left(t - \frac{\pi}{4}\right)$$
which is option (D).

KEY IDEA

For a stable LTI system, a sinusoid at ω produces a steady-state sinusoid at the same ω. Its amplitude is multiplied by $|G(j\omega)|$ and its phase is shifted by $\angle G(j\omega)$. Here $G(j1) = \frac{1}{1+j}$, so the amplitude becomes $\frac{1}{\sqrt{2}}$ and the phase shifts by $-\frac{\pi}{4}$.

WATCH OUT FOR

Getting the sign of the phase wrong and picking (C). A pole in the denominator makes the phase negative: $\angle \frac{1}{1+j} = -\tan^{-1} 1 = -\frac{\pi}{4}$, so the output lags the input. Picking (A) is another mistake: it uses the DC gain $G(0) = 1$, which only applies to a step input, not to a sinusoid.

Q.15 MCQ · 1 MARK

Engineering Mathematics › Complex analysis

Let $f(z) = \frac{1}{z+a}$, $a > 0$. The value of the integral $\oint f(z)\, dz$ over a circle C with center $(-a, 0)$ and radius $R > 0$ evaluated in the anti-clockwise direction is ___

(A) 0

(B) $2\pi i$

(C) $-2\pi i$

(D) $4\pi i$

Official answer: **B**

SOLUTION

1. **Singularity.** $f(z) = \dfrac{1}{z+a}$ has exactly one singularity: a simple pole at $z = -a$, which is the point $(-a, 0)$ in the complex plane.
2. **Where it lies.** The circle C is centred at $(-a, 0)$, so the pole sits at its centre. That means the pole is inside C for every radius $R > 0$, however small or large.
3. **Residue at the simple pole.** $\mathrm{Res}_{z=-a} f = \lim_{z\to -a}(z+a)\cdot\dfrac{1}{z+a} = 1.$
4. **Cauchy's residue theorem** (the contour is traversed anti-clockwise):
$$\oint_C \frac{dz}{z+a} = 2\pi i \times 1 = 2\pi i$$
5. **Check by parametrising.** Put $z = -a + Re^{i\theta}$ with $0 \le \theta \le 2\pi$. Then $dz = iRe^{i\theta}\,d\theta$ and
$$\oint_C \frac{dz}{z+a} = \int_0^{2\pi} \frac{iRe^{i\theta}}{Re^{i\theta}}\,d\theta = \int_0^{2\pi} i\,d\theta = 2\pi i,$$
which does not depend on R. The answer is option (B).

KEY IDEA

The only pole, $z = -a$, is the centre of the circle, so it is enclosed for every radius. Its residue is 1, and an anti-clockwise loop gives $2\pi i \times 1$. This is the standard result $\oint \frac{dz}{z-z_0} = 2\pi i$ for any positively oriented circle around z_0.

WATCH OUT FOR

Two traps. Some candidates pick 0 because they take the circle to be centred at the origin, or think a small R could leave the pole outside; the pole is the centre, so it is always inside. Others pick $-2\pi i$ by getting the orientation wrong: anti-clockwise is the positive direction and gives $+2\pi i$.

Q.16 NAT · 1 MARK

Engineering Mathematics › Probability & statistics

A player throws a ball at a basket kept at a distance. The probability that the ball falls into the basket in a single attempt is 0.1. The player attempts to throw the ball twice. Considering each attempt to be independent, the probability that this player puts the ball into the basket only in the second attempt (**rounded off to two decimal places**) is ____

Official answer: **0.09**

SOLUTION

1. Let S stand for a throw that goes in and F for a miss. For a single attempt $P(S) = 0.1$, so by the complement rule $P(F) = 1 - 0.1 = 0.9$.
2. "Only in the second attempt" describes exactly one outcome of the two throws: the first throw misses and the second goes in. That is the event $F_1 \cap S_2$. The outcomes S_1S_2 and S_1F_2 are excluded because the ball went in on the first throw, and F_1F_2 is excluded because it never went in.
3. The attempts are independent, so the multiplication rule gives
$$P(F_1 \cap S_2) = P(F_1)\,P(S_2) = 0.9 \times 0.1 = 0.09$$
4. 0.09 already has two decimal places, so no rounding is needed and the answer is 0.09. This is also the geometric-distribution probability that the first success comes on trial $k = 2$: $(1-p)^{k-1}p = 0.9 \times 0.1$.

KEY IDEA

Read "only in the second attempt" as one specific sequence: a miss on the first throw, then a basket on the second. Independence lets you multiply the two probabilities: $P(F_1)P(S_2) = (1-p)\,p$. This is the geometric probability that the first success comes on trial 2.

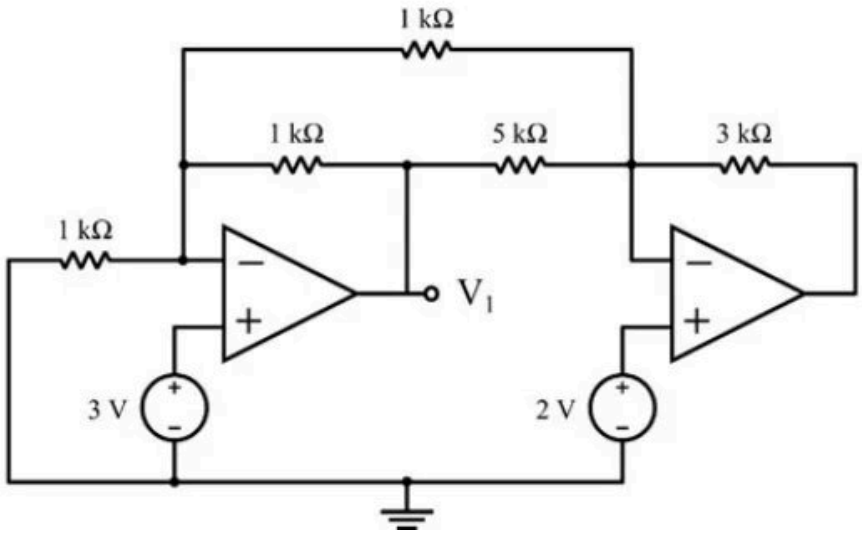
WATCH OUT FOR

Answering 0.1 by ignoring the requirement that the first throw misses. Another error is answering 0.18, which is $P(\text{exactly one success in two throws}) = 2 \times 0.9 \times 0.1$ and wrongly includes the case where the ball goes in on the first throw only. A third is answering 0.01, which is the probability of scoring on both throws.

Q.17 NAT · 1 MARK

Analog Electronics › Op-amps

Q.No. 17 Assuming ideal opamps, the output voltage at V_1 in the figure shown **(in volts)** is ___



Official answer: **7**

SOLUTION

1. Assume ideal op-amps: infinite open-loop gain, infinite input resistance and zero output resistance. Both op-amps have negative feedback. The left one feeds back from V_1 through a 1 kΩ resistor to its (−) input. The right one feeds back from its output through the 3 kΩ resistor to its (−) input. So each has a virtual short between its inputs, and no current flows into either input terminal.
2. Virtual short at the left op-amp: its (+) input sits on the 3 V source, so node A, its (−) input, is at $V_A = 3\ \text{V}$.
3. Virtual short at the right op-amp: its (+) input sits on the 2 V source, so node B, its (−) input, is at $V_B = 2\ \text{V}$. The right op-amp holds B at 2 V whatever V_1 is, so from node A's side, B behaves like a fixed 2 V source.
4. KCL at node A, with currents in mA and resistances in kΩ. Three currents leave A: through the 1 kΩ to ground, through the 1 kΩ to V_1, and through the top 1 kΩ to node B. No current enters the op-amp's input:
$$\frac{3-0}{1\,\text{k}\Omega} + \frac{3-V_1}{1\,\text{k}\Omega} + \frac{3-2}{1\,\text{k}\Omega} = 0$$
5. Solve: $3 + (3 - V_1) + 1 = 0 \Rightarrow V_1 = 7\ \text{V}$.
6. Check at node B, taking currents leaving B (in mA) and calling the right op-amp's output V_{o2}:
$$\frac{2-7}{5} + \frac{2-3}{1} + \frac{2-V_{o2}}{3} = 0 \Rightarrow -1 - 1 + \frac{2-V_{o2}}{3} = 0 \Rightarrow V_{o2} = -4\ \text{V}$$

This is finite, so the solution is consistent. The 5 kΩ and 3 kΩ resistors only set the right op-amp's output. They do not affect V_1.

KEY IDEA

An ideal op-amp with negative feedback forces its (−) input to the voltage on its (+) input. That fixes node A at 3 V and node B at 2 V. With both ends of every resistor at node A known except V_1, KCL at node A alone gives V_1. The 5 kΩ and 3 kΩ resistors only decide the right op-amp's output (-4 V).

WATCH OUT FOR

There are three common mistakes. (1) Reading the left stage as a plain non-inverting amplifier with gain $1 + 1/1 = 2$, which gives $V_1 = 6$ V. This ignores the top 1 kΩ, which carries 1 mA from A to B. (2) Treating node B as a virtual ground (0 V) instead of 2 V, which gives $V_1 = 9$ V. (3) Writing KCL at the V_1 node, where the op-amp's output current is unknown, instead of at the op-amp's input node, where the input current is zero.

Q.18 NAT · 1 MARK

Networks (Electric Circuits) › Three-phase (EE)

Three 400 Ω resistors are connected in delta and powered by a 400 V (rms), 50 Hz, balanced, symmetrical R-Y-B sequence, three-phase three-wire mains. The rms value of the **line current (in amperes, rounded off to one decimal place)** is _____.

Official answer: **1.7 to 1.8**

SOLUTION

1. Supply convention: a three-phase supply quoted as 400 V means the line-to-line voltage. So $V_L = 400$ V (rms).
2. In a delta connection each resistor is connected directly between two lines, so the phase voltage equals the line voltage: $V_{ph} = V_L = 400$ V.
3. Ohm's law for one phase: $I_{ph} = \dfrac{V_{ph}}{R} = \dfrac{400}{400} = 1$ A. The load is purely resistive, so this current is in phase with its phase voltage.
4. Line current in a balanced delta (KCL at a delta node): each line current is the phasor difference of two phase currents that are 120° apart, so its magnitude is $I_L = \sqrt{3}\, I_{ph}$.
5. $$I_L = \sqrt{3} \times 1 = 1.732 \text{ A} \approx 1.7 \text{ A}$$
6. Power check: $P = 3\dfrac{V_{ph}^2}{R} = 3 \times \dfrac{400^2}{400} = 1200$ W, and $P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 400 \times 1.732 \times 1 = 1200$ W. The two agree.

KEY IDEA

In a balanced delta load, each phase is connected across a line-to-line voltage, so $V_{ph} = V_L$. Each line current is the phasor difference of two phase currents that are 120° apart, which makes it $\sqrt{3}$ times as large: $I_L = \sqrt{3}\, I_{ph}$.

WATCH OUT FOR

Using the star relations for a delta load. Some candidates divide the voltage by $\sqrt{3}$ (taking $V_{ph} = 400/\sqrt{3} = 230.9$ V, which gives $I_{ph} = 0.577$ A). Others forget the $\sqrt{3}$ factor and report the phase current of 1 A as the line current. Both answers fall outside the key.

Q.19 NAT · 1 MARK

Signals and Systems › Fourier transform

Consider the signal $x(t) = e^{-|t|}$. Let $X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$ be the Fourier transform of $x(t)$. The value

of $X(j0)$ is ___

Official answer: **2**

SOLUTION

1. Put $\omega = 0$ in the definition. Then $e^{-j\omega t} = e^{0} = 1$, so the transform at zero frequency is the total area under the signal:
$$X(j0) = \int_{-\infty}^{\infty} x(t)\,dt = \int_{-\infty}^{\infty} e^{-|t|}\,dt$$
2. $e^{-|t|}$ is an even function of t, so the integral over the whole line is twice the integral over $t \geq 0$. There $|t| = t$:
$$X(j0) = 2\int_{0}^{\infty} e^{-t}\,dt$$
3. Evaluate:
$$2\int_{0}^{\infty} e^{-t}\,dt = 2\Big[-e^{-t}\Big]_{0}^{\infty} = 2\,(0 - (-1)) = 2$$
4. Cross-check with the standard pair $e^{-a|t|} \leftrightarrow \dfrac{2a}{a^2+\omega^2}$ for $a > 0$. With $a = 1$, $X(j\omega) = \dfrac{2}{1+\omega^2}$, and at $\omega = 0$ this gives $X(j0) = 2$.
5. Therefore $X(j0) = 2$.

KEY IDEA

A Fourier transform at zero frequency is the area under the signal: $X(j0) = \int_{-\infty}^{\infty} x(t)\,dt$. For the two-sided exponential $e^{-|t|}$, that area is 2. The pair $e^{-a|t|} \leftrightarrow \dfrac{2a}{a^2+\omega^2}$ confirms it.

WATCH OUT FOR

Integrating only over $t \geq 0$, as if the signal were $e^{-t}u(t)$. That gives 1 instead of 2. The absolute value makes the signal two-sided, so the left half adds an equal area. A second slip is using the one-sided pair $\dfrac{1}{1+j\omega}$, which also gives 1 at $\omega = 0$.

Q.20 NAT · 1 MARK

Control Systems › Time response

A second order system has closed loop poles located at $s = -3 \pm j4$. The time t at which the maximum value of the step response occurs (**in seconds, rounded off to two decimal places**) is ___

Official answer: **0.78 to 0.79**

SOLUTION

1. Match the poles to the standard form $s = -\zeta\omega_n \pm j\omega_d$. This gives $\zeta\omega_n = 3$ and $\omega_d = 4\ \text{rad/s}$.
2. Check that the system is underdamped: $\omega_n = \sqrt{3^2+4^2} = 5\ \text{rad/s}$ and $\zeta = 3/5 = 0.6 < 1$. The step response therefore overshoots, so it has a true maximum.
3. For a standard second-order system with no zeros, the unit step response is $c(t) = 1 - \dfrac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \theta)$. Its derivative is $\dfrac{dc}{dt} = \dfrac{\omega_n}{\sqrt{1-\zeta^2}}\,e^{-\zeta\omega_n t}\sin(\omega_d t)$. This is zero when $\omega_d t = n\pi$, and the first maximum is at $n = 1$. So $t_p = \dfrac{\pi}{\omega_d}$.
4. Substitute:
$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{4} = 0.7854\ \text{s}$$
5. Round to two decimal places: $t_p \approx 0.79\ \text{s}$.

KEY IDEA

Peak time depends only on the imaginary part of the closed-loop poles: $t_p = \pi/\omega_d$. For poles at $-3 \pm j4$, ω_d is simply 4, so there is no need to compute ζ or ω_n first.

WATCH OUT FOR

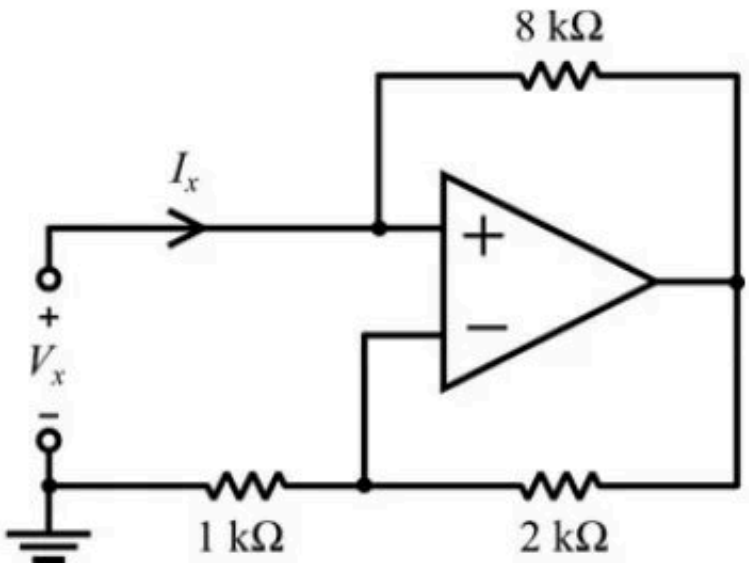
Using $\omega_n = 5$ in place of $\omega_d = 4$ gives $\pi/5 = 0.63$ s, which is wrong. The other common slip is using the real part: $\pi/3 = 1.05$ s is also wrong. Only the damped frequency ω_d, the imaginary part of the pole, sets the peak time.

Q.21 NAT · 1 MARK

Analog Electronics › Op-amps

Q.No. 21 Assume that the opamp in the circuit shown is ideal.

The value of $\frac{V_x}{I_x}$ **(in kΩ)** is ___



Official answer: **-4**

SOLUTION

1. Take the op-amp as ideal and operating in its linear region, as the question intends: no current flows into either input, and the two inputs are at the same voltage (virtual short), $V_+ = V_-$. I_x enters the node wired to the non-inverting input, so $V_+ = V_x$.
2. Voltage divider on the inverting side. No current enters the $-$ input, so the $2\,\text{k}\Omega$ and $1\,\text{k}\Omega$ resistors carry the same current and divide V_o:
$$V_- = V_o \frac{1\,\text{k}\Omega}{1\,\text{k}\Omega + 2\,\text{k}\Omega} = \frac{V_o}{3}$$
3. Virtual short: $\frac{V_o}{3} = V_x \Rightarrow V_o = 3V_x$. Seen from the $+$ input, this is a non-inverting amplifier with gain $1 + \frac{2}{1} = 3$.
4. KCL at the input node. The $+$ input draws no current, so all of I_x flows through the $8\,\text{k}\Omega$ resistor to the output:
$$I_x = \frac{V_x - V_o}{8\,\text{k}\Omega} = \frac{V_x - 3V_x}{8\,\text{k}\Omega} = -\frac{2V_x}{8\,\text{k}\Omega} = -\frac{V_x}{4\,\text{k}\Omega}$$
5. Input resistance:
$$\frac{V_x}{I_x} = -4\,\text{k}\Omega$$

The circuit is a negative-impedance converter. In general $R_{in} = -\dfrac{R_f}{A-1}$, and here $R_f = 8\,\text{k}\Omega$ and $A = 3$ give $-\dfrac{8}{2} = -4\,\text{k}\Omega$. Answer: -4.

KEY IDEA

With an ideal op-amp in linear operation, the input node sits at V_x and the non-inverting gain fixes $V_o = (1 + 2/1)V_x = 3V_x$. The $8\,\text{k}\Omega$ feedback resistor is the only path for I_x. The far end of that resistor is driven to three times the input voltage, so current flows out of the source terminal instead of in, and the resistance seen at the input is negative: $R_{in} = -R_f/(A-1)$.

WATCH OUT FOR

Seeing an 8 kΩ resistor from the input to the output and treating it as a load to ground, which gives $+8\,\text{k}\Omega$. Another slip is to use $V_o = -2V_x$, the inverting gain, because the divider looks like an inverting amplifier. The source drives the $+$ input, so the gain is $+3$, the resistor current is $(V_x - 3V_x)/8\,\text{k}\Omega$, and the sign of the answer must be negative.

Q.22 NAT · 1 MARK

Signals and Systems › Sampling

A sinusoid of 10 kHz is sampled at 15 k samples/s. The resulting signal is passed through an ideal low pass filter (LPF) with cut-off frequency of 25 kHz. The maximum frequency component at the output of the LPF (in kHz) is ____

Official answer: **25**

Q.23 NAT · 1 MARK

Digital Electronics › ADC & DAC

A 200 mV full-scale dual-slope analog to digital converter (DS-ADC) has a reference voltage of 100 mV. The first integration time is set as 100 ms. The DS-ADC is operated in the continuous conversion mode. The conversion time of the DS-ADC for an input voltage of 123.4 mV (in ms, rounded off to one decimal place) is ____

Official answer: **223.3 to 223.5**

SOLUTION

1. **Phase 1 (run-up, fixed time).** The integrator integrates the input V_{in} for a fixed time $T_1 = 100\ \text{ms}$. At the end of this phase its output has magnitude $V_o = \dfrac{V_{in}T_1}{RC}$.
2. **Phase 2 (run-down, variable time).** The integrator input is switched to the reference, which has the opposite polarity, and the output ramps back to zero in a time T_2. The two ramps cover the same voltage, so $\dfrac{V_{ref}T_2}{RC} = \dfrac{V_{in}T_1}{RC}$.
3. **Charge balance of the dual-slope ADC.** RC cancels, which gives
$$T_2 = T_1 \cdot \frac{V_{in}}{V_{ref}}$$
4. **Decisive computation.** $T_2 = 100\ \text{ms} \times \dfrac{123.4\ \text{mV}}{100\ \text{mV}} = 123.4\ \text{ms}$.
5. **Check against full scale.** The longest run-down happens at the full-scale input: $T_{2,\max} = 100\ \text{ms} \times \dfrac{200\ \text{mV}}{100\ \text{mV}} = 200\ \text{ms}$. Since $123.4\ \text{ms} < 200\ \text{ms}$, the input is within range.
6. **Conversion time.** In continuous conversion mode the next run-up starts as soon as the run-down ends, so one conversion takes the fixed run-up plus the variable run-down:
$$T_c = T_1 + T_2 = 100 + 123.4 = 223.4\ \text{ms}$$

KEY IDEA

A dual-slope ADC integrates the input for a fixed time T_1, then integrates the opposite-polarity reference until the integrator output returns to zero. Because both ramps cover the same voltage, $T_2 = T_1\, V_{in}/V_{ref}$, and RC drops out. In continuous mode, one conversion lasts the whole cycle $T_1 + T_2$, so the conversion time depends on the input voltage.

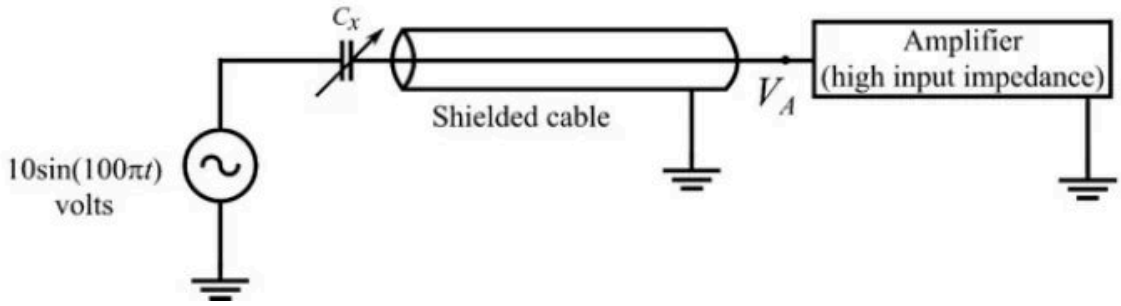
WATCH OUT FOR

Giving only the run-down time $T_2 = 123.4\ \text{ms}$, or only the fixed run-up $T_1 = 100\ \text{ms}$, as the conversion time. A conversion needs both phases, so the answer is $T_1 + T_2 = 223.4\ \text{ms}$. A second error is using the full-scale value (200 mV) in place of V_{ref} (100 mV) in $T_2 = T_1 V_{in}/V_{ref}$, which gives $T_2 = 61.7\ \text{ms}$ and a wrong total of 161.7 ms. Another is taking the worst case, $T_1 + T_{2,\max} = 300\ \text{ms}$, when the question asks about one particular input.

Q.24 NAT · 1 MARK

Sensors and Industrial Instrumentation › Sensing principles

Q.No. 24 The capacitance C_x of a capacitive type sensor is $(1000\,x)$ pF, where x is the input to the sensor. As shown in the figure, the sensor is excited by a voltage $10 \sin(100\pi\, t)$ V. The other terminal of the sensor is tied to the input of a high input impedance amplifier through a shielded cable, with shield connected to ground. The cable capacitance is 100 pF. The peak of the voltage V_A at the input of the amplifier when $x = 0.1$ **(in volts)** is ____



Official answer: **5**

SOLUTION

1. The cable shield is grounded, so the cable capacitance $C_c = 100\ \text{pF}$ sits between the amplifier input node V_A and ground.
2. The amplifier has high input impedance, so it draws negligible current and can be ignored. The source, C_x and C_c then form a series loop, with V_A the voltage across C_c.
3. At $x = 0.1$: $C_x = 1000 \times 0.1 = 100\ \text{pF}$.
4. The same charge Q flows through both series capacitors, so $Q = V_s \dfrac{C_x C_c}{C_x + C_c}$. The voltage across C_c is $V_A = \dfrac{Q}{C_c} = V_s \dfrac{C_x}{C_x + C_c}$.
5. The source peak is 10 V, so $V_{A,\text{peak}} = 10 \times \dfrac{100}{100 + 100} = 5\ \text{V}$.

KEY IDEA

A capacitive sensor feeding a high-impedance amplifier through a grounded-shield cable forms a capacitive divider: the cable capacitance acts as a shunt to ground, and $V_A = V_s \dfrac{C_x}{C_x + C_c}$.

WATCH OUT FOR

Inverting the divider ratio to $\dfrac{C_c}{C_x + C_c}$, as for resistors. For capacitors in series the voltage across one is proportional to the other's capacitance. Here the two values are equal, so the slip happens to give the same number. The ratio matters whenever $C_x \neq C_c$, and a changing C_x is how the sensor produces its output.

Q.25 NAT · 1 MARK

Electrical and Electronic Measurements › Errors & standards

Two $100\ \Omega$ resistors having tolerance $3\ \%$ and $4\ \%$ are connected in series. The effective tolerance of the series combination (**in %, rounded off to one decimal place**) is ___

Official answer: **2.5, or 3.5**

SOLUTION

1. A tolerance is a guaranteed (limiting) error, so it is a worst-case bound. First turn each percentage into an absolute error in ohms: $\delta R_1 = 0.03 \times 100 = 3\ \Omega$ and $\delta R_2 = 0.04 \times 100 = 4\ \Omega$.
2. Rule for a sum: in a series combination $R = R_1 + R_2$, the worst-case absolute errors add.
$$\delta R = \delta R_1 + \delta R_2 = 3 + 4 = 7\ \Omega$$
3. Nominal value of the combination: $R = 100 + 100 = 200\ \Omega$.
4. Effective tolerance of the combination:
$$\frac{\delta R}{R} \times 100 = \frac{7}{200} \times 100 = 3.5\ \%$$
5. Cross-check with the weighted-average form: $\dfrac{R_1}{R}\epsilon_1 + \dfrac{R_2}{R}\epsilon_2 = \frac{1}{2}(3) + \frac{1}{2}(4) = 3.5\ \%$. With equal resistors the result is simply the average of the two tolerances, so the combination is $200\ \Omega \pm 7\ \Omega$, that is $\pm 3.5\ \%$.
6. Why the official key also accepts 2.5: if the two errors are treated as independent random errors and combined by root-sum-square instead of worst case,
$$\delta R = \sqrt{3^2 + 4^2} = 5\ \Omega, \qquad \frac{5}{200} \times 100 = 2.5\ \%$$
The question does not say which convention to use, so both answers are accepted. For a tolerance (a guaranteed limit), the worst-case value $3.5\ \%$ is the standard answer.

KEY IDEA

For a sum of quantities, the absolute limiting errors add: 3 Ω + 4 Ω = 7 Ω on a 200 Ω total. So the relative error of a series combination is the resistance-weighted average of the individual tolerances, not their sum. Treating the errors as independent and combining them by root-sum-square (5 Ω) gives 2.5 % instead, and the key accepts both.

WATCH OUT FOR

Adding the percentages directly (3 % + 4 % = 7 %). Percentage errors add only for a product or a quotient. For a sum you add the absolute errors in ohms and then divide by the total resistance, which gives 7/200 = 3.5 %.

Q.26 MCQ · 2 MARKS

Engineering Mathematics › Linear algebra

Consider the matrix $M = \begin{bmatrix} 1 & -1 & 0 \\ 1 & -2 & 1 \\ 0 & -1 & 1 \end{bmatrix}$. One of the eigenvectors of M is

(A) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Official answer: **D**

SOLUTION

1. Definition: a nonzero vector v is an eigenvector of M if $Mv = \lambda v$ for some scalar λ. Multiply M into each option and check whether the result is a scalar multiple of that option.
2. Quick observation: every row of M sums to zero ($1 - 1 + 0 = 0$, $1 - 2 + 1 = 0$, $0 - 1 + 1 = 0$). Multiplying M by $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ adds up each row, so $M \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0 \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Option (D) is an eigenvector with $\lambda = 0$.
3. Cross-check with the determinant: $\det M = 1\left((-2)(1) - (1)(-1)\right) - (-1)\left((1)(1) - (1)(0)\right) + 0 = -1 + 1 = 0$. So $\lambda = 0$ is an eigenvalue, as expected.
4. Rule out (A): $M \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 + 1 + 0 \\ 1 + 2 + 1 \\ 0 + 1 + 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$. The first entry needs $\lambda(1) = 2$, so $\lambda = 2$. The second needs $\lambda(-1) = 4$, so $\lambda = -4$. No single λ works, so (A) is not an eigenvector.
5. Rule out (B): $M \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 - 1 + 0 \\ 1 - 2 - 1 \\ 0 - 1 - 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ -2 \end{bmatrix}$. The first entry forces $\lambda(1) = 0$, so $\lambda = 0$. But then the second entry would have to be 0, not -2. So (B) is not an eigenvector.
6. Rule out (C): $M \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 - 1 + 0 \\ -1 - 2 - 1 \\ 0 - 1 - 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \\ -2 \end{bmatrix}$. The first entry needs $\lambda(-1) = -2$, so $\lambda = 2$. The second needs $\lambda(1) = -4$, so $\lambda = -4$. That is a contradiction. (This fits, because (C) is just -1 times (A).)
7. Only $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ satisfies $Mv = \lambda v$ (with $\lambda = 0$). The answer is (D).

KEY IDEA

Test each option directly against $Mv = \lambda v$: one value of λ has to work in every entry. A useful shortcut: if every row of a matrix has the same sum s, then $\begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T$ is an eigenvector with eigenvalue s. Here $s = 0$.

WATCH OUT FOR

Stopping once one entry of Mv matches a multiple of v. For option (A) the first and third entries of $Mv = (2, 4, 2)^T$ fit $\lambda = 2$, but the middle entry does not. Every entry has to give the same λ. A second slip is to reject (D) because $Mv = 0$. The zero vector is not allowed as an eigenvector, but $\lambda = 0$ is a perfectly valid eigenvalue, and it occurs exactly when M is singular.

Q.27 MCQ · 2 MARKS

Engineering Mathematics › Differential equations

Consider the differential equation $\frac{dx}{dt} = \sin(x)$, with the initial condition $x(0) = 0$.

The solution to this ordinary differential equation is ___

(A) $x(t) = 0$

(B) $x(t) = \sin(t)$

(C) $x(t) = \cos(t)$

(D) $x(t) = \sin(t) - \cos(t)$

Official answer: **A**

SOLUTION

1. The right-hand side is $\sin(x)$, a function of the dependent variable x, not of t. So this is an autonomous equation $\frac{dx}{dt} = f(x)$ with $f(x) = \sin x$. It is not $\frac{dx}{dt} = \sin t$.
2. Equilibrium (constant) solutions satisfy $f(x) = 0$: $\sin x = 0 \Rightarrow x = n\pi$. The initial value $x(0) = 0$ is one of these equilibria.
3. Check $x(t) \equiv 0$: $\frac{dx}{dt} = 0$ and $\sin(0) = 0$, so the ODE holds. The initial condition $x(0) = 0$ holds too.
4. Uniqueness (Picard–Lindelöf theorem): $f(x) = \sin x$ has $|f'(x)| = |\cos x| \leq 1$, so f is Lipschitz. The initial value problem therefore has exactly one solution, and that solution is $x(t) = 0$.
5. Rule out the other options directly. (C): $x(0) = \cos 0 = 1 \neq 0$. (D): $x(0) = \sin 0 - \cos 0 = -1 \neq 0$. (B) meets the initial condition, but $\frac{dx}{dt} = \cos t$ while $\sin(x) = \sin(\sin t)$. These are not equal: at $t = 0$, $\cos 0 = 1$ but $\sin(\sin 0) = 0$.
6. So the solution is $x(t) = 0$, option (A).

KEY IDEA

In an autonomous equation $\frac{dx}{dt} = f(x)$, every root of $f(x) = 0$ gives a constant solution. If the initial value sits on such a root and f is Lipschitz, uniqueness says the solution never leaves it. Here $\sin 0 = 0$, so $x(t) \equiv 0$.

WATCH OUT FOR

Reading $\sin(x)$ as $\sin(t)$ and integrating gives $x = 1 - \cos t$. That is not an option, and it tempts students to pick (B) or (C) by pattern. A second trap: separating variables as $\frac{dx}{\sin x} = dt$ divides by $\sin x$, which is zero at $x = 0$. That throws away exactly the equilibrium solution the initial condition picks out, and $\ln\left|\tan\frac{x}{2}\right| = t + C$ then has no finite C for $x(0) = 0$.

Q.28 MCQ · 2 MARKS

Engineering Mathematics › Calculus

A straight line drawn on an x-y plane intercepts the x-axis at -0.5 and the y-axis at 1. The equation that describes this line is ___

(A) $y = -0.5\,x + 1$

(B) $y = x - 0.5$

(C) $y = 0.5x - 1$

(D) $y = 2x + 1$

Official answer: **D**

SOLUTION

1. Use the intercept form of a straight line, $\frac{x}{a} + \frac{y}{b} = 1$, where a is the x-intercept and b is the y-intercept. Here $a = -0.5$ and $b = 1$.

2. Substitute:
$$\frac{x}{-0.5}+\frac{y}{1}=1 \;\Rightarrow\; -2x+y=1 \;\Rightarrow\; y=2x+1$$
3. Cross-check with the slope-intercept form $y=mx+c$. The line passes through $(-0.5,\,0)$ and $(0,\,1)$, so $m=\dfrac{1-0}{0-(-0.5)}=2$, and $c=1$ (the y-intercept). Again $y=2x+1$.
4. Check the intercepts of $y=2x+1$: putting $x=0$ gives $y=1$, and putting $y=0$ gives $x=-0.5$. Both match the question.
5. Rule out the other options by their intercepts. (A) $y=-0.5x+1$ cuts the x-axis at $x=2$, not -0.5. (B) $y=x-0.5$ cuts the y-axis at -0.5, not 1. (C) $y=0.5x-1$ cuts the y-axis at -1, not 1. The answer is (D).

KEY IDEA
A line with x-intercept a and y-intercept b (both non-zero) is $\frac{x}{a}+\frac{y}{b}=1$. Its slope is $-\frac{b}{a}$ and the y-intercept is b itself. Here the slope is $-\dfrac{1}{-0.5}=2$ and $c=1$.

WATCH OUT FOR
Taking the slope as b/a or a/b without the minus sign, or using the x-intercept value -0.5 as the slope. Both lead to option (A) $y=-0.5x+1$. That option has the right y-intercept, but it cuts the x-axis at $x=2$. Always check both intercepts of the option you pick.

Q.29 MCQ · 2 MARKS

Control Systems › Nyquist

The loop transfer function of a negative feedback system is $G(s)H(s)=\dfrac{1}{s(s-2)}$. The Nyquist plot for the above system ____

(A) encircles $(-1+j0)$ point once in the clockwise direction

(B) encircles $(-1+j0)$ point once in the counterclockwise direction

(C) does not encircle $(-1+j0)$ point

(D) encircles $(-1+j0)$ point twice in the counterclockwise direction

Official answer: **A**

SOLUTION

1. **Open-loop poles:** $s=0$ and $s=+2$. The pole at $s=0$ lies on the $j\omega$ axis, so the Nyquist contour goes round it with a small indentation into the right half-plane, which leaves it outside the contour. The pole at $s=+2$ is inside. So $P=1$.
2. **Closed-loop poles (a quick shortcut):** $1+G(s)H(s)=0 \Rightarrow s(s-2)+1=0 \Rightarrow s^2-2s+1=0 \Rightarrow (s-1)^2=0$. Both closed-loop poles are at $s=+1$, in the right half-plane, so $Z=2$.
3. **Nyquist criterion:** $N=P-Z$, with N counted positive for counterclockwise encirclements of $-1+j0$. Here $N=1-2=-1$, which means **one clockwise** encirclement.
4. **Check with the frequency response:**
$$G(j\omega)H(j\omega)=\frac{1}{j\omega(j\omega-2)}=\frac{1}{-\omega^2-2j\omega}=\frac{-1}{\omega^2+4}+j\,\frac{2}{\omega(\omega^2+4)}$$
5. **Branch for $\omega: 0^+\to\infty$:** the real part rises steadily from $-\frac{1}{4}$ to 0, and the imaginary part stays positive, falling from $+\infty$ to 0. So the plot comes down from $+j\infty$ along the asymptote $\mathrm{Re}=-\frac{1}{4}$ and reaches the origin at -180° without crossing the real axis. The branch for $\omega:-\infty\to 0^-$ is its mirror image in the third quadrant, running from the origin down to $-j\infty$. Every point on both finite branches has $-\frac{1}{4}\le \mathrm{Re}<0$, so both lie to the right of -1.
6. **Indentation at the origin:** put $s=\varepsilon e^{j\theta}$, with θ going from -90° through 0° to $+90^\circ$. Near the origin, $s-2\approx -2$, so

$$GH \approx \frac{1}{-2s} = -\frac{1}{2\varepsilon}e^{-j\theta}$$

Its angle is $180^\circ - \theta$, which falls from 270° through 180° to 90°. That is an infinite arc running from $-j\infty$ through $-\infty$ to $+j\infty$, clockwise across the left half of the plane. The large semicircle $|s| \to \infty$ maps to the origin.

7. **Direction:** follow the whole contour in order. The $\omega < 0$ branch runs from the origin down to $-j\infty$ (right of -1). The infinite arc then climbs from $-j\infty$ round the left side to $+j\infty$. Finally, the $\omega > 0$ branch comes down from $+j\infty$ to the origin (right of -1). So the closed plot goes down on the right of -1 and back up on the left, which encloses $-1 + j0$ once, **clockwise**. This agrees with $N = -1$, and the closed-loop system is unstable, with two right-half-plane poles.

KEY IDEA

The pole at $s = 0$ maps to an infinite semicircle, and here that semicircle sweeps clockwise through the left half-plane, from $-j\infty$ via $-\infty$ to $+j\infty$. The finite branches never go left of $\mathrm{Re} = -\frac{1}{4}$, so $-1 + j0$ sits between them and the infinite arc and is enclosed once, clockwise. You can confirm this from $N = P - Z$ with $P = 1$ (the pole at $s = 2$) and $Z = 2$ (the closed-loop double pole at $s = 1$).

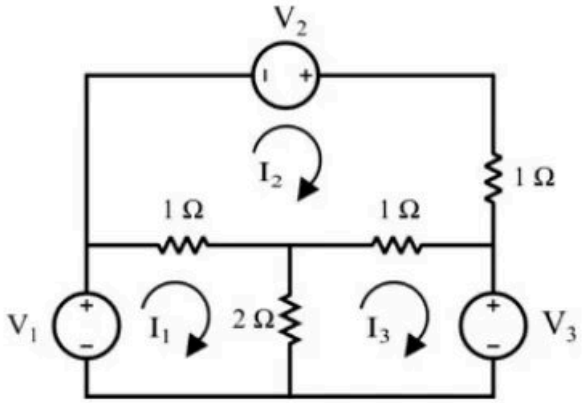
WATCH OUT FOR

The usual slip is to draw only the $\omega > 0$ polar plot. It never crosses the negative real axis, so people answer (C) "does not encircle". Another route to (C) is closing the plot through the right half-plane: for this type-1 system with a right-half-plane pole, the indentation maps to an arc through $-\infty$, not $+\infty$. Others see an open-loop right-half-plane pole ($P = 1$) and assume one counterclockwise encirclement (B). That is only what stability would need. The actual count is $N = P - Z = 1 - 2 = -1$.

Q.30 MCQ · 2 MARKS

Networks (Electric Circuits) › Laws & node/mesh

Q.No. 30 $\mathbf{I}_1$, $\mathbf{I}_2$ and $\mathbf{I}_3$ in the figure below are mesh currents. The correct set of mesh equations for these currents, in matrix form, is ___



(A) $\begin{bmatrix} 3 & -1 & -2 \\ -1 & 3 & -1 \\ -2 & -1 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ -V_3 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & -1 & -2 \\ -1 & 3 & -1 \\ -2 & -1 & -3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$

(C) $\begin{bmatrix} -3 & -1 & -2 \\ -1 & 3 & -1 \\ -2 & -1 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ -V_3 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & -1 & -2 \\ -1 & 2 & -1 \\ -2 & -1 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$

Official answer: **A**

SOLUTION

1. **Self-resistances (diagonal entries).** Mesh 1: $1 + 2 = 3\,\Omega$. Mesh 2: $1 + 1 + 1 = 3\,\Omega$ (the right $1\,\Omega$ plus the two middle $1\,\Omega$ resistors). Mesh 3: $1 + 2 = 3\,\Omega$.
2. **Mutual resistances (off-diagonal entries).** Meshes 1 and 2 share the L–M $1\,\Omega$. Meshes 2 and 3 share the M–R $1\,\Omega$. Meshes 1 and 3 share the $2\,\Omega$. All three currents are clockwise, so in every shared branch the two currents flow in opposite directions, and every mutual term is negative: $R_{12} = -1$, $R_{23} = -1$, $R_{13} = -2$.
3. **KVL in mesh 1 (clockwise).** Going up through V_1 from − to + is a rise of V_1:
$$V_1 = 1\,(I_1 - I_2) + 2\,(I_1 - I_3) \;\Rightarrow\; 3I_1 - I_2 - 2I_3 = V_1$$
4. **KVL in mesh 2 (clockwise).** Going left to right along the top through V_2 runs from − to +, a rise of V_2:
$$V_2 = 1\,I_2 + 1\,(I_2 - I_3) + 1\,(I_2 - I_1) \;\Rightarrow\; -I_1 + 3I_2 - I_3 = V_2$$
5. **KVL in mesh 3 (clockwise).** Going down the right side through V_3 runs from + to −, a drop of V_3:
$$0 = 1\,(I_3 - I_2) + V_3 + 2\,(I_3 - I_1) \;\Rightarrow\; -2I_1 - I_2 + 3I_3 = -V_3$$
6. **Matrix form.**
$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 3 & -1 \\ -2 & -1 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ -V_3 \end{bmatrix}$$

This is option (A). Option (C) has the wrong sign on R_{11}. Option (B) has the wrong sign on R_{33} and on V_3. Option (D) has the wrong self-resistances R_{11} and R_{22}, and the wrong sign on V_3.

KEY IDEA

When every mesh current in a resistive network without dependent sources is drawn in the same direction, you can write the mesh equations by inspection. The resistance matrix is symmetric. Each diagonal entry is the total resistance around that mesh and is always positive. Each off-diagonal entry is minus the resistance the two meshes share. Each right-hand-side entry is the net source voltage rise met when you go around the mesh in the direction of its current.

WATCH OUT FOR

Writing $+V_3$ instead of $-V_3$. The clockwise current I_3 goes down through V_3 from its + terminal to its – terminal, so V_3 is a drop in that mesh and appears as $-V_3$ on the source side. Another mistake is accepting a negative diagonal entry (options B and C). A self-resistance can never be negative, and the matrix must be symmetric. Those two checks rule out B and C straight away.

Q.31 NAT · 2 MARKS

Engineering Mathematics › Calculus

Consider the function $f(x, y) = x^2 + y^2$. The minimum value the function attains on the line $x + y = 1$ (**rounded off to one decimal place**) is ____

Official answer: **0.5**

SOLUTION

1. Use the constraint to remove a variable: $y = 1 - x$. On the line, f becomes a function of x alone:
$$g(x) = x^2 + (1 - x)^2 = 2x^2 - 2x + 1$$
2. Set the first derivative to zero: $g'(x) = 4x - 2 = 0 \Rightarrow x = \frac{1}{2}$, which gives $y = 1 - \frac{1}{2} = \frac{1}{2}$.
3. Second-derivative test: $g''(x) = 4 > 0$, so $x = \frac{1}{2}$ is a minimum. Because g is an upward-opening parabola, this is the global minimum over the whole line, not just a local one.
4. Evaluate: $f\left(\frac{1}{2}, \frac{1}{2}\right) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} = 0.5$.
5. Check with Lagrange multipliers. Write the constraint as $h(x, y) = x + y - 1 = 0$. Then $\nabla f = \lambda \nabla h$ gives $2x = \lambda$ and $2y = \lambda$, so $x = y$. With $x + y = 1$ this gives $x = y = \frac{1}{2}$ again.
6. Geometric check: $f = x^2 + y^2$ is the squared distance from the origin. The distance from the origin to the line $x + y - 1 = 0$ is
$$\frac{|0 + 0 - 1|}{\sqrt{1^2 + 1^2}} = \frac{1}{\sqrt{2}}$$
and its square is $\frac{1}{2} = 0.5$. All three methods give a minimum value of 0.5.

KEY IDEA

A constrained extremum on a line becomes a one-variable problem once you substitute the constraint ($y = 1 - x$), which gives a parabola whose vertex is the minimum. Lagrange multipliers give the same point. Geometrically, $x^2 + y^2$ is the squared distance from the origin, so the answer is the square of the perpendicular distance from the origin to the line, $\left(\frac{1}{\sqrt{2}}\right)^2 = 0.5$.

WATCH OUT FOR

Giving the distance $\frac{1}{\sqrt{2}} \approx 0.7$ instead of its square: f is the squared distance, so the answer is 0.5. Another slip is to find the critical point $x = y = \frac{1}{2}$ and then enter the coordinate 0.5 without checking that it is the value of f there. It happens to be the same number here, but only by coincidence.

Q.32 NAT · 2 MARKS

Engineering Mathematics › Probability & statistics

Consider two identical bags B1 and B2 each containing 10 balls of identical shapes and sizes. Bag B1 contains 7 Red and 3 Green balls, while bag B2 contains 3 Red and 7 Green balls. A bag is picked at random and a ball is drawn from it, which was found to be Red. The probability that the Red ball came from bag B1 (**rounded off to one decimal place**) is ___

Official answer: **0.7**

SOLUTION

1. Priors: the bag is picked at random, so $P(B_1) = P(B_2) = \frac{1}{2}$.
2. Likelihoods: each bag holds 10 balls, so $P(R \mid B_1) = \frac{7}{10} = 0.7$ and $P(R \mid B_2) = \frac{3}{10} = 0.3$.
3. Total probability theorem: $P(R) = P(B_1)P(R \mid B_1) + P(B_2)P(R \mid B_2) = \frac{1}{2}(0.7) + \frac{1}{2}(0.3) = 0.5$.
4. Bayes' theorem:
$$P(B_1 \mid R) = \frac{P(B_1)\,P(R \mid B_1)}{P(R)} = \frac{0.5 \times 0.7}{0.5} = 0.7$$
5. Check: $P(B_2 \mid R) = \frac{0.5 \times 0.3}{0.5} = 0.3$, and $0.7 + 0.3 = 1$. Rounded to one decimal place, the answer is 0.7.

KEY IDEA

This is Bayes' theorem. The two bags are equally likely to be picked, so the priors cancel. The posterior $P(B_1 \mid R)$ is then bag B1's share of all the red balls: $\frac{7}{7+3} = 0.7$.

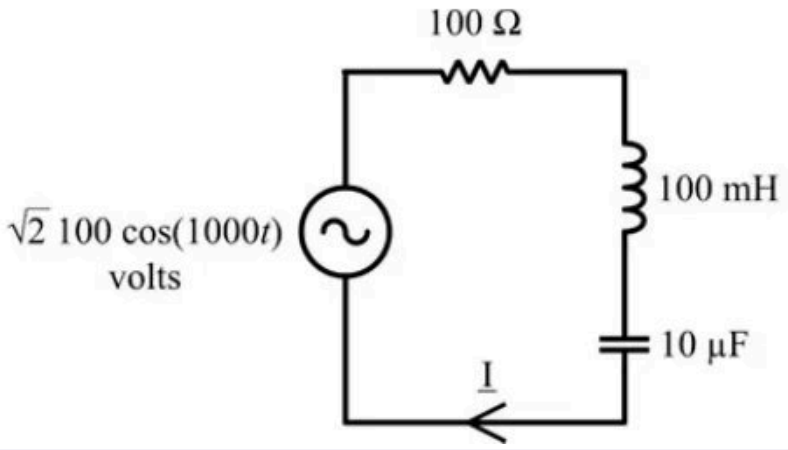
WATCH OUT FOR

The common slip is to answer $P(R) = 0.5$, the overall chance of drawing a red ball, or to multiply out only the numerator, $P(B_1)P(R \mid B_1) = 0.35$, and forget to divide by $P(R)$. Both confuse the joint or total probability with the conditional probability $P(B_1 \mid R)$.

Q.33 NAT · 2 MARKS

Networks (Electric Circuits) › AC, power & resonance

Q.No. 33 The rms value of the phasor current **I** in the circuit shown (**in amperes**) is ___



Official answer: **1**

SOLUTION

1. Source phasor: $v(t) = \sqrt{2}\,100\cos(1000t)$ V has a peak of $100\sqrt{2}$ V, so its rms phasor is $\mathbf{V} = 100\angle 0^\circ$ V (rms), with $\omega = 1000$ rad/s.
2. Inductive reactance: $X_L = \omega L = 1000 \times 100 \times 10^{-3} = 100\ \Omega$.

3. Capacitive reactance: $X_C = \dfrac{1}{\omega C} = \dfrac{1}{1000 \times 10 \times 10^{-6}} = 100\ \Omega$.
4. Series impedance: $Z = R + j(X_L - X_C) = 100 + j(100 - 100) = 100\ \Omega$. Because $X_L = X_C$, the circuit is at series resonance: $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{0.1 \times 10^{-5}}} = 1000\ \text{rad/s}$, which is the source frequency.
5. Ohm's law in phasor form: $\mathbf{I} = \dfrac{\mathbf{V}}{Z} = \dfrac{100\angle 0^\circ}{100} = 1\angle 0^\circ\ \text{A}$ (rms). The arrow's direction can only flip the sign, which shifts the phase by 180°. It cannot change the magnitude.
6. rms value of $\mathbf{I}$: $|\mathbf{I}| = 1\ \text{A}$.

KEY IDEA

At $\omega = 1000\ \text{rad/s}$ the series L and C have equal reactances ($100\ \Omega$ each), so the circuit is at series resonance. The two reactances cancel, only the $100\ \Omega$ resistor limits the current, and $I_{\text{rms}} = V_{\text{rms}}/R = 100/100 = 1\ \text{A}$.

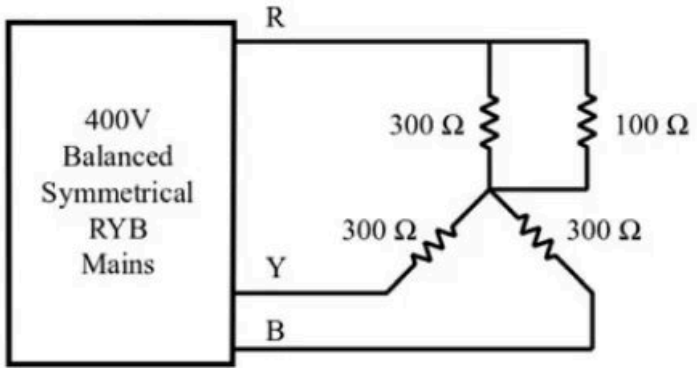
WATCH OUT FOR

Using the peak voltage $100\sqrt{2}\ \text{V}$ in place of the rms value gives $1.414\ \text{A}$, which is the peak current, not the rms current asked for. A second slip is adding the reactances ($X_L + X_C$) when they should be subtracted. That gives $|Z| = \sqrt{100^2 + 200^2}$ and a current that is far too small.

Q.34 NAT · 2 MARKS

Networks (Electric Circuits) › Three-phase (EE)

Q.No. 34 In the circuit shown, the rms value of the voltage across the $100\ \Omega$ resistor **(in volts)** is ___



Official answer: **115 to 116**

SOLUTION

1. **Reduce the R arm.** The 300 Ω and 100 Ω resistors are both connected between line R and the star point, so they are in parallel:
$$Z_R = \frac{300 \times 100}{300 + 100} = 75\ \Omega, \qquad Z_Y = Z_B = 300\ \Omega$$
The voltage across the 100 Ω resistor is therefore the whole R-arm voltage $V_{RN'}$.
2. **Source phase voltage.** Take 400 V as the line voltage of the balanced supply: $V_{ph} = \dfrac{400}{\sqrt{3}} = 230.94\ \text{V}$. Let $V_R = 230.94\angle 0^\circ\ \text{V}$ (measured from the source neutral N). Because the supply is balanced, $V_R + V_Y + V_B = 0$, so $V_Y + V_B = -V_R$.
3. **Neutral shift (Millman's theorem at the floating star point N').** There is no neutral wire, so
$$V_{N'N} = \frac{V_R/Z_R + V_Y/Z_Y + V_B/Z_B}{1/Z_R + 1/Z_Y + 1/Z_B} = \frac{(4V_R + V_Y + V_B)/300}{6/300} = \frac{3V_R}{6} = \frac{V_R}{2}$$
(using $1/75 = 4/300$).
4. **KVL across the R arm.**

$$V_{RN'} = V_R - V_{N'N} = \frac{V_R}{2} = \frac{230.94}{2}\angle 0^\circ = 115.47\angle 0^\circ \text{ V}$$

5. **Check with a Thevenin equivalent.** The Y and B arms are equal, so seen from the star point they reduce to $300 \parallel 300 = 150\ \Omega$ behind a source $\frac{V_Y + V_B}{2} = -\frac{V_R}{2}$ (the midpoint of the Y–B line voltage). The voltage from R to that point is $V_R + \frac{V_R}{2} = 1.5 \times 230.94 = 346.41$ V, i.e. $400 \times \frac{\sqrt{3}}{2}$. By the voltage divider:

$$V_{100\,\Omega} = 346.41 \times \frac{75}{75 + 150} = 115.47 \text{ V}$$

6. The rms voltage across the 100 Ω resistor is 115.47 V.

KEY IDEA

In an unbalanced star load with no neutral, the star point floats. Find its shift from the source neutral with Millman's theorem, then subtract it from the phase voltage. Here the R arm is 75 Ω against 300 Ω in each of the other two arms, which puts the star point exactly halfway to V_R, so the R arm gets V_R/2.

WATCH OUT FOR

The usual error is to assume the star point stays at the source neutral and answer 230.94 V (the full phase voltage). That ignores the neutral shift an unbalanced, ungrounded star produces. A second error is to put only part of the arm voltage across the 100 Ω by treating it as in series with the 300 Ω, when the two resistors are actually in parallel.

Q.35 NAT · 2 MARKS

Signals and Systems › LTI & convolution

Let $g[n] = \begin{cases} 1 & n = 0 \\ 0 & n = \pm 1, \pm 2, \pm 3, \ldots \end{cases}$ and $h[n] = \begin{cases} 1 & n = 0, 3, 6, 9, \ldots \\ 0 & \text{otherwise} \end{cases}$.

Consider $y[n] = h[n] \otimes g[n]$, where $\otimes$ denotes the convolution operator. The value of $y[2]$ is ____

Official answer: **0**

SOLUTION

1. Identify $g[n]$: it equals 1 at $n = 0$ and 0 at every other integer, so $g[n] = \delta[n]$, the unit impulse.
2. Write the convolution sum: $y[n] = \sum_{k=-\infty}^{\infty} h[k]\, g[n-k] = \sum_{k=-\infty}^{\infty} h[k]\, \delta[n-k]$.
3. Apply the sifting property. $\delta[n-k]$ is non-zero only when $k = n$, so only one term of the sum survives and $y[n] = h[n]$. Convolving with the unit impulse leaves any sequence unchanged.
4. Evaluate at $n = 2$: $y[2] = h[2]$.
5. $h[n] = 1$ only for $n \in \{0, 3, 6, 9, \ldots\}$, the non-negative multiples of 3. Since 2 is not a multiple of 3, $h[2] = 0$.
6. Therefore

$$y[2] = h[2] = 0$$

KEY IDEA

$g[n]$ is the unit impulse $\delta[n]$, and convolving any sequence with $\delta[n]$ gives the sequence back: $h[n] \otimes \delta[n] = h[n]$. So $y[2]$ is simply $h[2]$, which is 0 because 2 is not a multiple of 3.

WATCH OUT FOR

Not noticing that $g[n]$ is just $\delta[n]$ and working through the convolution sum term by term, which risks errors with the flipped index. A second mistake is misreading $h[n]$ as 1 at every n up to 9, or as periodic in both directions, and answering 1. It is non-zero only at $n = 0, 3, 6, 9, \ldots$

Q.36 NAT · 2 MARKS

Control Systems › Root locus

The loop transfer function of a negative feedback system is given by

$$G(s)H(s) = \frac{K}{s(s+2)(s+6)},$$

where $K > 0$. The value of K at the breakaway point of the root locus for the above system (**rounded off to one decimal place**) is ____

Official answer: **5.0 to 5.1**

SOLUTION

1. Open-loop poles are $0, -2, -6$, and there are no finite zeros. **Real-axis rule:** a point on the real axis is on the locus when an odd number of poles and zeros lie to its right. So the locus covers $-2 < s < 0$ and $s < -6$. The breakaway point lies in $(-2, 0)$, where the branches starting at 0 and -2 meet and leave the axis.
2. **Characteristic equation:** $1 + G(s)H(s) = 0 \Rightarrow s(s+2)(s+6) + K = 0 \Rightarrow s^3 + 8s^2 + 12s + K = 0$, so $K = -(s^3 + 8s^2 + 12s)$.
3. **Breakaway condition** $\dfrac{dK}{ds} = 0$:
$$3s^2 + 16s + 12 = 0 \;\Rightarrow\; s = \frac{-16 \pm \sqrt{256 - 144}}{6} = \frac{-16 \pm \sqrt{112}}{6}$$
4. The roots are $s = -0.9028$ and $s = -4.431$. The root $s = -4.431$ lies between -2 and -6, which is not part of the locus, so reject it. (It would also need $K \approx -16.9 < 0$.) The breakaway point is $s_b = -0.9028$.
5. **Magnitude condition** at s_b:
$$K = |s_b|\,|s_b + 2|\,|s_b + 6| = (0.9028)(1.0972)(5.0972)$$
6. $(0.9028)(1.0972) = 0.9906$, and $0.9906 \times 5.0972 = 5.049$. As a check, $-(s_b^3 + 8s_b^2 + 12s_b) = -(-0.7359 + 6.5208 - 10.8340) = 5.049$.
7. So $K \approx 5.049$, which is 5.0 to one decimal place.

KEY IDEA

Find the breakaway point from $dK/ds = 0$, where K comes from the characteristic equation. Keep only the root that lies on a real-axis segment of the locus. Then get the gain there from the magnitude condition $K = \prod |s_b - p_i|$.

WATCH OUT FOR

Stopping at the breakaway point $s_b \approx -0.90$ and giving that as the answer, when the question asks for K. Another slip is picking the root $s = -4.43$, which lies between -2 and -6. That segment is not on the locus, and that root gives a negative K.

Q.37 NAT · 2 MARKS

Control Systems › Time response

Q.No. 37 The system shown in Fig. (a) has a time response y(t) to an input $r(t) = 10\,u(t)$ as shown in Fig. (b), $u(t)$ being the unit step input. Both K, τ are positive. The gain K of the system is ___

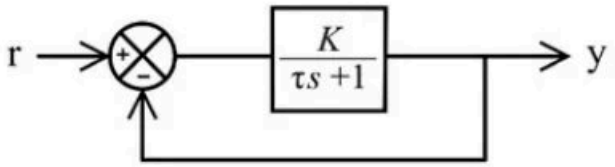


Fig. (a)

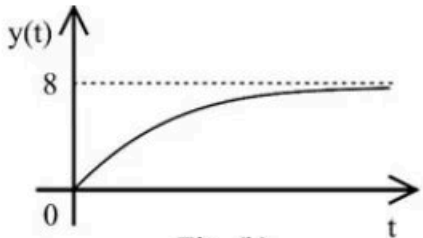


Fig. (b)

Official answer: **4**

SOLUTION

1. Closed-loop transfer function with unity negative feedback:
$$\frac{Y(s)}{R(s)} = \frac{\frac{K}{\tau s+1}}{1+\frac{K}{\tau s+1}} = \frac{K}{\tau s+1+K}$$
2. The input is $r(t) = 10\,u(t)$, so $R(s) = \dfrac{10}{s}$.
3. Final value theorem:
$$y(\infty) = \lim_{s\to 0} s \cdot \frac{K}{\tau s+1+K} \cdot \frac{10}{s} = \frac{10K}{1+K}$$
The theorem applies because the only closed-loop pole, $s = -\dfrac{1+K}{\tau}$, lies in the left half-plane when $K, \tau > 0$.
4. Fig. (b) shows a steady-state value of 8:
$$\frac{10K}{1+K} = 8 \;\Rightarrow\; 10K = 8+8K \;\Rightarrow\; K = 4$$
5. Check: the loop is type 0 with $K_p = \lim_{s\to 0} \dfrac{K}{\tau s+1} = K = 4$. So $e_{ss} = \dfrac{10}{1+K_p} = 2$ and $y(\infty) = 10-2 = 8$, which agrees with Fig. (b). The steady state does not depend on τ, so the figure gives no information about it and none is needed.

KEY IDEA

With unity feedback around a type-0 first-order plant, the closed-loop DC gain is $\frac{K}{1+K}$. The step response settles at $10 \cdot \frac{K}{1+K}$, not at $10K$. Set this equal to the asymptote in the plot, 8, and solve for K. The time constant τ only changes how fast the response rises, not where it settles.

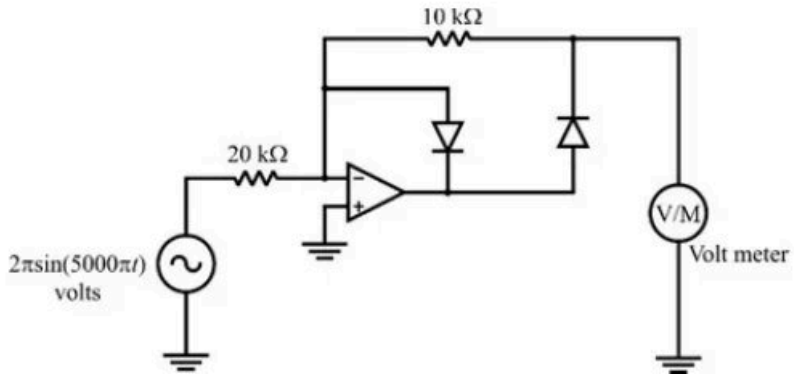
WATCH OUT FOR

Using the open-loop gain as though the loop were open, $10K = 8$ giving $K = 0.8$, which ignores the feedback path. Another error is reading the 8 as the steady-state error rather than the output, which gives $\frac{10}{1+K} = 8$ and $K = 0.25$. Some candidates also try to find τ from the curve, but the plot has no time scale and τ is not asked for.

Q.38 NAT · 2 MARKS

Analog Electronics › Op-amps

Q.No. 38 Assuming that the opamp used in the circuit shown is ideal, the reading of the 1 Hz bandwidth, permanent magnet moving coil (PMMC) type voltmeter (**in volts**) is ___



Official answer: **1**

SOLUTION

1. Assumptions: the op-amp is ideal. Its + input is grounded, so the virtual short holds the − input at $0\ \text{V}$, and no current flows into either input. The PMMC meter is treated as an ideal voltmeter that draws no current. The figure does not name the diodes, so call them D1 (anode at the − node, cathode at the op-amp output) and D2 (anode at the op-amp output, cathode at the meter node v_o). The $10\ \text{k}\Omega$ resistor runs from the − node to the meter node.
2. Positive half-cycle ($v_i > 0$): a current $v_i/20\ \text{k}\Omega$ flows into the − node, so the op-amp output swings negative. D1 conducts and closes the feedback loop. D2 has its anode at the negative op-amp output, so it is reverse biased.
3. With D2 off, the only things connected to the meter node are the $10\ \text{k}\Omega$ resistor and the meter. The meter draws no current, so no current flows in the $10\ \text{k}\Omega$ resistor either, and $v_o = v_- = 0\ \text{V}$.
4. Negative half-cycle ($v_i < 0$): current leaves the − node through the $20\ \text{k}\Omega$ resistor, and the op-amp output swings positive. D1 is reverse biased and D2 conducts, so the op-amp drives the meter node through D2 and feedback returns through the $10\ \text{k}\Omega$ resistor. KCL at the virtual-ground node gives an inverting amplifier:
$$v_o = -\frac{10\ \text{k}\Omega}{20\ \text{k}\Omega}\, v_i = -0.5\, v_i > 0$$
Both diodes sit inside the feedback loop, so their forward drops do not change v_o in either half-cycle.
5. So v_o is a half-wave rectified sine. It is non-zero only while $v_i < 0$, with peak $V_m = 0.5 \times 2\pi = \pi\ \text{V}$. The input has $\omega = 5000\pi\ \text{rad/s}$, so $f = 2500\ \text{Hz}$ and $T = 0.4\ \text{ms}$.
6. A PMMC meter responds to the average (DC) value. Its bandwidth is only $1\ \text{Hz}$, and every AC component of v_o is at $2500\ \text{Hz}$ or a multiple of it, so the meter shows only the DC component:

$$V_{\text{avg}} = \frac{1}{T}\int_{T/2}^{T} (-V_m \sin\omega t)\, dt = \frac{V_m}{T}\cdot\frac{2}{\omega} = \frac{V_m}{\pi} = \frac{\pi}{\pi} = 1\ \text{V}$$

KEY IDEA

This is an inverting precision half-wave rectifier. In one half-cycle the output is held at 0 V. In the other it is $-(R_f/R_1)v_i$, with gain $10/20 = 0.5$. The diode drops sit inside the feedback loop, so they have no effect. A PMMC meter with a 1 Hz bandwidth reads only the DC (average) value, V_m/π for a half-wave rectified sine.

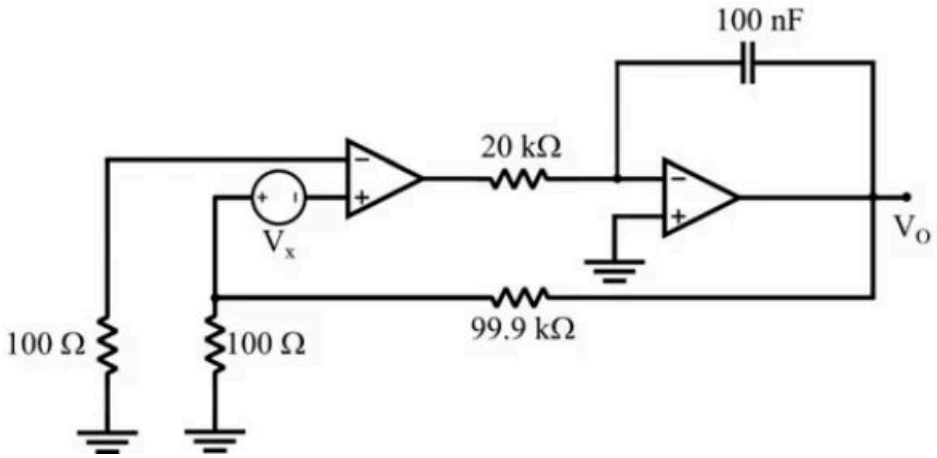
WATCH OUT FOR

Common mistakes: forgetting the gain of 0.5 and using the source peak 2π V, which gives 2 V; using the full-wave average $2V_m/\pi$ (2 V); or taking the RMS value $V_m/2$ ($\pi/2 \approx 1.57$ V). A PMMC meter reads the average, and a half-wave output averages to V_m/π.

Q.39 NAT · 2 MARKS

Analog Electronics › Op-amps

Q.No. 39 If the opamps in the circuit shown are ideal and $V_x = 0.5$ mV, the steady state value of V_O **(in volts, rounded off to two decimal places)** is ___



Official answer: **0.45 to 0.55**

SOLUTION

1. **Assumptions.** Both op-amps are ideal: open-loop gain $A \to \infty$ and zero input current. V_X is an ideal DC source. Op-amp 1's inverting input draws no current, so the left $100\ \Omega$ resistor carries no current and $V_- = 0$.
2. V_X connects only to op-amp 1's non-inverting input, which draws no current, so no current flows through V_X. The $100\ \Omega$ and $99.9\ \text{k}\Omega$ resistors are therefore an unloaded divider from V_O to node N:
$$V_N = V_O\,\frac{100}{100 + 99\,900} = \frac{V_O}{1000}$$
3. The $+$ terminal of V_X is at node N and its $-$ terminal is at op-amp 1's $+$ input, so going through the source gives
$$V_+ = V_N - V_X = \frac{V_O}{1000} - V_X$$
4. Op-amp 2 is an inverting integrator: $V_O(s) = -\dfrac{V_1(s)}{sRC}$ with $RC = 20\ \text{k}\Omega \times 100\ \text{nF} = 2\ \text{ms}$. Here $V_1 = A\,(V_+ - V_-)$ is op-amp 1's output.
5. **Check that the loop settles.** Combining steps 1 to 4:
$$sRC\,V_O(s) = -A\left(\frac{V_O(s)}{1000} - V_X(s)\right) \;\Rightarrow\; \frac{V_O(s)}{V_X(s)} = \frac{A}{sRC + A/1000}$$
This has a single pole at $s = -\dfrac{A}{1000\,RC}$, which is in the left half-plane for any $A > 0$. The loop is negative feedback: if V_O rises, V_+ rises, V_1 rises, and the inverting integrator pulls V_O back down. So a

steady state exists.

6. **Steady state.** Any non-zero constant V_1 at the integrator's input would make V_O ramp without limit, so in steady state $V_1 = 0$. Since $V_1 = A\,(V_+ - V_-)$, this means $V_+ = V_- = 0$.
7. So $V_N = V_X$, which gives $\dfrac{V_O}{1000} = 0.5\ \text{mV}$ and $V_O = 1000 \times 0.5\ \text{mV} = 0.50\ \text{V}$. Check with the final value theorem for a DC input $V_X(s) = V_X/s$:

$$V_O(\infty) = \lim_{s\to 0} s \cdot \frac{A}{sRC + A/1000} \cdot \frac{V_X}{s} = 1000\,V_X = 0.50\ \text{V}$$

KEY IDEA

An integrator inside a negative-feedback loop drives its own input to zero at DC. That input is op-amp 1's output, so op-amp 1's inputs end up at the same voltage: $V_+ = V_- = 0$. The divider voltage $V_O/1000$ must therefore exactly cancel V_X, so $V_O = 1000\,V_X$. The integrator enforces this even when A is finite, so the result does not depend on A.

WATCH OUT FOR

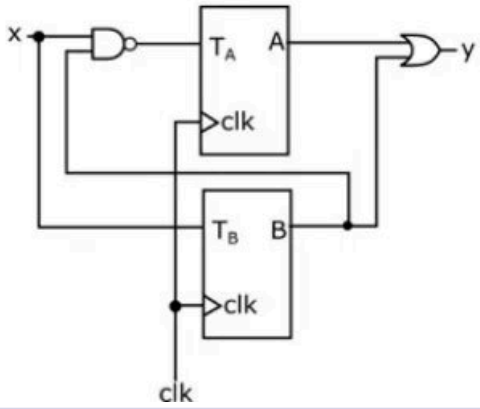
One mistake is reading the source polarity backwards. Its $-$ sign is drawn as a short vertical bar because the source is drawn horizontally, and a backwards reading gives $V_O = -0.50$ V. Another is treating op-amp 1 as an open-loop comparator that saturates. A third is assuming a DC input makes the integrator output ramp forever, when in fact the loop drives the integrator's input to zero. Students also sometimes load the divider with a current through V_X, but that current is zero because V_X feeds an op-amp input. Always check that the overall loop is negative feedback before using the virtual short.

Q.40 NAT · 2 MARKS

Digital Electronics › Sequential

Q.No. 40

Two T-flip flops are interconnected as shown in the figure. The present state of the flip flops are: A = 1, B = 1. The input x is given as 1, 0, 1 in the next three clock cycles. The decimal equivalent of $(ABy)_2$ with A being the MSB and y being the LSB, after the 3rd clock cycle is ___



Official answer: **7**

SOLUTION

1. Read the circuit (complement written with -): the NAND gate takes x and B, so $T_A = \overline{x\,B}$; the wire from the x junction goes straight to flip-flop B, so $T_B = x$; the OR gate takes A and B, so $y = A + B$.
2. T flip-flop rule: $Q^+ = T \oplus Q$. The flip-flop toggles when $T = 1$ and holds when $T = 0$. The T inputs are worked out from the present state, before the clock edge.

3. Clock 1 ($x = 1$, present $A = 1$, $B = 1$): $T_A = \overline{1 \cdot 1} = 0$, so A holds at 1. $T_B = 1$, so B toggles to 0. New state $AB = 10$.
4. Clock 2 ($x = 0$, present $A = 1$, $B = 0$): $T_A = \overline{0 \cdot 0} = 1$, so A toggles to 0. $T_B = 0$, so B holds at 0. New state $AB = 00$.
5. Clock 3 ($x = 1$, present $A = 0$, $B = 0$): $T_A = \overline{1 \cdot 0} = 1$, so A toggles to 1. $T_B = 1$, so B toggles to 1. New state $AB = 11$.
6. Output after the 3rd clock: y is a purely combinational function of the new state, so $y = A + B = 1 + 1 = 1$.
7. $(ABy)_2 = (111)_2 = 4 + 2 + 1 = 7$.

KEY IDEA

Write down the excitation equations from the gates ($T_A = \overline{x\,B}$, $T_B = x$). Then step through a state table one clock at a time, working each T input out from the present state and that cycle's x, and using $Q^+ = T \oplus Q$. The output $y = A + B$ is combinational, so read it from the state after the 3rd clock.

WATCH OUT FOR

The usual slips: (i) reading the gate as an AND and forgetting the bubble, which gives $T_A = x\,B$ and a different sequence; (ii) working a cycle's T_A out from the value B will have after the edge instead of its present value (at clock 1, for example, T_A uses $B = 1$, not the new $B = 0$); (iii) taking y from the state before the 3rd clock, or leaving y out of the 3-bit word.

Q.41 NAT · 2 MARKS

Digital Electronics › ADC & DAC

The address lines $A_9 \ldots A_2$ of a 10 bit, 1.023 V full-scale digital to analog converter (DAC) is connected to the data lines D_7 to D_0 of an 8-bit microprocessor, with A_1 and A_0 of the DAC grounded. Now, $D_7 \ldots D_0$ is changed from 1010 1010 to 1010 1011. The corresponding change in the output of the DAC (**in mV, rounded off to one decimal place**) is ___

Official answer: **3.5 to 4.5**

SOLUTION

1. **Step size of the DAC.** A 10-bit DAC has codes 0 to $2^{10} - 1 = 1023$. The full-scale (largest) output is 1023 steps, so the step is $\Delta = \dfrac{V_{FS}}{2^n - 1} = \dfrac{1.023\ \text{V}}{1023} = 1\ \text{mV}$.
2. **How the processor drives the DAC.** $D_7 \ldots D_0$ go to $A_9 \ldots A_2$, and $A_1 = A_0 = 0$. So the 10-bit DAC code is the 8-bit data word followed by two 0s: $N_{DAC} = (D_7 \ldots D_0\,00)_2 = 4 \times N_{data}$.
3. **Change in the data word.** $(1010\,1010)_2 = 170$ and $(1010\,1011)_2 = 171$, so $\Delta N_{data} = 1$.
4. **Change in the DAC code.** The DAC codes are $(1010101000)_2 = 680$ and $(1010101100)_2 = 684$, so $\Delta N_{DAC} = 4 \times 1 = 4$. Each LSB of the processor's data word is worth four DAC steps.
5. **Output change.**
$$\Delta V_o = \Delta N_{DAC} \times \Delta = 4 \times 1\ \text{mV} = 4.0\ \text{mV}$$
6. **Check with the other convention.** If you take the step as $V_{FS}/2^n = 1.023/1024\ \text{V} \approx 0.999\ \text{mV}$, you get $\Delta V_o = 4 \times 0.999\ \text{mV} \approx 3.996\ \text{mV}$. That still rounds to $4.0\ \text{mV}$ at one decimal place, so the answer is the same either way.

KEY IDEA

The two DAC LSBs (A_1, A_0) are grounded and the processor drives the upper eight bits, so the 10-bit code is always $4\times$ the data word. A one-count change in the data therefore moves the DAC by $2^2 = 4$ steps of $1.023/1023\ \text{V} = 1\ \text{mV}$ each.

WATCH OUT FOR

Treating the change as one DAC step (1 mV), which ignores that D_0 drives A_2, not A_0. Another slip is to work out the step from an 8-bit DAC ($1.023/255\ \text{V} \approx 4.01\ \text{mV}$). That lands inside the key range by luck, but it describes the wrong hardware.

Q.42 NAT · 2 MARKS

Networks (Electric Circuits) › Three-phase (EE)

The real power drawn by a balanced load connected to a 400 V, 50 Hz, balanced, symmetrical 3-phase, 3-wire, RYB sequence mains is measured using the two-wattmeter method. Wattmeter W_1 is connected in the R line and wattmeter W_2 is connected in the B line. The line current is measured as $\frac{1}{\sqrt{3}}$ A. If the wattmeter W_1 reads zero, the reading on W_2 (in watts) is ____

Official answer: **199 to 201**

SOLUTION

1. Set up the connection. In the two-wattmeter method, the pressure coils of both meters return to the third line, Y. The current coil of W_1 is in R and the current coil of W_2 is in B. So W_1 measures V_{RY} against I_R, and W_2 measures V_{BY} against I_B.
2. Write the phasors for RYB sequence, taking $V_{RN} = V_p\angle 0^\circ$ as reference, with $V_{YN} = V_p\angle -120^\circ$ and $V_{BN} = V_p\angle 120^\circ$. Let the load power-factor angle be ϕ, with lagging taken as positive:
$$V_{RY} = V_{RN} - V_{YN} = \sqrt{3}V_p\angle 30^\circ, \quad I_R = I_L\angle -\phi$$
$$V_{BY} = V_{BN} - V_{YN} = \sqrt{3}V_p\angle 90^\circ, \quad I_B = I_L\angle (120^\circ - \phi)$$
3. Wattmeter readings, using $W = VI\cos(\text{angle between } V \text{ and } I)$ with $V_L = \sqrt{3}V_p$:
$$W_1 = V_L I_L \cos\big(30^\circ - (-\phi)\big) = V_L I_L \cos(30^\circ + \phi)$$
$$W_2 = V_L I_L \cos\big(90^\circ - (120^\circ - \phi)\big) = V_L I_L \cos(30^\circ - \phi)$$
4. Apply the given condition. $W_1 = 0 \Rightarrow \cos(30^\circ + \phi) = 0 \Rightarrow \phi = 60^\circ$ lagging, i.e. power factor 0.5 lagging. The other root, $\phi = -120^\circ$, would put $|\phi| > 90^\circ$, meaning the load delivers power instead of drawing it. That is excluded.
5. Reading of W_2, with $V_L = 400$ V and $I_L = \frac{1}{\sqrt{3}}$ A:
$$W_2 = 400 \times \frac{1}{\sqrt{3}} \times \cos(30^\circ - 60^\circ) = \frac{400}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 200 \text{ W}$$
6. Check: total power $P = \sqrt{3}V_L I_L \cos\phi = \sqrt{3} \times 400 \times \frac{1}{\sqrt{3}} \times 0.5 = 200 \text{ W} = W_1 + W_2$, as expected.

KEY IDEA

In the two-wattmeter method, $W_1 = V_L I_L \cos(30^\circ + \phi)$ and $W_2 = V_L I_L \cos(30^\circ - \phi)$. If one meter reads zero, then $\phi = 60^\circ$ (power factor 0.5), and the other meter carries all the power: $W_2 = V_L I_L \cos 30^\circ = \sqrt{3}V_L I_L \cos 60^\circ$.

WATCH OUT FOR

One common mistake is using the phase voltage $400/\sqrt{3}$ in the meter formula instead of the line voltage. Each pressure coil sees a line-to-line voltage, and using phase voltage gives about 115.5 W. Another is forgetting that one meter at zero means $\phi = 60^\circ$ and the other meter reads the whole power. That leads to wrongly halving the total, or using $\cos\phi = 1$ and getting 400 W.

Q.43 NAT · 2 MARKS

Electrical and Electronic Measurements › Time, frequency & scopes

A $6\frac{1}{2}$ digit timer-counter is set in the 'time period' mode of operation and the range is set as 'ns'. For an input signal, the timer-counter displays **1000000**. With the same input signal, the timer-counter is changed to 'frequency' mode of operation and the range is set as 'Hz'. The display will show the number ____

Official answer: **999 to 1001**

SOLUTION

1. In period mode the display gives the period as a count in the selected unit. With the range at ns, the period is $T = 1000000$ ns.

2. Convert to seconds: $T = 10^{6} \times 10^{-9}\ \text{s} = 10^{-3}\ \text{s} = 1\ \text{ms}$.
3. The input signal has not changed, and frequency is the reciprocal of period:
$$f = \frac{1}{T} = \frac{1}{10^{-3}\ \text{s}} = 1000\ \text{Hz}$$
4. In frequency mode with the range at Hz, the display gives the frequency in hertz, so it reads 1000. That fits easily on a $6\frac{1}{2}$ digit display, which can show up to 1999999.

KEY IDEA

A timer-counter shows its reading as a number in the selected range unit. Turn the period reading into seconds, take the reciprocal to get the frequency, then write that frequency in the new range unit (Hz).

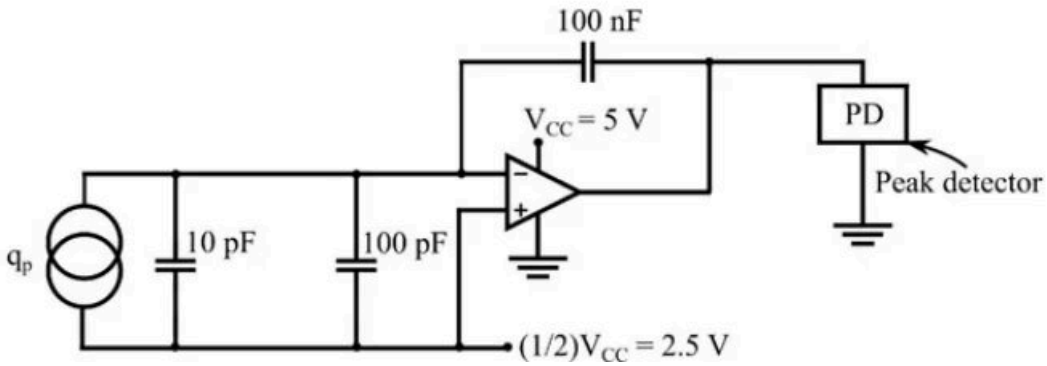
WATCH OUT FOR

Using the displayed number without its unit and writing $f = 1/10^{6} = 10^{-6}$. Another error is treating ns as 10^{-6} s (that is microseconds), which gives 1 Hz. The reading 10^{6} ns is 10^{-3} s, so the frequency is 1000 Hz.

Q.44 NAT · 2 MARKS

Analog Electronics › Op-amps

Q.No. 44 The circuit shown uses ideal opamp powered from a supply $V_{CC} = 5$ V. If the charge q_p generated by the piezoelectric sensor is of the form $q_p = 0.1 \sin(10000\pi t)$ μC, the peak detector output after 10 cycles of q_p **(in volts, rounded off to one decimal place)** is ___



Official answer: **3.4 to 3.6**

SOLUTION

1. **Read the circuit correctly.** The non-inverting input is wired to the bottom rail, so $v_+ = \frac{1}{2}V_{CC} = 2.5$ V. The ground under the op-amp is its negative supply pin, so the output can swing only between 0 and 5 V. The sensor, the 10 pF and the 100 pF all sit between the inverting input and this same 2.5 V rail. The feedback element is $C_f = 100$ nF.
2. **Virtual short (ideal op-amp, linear operation, zero input current).** Negative feedback through C_f gives $v_- = v_+ = 2.5$ V.
3. **The input capacitors carry no current.** The 10 pF and 100 pF capacitors are connected between $v_- = 2.5$ V and the 2.5 V rail, so the voltage across them is always 0:
$$i_C = (10 + 100)\ \text{pF} \cdot \frac{d(v_- - 2.5)}{dt} = 0$$
4. **KCL at the inverting node.** Take $i_p = \dfrac{dq_p}{dt}$ as the sensor current entering the node. No current flows into the op-amp or the input capacitors, so all of it flows through C_f towards the output:
$$\frac{dq_p}{dt} = C_f \frac{d(v_- - v_o)}{dt} = -C_f \frac{dv_o}{dt} \quad (v_- \text{ is constant})$$
5. **Integrate.** Assume C_f starts uncharged, so $v_o(0) = v_- = 2.5$ V. Also $q_p(0) = 0$. Then

$$v_o(t) = 2.5\ \text{V} - \frac{q_p(t)}{C_f}$$

6. **Decisive computation.**

$$\frac{q_{p,\max}}{C_f} = \frac{0.1 \times 10^{-6}\ \text{C}}{100 \times 10^{-9}\ \text{F}} = 1\ \text{V} \ \Rightarrow\ v_o(t) = 2.5 - \sin(10000\pi t)\ \text{V}$$

   The sensor's polarity is not marked. Reversing it only flips the sign of the sine term, so the output range is the same either way.

7. **Check that the op-amp stays linear.** v_o swings between $2.5 - 1 = 1.5$ V and $2.5 + 1 = 3.5$ V. That is well inside the 0 to 5 V supply range, so the output never clips and the virtual-short assumption holds for the whole time.

8. **Peak detector after 10 cycles.** $\omega = 10000\pi$ rad/s, so $f = 5$ kHz and $T = 0.2$ ms; 10 cycles take 2 ms. Since q_p is periodic with zero average, v_o repeats the same 1.5 to 3.5 V swing every cycle, and nothing builds up. An ideal peak detector holds the largest value reached:

$$V_{PD} = 2.5 + 1 = 3.5\ \text{V}$$

KEY IDEA

This is a charge amplifier. With the + input biased at $V_{CC}/2 = 2.5$ V, the virtual short puts the sensor's own capacitance and the cable capacitance ($10 + 100$ pF) at zero volts across, so they carry no current. All of the sensor charge goes onto C_f, and $v_o = V_{CC}/2 - q_p/C_f$. Biasing at mid-supply lets a single-supply op-amp handle a bipolar charge signal. The peak is the bias plus $q_{p,\max}/C_f = 1$ V.

WATCH OUT FOR

Reading the ground symbol under the op-amp as the + input's reference. That ground is the op-amp's negative supply pin; the + input is wired to the 2.5 V rail. With the + input taken as 0 V you get $v_o = \mp\sin(10000\pi t)$, which clips at the 0 V rail and gives a wrong 1.0 V. Another mistake is dividing by $C_f + 110$ pF: the input capacitors have zero voltage across them under the virtual short, so they take no charge. The phrase '10 cycles' is a distractor. With zero-mean charge nothing accumulates, and the peak is the same in every cycle.

Q.45 NAT · 2 MARKS

Sensors and Industrial Instrumentation › Sensing principles

A metallic strain gauge of resistance R_x with a gauge factor of 2 is bonded to a structure made of a metal with modulus of elasticity of 200 GN/m^2. The value of R_x is 1 kΩ when no stress is applied. R_x is a part of a quarter bridge with three identical fixed resistors of 1 kΩ each. The bridge is excited from a DC voltage of 4 V. The structure is subjected to a stress of 100 MN/m^2. Magnitude of the output of the bridge (in mV, rounded off to two decimal places) is ___

Official answer: **0.95 to 1.05**

SOLUTION

1. Strain from Hooke's law: $\varepsilon = \dfrac{\sigma}{E} = \dfrac{100 \times 10^6\ \text{N/m}^2}{200 \times 10^9\ \text{N/m}^2} = 5 \times 10^{-4}$.
2. Gauge factor $G = \dfrac{\Delta R/R}{\varepsilon}$, so $\dfrac{\Delta R}{R} = 2 \times 5 \times 10^{-4} = 1 \times 10^{-3}$. With $R_x = 1$ kΩ this gives $\Delta R = 1\ \Omega$.
3. Quarter bridge: the strain gauge R_x sits in one arm and the other three arms are fixed $R = 1$ kΩ resistors. The output is the difference of the two divider voltages:

$$V_o = V\left[\frac{R + \Delta R}{2R + \Delta R} - \frac{1}{2}\right] = V\,\frac{\Delta R}{2\,(2R + \Delta R)}$$

4. Substitute $V = 4$ V, $R = 1000\ \Omega$, $\Delta R = 1\ \Omega$:

$$V_o = 4 \times \frac{1}{2 \times 2001} = 0.9995 \times 10^{-3}\ \text{V} = 0.9995\ \text{mV}$$

5. Check with the small-change form for a quarter bridge: $V_o \approx \frac{V}{4}\frac{\Delta R}{R} = \frac{4}{4} \times 10^{-3}\ \text{V} = 1\ \text{mV}$, which agrees.
6. Rounded to two decimal places, the magnitude of the output is $1.00\ \text{mV}$.

KEY IDEA

Convert stress to strain with Hooke's law, strain to $\Delta R/R$ with the gauge factor, then use the quarter-bridge output $V_o \approx \frac{V}{4}\frac{\Delta R}{R}$ (exact form $V\frac{\Delta R}{2(2R+\Delta R)}$).

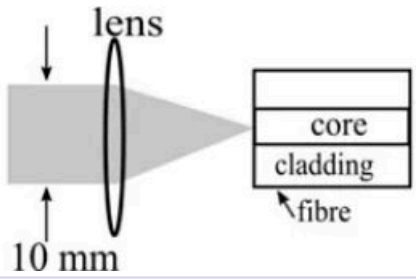
WATCH OUT FOR

Using the half-bridge or full-bridge factor (V/2 or V instead of V/4) for a quarter bridge, or mixing units so that GN/m^2 and MN/m^2 are off by a factor of 1000 in the strain.

Q.46 NAT · 2 MARKS

Electromagnetics › Waveguides & fibres

Q.No. 46 A laser beam of 10 mm beam diameter is focused onto an optical fibre using a thin biconvex lens as shown in the figure. The refractive index of the lens is 1.5. The refractive indices of the core and cladding of the fibre are 1.55 and 1.54 respectively. The minimum value of the focal length of the lens to attain the maximum coupling to the fibre **(in mm, rounded off to one decimal place)** is ___



Official answer: **27.5 to 28.5**

SOLUTION

1. **Numerical aperture of the fibre.** The medium around the fibre is not stated, so take it as air ($n_0 = 1$). With core index $n_1 = 1.55$ and cladding index $n_2 = 1.54$:
$$\text{NA} = n_0 \sin\theta_a = \sqrt{n_1^2 - n_2^2} = \sqrt{(1.55-1.54)(1.55+1.54)} = \sqrt{(0.01)(3.09)} = \sqrt{0.0309} = 0.17578$$
2. **Acceptance half-angle.** $\theta_a = \sin^{-1}(0.17578) \approx 10.12^\circ$. A ray that reaches the core face at a larger angle to the axis is not totally internally reflected at the core–cladding boundary, so it leaks out of the core and is lost.
3. **Geometry of the focused beam.** The collimated beam has diameter $10\ \text{mm}$, so its radius is $a = 5\ \text{mm}$. A thin lens brings a collimated beam to a focus at distance f. The edge (marginal) ray therefore meets the axis at a half-angle θ given by
$$\tan\theta = \frac{a}{f} = \frac{5}{f}$$
4. **Condition for maximum coupling.** The whole beam enters the guided modes only if the marginal ray, which is the steepest ray, lies inside the acceptance cone: $\theta \le \theta_a$. Since $\tan\theta = 5/f$ gets smaller as f grows, this means $f \ge \dfrac{5}{\tan\theta_a}$. The minimum focal length is the one that puts the marginal ray exactly at θ_a.
5. **Convert NA to a tangent.**
$$\tan\theta_a = \frac{\sin\theta_a}{\sqrt{1-\sin^2\theta_a}} = \frac{0.17578}{\sqrt{1-0.0309}} = \frac{0.17578}{0.98443} = 0.17856$$

6. **Minimum focal length.**

$$f_{\min} = \frac{a}{\tan\theta_a} = \frac{5\ \text{mm}}{0.17856} = 28.00\ \text{mm} \approx 28.0\ \text{mm}$$

7. **The lens index is not needed.** The refractive index of the lens (1.5) would matter only for working out f from the radii of curvature (lensmaker's equation). No radii are given and the question asks for f itself, so the index plays no part.

KEY IDEA

For maximum coupling, every ray of the focused cone must lie inside the fibre's acceptance cone. The steepest ray is the marginal ray, at $\tan\theta = a/f$ where a is the beam radius. Setting $\theta = \theta_a$ with $\sin\theta_a = \text{NA} = \sqrt{n_1^2 - n_2^2}$ gives the shortest focal length that still couples the whole beam: $f_{\min} = a/\tan\theta_a$.

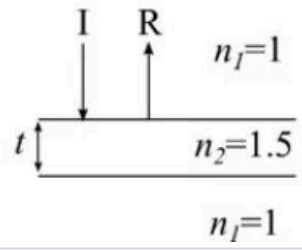
WATCH OUT FOR

Using the full diameter (10 mm) where the geometry needs the radius (5 mm). That doubles the answer to about 56 mm. Other slips: forgetting the square root in $\text{NA} = \sqrt{n_1^2 - n_2^2}$; writing $\text{NA} = n_1 - n_2$; treating the lens index 1.5 as part of the NA. Setting $\sin\theta_a = a/f$ instead of $\tan\theta_a = a/f$ gives about 28.4 mm, which happens to fall inside the key range, but the ray geometry is a tangent, not a sine.

Q.47 NAT · 2 MARKS

Electromagnetics › Plane waves

Q.No. 47 As shown in the figure, a slab of finite thickness t with refractive index $n_2 = 1.5$, has air ($n_1 = 1$) above and below it. Light of free space wavelength 600 nm is incident normally from air as shown. For a destructive interference to be observed at R, the minimum value of thickness of the slab t **(in nm)** is ___



Official answer: **200**

SOLUTION

1. **Phase change on reflection.** At the top face the light goes from air ($n_1 = 1$) into the slab ($n_2 = 1.5$). That is a rarer-to-denser reflection, so ray 1 picks up a phase shift of π, which is the same as an extra path of $\lambda/2$. At the bottom face the light goes from the slab into air, a denser-to-rarer reflection, so ray 2 picks up no phase shift.
2. **Path difference from the slab.** At normal incidence, ray 2 crosses the slab twice, so it travels an extra optical path of $2n_2t$.
3. **Total effective path difference** between the two reflected rays. A π shift can be written as either $+\lambda/2$ or $-\lambda/2$, and both give the same interference condition. Taking $+\lambda/2$:

$$\Delta = 2n_2t + \frac{\lambda}{2}$$

4. **Condition for destructive interference:** $\Delta = \left(m + \frac{1}{2}\right)\lambda$. The reflection term $\lambda/2$ cancels on both sides, leaving

$$2n_2t = m\lambda, \quad m = 0, 1, 2, \ldots$$

$m = 0$ gives $t = 0$, which means there is no slab, so the smallest real slab comes from $m = 1$.

5. **Minimum thickness** ($m = 1$):

$$t_{\min} = \frac{\lambda}{2n_2} = \frac{600\ \text{nm}}{2 \times 1.5} = 200\ \text{nm}$$

6. **Check.** Inside the slab the wavelength is $600/1.5 = 400$ nm, so $t = 200$ nm is half a wavelength in the slab. The round trip adds exactly one wavelength, so the only phase difference left is the π from reflection, and the two rays cancel. The same result holds if you count every multiple reflection: a half-wave layer with the same medium on both sides reflects nothing at that wavelength.

KEY IDEA

When the slab has the same medium (air) on both sides, only one of the two reflections picks up a π phase shift. That shift already makes the two rays opposite in phase, so for destructive interference the round trip $2n_2t$ must be a whole number of wavelengths: $2n_2t = m\lambda$. The smallest non-zero thickness is a half-wave layer, $t = \lambda/(2n_2)$.

WATCH OUT FOR

One mistake is to forget the π shift at the top face, or to count a π shift at both faces. Either way you get $2n_2t = \lambda/2$ and the wrong answer $t = \lambda/(4n_2) = 100$ nm, which is the condition for constructive reflection here. Another is to use the free-space path $2t$ instead of the optical path $2n_2t$, which gives 300 nm.

Q.48 NAT · 2 MARKS

Signals and Systems › DTFT & DFT

Consider the finite sequence $X = (1, 1, 1)$. The Inverse Discrete Fourier Transform (IDFT) of X is given as $(x(0),\ x(1),\ x(2))$. The value of $x(2)$ is ____

Official answer: **0**

SOLUTION

1. Here $N = 3$ and $X(0) = X(1) = X(2) = 1$. Use the IDFT definition:
$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j2\pi kn/N}$$
2. Put $n = 2$ and $N = 3$. The exponents are $j2\pi k \cdot 2/3$ for $k = 0, 1, 2$:
$$x(2) = \frac{1}{3}\left(1 + e^{j4\pi/3} + e^{j8\pi/3}\right)$$
3. Because $e^{j8\pi/3} = e^{j2\pi}\, e^{j2\pi/3} = e^{j2\pi/3}$, the bracket is $1 + e^{j2\pi/3} + e^{j4\pi/3}$. This is the sum of the three cube roots of unity, which is 0.
4. Check it directly:
$$1 + \left(-\tfrac{1}{2} + j\tfrac{\sqrt{3}}{2}\right) + \left(-\tfrac{1}{2} - j\tfrac{\sqrt{3}}{2}\right) = 0$$
so $x(2) = \dfrac{1}{3} \cdot 0 = 0$.
5. Shortcut: the N-point DFT of $\delta(n) = (1, 0, 0)$ is $X(k) = 1$ for every k. The DFT is one-to-one, so the IDFT of the flat spectrum $(1, 1, 1)$ must be $(1, 0, 0)$. That gives $x(0) = 1$ and $x(1) = x(2) = 0$.

KEY IDEA

A flat N-point spectrum $X(k) = 1$ is the DFT of the unit impulse $\delta(n)$, so its IDFT is $(1, 0, \ldots, 0)$. Worked out term by term, $x(2)$ is $\frac{1}{3}$ times the sum of the three cube roots of unity, and that sum is zero.

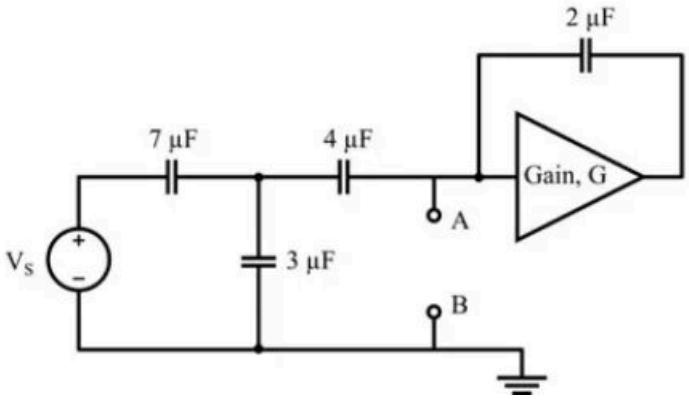
WATCH OUT FOR

Leaving out the $\frac{1}{N}$ factor or using the forward-DFT sign $e^{-j2\pi kn/N}$. For this spectrum neither changes $x(2)$, but both give the wrong $x(0)$ (3 instead of 1 without the factor). Another mistake is to assume that a constant spectrum gives a constant sequence $(1, 1, 1)$, or $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. A constant sequence has a spectrum that is an impulse. A constant spectrum belongs to an impulse sequence.

Q.49 NAT · 2 MARKS

Networks (Electric Circuits) › Theorems

Q.No. 49 A circuit consisting of capacitors, DC voltage source and an amplifier having a voltage gain G = −5 is shown in the figure. The effective capacitance across the nodes A and B **(in μF, rounded off to one decimal place)** is ___



Official answer: **14.5 to 15.0**

SOLUTION

1. **Deactivate the source.** The capacitance seen between A and B is found with the independent source switched off, so the DC source V_s is replaced by a short circuit. The left end of the 7 μF capacitor is then tied to B (ground). The amplifier is taken as ideal: its input draws no current and its output is a stiff voltage $V_o = G\,V_A$.
2. **Passive branch into A.** With the source shorted, the 7 μF and 3 μF capacitors both run from node X to ground, so they are in parallel: $7 + 3 = 10\ \mu\text{F}$. That combination is in series with the 4 μF capacitor:
$$C_1 = \frac{4 \times 10}{4 + 10} = \frac{40}{14} = 2.857\ \mu\text{F}$$
3. **Miller's theorem for the 2 μF feedback capacitor.** The capacitor sits between the amplifier input (A) and its output, where $V_o = G\,V_A$. Its current, flowing out of A, is
$$i = C\,\frac{d}{dt}\,(V_A - GV_A) = C(1 - G)\,\frac{dV_A}{dt}$$
Seen from A, it therefore behaves as a capacitance to ground of
$$C_M = C(1 - G) = 2\ (1 - (-5)) = 12\ \mu\text{F}$$
4. **Combine at node A.** Both C_1 and C_M run from A to ground (B), so they are in parallel. The ideal amplifier input draws no current, so nothing else loads node A:
$$C_{AB} = C_1 + C_M = 2.857 + 12 = 14.857\ \mu\text{F}$$
5. **Check with a test voltage V applied at A.** The charge drawn through the 2 μF capacitor is $2(V - GV) = 2(1 + 5)V = 12V$, and through the passive branch it is $\frac{40}{14}V$. So
$$Q = (12 + 2.857)\,V \quad \Rightarrow \quad C_{AB} = \frac{Q}{V} = 14.857\ \mu\text{F} \approx 14.9\ \mu\text{F}$$

KEY IDEA

Short the independent DC source, reduce the passive ladder (7 µF parallel with 3 µF, then in series with 4 µF, giving 40/14 µF), and use Miller's theorem on the feedback capacitor: at the input side it looks like $C(1-G)$ to ground, which is 12 µF for G = −5. The two capacitances at A are in parallel.

WATCH OUT FOR

Three slips are common. (1) Getting the Miller sign wrong. Writing $C(1+G)$ with G = −5 gives a negative capacitance, and using the output-side value $C(1-1/G) = 2.4\ \mu\text{F}$ gives about 5.3 µF. (2) Treating the source as an open circuit, so the 7 µF branch drops out and the passive part becomes $4 \times 3/7 = 1.71\ \mu\text{F}$, for a total of about 13.7 µF. (3) Putting the 7 µF and 3 µF in series instead of in parallel once the source is shorted.

Q.50 NAT · 2 MARKS

Control Systems › State space

Consider the following state variable equations:

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = -6x_1(t) - 5x_2(t)$$

The initial conditions are $x_1(0) = 0$ and $x_2(0) = 1$. At $t = 1$ second, the value of $x_2(1)$ (rounded off to two decimal places) is ____

Official answer: **-0.13 to -0.11**

SOLUTION

1. Write the system as $\dot{x} = Ax$ with $A = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix}$ and $x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. There is no input, so we want the zero-input (free) response.
2. Take the Laplace transform of $\dot{x} = Ax$: $sX(s) - x(0) = AX(s)$, so $X(s) = (sI - A)^{-1}x(0)$, where $sI - A = \begin{bmatrix} s & -1 \\ 6 & s+5 \end{bmatrix}$.
3. Find the resolvent matrix. The determinant is $s(s+5) + 6 = s^2 + 5s + 6 = (s+2)(s+3)$, so
$$(sI - A)^{-1} = \frac{1}{(s+2)(s+3)} \begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix}$$
4. Because $x(0) = [0\ \ 1]^T$, only the second column of the resolvent matters: $X_1(s) = \dfrac{1}{(s+2)(s+3)}$ and $X_2(s) = \dfrac{s}{(s+2)(s+3)}$.
5. Split $X_2(s)$ into partial fractions using residues. At $s = -2$: $\left.\dfrac{s}{s+3}\right|_{s=-2} = \dfrac{-2}{1} = -2$. At $s = -3$: $\left.\dfrac{s}{s+2}\right|_{s=-3} = \dfrac{-3}{-1} = 3$. So $X_2(s) = \dfrac{-2}{s+2} + \dfrac{3}{s+3}$.
6. Take the inverse Laplace transform: $x_2(t) = 3e^{-3t} - 2e^{-2t}$. Checks: $x_2(0) = 3 - 2 = 1$, which is correct. Also $x_1(t) = e^{-2t} - e^{-3t}$ gives $x_1(0) = 0$, and differentiating it gives $\dot{x}_1 = -2e^{-2t} + 3e^{-3t} = x_2$, which agrees with the first state equation.
7. Put in $t = 1\,\text{s}$:
$$x_2(1) = 3e^{-3} - 2e^{-2} = 3(0.04979) - 2(0.13534) = 0.14936 - 0.27067 = -0.1213$$
8. Rounded to two decimal places, $x_2(1) \approx -0.12$.

KEY IDEA

With no input, the state response is $X(s) = (sI - A)^{-1}x(0)$, or equivalently $x(t) = e^{At}x(0)$. Since $x(0) = [0 \;\; 1]^T$, $x(t)$ is just the second column of the state-transition matrix $\Phi(t)$. The eigenvalues -2 and -3 (the roots of $s^2 + 5s + 6$) tell you in advance which exponentials will appear.

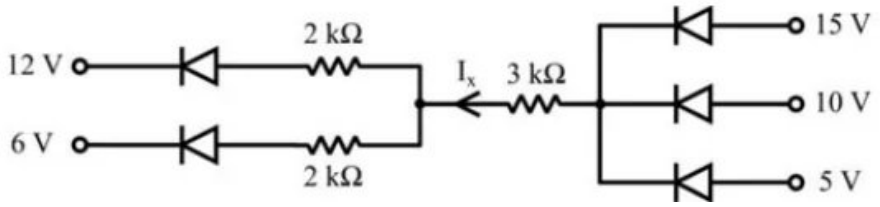
WATCH OUT FOR

The common mistakes are these: picking the first column of $(sI - A)^{-1}$ instead of the second, which gives $x_2 = -6/((s+2)(s+3))$ and $x_2(1) \approx -0.51$; getting a sign wrong in the adjugate (the off-diagonal entries must become $+1$ and -6); and reporting $x_1(1) = e^{-2} - e^{-3} \approx 0.09$ when the question asks for $x_2(1)$. Also note that $x_2(1)$ comes out negative even though $x_2(0) = 1$. The fast e^{-3t} term dies away before the slower $-2e^{-2t}$ term does, so the answer really is negative. Do not drop the sign.

Q.51 NAT · 2 MARKS

Analog Electronics › Diode circuits

Q.No. 51 Assume the diodes in the circuit shown are ideal. The current I_x flowing through the 3 kΩ resistor (**in mA, rounded off to one decimal place**) is ___



Official answer: **1.8**

SOLUTION

1. **Assumptions.** Every diode is ideal: it is a short circuit when it conducts forward current (anode to cathode) and an open circuit otherwise. All terminal voltages are measured from a common ground. Resistances are in $\mathrm{k\Omega}$, so a voltage in volts divided by a resistance in $\mathrm{k\Omega}$ gives a current in mA.
2. **The diode OR on the right.** All three right-hand diodes have their anodes on the sources and their cathodes on node Y, so current can only flow from a source into Y. The diode on the highest source conducts: the 15 V diode turns on and $V_Y = 15\ \mathrm{V}$. The 10 V and 5 V diodes then have their cathodes at 15 V, so they are reverse biased (by 5 V and 10 V) and off.
3. **Which way the left-hand diodes point.** Each left-hand diode has its anode on the node-X side and its cathode on its source. So current can only flow out of X, through a $2\ \mathrm{k\Omega}$ resistor, into the 12 V or the 6 V source. That agrees with the marked direction of I_x, from Y to X.
4. **First guess: both left-hand diodes ON.** KCL at X: $\dfrac{15 - V_X}{3} = \dfrac{V_X - 12}{2} + \dfrac{V_X - 6}{2}$. Multiplying by 6 gives $30 - 2V_X = 6V_X - 54$, so $V_X = 10.5\ \mathrm{V}$.
5. **The check fails.** With $V_X = 10.5\ \mathrm{V} < 12\ \mathrm{V}$, the 12 V branch would carry $(10.5 - 12)/2 = -0.75\ \mathrm{mA}$, which is against the diode's direction. An ideal diode cannot do that, so the 12 V diode must be OFF.
6. **Second guess: only the 6 V diode ON.** KCL at X: $\dfrac{15 - V_X}{3} = \dfrac{V_X - 6}{2}$, so $30 - 2V_X = 3V_X - 18$ and $V_X = \dfrac{48}{5} = 9.6\ \mathrm{V}$.
7. **This state is consistent.** No current flows in the 12 V branch, so that diode's anode sits at $V_X = 9.6\ \mathrm{V}$ and its cathode at 12 V. It is reverse biased by 2.4 V and stays OFF. The 6 V diode carries $(9.6 - 6)/2 = 1.8\ \mathrm{mA} > 0$, so it is ON.
8. **Current through the $3\ \mathrm{k\Omega}$ resistor:**

$$I_x = \frac{V_Y - V_X}{3\ \mathrm{k\Omega}} = \frac{15 - 9.6}{3}\ \mathrm{mA} = 1.8\ \mathrm{mA}$$

This current is positive, so it flows forward through the 15 V diode, which confirms the assumption in the second step that this diode is ON. It is also equal to the 6 V branch current, as KCL at X requires. Hence $I_x = 1.8\ \text{mA}$.

KEY IDEA

On the right, the three diodes form an OR circuit that holds node Y at the highest source, 15 V. On the left, guess which diodes are ON, solve KCL at node X, then check every diode: an ON diode must carry forward current, and an OFF diode must be reverse biased. With both left diodes ON, the 12 V branch would need negative current. So only the 6 V diode conducts, $V_X = 9.6\ \text{V}$ and $I_x = (15 - 9.6)/3 = 1.8\ \text{mA}$.

WATCH OUT FOR

Assuming both left-hand diodes conduct and stopping at $V_X = 10.5\ \text{V}$, which gives $I_x = 1.5\ \text{mA}$. That state needs $-0.75\ \text{mA}$ through the 12 V diode, which an ideal diode cannot carry. A second mistake is misreading the diode directions: the right-hand diodes pass current INTO node Y (so V_Y is set by the highest source, 15 V, not the lowest), and the left-hand diodes pass current OUT of node X.

Q.52 NAT · 2 MARKS

Electrical and Electronic Measurements › Instrument transformers

A 1000/1 A, 5 VA, UPF bar-primary measuring current transformer has 1000 secondary turns. The current transformer exhibits a ratio error of $-0.1\ \%$ and a phase error of 3.438 minutes when the primary current is 1000 A. At this operating condition, the rms value of the magnetization current of the current transformer (in amperes, rounded off to two decimal places) is ____

Official answer: **0.95 to 1.05**

SOLUTION

1. Bar primary means $N_p = 1$, so the turns ratio is $n = N_s/N_p = 1000$. The nominal ratio is $K_n = 1000/1 = 1000$.
2. Ratio error $= \dfrac{K_n - R}{R} = -0.1\ \% = -0.001$. This gives the actual ratio $R = \dfrac{1000}{0.999} = 1001.001$. The secondary current is therefore $I_s = \dfrac{I_p}{R} = \dfrac{1000 \times 0.999}{1000} = 0.999\ \text{A}$. Referred to the primary, this is $nI_s = 999\ \text{A}$.
3. The burden is at UPF, and the secondary winding impedance is taken as resistive because no leakage reactance is given. So the secondary circuit angle is $\delta = 0$, and I_s is in phase with E_s. In the phasor diagram the flux is at 90° to the induced emf. The reflected secondary current nI_s and the core-loss component I_e of the exciting current therefore lie along the same line. The magnetizing component I_m, which is in phase with the flux, is at 90° to them.
4. Take nI_s as the reference direction. The primary current is the phasor sum $\vec{I_p} = (nI_s + I_e) + jI_m$. The phase error θ is the angle between $\vec{I_p}$ and nI_s, so $I_p \sin\theta = I_m$ and $I_p \cos\theta = nI_s + I_e$.
5. Convert the phase error to radians: $\theta = 3.438\ \text{min} = \dfrac{3.438}{60} \times \dfrac{\pi}{180} = 1.000 \times 10^{-3}\ \text{rad}$.
6. The quadrature component gives $I_m = I_p \sin\theta = 1000 \times \sin(1.000 \times 10^{-3}) = 1.00\ \text{A}$. The standard approximation with $\delta = 0$, $\theta \approx \dfrac{I_m}{nI_s}$, gives the same value to two decimals: $I_m = 10^{-3} \times 999 = 0.999 \approx 1.00\ \text{A}$.
7. Consistency check using the in-phase component: $I_e = I_p \cos\theta - nI_s = 999.9995 - 999 = 0.9995\ \text{A}$. The approximate formula $R \approx n + \dfrac{I_e}{I_s}$ agrees: $I_e = (1001.001 - 1000)(0.999) = 1.00\ \text{A}$. This is the loss component, not the magnetizing component, so it does not change the answer.
8. Therefore the rms magnetizing current is $I_m \approx 1.00\ \text{A}$.

KEY IDEA

A UPF burden with a resistive secondary gives $\delta = 0$. The reflected secondary current nI_s then lies along the core-loss component I_e, and only the magnetizing component I_m sits in quadrature. So the phase error fixes I_m directly: $I_m = I_p \sin\theta \approx nI_s\,\theta$. The ratio error fixes I_e. 3.438 minutes is exactly 1 mrad.

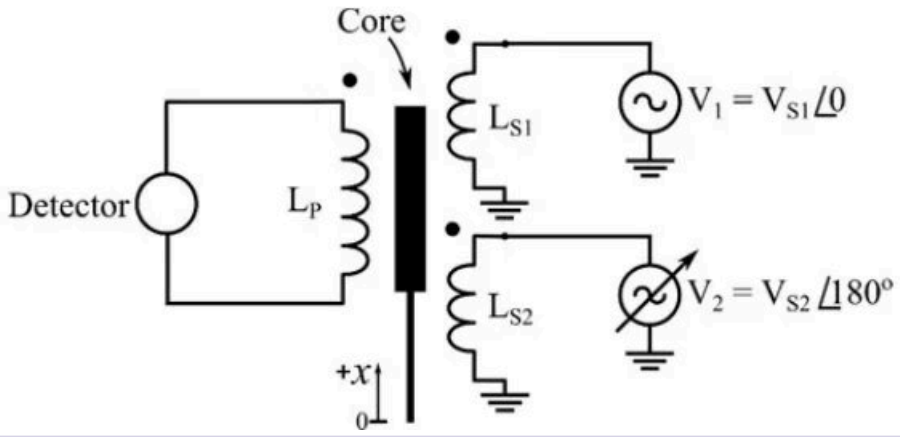
WATCH OUT FOR

The usual slips are computing I_e from the ratio error and reporting it as the magnetizing current, and plugging 3.438 minutes (or degrees) straight into the formula without converting to radians. With $\delta = 0$, the ratio error comes from I_e and the phase error comes from I_m. The two values are nearly equal here (about 1 A each), which hides the mix-up but does not justify it.

Q.53 NAT · 2 MARKS

Sensors and Industrial Instrumentation › Motion & force

Q.No. 53 The mutual inductances between the primary coil and the secondary coils of a linear variable differential transformer (LVDT) shown in the figure are M_1 and M_2. Assume that the self-inductances L_{S1} and L_{S2} remain constant and are independent of x. When $x = 0$, $M_1 = M_2 = M_0$. When x is in the range ± 10 mm, M_1 and M_2 change linearly with x. At $x = +10$ mm or -10 mm, the change in the magnitudes of M_1 and M_2 is $0.25\,M_0$. For a particular displacement $x = D$, the voltage across the detector becomes zero when $|V_2| = 1.25|V_1|$. The value of D **(in mm, rounded off to one decimal place)** is ___



Official answer: **4.3 to 4.6**

SOLUTION

1. Linear variation with a change of $0.25M_0$ at $|x| = 10$ mm gives the slope $k = \dfrac{0.25}{10} = 0.025\ \text{mm}^{-1}$, as a fraction of M_0 per mm.
2. In the figure $+x$ points upward, towards the upper secondary L_{s1}. So $M_1 = M_0(1 + kx)$ and $M_2 = M_0(1 - kx)$.
3. Each secondary carries a current set by its own source and self-inductance: $I_1 = \dfrac{V_1}{j\omega L_{s1}}$ and $I_2 = \dfrac{V_2}{j\omega L_{s2}}$. Each current induces a voltage in the primary: $M_1 I_1$ from the first and $M_2 I_2$ from the second, times $j\omega$. Both secondaries have the same dot position, and the sources are $180°$ apart, so the two induced voltages oppose.

4. The detector voltage is zero when the magnitudes are equal: $\frac{M_1|V_1|}{L_{s1}} = \frac{M_2|V_2|}{L_{s2}}$. The two secondaries are identical coils in the symmetric figure, so take $L_{s1} = L_{s2}$. Then $M_1|V_1| = M_2|V_2|$.
5. With $|V_2| = 1.25|V_1|$ this gives $\frac{M_1}{M_2} = 1.25$.
6. Substitute: $\frac{1+kx}{1-kx} = 1.25 \Rightarrow 1 + kx = 1.25 - 1.25kx \Rightarrow 2.25\,kx = 0.25 \Rightarrow kx = \frac{1}{9}$.
7. $$D = \frac{1}{9 \times 0.025}\ \text{mm} = 4.44\ \text{mm} \approx 4.4\ \text{mm}$$

   This lies inside the ± 10 mm linear range and inside the key range 4.3 to 4.6.

KEY IDEA

The detector reads zero when the two opposing induced voltages are equal in magnitude. With equal self-inductances this means $M_1|V_1| = M_2|V_2|$. Since M_1 and M_2 vary linearly and oppositely with x, the ratio $M_1/M_2 = 1.25$ fixes the displacement.

WATCH OUT FOR

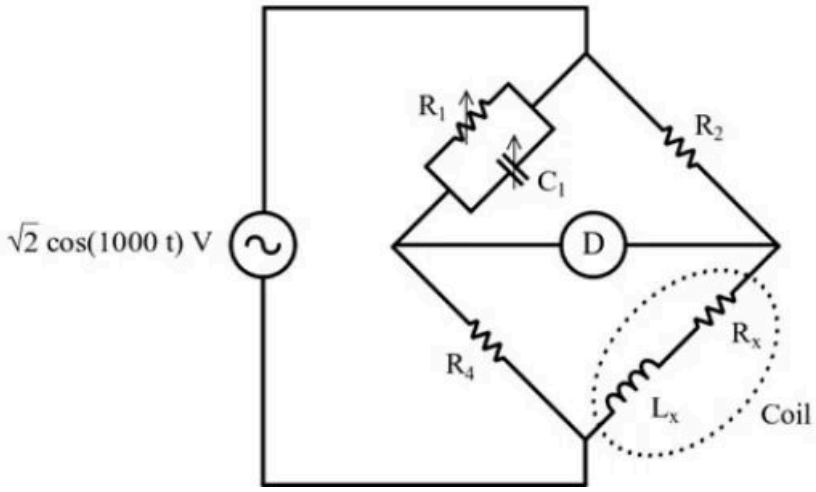
Using $M_1/M_2 = |V_1|/|V_2| = 0.8$ (the inverted ratio) gives a negative displacement. Another slip is to take the slope as 0.25 per 10 mm of M_0 but apply it to M rather than to the fraction of M_0.

Q.54 NAT · 2 MARKS

Electrical and Electronic Measurements › Bridges

Q.No. 54

In the Maxwell-Wien bridge shown, the detector D reads zero when $C_1 = 100$ nF and $R_1 = 100$ kΩ. The Q factor of the coil is ___



Official answer: **10**

SOLUTION

1. Identify the arms. The source $\sqrt{2}\cos(1000\,t)$ V feeds the top and bottom nodes, and D sits between the left and right nodes. The opposite pairs are therefore ($R_1 \parallel C_1$, coil) and (R_2, R_4).
2. Balance condition (D reads zero): the products of opposite-arm impedances are equal, $Z_1 Z_x = R_2 R_4$, where $\frac{1}{Z_1} = \frac{1}{R_1} + j\omega C_1$ and $Z_x = R_x + j\omega L_x$.
3. Solve for the coil impedance:
$$R_x + j\omega L_x = \frac{R_2 R_4}{Z_1} = R_2 R_4 \left(\frac{1}{R_1} + j\omega C_1\right)$$
4. Equate real parts: $R_x = \frac{R_2 R_4}{R_1}$. Equate imaginary parts: $\omega L_x = \omega R_2 R_4 C_1$, so $L_x = R_2 R_4 C_1$.

5. Q factor of the coil:

$$Q = \frac{\omega L_x}{R_x} = \frac{\omega R_2 R_4 C_1}{R_2 R_4 / R_1} = \omega C_1 R_1$$

R_2 and R_4 cancel, which is why the question does not give their values.

6. The source $\sqrt{2}\cos(1000\,t)$ gives $\omega = 1000\ \text{rad/s}$. This is the angular frequency, not $1000\ \text{Hz}$.

7. $$Q = 1000 \times (100 \times 10^{-9}) \times (100 \times 10^{3}) = 10$$

KEY IDEA

At balance in a Maxwell-Wien (inductance-capacitance) bridge, the coil's L_x/R_x ratio equals C_1R_1 of the parallel RC arm opposite it. So $Q = \omega L_x/R_x = \omega C_1 R_1$ depends only on ω, C_1 and R_1, and the ratio arms R_2, R_4 cancel.

WATCH OUT FOR

Using $\omega = 2\pi \times 1000$, which gives $Q \approx 62.8$. The source is written as $\cos(1000\,t)$, so 1000 is already ω in rad/s. A second mistake is to treat R_1 and C_1 as a series combination instead of a parallel one, which gives $Q = 1/(\omega C_1 R_1) = 0.1$.

Q.55 NAT · 2 MARKS

Control Systems › Bode & margins

The loop transfer function of a negative feedback system is $G(s)H(s) = \dfrac{2(s+1)}{s^2}$. The phase margin of the system (**in degrees, rounded off to one decimal place**) is ____

Official answer: **65.4 to 65.6**

SOLUTION

1. Put $s = j\omega$: $G(j\omega)H(j\omega) = \dfrac{2(1+j\omega)}{(j\omega)^2} = -\dfrac{2(1+j\omega)}{\omega^2}$.
2. Magnitude: $|GH| = \dfrac{2\sqrt{1+\omega^2}}{\omega^2}$. Phase: the double pole at the origin contributes -180° and the zero at $s = -1$ adds $\tan^{-1}\omega$, so $\angle GH = -180^\circ + \tan^{-1}\omega$.
3. Gain crossover, where $|GH| = 1$: $2\sqrt{1+\omega^2} = \omega^2 \ \Rightarrow\ 4(1+\omega^2) = \omega^4 \ \Rightarrow\ \omega^4 - 4\omega^2 - 4 = 0$.
4. Solve the quadratic in ω^2 and keep the positive root:

$$\omega_{gc}^2 = \frac{4 + \sqrt{16+16}}{2} = 2 + 2\sqrt{2} = 4.828 \ \Rightarrow\ \omega_{gc} = 2.197\ \text{rad/s}$$

Check: $\dfrac{2\sqrt{1+4.828}}{4.828} = \dfrac{2(2.414)}{4.828} = 1$.

5. Phase margin definition: $\text{PM} = 180^\circ + \angle GH(j\omega_{gc}) = 180^\circ - 180^\circ + \tan^{-1}\omega_{gc} = \tan^{-1}(2.197)$.
6. $\text{PM} = \tan^{-1}(2.197) = 65.53^\circ \approx 65.5^\circ$. (The phase stays above -180° for every $\omega > 0$, so there is no phase crossover; consistently, the closed-loop characteristic equation $s^2 + 2s + 2 = 0$ has both roots in the left half-plane.)

KEY IDEA

The double pole at the origin holds the phase at exactly -180°, so the phase margin is simply the phase lead of the zero, $\tan^{-1}\omega_{gc}$, measured at the gain crossover frequency. Setting $|GH| = 1$ gives a quadratic in ω^2.

WATCH OUT FOR

Students often misplace the zero's phase lead, or read $\angle GH = -180^\circ + \tan^{-1}\omega$ as meaning the system sits at the stability limit and write a phase margin of 0°. Some also take the negative root of the quadratic in ω^2, or leave the calculator in radians and write $\tan^{-1}(2.197) = 1.14$.

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