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GATE IN · 2018

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# GATE IN 2018 Solved Paper

INSIDE

- All 65 questions, in the order of the paper
- The official answer to every question
- Step-by-step worked solutions
- The answer key on one page

Question paper and official answer key: IIT Guwahati

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## About this paper

**65 questions, 100 marks.** Question types: 37 MCQ · 28 NAT.

**Official answer key:** IIT Guwahati

This paper numbers General Aptitude GA Q.1 to GA Q.10 and the subject questions Q.1 to Q.55: 65 questions in all.

We placed each question under a subject and topic ourselves; the exam paper does not label them.

This paper online, with the questions linked to their topic notes:
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## Answer key

| Q | TYPE | MARKS | ANSWER |
|---|---|---|---|
| GA Q.1 | MCQ | 1 | **A** |
| GA Q.2 | MCQ | 1 | **C** |
| GA Q.3 | MCQ | 1 | **B** |
| GA Q.4 | MCQ | 1 | **C** |
| GA Q.5 | MCQ | 1 | **D** |
| GA Q.6 | MCQ | 2 | **A** |
| GA Q.7 | MCQ | 2 | **C** |
| GA Q.8 | MCQ | 2 | **D** |
| GA Q.9 | MCQ | 2 | **C** |
| GA Q.10 | MCQ | 2 | **A** |
| Q.1 | MCQ | 1 | **A** |
| Q.2 | MCQ | 1 | **B** |
| Q.3 | MCQ | 1 | **D** |
| Q.4 | NAT | 1 | **1** |
| Q.5 | MCQ | 1 | **B** |
| Q.6 | MCQ | 1 | **A** |
| Q.7 | MCQ | 1 | **C** |
| Q.8 | NAT | 1 | **1009 to 1011** |
| Q.9 | MCQ | 1 | **D** |
| Q.10 | MCQ | 1 | **A** |
| Q.11 | MCQ | 1 | **C** |
| Q.12 | NAT | 1 | **90, or -270** |
| Q.13 | MCQ | 1 | **A** |
| Q.14 | NAT | 1 | **180, or -180** |
| Q.15 | NAT | 1 | **0** |
| Q.16 | NAT | 1 | **200** |
| Q.17 | NAT | 1 | **10** |
| Q.18 | MCQ | 1 | **C** |
| Q.19 | MCQ | 1 | **C** |
| Q.20 | MCQ | 1 | **D** |
| Q.21 | NAT | 1 | **200** |
| Q.22 | NAT | 1 | **95 to 97** |
| Q.23 | MCQ | 1 | **A** |
| Q.24 | NAT | 1 | **0.3** |
| Q.25 | NAT | 1 | **75** |
| Q.26 | MCQ | 2 | **D** |
| Q.27 | MCQ | 2 | **B** |
| Q.28 | MCQ | 2 | **C** |
| Q.29 | MCQ | 2 | **D** |
| Q.30 | NAT | 2 | **0.95 to 1.05** |
| Q.31 | NAT | 2 | **66 to 68** |
| Q.32 | MCQ | 2 | **A** |
| Q.33 | MCQ | 2 | **C** |
| Q.34 | MCQ | 2 | **D** |
| Q.35 | MCQ | 2 | **C** |
| Q.36 | NAT | 2 | **0** |
| Q.37 | MCQ | 2 | **D** |
| Q.38 | NAT | 2 | **8.9 to 9.1** |
| Q.39 | MCQ | 2 | **D** |
| Q.40 | NAT | 2 | **Marks to all** |
| Q.41 | NAT | 2 | **17.5 to 17.9** |
| Q.42 | NAT | 2 | **0.98 to 1.00** |
| Q.43 | MCQ | 2 | **B** |
| Q.44 | MCQ | 2 | **C** |
| Q.45 | NAT | 2 | **279 to 282** |
| Q.46 | MCQ | 2 | **A** |
| Q.47 | NAT | 2 | **2.49 to 2.53** |
| Q.48 | NAT | 2 | **0.9 to 1.1** |
| Q.49 | NAT | 2 | **221 to 223** |
| Q.50 | NAT | 2 | **3.5 to 5.7** |
| Q.51 | NAT | 2 | **0.48 to 0.51** |
| Q.52 | NAT | 2 | **10.6 to 11.0** |
| Q.53 | NAT | 2 | **352000** |
| Q.54 | NAT | 2 | **249 to 251** |
| Q.55 | NAT | 2 | **29 to 31** |

# General Aptitude GA Q.1 to Q.10

## GA Q.1 MCQ · 1 MARK

General Aptitude › Grammar & vocabulary

"When she fell down the _____, she received many _____ but little help."

The words that best fill the blanks in the above sentence are

**(A)** stairs, stares

**(B)** stairs, stairs

**(C)** stares, stairs

**(D)** stares, stares

Official answer: **A**

**SOLUTION**

1. **Spot the homophones.** Every option is built from the same two words, which sound alike but mean different things: *stairs* (a flight of steps) and *stares* (long, fixed looks).
2. **Blank 1.** A person falls down a flight of steps, so the phrase must be *fell down the stairs*. Nobody can fall down looks, so *stares* does not fit here. This rules out (C) and (D).
3. **Blank 2.** After *received many*, the word must name something a person gets from the people around her. *Received many stares* means people looked at her. Nobody receives *stairs*, so (B) is out.
4. **Check the contrast.** *But little help* sets what she got against what she needed: plenty of looks, hardly any help. Only *stares* makes this contrast work.
5. The sentence reads: "When she fell down the **stairs**, she received many **stares** but little help." So the answer is (A).

**KEY IDEA**

This is a homophone question. *Stairs* (steps) and *stares* (fixed looks) sound the same, so pick each word from its meaning. You fall down *stairs*, and onlookers give you *stares*. The word *but* sets the many looks against the little help she got.

**WATCH OUT FOR**

Picking by sound, or assuming the same word goes in both blanks (option B or D). Another slip is swapping the two words (option C). Test each blank on its own: can you fall down it, and can you receive many of it?

## GA Q.2 MCQ · 1 MARK

General Aptitude › Grammar & vocabulary

"In spite of being warned repeatedly, he failed to correct his _____ behaviour."

The word that best fills the blank in the above sentence is

**(A)** rational

**(B)** reasonable

**(C)** errant

**(D)** good

Official answer: **C**

**SOLUTION**

1. Start with the clues. "Warned repeatedly" and "failed to correct" both say the behaviour was wrong, because you only correct something that is faulty.
2. So the blank needs an adjective with a negative meaning, something like *wrong* or *faulty*.
3. (A) *rational* means based on reason, and (B) *reasonable* means sensible or fair. Both are positive words, and nobody gets warned again and again to correct sensible behaviour.

4. (D) *good* is also positive, so "correct his good behaviour" contradicts itself.
5. (C) *errant* means straying from the proper course, that is, misbehaving or doing wrong. "Correct his errant behaviour" makes sense.
6. *Errant* is the only option with the negative meaning the sentence needs, so the answer is (C).

**KEY IDEA**

Let the context decide whether the word is positive or negative. "Warned repeatedly" and "failed to correct" mean the behaviour was wrong, so the blank needs a negative adjective. Of the four options, only *errant* (straying from what is right, misbehaving) is negative.

**WATCH OUT FOR**

Choosing *rational* or *reasonable* because they sound formal, or because the student half-remembers them as words used about behaviour. Both are positive, and you do not correct sensible behaviour. Some students also mix up *errant* with *arrogant* or *errand*. *Errant* comes from the same root as *err* and *error* and means going wrong.

## GA Q.3 MCQ · 1 MARK

General Aptitude › Numerical

For $0 \leq x \leq 2\pi$, $\sin x$ and $\cos x$ are both decreasing functions in the interval _____.

**(A)** $\left(0, \frac{\pi}{2}\right)$
**(B)** $\left(\frac{\pi}{2}, \pi\right)$
**(C)** $\left(\pi, \frac{3\pi}{2}\right)$
**(D)** $\left(\frac{3\pi}{2}, 2\pi\right)$

Official answer: **B**

**SOLUTION**

1. A differentiable function decreases on an interval where its derivative is negative there. $\frac{d}{dx}\sin x = \cos x$ and $\frac{d}{dx}\cos x = -\sin x$.
2. $\sin x$ decreases where $\cos x < 0$. In $[0, 2\pi]$ that is the 2nd and 3rd quadrants, $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$. Over this interval $\sin x$ falls from $1$ at $\frac{\pi}{2}$ to $-1$ at $\frac{3\pi}{2}$.
3. $\cos x$ decreases where $-\sin x < 0$, i.e. where $\sin x > 0$. In $[0, 2\pi]$ that is the 1st and 2nd quadrants, $(0, \pi)$. Over this interval $\cos x$ falls from $1$ at $0$ to $-1$ at $\pi$.
4. Both functions decrease on the overlap of the two intervals:
$$\left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \cap (0, \pi) = \left(\frac{\pi}{2}, \pi\right)$$
5. Check the other options. (A) $\left(0, \frac{\pi}{2}\right)$: $\sin x$ rises from $0$ to $1$. (C) $\left(\pi, \frac{3\pi}{2}\right)$: $\sin x$ falls, but $\cos x$ rises from $-1$ to $0$. (D) $\left(\frac{3\pi}{2}, 2\pi\right)$: $\sin x$ rises from $-1$ to $0$ and $\cos x$ rises from $0$ to $1$. Only (B) works.

**KEY IDEA**

Each function decreases where its derivative is negative: $\sin x$ where $\cos x < 0$, which is $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$, and $\cos x$ where $\sin x > 0$, which is $(0, \pi)$. The answer is where the two intervals overlap. You can also check each quadrant on the unit circle: the second quadrant is the only one where both coordinates are shrinking.

**WATCH OUT FOR**

Picking (C) because $\sin x$ and $\cos x$ are both negative in the third quadrant. Being negative is not the same as decreasing: in $\left(\pi, \frac{3\pi}{2}\right)$, $\cos x$ climbs from $-1$ to $0$. Another slip is mixing up which derivative controls which function, for example deciding that $\sin x$ decreases wherever $\sin x > 0$.

## GA Q.4 MCQ · 1 MARK

General Aptitude › Geometry

The area of an equilateral triangle is $\sqrt{3}$. What is the perimeter of the triangle?

**(A)** 2
**(B)** 4

**(C)** 6

**(D)** 8

Official answer: **C**

**SOLUTION**

1. For an equilateral triangle of side $a$, the area is $A = \frac{\sqrt{3}}{4}a^2$.
2. Set this equal to the given area: $\frac{\sqrt{3}}{4}a^2 = \sqrt{3}$.
3. Divide both sides by $\sqrt{3}$ to get $\frac{a^2}{4} = 1$, so $a^2 = 4$ and $a = 2$. Take the positive root because a length cannot be negative.
4. All three sides of an equilateral triangle are equal, so the perimeter is $3a = 3 \times 2 = 6$.
5. Check: $\frac{\sqrt{3}}{4}(2)^2 = \frac{4\sqrt{3}}{4} = \sqrt{3}$, which matches the given area. The answer is (C) 6.

**KEY IDEA**

The area of an equilateral triangle depends only on its side: $A = \frac{\sqrt{3}}{4}a^2$. Solve this for $a$, then multiply by 3 to get the perimeter.

**WATCH OUT FOR**

Stopping at the side length, which gives $a = 2$ and wrongly picks (A) 2. Another mistake is using $A = \frac{\sqrt{3}}{2}a^2$ (mixing up the altitude $\frac{\sqrt{3}}{2}a$ with the area). That gives $a = \sqrt{2}$, and the perimeter $3\sqrt{2}$ is not among the options.

## GA Q.5 MCQ · 1 MARK

General Aptitude › Geometry

Arrange the following three-dimensional objects in the descending order of their volumes:

(i) A cuboid with dimensions 10 cm, 8 cm and 6 cm

(ii) A cube of side 8 cm

(iii) A cylinder with base radius 7 cm and height 7 cm

(iv) A sphere of radius 7 cm

**(A)** (i), (ii), (iii), (iv)

**(B)** (ii), (i), (iv), (iii)

**(C)** (iii), (ii), (i), (iv)

**(D)** (iv), (iii), (ii), (i)

Official answer: **D**

**SOLUTION**

1. Cuboid, volume $= l \times b \times h$: $V_1 = 10 \times 8 \times 6 = 480\ \text{cm}^3$.
2. Cube, volume $= a^3$: $V_2 = 8^3 = 512\ \text{cm}^3$.
3. Cylinder, volume $= \pi r^2 h$: $V_3 = \pi \times 7^2 \times 7 = 343\pi \approx \frac{22}{7} \times 343 = 22 \times 49 = 1078\ \text{cm}^3$.
4. Sphere, volume $= \frac{4}{3}\pi r^3$: $V_4 = \frac{4}{3}\pi \times 7^3 = \frac{4}{3} \times 343\pi \approx \frac{4}{3} \times 1078 \approx 1437.3\ \text{cm}^3$.
5. Shortcut for (iii) and (iv): the cylinder has $h = r$, so $V_3 = \pi r^3$, and $V_4 = \frac{4}{3}\pi r^3$ is larger than it by a factor of $\frac{4}{3}$. Since $\pi r^3 = 343\pi > 3 \times 343 = 1029 > 512$, both are larger than the cube and the cuboid, whatever approximation of $\pi$ is used.

6. Descending order: $V_4 > V_3 > V_2 > V_1$, i.e. $1437.3 > 1078 > 512 > 480$, which is (iv), (iii), (ii), (i). This is option (D).

**KEY IDEA**

Work out each volume with its standard formula: cuboid $lbh$, cube $a^3$, cylinder $\pi r^2 h$, sphere $\frac{4}{3}\pi r^3$. The cylinder has $h = r = 7$, so its volume is $\pi r^3$ and the sphere of the same radius is exactly $\frac{4}{3}$ times it. Both are more than $3 \times 343 = 1029\ \text{cm}^3$, far above the cube (512) and the cuboid (480), so the only close comparison is cube against cuboid.

**WATCH OUT FOR**

Guessing that the cuboid is bigger than the cube because its longest side (10 cm) is longer than the cube's. In fact $10 \times 8 \times 6 = 480 < 512 = 8^3$. Another slip is using $\pi r^2$ or $\frac{4}{3}\pi r^2$ for the sphere and ending up with the sphere smaller than the cylinder. Also, the question asks for descending order, so reading it as ascending picks the reverse of the right sequence.

## GA Q.6 MCQ · 2 MARKS

General Aptitude › Numerical

An automobile travels from city A to city B and returns to city A by the same route. The speed of the vehicle during the onward and return journeys were constant at 60 km/h and 90 km/h, respectively. What is the average speed in km/h for the entire journey?

**(A)** 72

**(B)** 73

**(C)** 74

**(D)** 75

Official answer: **A**

**SOLUTION**

1. Let the one-way distance from A to B be $d$. The car covers that distance twice, so the total distance is $2d$.
2. Time is distance divided by speed. Onward time $= \dfrac{d}{60}$ h and return time $= \dfrac{d}{90}$ h.
3. Total time $= \dfrac{d}{60} + \dfrac{d}{90} = \dfrac{3d + 2d}{180} = \dfrac{5d}{180} = \dfrac{d}{36}$ h.
4. Average speed is total distance divided by total time:
$$v_{\text{avg}} = \frac{2d}{d/36} = 72\ \text{km/h}$$
5. Check with the equal-distance shortcut (the harmonic mean of the two speeds):
$$v_{\text{avg}} = \frac{2v_1 v_2}{v_1 + v_2} = \frac{2 \times 60 \times 90}{60 + 90} = \frac{10800}{150} = 72\ \text{km/h}$$
6. 72 km/h is option (A).

**KEY IDEA**

Average speed means total distance divided by total time. When two legs cover the same distance at speeds $v_1$ and $v_2$, the average is their harmonic mean, $\dfrac{2v_1 v_2}{v_1 + v_2}$. The car spends longer on the slower leg, so that speed counts for more.

**WATCH OUT FOR**

Taking the arithmetic mean of the two speeds, $\dfrac{60 + 90}{2} = 75$ km/h, and picking option (D). That would only be right if the car spent equal TIME at each speed. Here it covers equal DISTANCES, so it spends 1.5 times as long at 60 km/h as at 90 km/h, and the true average falls below 75.

## GA Q.7 MCQ · 2 MARKS

General Aptitude › Numerical

A set of 4 parallel lines intersect with another set of 5 parallel lines. How many parallelograms are formed?

**(A)** 20

**(B)** 48

**(C)** 60

**(D)** 72

Official answer: **C**

**SOLUTION**

1. A parallelogram needs two pairs of parallel sides: one pair of lines from the first set and one pair from the second set. Any two lines of one set together with any two lines of the other set enclose exactly one parallelogram, and different choices give different parallelograms.
2. Ways to pick 2 of the 4 parallel lines: $\binom{4}{2} = \dfrac{4 \times 3}{2} = 6.$
3. Ways to pick 2 of the 5 parallel lines: $\binom{5}{2} = \dfrac{5 \times 4}{2} = 10.$
4. The two choices are independent, so by the multiplication principle:

$$\text{Parallelograms} = \binom{4}{2} \times \binom{5}{2} = 6 \times 10 = 60$$

5. This matches option (C).

**KEY IDEA**

A parallelogram is fixed by choosing its two sides from each set of parallel lines, so the count is $\binom{m}{2} \times \binom{n}{2}$ for $m$ lines crossing $n$ lines. This counts every parallelogram of every size, not only the smallest cells of the grid.

**WATCH OUT FOR**

Counting only the smallest cells of the grid, $(4-1) \times (5-1) = 12$, which leaves out the larger parallelograms made up of several cells. Another mistake is multiplying $4 \times 5 = 20$ (option A), which is the number of intersection points, not the number of parallelograms.

## GA Q.8 MCQ · 2 MARKS

General Aptitude › Numerical

To pass a test, a candidate needs to answer at least 2 out of 3 questions correctly. A total of 6,30,000 candidates appeared for the test. Question A was correctly answered by 3,30,000 candidates. Question B was answered correctly by 2,50,000 candidates. Question C was answered correctly by 2,60,000 candidates. Both questions A and B were answered correctly by 1,00,000 candidates. Both questions B and C were answered correctly by 90,000 candidates. Both questions A and C were answered correctly by 80,000 candidates. If the number of students answering all questions correctly is the same as the number answering none, how many candidates failed to clear the test?

**(A)** 30,000

**(B)** 2,70,000

**(C)** 3,90,000

**(D)** 4,20,000

Official answer: **D**

**SOLUTION**

1. Let $x$ be the number of candidates who answered all three questions correctly. By the given condition, $x$ candidates also answered none correctly. Total $N = 6{,}30{,}000$.
2. Candidates with at least one correct answer: $n(A \cup B \cup C) = N - x = 6{,}30{,}000 - x$.
3. Inclusion-exclusion for three sets: $n(A \cup B \cup C) = \sum n(A) - \sum n(A \cap B) + n(A \cap B \cap C)$, so $n(A \cup B \cup C) = (3{,}30{,}000 + 2{,}50{,}000 + 2{,}60{,}000) - (1{,}00{,}000 + 90{,}000 + 80{,}000) + x = 5{,}70{,}000 + x$
4. Equate the two expressions: $6{,}30{,}000 - x = 5{,}70{,}000 + x \Rightarrow 2x = 60{,}000 \Rightarrow x = 30{,}000.$

5. Each pairwise count includes the all-three group, so the number with exactly two correct is $(1{,}00{,}000 + 90{,}000 + 80{,}000) - 3x = 2{,}70{,}000 - 90{,}000 = 1{,}80{,}000$.
6. Passed (at least 2 correct) = exactly two + all three $= 1{,}80{,}000 + 30{,}000 = 2{,}10{,}000$.
7. Failed $= 6{,}30{,}000 - 2{,}10{,}000 = 4{,}20{,}000$, which is option (D).
8. Check: exactly one correct $= \sum n(A) - 2\sum n(A \cap B) + 3x = 8{,}40{,}000 - 5{,}40{,}000 + 90{,}000 = 3{,}90{,}000$. Then $3{,}90{,}000 + 1{,}80{,}000 + 30{,}000 + 30{,}000 = 6{,}30{,}000$, so every candidate is counted. The failures are the exactly-one group plus the none group: $3{,}90{,}000 + 30{,}000 = 4{,}20{,}000$, which agrees.

**KEY IDEA**

Three-set inclusion-exclusion with one unknown. Call the all-three count $x$; it is also the none count. Writing the union two ways, as $N - x$ and as singles minus pairs plus $x$, fixes $x$. Passing means exactly two correct plus all three correct, and exactly two correct is the sum of the pairwise counts minus $3x$.

**WATCH OUT FOR**

There are three common traps. (1) Taking the passers to be the sum of the pairwise counts (2,70,000, option B). That counts the all-three group three times. (2) Counting only the exactly-one group as failures (3,90,000, option C) and forgetting the 30,000 who got none right. (3) Reporting $x = 30{,}000$ (option A), which is the all-three count and not the number who failed.

## GA Q.9 MCQ · 2 MARKS

General Aptitude › Numerical

If $x^2 + x - 1 = 0$ what is the value of $x^4 + \dfrac{1}{x^4}$?

**(A)** 1

**(B)** 5

**(C)** 7

**(D)** 9

Official answer: **C**

**SOLUTION**

1. $x = 0$ does not satisfy the equation (it would give $-1 = 0$), so you can divide every term by $x$: $x + 1 - \dfrac{1}{x} = 0$, which gives $x - \dfrac{1}{x} = -1$.
2. Square both sides using $(a-b)^2 = a^2 - 2ab + b^2$. The cross term is $2 \cdot x \cdot \frac{1}{x} = 2$:
$$x^2 - 2 + \frac{1}{x^2} = 1 \;\Rightarrow\; x^2 + \frac{1}{x^2} = 3$$
3. Square again using $(a+b)^2 = a^2 + 2ab + b^2$. The cross term is $2 \cdot x^2 \cdot \frac{1}{x^2} = 2$:
$$x^4 + 2 + \frac{1}{x^4} = 9 \;\Rightarrow\; x^4 + \frac{1}{x^4} = 7$$
4. Check with an actual root: $x = \dfrac{-1+\sqrt{5}}{2} \approx 0.618$ gives $x^4 \approx 0.1459$ and $\dfrac{1}{x^4} \approx 6.854$, which add up to $\approx 7.000$. The other root, $x = \dfrac{-1-\sqrt{5}}{2} \approx -1.618$, is the negative reciprocal of the first, so it gives the same two numbers the other way round and the same sum.
5. So $x^4 + \dfrac{1}{x^4} = 7$, which is option (C).

**KEY IDEA**

Divide the quadratic by $x$ to get $x - \frac{1}{x}$, then square twice. Each squaring turns $x^n \pm \frac{1}{x^n}$ into $x^{2n} + \frac{1}{x^{2n}}$ plus or minus 2, because $x^n \cdot \frac{1}{x^n} = 1$. You never have to solve for $x$.

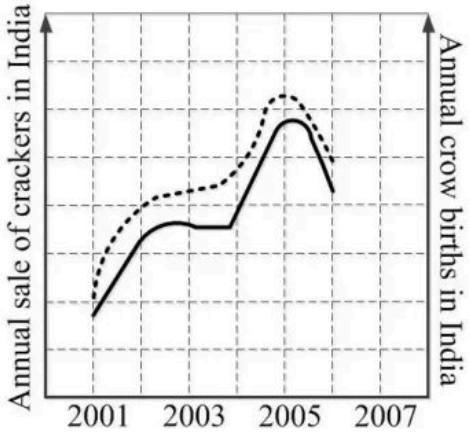
**WATCH OUT FOR**

Getting the sign of $x - \frac{1}{x}$ wrong. Dividing by $x$ gives $x + 1 - \frac{1}{x} = 0$, so $x - \frac{1}{x} = -1$, not $x + \frac{1}{x} = -1$. If you take $x + \frac{1}{x} = -1$, the first squaring gives $x^2 + \frac{1}{x^2} = 1 - 2 = -1$, which is impossible for real $x$ and leads to $x^4 + \frac{1}{x^4} = -1$, not any of the options. A second slip is forgetting the $\pm 2$ cross term when squaring, which gives 1 or 9 (options A and D) instead of 7.

## GA Q.10 MCQ · 2 MARKS

General Aptitude › Data interpretation

Q.10 In a detailed study of annual crow births in India, it was found that there was relatively no growth during the period 2002 to 2004 and a sudden spike from 2004 to 2005. In another unrelated study, it was found that the revenue from cracker sales in India which remained fairly flat from 2002 to 2004, saw a sudden spike in 2005 before declining again in 2006. The solid line in the graph below refers to annual sale of crackers and the dashed line refers to the annual crow births in India. Choose the most appropriate inference from the above data.




(A) There is a strong correlation between crow birth and cracker sales.
(B) Cracker usage increases crow birth rate.
(C) If cracker sale declines, crow birth will decline.
(D) Increased birth rate of crows will cause an increase in the sale of crackers.

Official answer: **A**

**SOLUTION**

1. Read the graph first. The solid line (cracker sales) and the dashed line (crow births) both rise from 2001 to 2002, stay nearly flat from 2002 to about 2004, jump to a peak at 2005 and fall in 2006. The stem's own text says the same about both studies: flat from 2002 to 2004, then a sudden spike in 2005. The two series move up and down together.
2. Two quantities that consistently rise and fall together are **correlated**. Here the match holds in every stretch (the rise, the plateau, the spike and the fall), so the correlation is strong. That is exactly what option (A) says, and it only describes the data.
3. Now test the other options. The stem says the two sets of figures come from **unrelated** studies. Nothing in them links crows to crackers or explains how one could affect the other.
4. **Correlation does not imply causation.** Moving together does not tell us which quantity drives the other, or whether either one does. A shared outside factor, or plain coincidence, could produce the same curves.

5. (B) says cracker usage increases crow births, and (D) says crow births increase cracker sales. Each claims a cause in one direction, which the data cannot show, so both are rejected. (B) also talks about cracker *usage*, while the graph shows only *sales*.
6. (C) says that whenever cracker sales fall, crow births will fall too. That claims crow births depend on cracker sales and must follow them in future. It is a cause-and-effect or predictive claim, and six years of co-movement between two unrelated series cannot establish it. Rejected.
7. The only claim the data actually support is the co-movement itself, so the answer is **(A)**.

**KEY IDEA**

When two series move up and down together on a graph, that shows correlation and nothing more. It cannot show that either one causes the other, in either direction, and it gives no rule for predicting one from the other. Here the two series also come from unrelated studies, which makes this even clearer.

**WATCH OUT FOR**

Picking (B) or (C) because the two curves match so closely. That reads a cause-and-effect link, or a rule for predicting one series from the other, into what is only co-movement between two series from unrelated studies.

## Instrumentation Engineering Q.1 to Q.55

### Q.1 MCQ · 1 MARK

Engineering Mathematics › Linear algebra

Let $N$ be a 3 by 3 matrix with real number entries. The matrix $N$ is such that $N^2 = 0$. The eigen values of $N$ are

**(A)** 0, 0, 0

**(B)** 0,0,1

**(C)** 0,1,1

**(D)** 1,1,1

Official answer: **A**

**SOLUTION**

1. Let $\lambda$ be any eigenvalue of $N$, with eigenvector $v \neq 0$ (real or complex), so $Nv = \lambda v$.
2. Apply $N$ again: $N^2 v = N(\lambda v) = \lambda(Nv) = \lambda^2 v$. In general, if $\lambda$ is an eigenvalue of $N$, then $\lambda^k$ is an eigenvalue of $N^k$.
3. Since $N^2 = 0$, the left side is $0$, so $\lambda^2 v = 0$. Because $v \neq 0$, this forces $\lambda^2 = 0$, so $\lambda = 0$.
4. This holds for every eigenvalue, real or complex. The characteristic polynomial of the $3 \times 3$ matrix $N$ has degree 3, so it has three roots counted with multiplicity, and each of them is $0$. The eigenvalues are therefore $0, 0, 0$. (Equivalently: $x^2$ annihilates $N$, so the minimal polynomial divides $x^2$, and its only possible root is $0$.)
5. Check with an example: $N = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ satisfies $N^2 = 0$. The matrix $N - \lambda I$ is upper triangular, so $\det(N - \lambda I) = (-\lambda)^3 = -\lambda^3$, and the eigenvalues are $0, 0, 0$.
6. An eigenvalue of $1$ would mean $1^2 = 1$ is an eigenvalue of $N^2 = 0$, which is impossible because the zero matrix has only the eigenvalue $0$. This rules out options (B), (C) and (D). The answer is (A).

**KEY IDEA**

If $Nv = \lambda v$, then $N^k v = \lambda^k v$. So a matrix with $N^k = 0$ (a nilpotent matrix) can only have the eigenvalue $0$, repeated $n$ times for an $n \times n$ matrix, whatever the size of $k$.

**WATCH OUT FOR**

One mistake is to think $N^2 = 0$ forces $N = 0$ and then stop. Another is to allow eigenvalues of $1$ because $N$ is not the zero matrix. $N$ need not be zero (for example, a single $1$ above the diagonal), but every eigenvalue still satisfies $\lambda^2 = 0$. A nonzero nilpotent matrix still has only the eigenvalue $0$.

### Q.2 MCQ · 1 MARK

Engineering Mathematics › Complex analysis

Let $f_1(z) = z^2$ and $f_2(z) = \bar{z}$ be two complex variable functions. Here $\bar{z}$ is the complex conjugate of $z$. Choose the correct answer.

**(A)** Both $f_1(z)$ and $f_2(z)$ are analytic

**(B)** Only $f_1(z)$ is analytic

**(C)** Only $f_2(z)$ is analytic

**(D)** Both $f_1(z)$ and $f_2(z)$ are not analytic

Official answer: **B**

**SOLUTION**

1. Write $z = x + iy$ and split each function into $f = u + iv$. Then test the Cauchy-Riemann equations $u_x = v_y$ and $u_y = -v_x$.

2. For $f_1(z) = z^2 = (x+iy)^2 = (x^2 - y^2) + i(2xy)$, we have $u = x^2 - y^2$ and $v = 2xy$.
3. $u_x = 2x = v_y$ and $u_y = -2y = -v_x$. The Cauchy-Riemann equations hold at every point and all four partial derivatives are continuous, so $f_1$ is analytic everywhere. It is entire, with $f_1'(z) = 2z$.
4. For $f_2(z) = \bar{z} = x - iy$, we have $u = x$ and $v = -y$.
5. $u_x = 1$ but $v_y = -1$, so $u_x \neq v_y$ at every point. The Cauchy-Riemann equations fail everywhere, so $f_2$ is not differentiable at any point and cannot be analytic.
6. Quick check: in complex form the Cauchy-Riemann equations read $\dfrac{\partial f}{\partial \bar{z}} = 0$. For $z^2$ this derivative is $0$. For $\bar{z}$ it is $1 \neq 0$.
7. So only $f_1(z)$ is analytic, which is option (B).

**KEY IDEA**

A function is analytic in a region when it satisfies the Cauchy-Riemann equations $u_x = v_y$, $u_y = -v_x$ there and its partial derivatives are continuous. Equivalently, it must not depend on $\bar{z}$: $\partial f / \partial \bar{z} = 0$. Every polynomial in $z$ is entire. $\bar{z}$ is the standard example of a function that is continuous everywhere but differentiable nowhere.

**WATCH OUT FOR**

Assuming $\bar{z}$ is analytic because it is continuous and "simple" (it is linear in $x$ and $y$). Continuity is not enough. For $\bar{z} = x - iy$, $u_x = 1$ while $v_y = -1$, so the Cauchy-Riemann equations fail at every point. Some students also get a sign wrong when expanding $z^2$ and wrongly conclude that $u_y = -v_x$ fails for $z^2$.

## Q.3 MCQ · 1 MARK

Engineering Mathematics › Probability & statistics

$X$ and $Y$ are two independent random variables with variances 1 and 2, respectively. Let $Z = X - Y$. The variance of $Z$ is

**(A)** 0

**(B)** 1

**(C)** 2

**(D)** 3

Official answer: **D**

**SOLUTION**

1. Given: $\mathrm{Var}(X) = 1$ and $\mathrm{Var}(Y) = 2$. Because $X$ and $Y$ are independent, $\mathrm{Cov}(X, Y) = 0$.
2. The variance of a linear combination is $\mathrm{Var}(aX + bY) = a^2\,\mathrm{Var}(X) + b^2\,\mathrm{Var}(Y) + 2ab\,\mathrm{Cov}(X, Y)$.
3. For $Z = X - Y$, take $a = 1$ and $b = -1$. Then $b^2 = 1$, and the covariance term is zero because $X$ and $Y$ are independent.
4. $$\mathrm{Var}(Z) = (1)^2(1) + (-1)^2(2) + 0 = 1 + 2 = 3$$
5. The variance of $Z$ is 3, which is option (D).

**KEY IDEA**

When two random variables are independent, their variances add, whether you add or subtract the variables: $\mathrm{Var}(X \pm Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$. The coefficient $-1$ is squared, so the minus sign disappears.

**WATCH OUT FOR**

Subtracting the variances ($1 - 2 = -1$, or $|1 - 2| = 1$, which gives option B) because $Z$ is a difference. Variance cannot be negative. The coefficient of $Y$ is squared ($(-1)^2 = 1$), so the variances add. Only the means subtract: $E[Z] = E[X] - E[Y]$.

## Q.4 NAT · 1 MARK

Engineering Mathematics › Calculus

Consider two functions $f(x) = (x-2)^2$ and $g(x) = 2x - 1$, where $x$ is real. The smallest value of $x$ for

which $f(x) = g(x)$ is _____.

Official answer: **1**

**SOLUTION**

1. Set the two functions equal: $(x-2)^2 = 2x - 1$.
2. Expand the left side: $x^2 - 4x + 4 = 2x - 1$.
3. Move every term to one side to get a quadratic: $x^2 - 6x + 5 = 0$.
4. Factorise: $(x-1)(x-5) = 0$, so $x = 1$ or $x = 5$. Both roots are real, which the question requires.
5. Check both roots: at $x = 1$, $f(1) = (1-2)^2 = 1$ and $g(1) = 2(1) - 1 = 1$. At $x = 5$, $f(5) = (5-2)^2 = 9$ and $g(5) = 2(5) - 1 = 9$. Both are genuine intersection points.
6. The smaller of the two roots is $x = 1$.

**KEY IDEA**

Where two curves meet, their values are equal, so setting $f(x) = g(x)$ gives the quadratic $x^2 - 6x + 5 = 0$. It has two real roots, $1$ and $5$, and the question asks for the smaller one.

**WATCH OUT FOR**

Three slips are common. Some students answer $5$, the larger root. Others answer $3$, which is where $f(x) - g(x)$ is smallest (the vertex of $x^2 - 6x + 5$), not where it equals zero. Others expand $(x-2)^2$ wrongly, as $x^2 + 4$ or $x^2 - 4x - 4$, and end up with the wrong quadratic.

## Q.5 MCQ · 1 MARK

Engineering Mathematics › Probability & statistics

Consider a sequence of tossing of a fair coin where the outcomes of tosses are independent. The probability of getting the head for the third time in the fifth toss is

(A) $\frac{5}{16}$

(B) $\frac{3}{16}$

(C) $\frac{3}{5}$

(D) $\frac{9}{16}$

Official answer: **B**

**SOLUTION**

1. "Head for the third time on the fifth toss" means two things happen together: the fifth toss is a head, and the first four tosses contain exactly 2 heads.
2. The tosses are independent, so the probability that both happen is the product of their probabilities.
3. Binomial probability for the first four tosses: $P(\text{exactly 2 heads in 4}) = \binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = \frac{6}{16} = \frac{3}{8}$.
4. The fifth toss is a head with probability $\frac{1}{2}$.
5. Multiply. This is the negative binomial probability of the $r$-th success on the $n$-th trial, $\binom{n-1}{r-1}p^r(1-p)^{n-r}$, with $r = 3$, $n = 5$, $p = \frac{1}{2}$:
$$P = \frac{3}{8} \times \frac{1}{2} = \binom{4}{2}\left(\tfrac{1}{2}\right)^5 = \frac{6}{32} = \frac{3}{16}$$
6. Answer: option (B), $\frac{3}{16}$.

**KEY IDEA**

For the $r$-th success to land exactly on trial $n$, trial $n$ must be a success and the first $n-1$ trials must hold exactly $r-1$ successes. That gives the negative binomial probability $\binom{n-1}{r-1}p^r(1-p)^{n-r}$. Here it is $\binom{4}{2}\left(\frac{1}{2}\right)^5 = \frac{3}{16}$.

**WATCH OUT FOR**

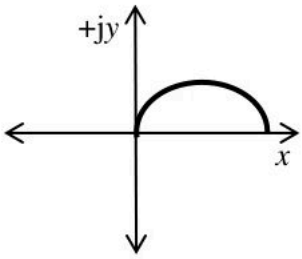
Using $\binom{5}{3}\left(\frac{1}{2}\right)^5 = \frac{10}{32} = \frac{5}{16}$ (option A). That is the chance of exactly 3 heads somewhere in 5 tosses. It includes sequences such as HHHTT, where the third head comes before the fifth toss. The fifth toss has to be fixed as the third head, so only the first four tosses are free to arrange, giving $\binom{4}{2}$.

## Q.6 MCQ · 1 MARK

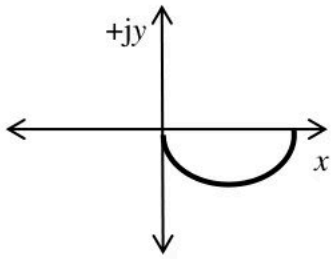
Networks (Electric Circuits) › AC, power & resonance

Q.6 A series R-C circuit is excited by a $1\angle 0$ V sinusoidal ac voltage source. The locus diagram of the phasor current $\underline{I} = (x + jy)$ A, when $C$ is varied, while keeping $R$ fixed, is

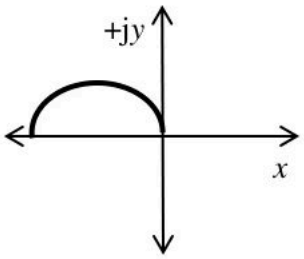
(A)



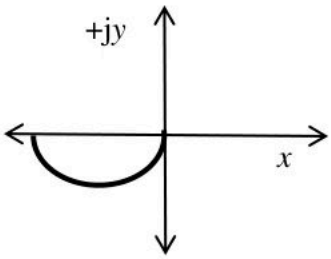

(B)




(C)




(D)




IN 1/14

Official answer: **A**

**SOLUTION**

1. Impedance of the series R-C branch: $Z = R - jX_C$, where $X_C = \dfrac{1}{\omega C}$. As $C$ goes from $0$ to $\infty$, $X_C$ falls from $\infty$ to $0$.
2. Ohm's law in phasor form with $V = 1\angle 0^\circ$ V:
$$\underline{I} = \frac{1}{R - jX_C} = \frac{R + jX_C}{R^2 + X_C^2}$$
so $x = \dfrac{R}{R^2 + X_C^2}$ and $y = \dfrac{X_C}{R^2 + X_C^2}$.
3. Signs: $R > 0$ and $X_C \geq 0$, so $x \geq 0$ and $y \geq 0$. The current leads the voltage (the circuit is capacitive), so the whole locus lies in the first quadrant. That rules out (B), (C) and (D).
4. Shape: $x^2 + y^2 = \dfrac{R^2 + X_C^2}{(R^2 + X_C^2)^2} = \dfrac{1}{R^2 + X_C^2} = \dfrac{x}{R}$. Completing the square gives
$$\left(x - \frac{1}{2R}\right)^2 + y^2 = \left(\frac{1}{2R}\right)^2$$
This is a circle with centre $\left(\frac{1}{2R}, 0\right)$ and radius $\frac{1}{2R}$, so it passes through the origin.
5. End points: as $C \to 0$ ($X_C \to \infty$), $\underline{I} \to 0$, the origin. As $C \to \infty$ ($X_C \to 0$), $\underline{I} \to \dfrac{1}{R}\angle 0^\circ$ A, a point on the positive x-axis. The top of the arc is at $X_C = R$, where $\underline{I} = \dfrac{1+j}{2R} = \dfrac{1}{\sqrt{2}\,R}\angle 45^\circ$ A.

6. So the locus is the upper semicircle of diameter $1/R$ standing on the positive real axis, running from the origin to $1/R$. That is option (A).

**KEY IDEA**

With $R$ fixed and $C$ varied, the impedance $Z = R - jX_C$ moves along the vertical half-line $\mathrm{Re}\, Z = R$ below the real axis. Inverting a straight line that does not pass through the origin gives a circle that does pass through the origin, so $\underline{I} = V/Z$ traces a circle through the origin with diameter $1/R$ on the real axis. Because the circuit is capacitive, the current leads the voltage, so only the upper half ($y > 0$) is traced.

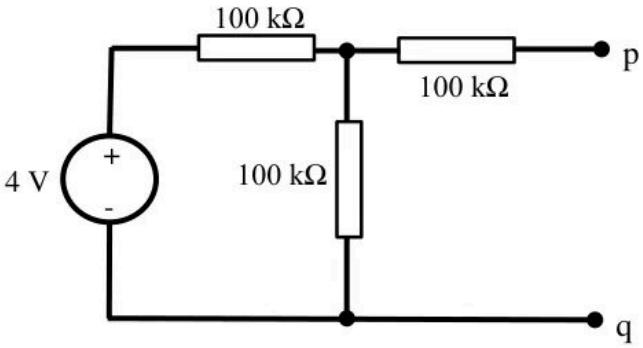
**WATCH OUT FOR**

Picking (B) by mixing up leading and lagging. The fourth-quadrant semicircle is the current locus of a series R-L circuit with $L$ varied, where the current lags. A related slip is to sketch the impedance locus, which for R-C lies below the real axis, and take it for the current locus: inverting $Z$ flips the sign of the angle. Options (C) and (D) need a negative real part of the current, which a positive resistance cannot give.

## Q.7 MCQ · 1 MARK

Networks (Electric Circuits) › Theorems

Q.7 The Thevenin equivalent circuit representation across terminals p-q of the circuit shown in the figure is a




(A) 1 V source in series with 150 kΩ
(B) 1 V source in parallel with 100 kΩ
(C) 2 V source in series with 150 kΩ
(D) 2 V source in parallel with 200 kΩ

Official answer: **C**

**SOLUTION**

1. **Open-circuit voltage.** With p-q open, no current flows in the 100 kΩ resistor leading to p, so there is no voltage drop across it. $V_{th}$ is therefore the voltage across the shunt 100 kΩ resistor, with p positive with respect to q.
2. **Voltage divider.** The 4 V source drives the first series 100 kΩ resistor and the shunt 100 kΩ resistor in series:
$$V_{th} = 4 \times \frac{100}{100 + 100} = 2 \text{ V}$$
3. **Thevenin resistance.** Set the independent 4 V source to zero, which means replacing it with a short circuit. Looking in from p-q, the first 100 kΩ resistor is now in parallel with the shunt 100 kΩ resistor:
$$100 \parallel 100 = \frac{100 \times 100}{100 + 100} = 50 \text{ kΩ}$$
4. That 50 kΩ is in series with the 100 kΩ resistor leading to p:
$$R_{th} = 50 + 100 = 150 \text{ kΩ}$$
5. **Form of the equivalent.** A Thevenin equivalent is always an ideal voltage source $V_{th}$ in series with $R_{th}$. A voltage source in parallel with a resistor, as in (B) and (D), behaves at its terminals like the bare source ($R_{th} = 0$), so neither of those can be right. The equivalent is a 2 V source in series with 150 kΩ, which is option (C).

**KEY IDEA**

$V_{th}$ is the open-circuit voltage. No current flows in the resistor leading to the open terminal, so it is just the divider voltage across the shunt resistor, $4 \times \frac{1}{2} = 2$ V. For $R_{th}$, short the voltage source: $(100 \parallel 100) + 100 = 150\ \text{k}\Omega$. A Thevenin equivalent is always a source in series with a resistance.

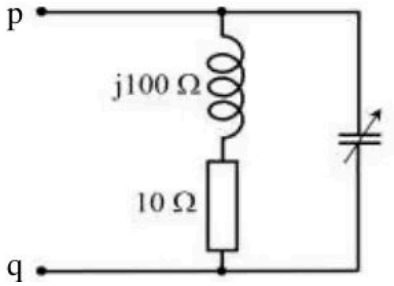
**WATCH OUT FOR**

Three slips are common. (1) Treating the resistor leading to p as part of the divider, which gives $V_{th} = 4/3$ V or a similar wrong value, even though it carries no current when p-q is open. (2) Forgetting that same series resistor when finding $R_{th}$, which gives 50 kΩ. (3) Opening the voltage source instead of shorting it, which gives $100 + 100 = 200\ \text{k}\Omega$ and makes (D) look plausible. Separately, any option with a voltage source in parallel with a resistor has the wrong form for a Thevenin equivalent.

## Q.8 NAT · 1 MARK

Networks (Electric Circuits) › AC, power & resonance

Q.8 A coil having an impedance of $(10 + \text{j}100)\ \Omega$ is connected in parallel to a variable capacitor as shown in figure. Keeping the excitation frequency unchanged, the value of the capacitor is changed so that parallel resonance occurs. The impedance across terminals p-q at resonance (in Ω) is _____.




Official answer: **1009 to 1011**

**SOLUTION**

1. Frequency is fixed, so the coil stays at $Z_L = 10 + j100\ \Omega$. Only the capacitor's susceptance $B_C = \omega C$ changes.
2. Admittance of the coil:
$$Y_L = \frac{1}{10 + j100} = \frac{10 - j100}{10^2 + 100^2} = \frac{10}{10100} - j\frac{100}{10100}\ \text{S}$$
3. Parallel resonance means the total admittance $Y = Y_L + j\omega C$ is purely real, so the capacitor has to cancel the coil's susceptance: $\omega C = \dfrac{100}{10100}$ S.
4. At resonance only the coil's conductance is left: $Y = \dfrac{10}{10100}$ S.
5. Impedance across p-q:
$$Z_{pq} = \frac{1}{Y} = \frac{10100}{10} = 1010\ \Omega$$
6. Check with the dynamic-resistance form: $Z = \dfrac{R^2 + X_L^2}{R} = R\,(1 + Q^2)$ with $Q = X_L/R = 10$, which gives $10 \times 101 = 1010\ \Omega$, purely resistive.

**KEY IDEA**

A lossy coil (R in series with L) in parallel with a capacitor resonates when the capacitor's susceptance cancels the coil's susceptance. This is the unity-power-factor condition: total admittance purely real. Only the coil's conductance $R/(R^2 + X_L^2)$ is left, so the circuit looks like the dynamic resistance $Z_D = (R^2 + X_L^2)/R = R(1 + Q^2)$. For a high-Q coil this is roughly $L/(CR)$.

**WATCH OUT FOR**

Treating resonance as $X_C = X_L = 100\ \Omega$ and then combining $(10 + j100)$ with $-j100$ in parallel gives $1000 - j100\ \Omega$. That is not purely resistive, so it is not resonance. With the coil resistance present, the capacitor has to cancel the coil's susceptance ($\omega C = 100/10100$ S), not its reactance. A second slip is answering $10\ \Omega$, as if the circuit were series resonant: at parallel resonance the impedance is at its largest, not its smallest.

## Q.9 MCQ · 1 MARK

Signals and Systems › Signals basics

Two periodic signals $x(t)$ and $y(t)$ have the same fundamental period of 3 seconds. Consider the signal $z(t) = x(-t) + y(2t + 1)$. The fundamental period of $z(t)$ in seconds is

**(A)** 1

**(B)** 1.5

**(C)** 2

**(D)** 3

Official answer: **D**

**SOLUTION**

1. **Time reversal keeps the period unchanged.** Shifting $t$ by $T$ gives $x(-(t + T)) = x(-t - T)$, which equals $x(-t)$ for every $t$ exactly when $T$ is a period of $x$. So $x(-t)$ has the same fundamental period as $x(t)$: $T_1 = 3$ s.
2. **A time shift keeps the period unchanged; time scaling by $a$ divides it by $|a|$.** Shifting $t$ by $T$ gives $y(2(t + T) + 1) = y(2t + 1 + 2T)$, which equals $y(2t + 1)$ for every $t$ exactly when $2T$ is a period of $y$. The smallest such $T$ satisfies $2T = 3$, so $y(2t + 1)$ has fundamental period $T_2 = \dfrac{3}{2} = 1.5$ s. The $+1$ only shifts the signal and does not affect the period.
3. **Periodicity of the sum.** The ratio $\dfrac{T_1}{T_2} = \dfrac{3}{1.5} = 2$ is rational, so $z(t)$ is periodic. Since $3 = 1 \times T_1 = 2 \times T_2$, both terms repeat after 3 s, so 3 s is a period of $z(t)$. It is the LCM: $\mathrm{LCM}(3,\ 1.5) = 3$ s.
4. **Rule out 1.5 s and 2 s.** Every period of $z$ is an integer multiple of its fundamental period $T_0$, so $3 = nT_0$ for some integer $n$. If $n$ were even, $1.5 = \frac{n}{2}T_0$ would also be a period of $z$. But the $y(2t + 1)$ term repeats every 1.5 s, so
$$z(t + 1.5) - z(t) = x(-t - 1.5) - x(-t),$$
and this is not zero for all $t$, because 1.5 s is shorter than 3 s, the fundamental period of $x$, so it is not a period of $x$. Hence $n$ is odd, and 1.5 s (B) is ruled out. 2 s (C) is also impossible, because $2 \neq \dfrac{3}{n}$ for any integer $n$.
5. **Rule out 1 s: it needs an exact cancellation.** The only remaining option shorter than 3 s is 1 s ($n = 3$). It would require $x(-t - 1) - x(-t) = -\big[y(2t + 3) - y(2t + 1)\big]$ for every $t$. That means the change in one term exactly cancels the change in the other, and the question gives no such relation between $x$ and $y$. For ordinary signals it fails. For example, $x(t) = y(t) = \cos\dfrac{2\pi t}{3}$ gives $z(t) = \cos\dfrac{2\pi t}{3} + \cos\dfrac{2\pi(2t + 1)}{3}$. The two terms have different frequencies, so they cannot cancel, and the $\frac{1}{3}$ Hz term does not repeat after 1 s. (A pair can be built to cancel on purpose. $x(t) = \cos\dfrac{4\pi t}{3} + \cos 2\pi t$ and

$y(t) = -\cos\dfrac{2\pi(t-1)}{3}$ both have fundamental period 3 s, yet they give $z(t) = \cos 2\pi t$, which has period 1 s. This is why the question can only be asking for the period that holds for any $x$ and $y$.)

6. **Answer.** For $x(t)$ and $y(t)$ with fundamental period 3 s and no special relation between them, the fundamental period of $z(t)$ is the LCM, $T_0 = 3$ s. **Option (D).**

**KEY IDEA**

Time reversal ($t \to -t$) and time shift ($t \to t + t_0$) leave the period unchanged. Time scaling $t \to at$ divides it by $|a|$. When the ratio of two periods is rational, the sum is periodic with period equal to the LCM of the two periods. That holds unless parts of the two terms cancel exactly, and the question gives no relation between x and y that could make them cancel.

**WATCH OUT FOR**

Answering 1.5 s (B). Some students apply the compression by 2 to the whole of z(t), or take the smaller of the two periods instead of their LCM. Only the y term is compressed. Its 1.5 s repetition does not make x(-t) repeat, so the sum repeats only at 3 s. Another slip is thinking the +1 in y(2t+1) or the time reversal in x(-t) changes the period. Neither does.

## Q.10 MCQ · 1 MARK

Signals and Systems › Signals basics

Consider signal $x(t) = \begin{cases} 1, & |t| \le 2 \\ 0, & |t| > 2 \end{cases}$. Let $\delta(t)$ denote the unit impulse (Dirac-delta) function. The value of the integral $\int_0^5 2x(t-3)\,\delta(t-4)\,dt$ is

**(A)** 2

**(B)** 1

**(C)** 0

**(D)** 3

Official answer: **A**

**SOLUTION**

1. Sifting property of the unit impulse: for a function $f(t)$ continuous at $t_0$,
$$\int_a^b f(t)\,\delta(t-t_0)\,dt = \begin{cases} f(t_0), & a < t_0 < b \\ 0, & t_0 < a \text{ or } t_0 > b \end{cases}$$
2. Here the impulse $\delta(t-4)$ sits at $t_0 = 4$, which lies strictly inside the limits $0 < 4 < 5$, so the integral picks up the value of the multiplying function at $t = 4$.
3. Take $f(t) = 2x(t-3)$ and sift:
$$\int_0^5 2x(t-3)\,\delta(t-4)\,dt = 2x(4-3) = 2x(1)$$
4. $x(t-3)$ is the pulse shifted right by 3, so it equals 1 for $1 \le t \le 5$; at $t = 4$ it is inside this range and away from the edges, so $f(t)$ is continuous there. Equivalently, $|1| \le 2$, so $x(1) = 1$.
5. Result: $2 \times 1 = 2$, which is option (A).

**KEY IDEA**

Sifting property: an impulse at $t_0$ inside the limits of integration picks out the value of the multiplying function at $t_0$. Here that gives $2x(t-3)$ evaluated at $t = 4$, which is $2x(1)$.

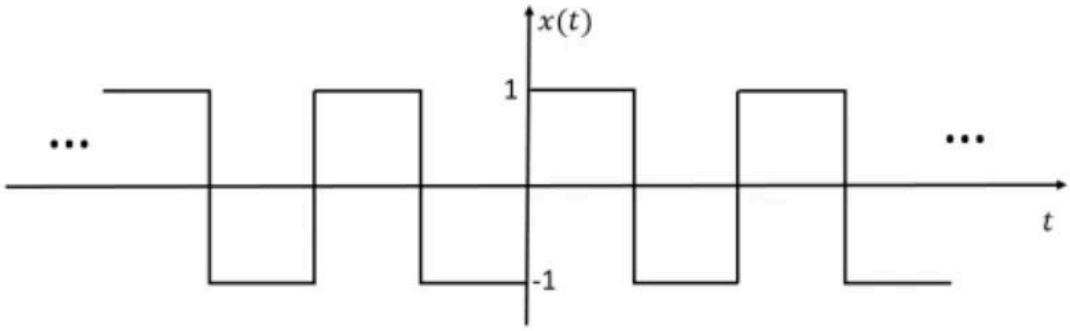
**WATCH OUT FOR**

Substituting the impulse location into the shift the wrong way, for example evaluating $x(4)$ or $x(4+3)$ instead of $x(4-3) = x(1)$. Since $|4| > 2$ and $|7| > 2$, this gives 0 (option C). The other slip is dropping the factor 2 and answering 1 (option B).

## Q.11 MCQ · 1 MARK

Signals and Systems › Fourier series

Q.11 An ideal square wave with period of 20 ms shown in the figure, is passed through an ideal low pass filter with cut-off frequency 120 Hz. Which of the following is an accurate description of the output?




(A) Output is zero.
(B) Output consists of both 50 Hz and 100 Hz frequency components.
(C) Output is a pure sinusoid of frequency 50 Hz.
(D) Output is a square wave of fundamental frequency 50 Hz.

Official answer: **C**

**SOLUTION**

1. Fundamental frequency: $f_0 = \dfrac{1}{T} = \dfrac{1}{20\ \text{ms}} = 50\ \text{Hz}$, so $\omega_0 = 2\pi f_0 = 100\pi\ \text{rad/s}$.
2. DC term: the wave spends equal time at $+1$ and $-1$ in every period, so $a_0 = \dfrac{1}{T}\displaystyle\int_0^T x(t)\,dt = 0$.
3. Symmetry. From the figure, $x(t) = +1$ for $0 < t < T/2$ and $x(t) = -1$ for $-T/2 < t < 0$, so $x(-t) = -x(t)$. The function is odd, so every cosine coefficient $a_n$ is zero. It also has half-wave symmetry, $x(t - T/2) = -x(t)$, so every even harmonic is zero and the series has only odd harmonics.
4. Sine coefficients:
$$b_n = \frac{2}{T}\int_0^T x(t)\sin(n\omega_0 t)\,dt = \frac{4}{T}\int_0^{T/2} \sin(n\omega_0 t)\,dt = \frac{2}{n\pi}\left(1 - \cos n\pi\right)$$
This gives $b_n = \dfrac{4}{n\pi}$ for odd $n$ and $b_n = 0$ for even $n$.
5. Fourier series:
$$x(t) = \frac{4}{\pi}\left[\sin(2\pi\,50\,t) + \frac{1}{3}\sin(2\pi\,150\,t) + \frac{1}{5}\sin(2\pi\,250\,t) + \cdots\right]$$
6. Components present: $50\ \text{Hz}, 150\ \text{Hz}, 250\ \text{Hz}, \ldots$ There is no $100\ \text{Hz}$ term and no DC term.
7. Ideal LPF with cut-off $120\ \text{Hz}$: only the $50\ \text{Hz}$ component is below the cut-off. The next component, $150\ \text{Hz}$, is above it and is removed completely, and so is every higher harmonic.
8. Output (taking the filter's passband gain as 1): $y(t) = \dfrac{4}{\pi}\sin(100\pi t)$. This is a pure $50\ \text{Hz}$ sinusoid, which is option (C). It is not zero, which rules out (A). It has no $100\ \text{Hz}$ term, which rules out (B). It has lost every harmonic that makes up a square shape, which rules out (D).

**KEY IDEA**

A zero-mean square wave with a 50% duty cycle has half-wave symmetry, so it contains only odd harmonics of its fundamental: $f_0, 3f_0, 5f_0, \ldots$ With $f_0 = 50$ Hz these are $50, 150, 250, \ldots$ Hz. An ideal LPF cutting off at $120$ Hz passes only the $50$ Hz fundamental, so the output is a single sinusoid.

**WATCH OUT FOR**

Assuming every harmonic is present and counting $100$ Hz as a component below the cut-off, which leads to (B). A symmetric square wave has no even harmonics. A second trap is (D): reasoning that the square wave's fundamental is below the cut-off, so the square wave passes. A square shape needs the higher odd harmonics, and the filter removes all of them.

## Q.12 NAT · 1 MARK

Control Systems › Bode & margins

An input $p(t) = \sin(t)$ is applied to the system $G(s) = \dfrac{s-1}{s+1}$. The corresponding steady state output is $y(t) = \sin(t + \varphi)$, where the phase $\varphi$ (in degrees), when restricted to $0^\circ \le \varphi \le 360^\circ$, is _____.

Official answer: **90, or -270**

**SOLUTION**

1. Sinusoidal steady state: $G(s)$ has its only pole at $s = -1$, so it is stable and the steady state exists. An input $\sin(\omega t)$ then gives the output $|G(j\omega)| \sin\left(\omega t + \angle G(j\omega)\right)$. Here $\omega = 1$ rad/s.
2. Substitute $s = j$:
$$G(j1) = \frac{-1+j}{1+j}$$
3. Rationalise:
$$G(j1) = \frac{(-1+j)(1-j)}{(1+j)(1-j)} = \frac{-1+j+j-j^2}{1-j^2} = \frac{-1+2j+1}{2} = \frac{2j}{2} = j$$
4. Magnitude: $|G(j1)| = 1$. This is an all-pass network ($|G(j\omega)| = 1$ at every $\omega$), which matches the unit amplitude of $y(t)$ given in the question.
5. Phase: $\angle G(j1) = \angle j = 90^\circ$. Check by quadrants: the numerator $-1 + j$ lies in the second quadrant, so its angle is $180^\circ - 45^\circ = 135^\circ$. The denominator $1 + j$ has angle $45^\circ$. Difference: $135^\circ - 45^\circ = 90^\circ$. In general,
$$\angle G(j\omega) = \left(180^\circ - \tan^{-1}\omega\right) - \tan^{-1}\omega = 180^\circ - 2\tan^{-1}\omega,$$
which is $180^\circ - 90^\circ = 90^\circ$ at $\omega = 1$ rad/s.
6. So $y(t) = \sin(t + 90^\circ)$ and $\varphi = 90^\circ$. This already lies in $[0^\circ, 360^\circ]$; the equivalent angle $-270^\circ$ is the same phase.

**KEY IDEA**

For a stable LTI system, a sinusoid of frequency $\omega$ comes out scaled by $|G(j\omega)|$ and shifted by $\angle G(j\omega)$. $\dfrac{s-1}{s+1}$ is an all-pass network with phase $180^\circ - 2\tan^{-1}\omega$, so at $\omega = 1$ rad/s you evaluate $G(j1) = j$ and read off $90^\circ$.

**WATCH OUT FOR**

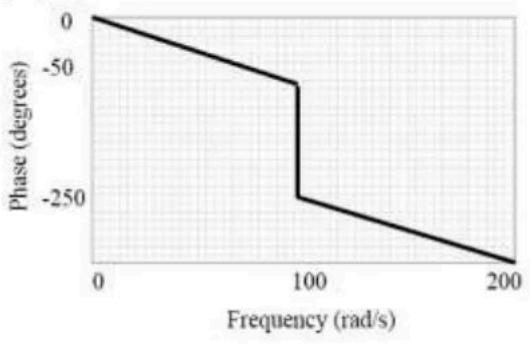
Taking the angle of the right-half-plane zero factor $(j\omega - 1)$ as $\tan^{-1}(\omega/(-1)) = -45^\circ$ from a calculator, when $-1 + j$ actually sits in the second quadrant at $135^\circ$. That slip gives a total of $-90^\circ$, which becomes $270^\circ$ in the requested range, and that is wrong. Another error is to treat the numerator like a left-half-plane zero ($+\tan^{-1}\omega$), which gives a phase of $0^\circ$.

## Q.13 MCQ · 1 MARK

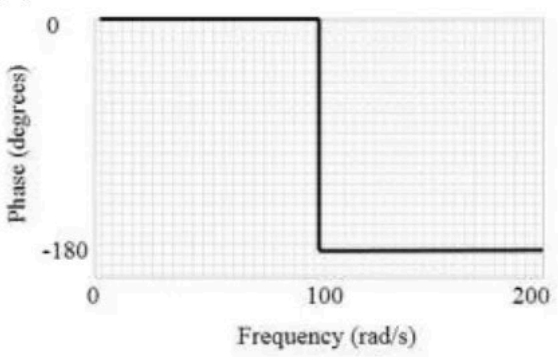
Control Systems › Bode & margins

Q.13 The approximate phase response of $\frac{100^2\, e^{-0.01\, s}}{s^2+0.2\, s+\, 100^2}$ is

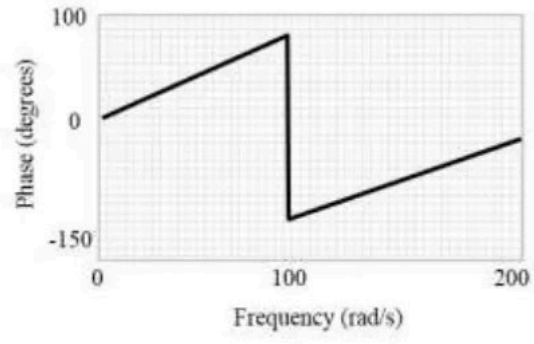
(A)



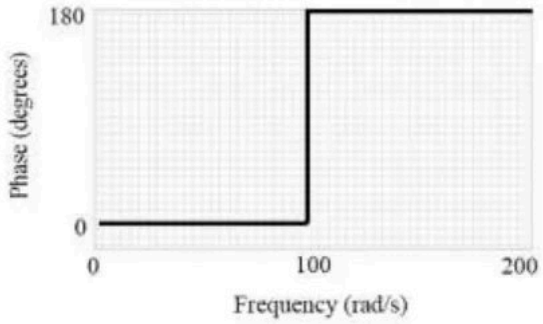

(B)




(C)




(D)




IN 3/14

Official answer: **A**

### SOLUTION

1. Split the phase into two parts. When factors are multiplied, their phases add. The constant $100^2$ in the numerator adds nothing.

$$\angle G(j\omega) = \angle e^{-j0.01\omega} + \angle \frac{100^2}{(j\omega)^2 + 0.2\, j\omega + 100^2}$$

2. Match the quadratic to the standard form $s^2 + 2\zeta\omega_n s + \omega_n^2$. This gives $\omega_n = 100$ rad/s and $2\zeta\omega_n = 0.2$, so $\zeta = \dfrac{0.2}{2 \times 100} = 0.001$. The poles are almost undamped.

3. Phase of the quadratic. At $s = j\omega$ the denominator is $(100^2 - \omega^2) + j\, 0.2\, \omega$. Its imaginary part is always positive. Its real part changes sign at $\omega = 100$. So

$$\omega < 100 : \ \phi_2 = -\tan^{-1} \frac{0.2\, \omega}{100^2 - \omega^2} \approx 0^\circ$$

$$\omega = 100 : \ \phi_2 = -90^\circ$$

$$\omega > 100 : \ \phi_2 = -180^\circ + \tan^{-1} \frac{0.2\, \omega}{\omega^2 - 100^2} \approx -180^\circ$$

   The phase passes $-45^\circ$ at $\omega \approx 99.9$ rad/s and $-135^\circ$ at $\omega \approx 100.1$ rad/s. The whole $180^\circ$ fall therefore happens within about $0.2$ rad/s. On a 0 to 200 rad/s axis that is a vertical downward step at $\omega_n = 100$ rad/s.

4. Phase of the pure delay. $|e^{-j\omega T}| = 1$, but $\angle e^{-j\omega T} = -\omega T$. With $T = 0.01$ s, $\phi_1 = -0.01\, \omega$ rad $= -0.573\, \omega$ degrees. This is a straight line through the origin with a negative slope, and it continues on both sides of 100 rad/s.

5. Add the two parts at the key frequencies:

$$\omega \to 0:\ \phi = 0^\circ + 0^\circ = 0^\circ$$
$$\omega = 100^-:\ \phi = -57.3^\circ + 0^\circ = -57.3^\circ$$
$$\omega = 100^+:\ \phi = -57.3^\circ - 180^\circ = -237.3^\circ$$
$$\omega = 200:\ \phi = -114.6^\circ - 180^\circ = -294.6^\circ$$

6. So the phase has three parts: a falling straight line, a $180^\circ$ downward drop at 100 rad/s, and then a falling line with the same slope. Option (A) is the only plot with this shape. Its axis labels are not evenly spaced, so it is a sketch and its corner values cannot be read off it. The shape identifies it.
7. Rule out the others. (B) and (D) are flat except for the step, so they leave out the delay's linear phase lag. (D) also steps up to $+180^\circ$. Tracked continuously, the phase of a pair of left-half-plane poles moves from $0^\circ$ down to $-180^\circ$, never up. (C) has a rising slope, which would need a lead term $e^{+0.01s}$ instead of a delay.

**KEY IDEA**

When factors are multiplied, their phases add. A pure delay $e^{-sT}$ has magnitude 1 but phase $-\omega T$, which is a straight falling line on a linear frequency axis. A second-order pair with $\zeta \approx 0$ has a phase that jumps almost vertically from $0^\circ$ to $-180^\circ$ at $\omega_n$. The answer is the delay's line with a $-180^\circ$ step cut into it at $\omega_n = 100\ \text{rad/s}$.

**WATCH OUT FOR**

The most common mistake is leaving out the delay because $|e^{-j\omega T}| = 1$, which leads to (B). The delay's magnitude is 1, but its phase is not zero. Two other slips: taking the delay's phase as $+\omega T$, which gives the rising slope of (C); and evaluating $\tan^{-1}\dfrac{0.2\,\omega}{100^2 - \omega^2}$ on a calculator above $\omega_n$. There the denominator is negative, so the angle lies in the second quadrant and the phase is near $-180^\circ$, not near $0^\circ$.

## Q.14 NAT · 1 MARK

Control Systems › Bode & margins

Consider the transfer function $G(s) = \dfrac{2}{(s+1)(s+2)}$. The phase margin of $G(s)$ in degrees is _____.

Official answer: **180, or -180**

**SOLUTION**

1. Frequency response: $G(j\omega) = \dfrac{2}{(1+j\omega)(2+j\omega)}$, so
$$|G(j\omega)| = \frac{2}{\sqrt{\omega^2+1}\sqrt{\omega^2+4}}, \qquad \angle G(j\omega) = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$
2. Gain crossover condition $|G(j\omega_{gc})| = 1$:
$$(\omega^2+1)(\omega^2+4) = 4 \ \Rightarrow\ \omega^4 + 5\omega^2 + 4 = 4 \ \Rightarrow\ \omega^2(\omega^2+5) = 0$$
3. The only real, non-negative root is $\omega_{gc} = 0\ \text{rad/s}$ (the other factor gives $\omega^2 = -5$, which has no real solution). Check: the DC gain is $G(0) = \dfrac{2}{1\cdot 2} = 1$, i.e. $0\ \text{dB}$, and since $(\omega^2+1)(\omega^2+4)$ increases for every $\omega > 0$, $|G(j\omega)|$ falls strictly below 1 after that. The magnitude curve therefore touches $0\ \text{dB}$ only at $\omega = 0$.
4. Phase at gain crossover: $\angle G(j0) = -\tan^{-1}0 - \tan^{-1}0 = 0^\circ$. (On the Nyquist plot, the curve starts at the point $1 + j0$, which is exactly $180^\circ$ away from the critical point $-1 + j0$ on the unit circle.)
5. Phase margin definition:
$$\text{PM} = 180^\circ + \angle G(j\omega_{gc}) = 180^\circ + 0^\circ = 180^\circ$$

**KEY IDEA**

Phase margin is measured where $|G(j\omega)| = 1$. Here the DC gain $\frac{2}{1 \cdot 2}$ is exactly $1$ and the magnitude only falls from there, so the gain crossover sits at $\omega = 0$, where the phase is $0^\circ$. That gives $\mathrm{PM} = 180^\circ + 0^\circ = 180^\circ$.

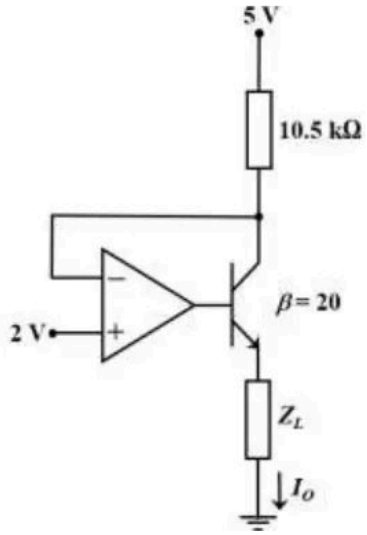
**WATCH OUT FOR**

Seeing that $|G(j\omega)|$ never goes above $0\ \mathrm{dB}$ and deciding there is no gain crossover, then answering "infinite" or "undefined". The magnitude equals $1$ at $\omega = 0$, and that point is the gain crossover. Another mistake is to mix up phase margin with gain margin. The phase only reaches $-180^\circ$ as $\omega \to \infty$, so there is no finite phase crossover and the gain margin is infinite, but that is a separate quantity from the phase margin. A third slip is dropping the constant term when expanding $(\omega^2 + 1)(\omega^2 + 4) = 4$, which gives a spurious non-zero crossover frequency.

## Q.15 NAT · 1 MARK

Analog Electronics › Biasing

Q.15 In the given circuit, assume that the opamp is ideal and the transistor has a $\beta$ of 20. The current $I_o$ (in μA) flowing through the load $Z_L$ is ____.




Official answer: **0**

**SOLUTION**

1. **Assumptions.** The op-amp is ideal: infinite open-loop gain $A$, zero input current, and output $v_o = A(V_+ - V_-)$, limited to its supply rails. The rails are not drawn. The transistor is an NPN with $\beta = 20$ and conducts only when its base–emitter junction is forward biased.
2. **Check the feedback sign before using a virtual short.** $V_- = V_+$ holds only if the loop gives negative feedback. Follow a small rise in the op-amp output around the loop:
$$v_o \uparrow \Rightarrow\ I_C \uparrow \Rightarrow\ V_C = 5 - (10.5\ \mathrm{k\Omega})\, I_C \downarrow \Rightarrow\ V_- \downarrow \Rightarrow\ (V_+ - V_-) \uparrow \Rightarrow\ v_o \uparrow$$
The disturbance reinforces itself.
3. **What that means.** The common-emitter stage inverts from base to collector. The collector goes back to the op-amp's inverting (–) input, which inverts a second time. Two inversions make the loop non-inverting, so the feedback is positive (regenerative). With the ideal op-amp's infinite gain the loop gain is far above 1, so the op-amp cannot stay in its linear region. The point $V_C = 2\ \mathrm{V}$ would give $I_C = 3/10.5\ \mathrm{mA} = 285.71\ \mu\mathrm{A}$ and $I_E = 300\ \mu\mathrm{A}$, but it is an unstable balance point. Any disturbance drives the output to a rail.
4. **Find the saturated state that is self-consistent.** Suppose the transistor carries no current. The (–) input draws no current either, so no current flows in the 10.5 kΩ resistor and

$$V_C = 5 - (10.5\ \text{k}\Omega)(0) = 5\ \text{V}$$

Then $V_- = 5\ \text{V} > V_+ = 2\ \text{V}$, so $v_o = A(2-5)$ goes to the negative rail (or to 0 V on a single supply).

5. **Check the assumption.** With no current in $Z_L$ the emitter is at 0 V and the base is at $v_o \le 0$. So $V_{BE} \le 0$ and the transistor is cut off, as assumed. The circuit also starts in this state. At switch-on no collector current has flowed yet, so the collector sits at the +5 V rail, above the 2 V reference. The output swings negative, and the positive feedback keeps the transistor off.
6. **Load current.** In cut-off $I_B = I_C = 0$, so
$$I_o = I_E = I_B + I_C = 0\ \mu\text{A}$$
7. **Remark.** 300 µA is what you get if the op-amp inputs are swapped (collector to (+), 2 V to (−)). Then a rise in $v_o$ lowers $V_+$, the feedback is negative and the virtual short holds. So $V_C = 2$ V and $I_o = I_C(\beta+1)/\beta = 285.71 \times 21/20 = 300\ \mu\text{A}$. If the op-amp output sat at its positive limit instead, $I_o$ would depend on $Z_L$ and on the op-amp's rail and current limit. None of these is given, so that state cannot produce the single number the question asks for.

**KEY IDEA**

Check the sign of the feedback before assuming $V_- = V_+$. The CE stage inverts from base to collector, and the feedback goes to the (−) input, which inverts again. Two inversions make the feedback positive, so the op-amp saturates. The consistent state has the collector at 5 V, above the 2 V reference, with the op-amp output at its negative limit and the transistor cut off. So $I_o = 0$.

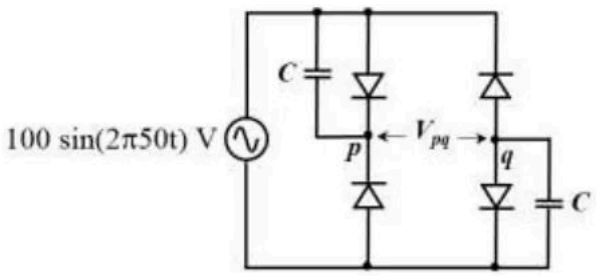
**WATCH OUT FOR**

Using the virtual short straight away ($V_C = 2$ V, so $I_C = 285.7\ \mu$A and $I_o = I_E = 300\ \mu$A) without checking that the feedback is negative. That answer holds only if the op-amp inputs are swapped. As drawn, the loop is regenerative and the transistor latches off.

## Q.16 NAT · 1 MARK

Analog Electronics › Diode circuits

Q.16 The diodes given in the circuit are ideal. At $t = 60$ ms, $V_{pq}$ (in Volts) is ____.




IN 4/14

Official answer: **200**

**SOLUTION**

1. **Reference and timing.** Take the bottom rail B as ground, so the top rail is at $v_A = 100\sin(100\pi t)$ V, with $V_m = 100$ V and $T = 20$ ms. Assume both capacitors start uncharged and nothing loads p or q. Then $t = 60\ \text{ms} = 3T$, so $v_A(60\ \text{ms}) = 100\sin 6\pi = 0$. Both capacitors have already been through the negative peak at $t = 15$ ms.
2. **Left branch: the diode constraints.** An ideal diode can never be forward biased. The upper-left diode (anode at A, cathode at p) sits directly across the left capacitor, so $V_p \ge v_A$. The lower-left diode (anode at ground, cathode at p) gives $V_p \ge 0$.
3. **Left branch: how the capacitor charges (a clamper).** Both left diodes can only push current *into* p, and the only way out of p is through C to A. So C can charge only around the loop ground → lower diode → p → C → A → source, and it can never discharge. It charges while $v_A$ is falling to a new minimum below

zero (10 to 15 ms in the first cycle). During that time the lower diode holds $V_p = 0$, so $V_C = V_p - v_A = -v_A$. At the negative peak C has charged to 100 V, with p positive relative to A. It keeps that charge, because both diodes block any discharge current and p is otherwise open.

4. **So p rides on the source (positive clamper):** $V_p = v_A + 100$ V, which swings between 0 and 200 V.
5. **Right branch: a negative peak detector.** The lower-right diode (anode at q, cathode at ground) sits directly across the right capacitor, so $V_q \le 0$. The upper-right diode (anode at q, cathode at A) gives $V_q \le v_A$. Both right diodes can only draw current *out of* q, so the right capacitor can only charge with q negative. The upper-right diode conducts and pulls q down whenever $v_A$ falls below $V_q$. At $t = 15$ ms, q reaches $-100$ V. After that $v_A \ge -100$ V always, so neither diode is forward biased again, and the capacitor holds $V_q = -100$ V.
6. **Evaluate at $t = 60$ ms:**

$$V_{pq} = V_p - V_q = (0 + 100) - (-100) = 200 \text{ V}$$

7. **Check.** In steady state $V_{pq}(t) = 200 + 100\sin(100\pi t)$ V. The source polarity is not marked, but reversing it would not change the value at $t = 60$ ms. The source is at 0 then, and both capacitors have already charged to 100 V (the negative peak would simply come at 5 ms instead). The value of C does not matter, because ideal diodes charge the capacitors instantly.

**KEY IDEA**

The left half is a positive clamper: its capacitor charges to $V_m$, and node p rides on the source at $v + V_m$ (between 0 and $2V_m$). The right half is a negative peak detector: its capacitor holds q at $-V_m$. So $V_{pq} = v + 2V_m$. At $t = 60$ ms $= 3T$ the source is exactly zero, which leaves $V_{pq} = 2V_m = 200$ V.

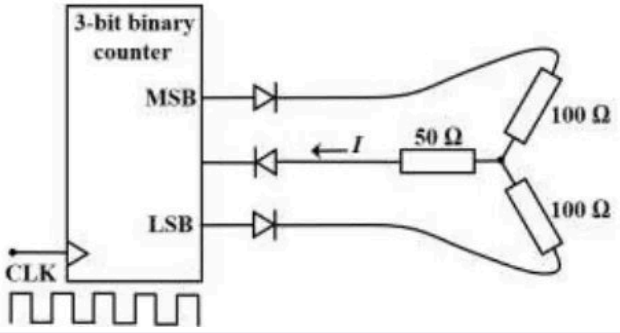
**WATCH OUT FOR**

The usual slip is treating p as a peak detector held at a constant value. p is a clamper output that swings from 0 to 200 V, so its value depends on the instant. Two common wrong answers come from evaluating at a source peak instead of at $t = 60$ ms: 300 V (positive peak) and 100 V (negative peak). The other slip is giving q the wrong sign: taking $V_q = +100$ V gives 0 V, and computing $V_q - V_p$ instead of $V_p - V_q$ gives $-200$ V.

## Q.17 NAT · 1 MARK

Digital Electronics › Sequential

Q.17 For the 3-bit binary counter shown in the figure, the output increments at every positive transition in the clock (CLK). Assume ideal diodes and the starting state of the counter as 000. If output high is 1 V and output low is 0 V, the current $I$ (in mA) flowing through the 50 Ω resistor during the 5th clock cycle is (up to one decimal place) _____.




Official answer: **10**

**SOLUTION**

1. **Reading the circuit.** Call the counter outputs $Q_2$ (MSB), $Q_1$ (middle bit, unlabelled in the figure) and $Q_0$ (LSB). The MSB and LSB diodes point away from the counter, so each can only push current from a HIGH output through its 100 Ω resistor to the junction X. The middle-bit diode points into the counter, so

the $50\,\Omega$ branch carries current from X into the middle bit only when that bit is LOW. Current $I$ therefore flows only when $Q_1 = 0$ and at least one of $Q_2$ or $Q_0$ is 1.

2. **Finding the counter state.** The counter starts at $(000)_2$ and goes up by one on every positive clock edge. Each clock cycle begins with a rising edge, so the edge that starts clock cycle $n$ sets the count to $n$. During the 5th clock cycle the state is therefore $(101)_2$: $Q_2 = 1$, $Q_1 = 0$, $Q_0 = 1$.
3. **Diode states for 101.** The MSB and LSB diodes each have 1 V on the anode side, so both conduct (ideal diodes, 0 V drop). The middle-bit diode has its cathode at 0 V and conducts. The two $100\,\Omega$ branches are then two 1 V sources in parallel feeding X.
4. **KCL at X:**
$$2 \cdot \frac{1 - V_X}{100} = \frac{V_X - 0}{50} \;\Rightarrow\; \frac{1 - V_X}{50} = \frac{V_X}{50} \;\Rightarrow\; V_X = 0.5\ \text{V}$$
5. **Current in the 50 Ω resistor:**
$$I = \frac{V_X}{50} = \frac{0.5}{50}\ \text{A} = 10\ \text{mA}$$
Check with an equivalent resistance: $(100 \parallel 100) + 50 = 100\,\Omega$, so $I = 1\ \text{V}/100\,\Omega = 10\ \text{mA}$. The assumed diode states hold: each $100\,\Omega$ branch carries $(1 - 0.5)/100 = 5\ \text{mA}$ forward, and the middle diode carries the full $10\ \text{mA}$ forward. To one decimal place, $I = 10.0\ \text{mA}$.
6. **Sanity check against the other states.** For 001 and 100 only one diode conducts, so $I = 1/(100 + 50)\ \text{A} = 6.67\ \text{mA}$. For 000, 010, 011, 110 and 111, $I = 0$: either no output is HIGH, or the middle-bit diode's cathode sits at 1 V and blocks, because X can never rise above 1 V. Only 101 gives the parallel pair, which is why the state during the 5th cycle decides this question.

**KEY IDEA**

Work out the counter state during the asked-for clock cycle, then use the diode directions to find which branches conduct. The MSB and LSB diodes can only source current from a HIGH output. The middle diode can only sink current into a LOW middle bit. In state 101 two 1 V sources, each behind $100\,\Omega$, act in parallel through $50\,\Omega$ into 0 V.

**WATCH OUT FOR**

Taking the state during the 5th clock cycle to be $(100)_2$ (the count before the 5th edge) instead of $(101)_2$. That leaves only the MSB branch conducting and gives $1/150\ \text{A} = 6.7\ \text{mA}$. A second slip is misreading the middle diode's direction. It points INTO the counter, so current flows only when the middle bit is 0; if you treat it as a source like the other two you get the wrong network.

## Q.18 MCQ · 1 MARK

Digital Electronics › Number systems

The representation of the decimal number $(27.625)_{10}$ in base-2 number system is

**(A)** 11011.110

**(B)** 11101.101

**(C)** 11011.101

**(D)** 10111.110

Official answer: **C**

**SOLUTION**

1. Convert the integer part and the fractional part separately: $(27.625)_{10} = 27 + 0.625$.
2. Integer part: divide repeatedly by 2 and read the remainders from bottom to top. $27 \to 13\,r\,1$, $13 \to 6\,r\,1$, $6 \to 3\,r\,0$, $3 \to 1\,r\,1$, $1 \to 0\,r\,1$, so $(27)_{10} = (11011)_2$.
3. Check with place values: $16 + 8 + 0 + 2 + 1 = 27$.
4. Fractional part: multiply repeatedly by 2 and read the integer carries from top to bottom. $0.625 \times 2 = 1.25 \to 1$, $0.25 \times 2 = 0.5 \to 0$, $0.5 \times 2 = 1.0 \to 1$. The fraction is now zero, so the conversion ends.

5. So $(0.625)_{10} = (0.101)_2$. Check: $\frac{1}{2} + 0 + \frac{1}{8} = 0.625$.
6. Put the two parts together: $(27.625)_{10} = (11011.101)_2$, which is option (C).

**KEY IDEA**

Convert the integer part and the fractional part separately. For the integer part, divide repeatedly by 2 and read the remainders from the last one up. For the fractional part, multiply repeatedly by 2 and read the carries from the first one down. $0.625 = \frac{1}{2} + \frac{1}{8}$ terminates in binary, so the result is exact.

**WATCH OUT FOR**

The usual slip is reading the fractional carries in the wrong order. Reading them bottom to top gives .101 again only because it is a palindrome, but on other fractions it gives a wrong answer. The distractors come from these slips: (A) $11011.110 = 27.75$ gets the fraction wrong, (B) $11101.101 = 29.625$ swaps bits in the integer part, and (D) $10111.110 = 23.75$ reverses the integer remainders (reading them top to bottom) and also gets the fraction wrong. A quick place-value check of the chosen option catches all three.

## Q.19 MCQ · 1 MARK

Digital Electronics › ADC & DAC

The number of comparators required for implementing an 8-bit flash analog-to-digital converter is

**(A)** 8

**(B)** 128

**(C)** 255

**(D)** 256

Official answer: **C**

**SOLUTION**

1. A flash (parallel) ADC compares the analog input with every quantisation threshold at the same time. Each threshold needs its own comparator.
2. An $n$-bit converter has $2^n$ output codes, from $(00\ldots0)_2$ to $(11\ldots1)_2$. To separate $2^n$ adjacent levels you need $2^n - 1$ boundaries between them, so the converter needs $2^n - 1$ comparators. A resistor ladder of $2^n$ resistors across the reference voltage produces those $2^n - 1$ threshold voltages.
3. The comparator outputs form a thermometer code: every comparator whose threshold is below the input reads 1, and every one above it reads 0. A $(2^n - 1)$-input priority encoder converts this into the $n$-bit binary output.
4. For $n = 8$:

$$N_{\text{comp}} = 2^n - 1 = 2^8 - 1 = 256 - 1 = 255$$

5. The answer is option (C), 255.

**KEY IDEA**

A flash ADC uses one comparator for each boundary between adjacent output codes. An $n$-bit converter has $2^n$ codes and so $2^n - 1$ boundaries, which means it needs $2^n - 1$ comparators. That number doubles with every extra bit, which is why flash converters are fast but large.

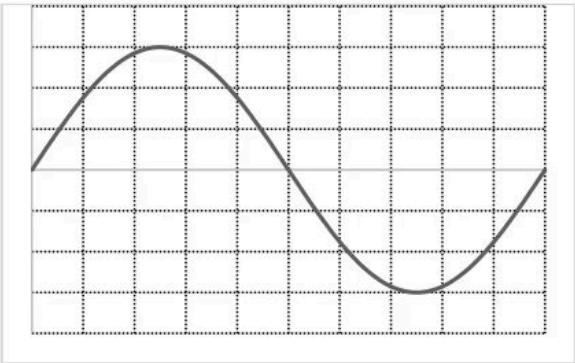
**WATCH OUT FOR**

Picking 256 ($2^n$) by counting output levels or ladder resistors when you should count the boundaries between levels. The other trap is picking 8 ($n$) by confusing a flash ADC with a successive-approximation ADC, which uses a single comparator for $n$ clock cycles.

## Q.20 MCQ · 1 MARK

Electrical and Electronic Measurements › Time, frequency & scopes

Q.20 A voltage of 6 cos(100πt) V is fed as y-input to a CRO. The waveform seen on the screen of the CRO is shown in the figure. The Y and X axes settings for the CRO are respectively



(A) 1 V/div and 1 ms/div
(B) 1 V/div and 2 ms/div
(C) 2 V/div and 1 ms/div
(D) 2 V/div and 2 ms/div

Official answer: **D**

### SOLUTION

1. Read the signal: $v(t) = 6\cos(100\pi t)$ V, so the peak value is $V_m = 6$ V and the angular frequency is $\omega = 100\pi$ rad/s.
2. Frequency and period from $\omega = 2\pi f$: $f = \dfrac{100\pi}{2\pi} = 50$ Hz, so $T = \dfrac{1}{f} = \dfrac{1}{50}$ s $= 20$ ms.
3. Read the screen vertically: the positive peak is 3 divisions above the centre line and the negative peak is 3 divisions below it, so the amplitude spans 3 vertical divisions (6 divisions peak-to-peak).
4. Vertical sensitivity = volts per division: $\dfrac{V_m}{3\text{ div}} = \dfrac{6\text{ V}}{3\text{ div}} = 2\text{ V/div}$. Check with peak-to-peak values: $\dfrac{12\text{ V}}{6\text{ div}} = 2\text{ V/div}$.
5. Read the screen horizontally: the trace goes from the centre line up to the peak, down through the centre line at the middle of the screen, down to the trough and back to the centre line at the right edge. That is exactly one complete cycle across all 10 horizontal divisions.
6. Time base = time per division: $\dfrac{T}{10\text{ div}} = \dfrac{20\text{ ms}}{10\text{ div}} = 2\text{ ms/div}$.
7. The trace starts at a zero crossing, not at a peak, only because of where the sweep is triggered. The trigger point sets the starting phase on screen but does not change the amplitude or the period. So the settings are $2\text{ V/div}$ and $2\text{ ms/div}$, which is option (D).

**KEY IDEA**

On a CRO, volts per division = voltage ÷ vertical divisions it covers, and time per division = time ÷ horizontal divisions it covers. Take the peak (or peak-to-peak) voltage and the period $T = 2\pi/\omega$ from the signal, count their divisions on the graticule, and divide.

**WATCH OUT FOR**

Three common slips. (1) Dividing the peak voltage by the peak-to-peak height: $6\text{ V}/6\text{ div}$ gives the wrong answer of 1 V/div. (2) Taking $100\pi$ as the frequency in hertz, or using $f = 100$ Hz ($T = 10$ ms, which gives 1 ms/div), when in fact $\omega = 2\pi f$ gives $f = 50$ Hz. (3) Thinking that because the input is a cosine, the trace cannot start at a zero crossing. Where the trace starts depends only on the trigger point.

## Q.21 NAT · 1 MARK

Electrical and Electronic Measurements › V, I, power & energy

A 300 V, 5A, 0.2 pf low power factor wattmeter is used to measure the power consumed by a load. The wattmeter scale has 150 divisions and the pointer is on the $100^{\text{th}}$ division. The power consumed by the load (in Watts) is _____ .

Official answer: **200**

**SOLUTION**

1. A low power factor (LPF) wattmeter is calibrated so that full-scale deflection corresponds to rated voltage × rated current × rated power factor, not to the product $VI$ alone.
2. Full-scale power:
$$P_{fs} = V_{rated}\, I_{rated} \cos\phi_{rated} = 300 \times 5 \times 0.2 = 300 \text{ W}$$
3. The scale has 150 divisions, so the multiplying factor (watts per division) is
$$k = \frac{300 \text{ W}}{150 \text{ div}} = 2 \text{ W/div}$$
4. Wattmeter deflection is proportional to power, so the reading at the 100th division is
$$P = 100 \text{ div} \times 2 \text{ W/div} = 200 \text{ W}$$
5. Power consumed by the load $= 200$ W.

**KEY IDEA**

For an LPF wattmeter, full-scale reading = rated $V\times$ rated $I\times$ rated power factor. Here that is $300 \times 5 \times 0.2 = 300$ W across 150 divisions, so each division is worth 2 W.

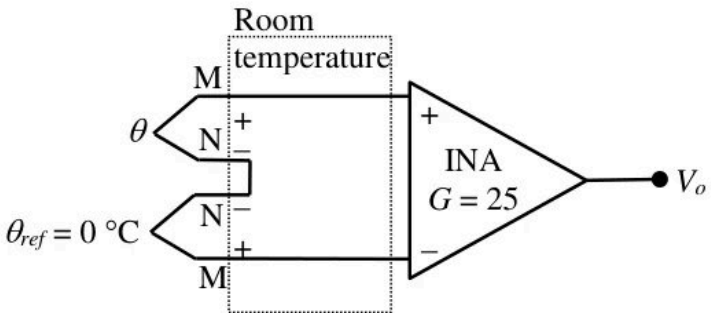
**WATCH OUT FOR**

Leaving out the rated power factor and taking full scale as $VI = 1500$ W. That gives 10 W/div and a wrong answer of 1000 W. A second slip is to multiply the reading by 0.2 again. The rated power factor only sets the full-scale value; it is not applied to the reading.

## Q.22 NAT · 1 MARK

Sensors and Industrial Instrumentation › Temperature

Q.22 As shown in the figure, temperature $\theta$ is measured using a K type thermocouple. It has a sensitivity of 40 μV/°C. The gain ($G$) of the ideal instrumentation amplifier is 25. If the output $V_o$ is 96 mV, then the value of $\theta$ (in °C) is ____.




Official answer: **95 to 97**

**SOLUTION**

1. The two thermocouple junctions (M-N at $\theta$ and N-M at $\theta_{ref} = 0\,^{\circ}\text{C}$) are connected in series opposition through the joined N legs. Every other joint (the N-N link and the M leads meeting the amplifier terminals) is at room temperature, and by the law of intermediate metals and temperatures these add no net emf. The net emf is therefore $E = S(\theta - \theta_{ref})$.
2. The reference junction is at $0\,^{\circ}\text{C}$ and $S = 40\ \mu\text{V}/^{\circ}\text{C}$, so $E = S\,\theta$.

3. The ideal instrumentation amplifier has gain $G = 25$ and the emf is applied across its inputs, so $V_o = G\,E$.
4. The input emf is
$$E = \frac{V_o}{G} = \frac{96\ \text{mV}}{25} = 3.84\ \text{mV}$$
5. The temperature is
$$\theta = \frac{E}{S} = \frac{3.84 \times 10^{-3}\ \text{V}}{40 \times 10^{-6}\ \text{V}/^{\circ}\text{C}} = 96\ ^{\circ}\text{C}$$

**KEY IDEA**
With the reference junction at $0\ ^{\circ}\text{C}$, the thermocouple emf is $S\,\theta$. An ideal amplifier of gain $G$ gives $V_o = G\,S\,\theta$, so $\theta = V_o/(G\,S)$. Joints at room temperature in the circuit add no net emf.

**WATCH OUT FOR**
Forgetting to divide $V_o$ by the amplifier gain, or mixing mV with $\mu$V and being off by a factor of 1000. Another slip is to subtract room temperature from $\theta$, even though the reference junction here is held at $0\ ^{\circ}\text{C}$.

## Q.23 MCQ · 1 MARK

Sensors and Industrial Instrumentation › Sensing principles

A piezoelectric pressure sensor has a bandpass characteristic with cut-off frequencies of 0.1 Hz and 1 MHz, and a sensitivity of 100 mV/kPa. The sensor is subjected to a static constant pressure of 100 kPa. Its steady-state output will be

**(A)** 0 V
**(B)** 0.1 V
**(C)** 1 V
**(D)** 10 V

Official answer: **A**

**SOLUTION**

1. A piezoelectric sensor produces charge only when the applied force changes. The charge leaks away through the finite insulation resistance and the amplifier input resistance. So the sensor behaves as a high-pass element at low frequency.
2. The sensor has a bandpass characteristic with a lower cut-off of $0.1\ \text{Hz}$ and an upper cut-off of $1\ \text{MHz}$. The gain therefore falls to zero as $f \to 0$.
3. A static constant pressure has frequency $f = 0\ \text{Hz}$. This is far below the lower cut-off, so the transfer function gives $H(0) = 0$.
4. The sensitivity applies only inside the passband. For a dynamic pressure of $100\ \text{kPa}$ the output would be $100\ \text{mV/kPa} \times 100\ \text{kPa} = 10\ \text{V}$.
5. For the static input the steady-state output is $V_{ss} = H(0) \times 10\ \text{V} = 0\ \text{V}$. The charge produced when the pressure is first applied decays to zero. The answer is option (A).

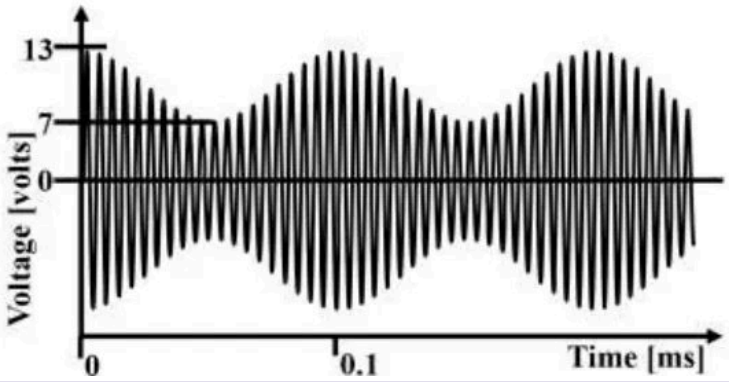
**KEY IDEA**
A piezoelectric sensor responds only to changing pressure. Its response at DC is zero, so a static pressure gives zero steady-state output, whatever the sensitivity.

**WATCH OUT FOR**
Multiplying the sensitivity by the pressure ($100\ \text{mV/kPa} \times 100\ \text{kPa} = 10\ \text{V}$) and choosing (D). That figure holds only for a signal inside the passband, not for a constant input.

## Q.24 NAT · 1 MARK

Communications › Analog modulation

Q.24 An amplitude modulated signal is shown in the figure. The modulation index is (up to one decimal place) _____.




Official answer: **0.3**

**SOLUTION**

1. Read the envelope from the figure. The envelope peaks (the horizontal line from the 13 label) at $A_{max} = 13$ V. It dips at its troughs (the horizontal line from the 7 label) to $A_{min} = 7$ V. The waveform is symmetric about the zero line.
2. For standard AM, $s(t) = A_c\,[1 + \mu\cos(2\pi f_m t)]\cos(2\pi f_c t)$. The envelope is $A_c[1 + \mu\cos(2\pi f_m t)]$, so $A_{max} = A_c(1 + \mu)$ and $A_{min} = A_c(1 - \mu)$.
3. Add and subtract: $A_c = \dfrac{A_{max} + A_{min}}{2} = \dfrac{13 + 7}{2} = 10$ V and $\mu A_c = \dfrac{A_{max} - A_{min}}{2} = \dfrac{13 - 7}{2} = 3$ V.
4. Modulation index: $\mu = \dfrac{A_{max} - A_{min}}{A_{max} + A_{min}} = \dfrac{13 - 7}{13 + 7} = \dfrac{6}{20} = 0.3.$
5. Since $\mu < 1$, the envelope never reaches zero. This agrees with the figure, where the envelope bottoms out at 7 V and the signal is not over-modulated.

**KEY IDEA**

The modulation index of an AM wave is $\mu = \dfrac{A_{max} - A_{min}}{A_{max} + A_{min}}$, found from the largest and smallest envelope values. Here the envelope runs from 7 V to 13 V.

**WATCH OUT FOR**

Using $\dfrac{A_{max} - A_{min}}{A_{max}} = \dfrac{6}{13} \approx 0.46$ or $\dfrac{A_{max} - A_{min}}{2} = 3$. The denominator must be $A_{max} + A_{min} = 2A_c$, the sum of the two envelope levels, and not $A_{max}$ alone.

## Q.25 NAT · 1 MARK

Sensors and Industrial Instrumentation › Optical

An optical pulse containing $6 \times 10^6$ photons is incident on a photodiode and $4.5 \times 10^6$ electron-hole pairs are created. The maximum possible quantum efficiency (in %) of the photodiode is _____ .

Official answer: **75**

**SOLUTION**

1. Quantum efficiency is the ratio of the number of electron-hole pairs generated to the number of photons incident: $\eta = \dfrac{\text{electron-hole pairs generated}}{\text{incident photons}}$.
2. Each absorbed photon creates at most one electron-hole pair, so the $4.5 \times 10^6$ pairs created are the most that can be credited to the $6 \times 10^6$ incident photons. The word 'maximum' means every created pair is counted, with none lost to recombination.

3. $$\eta = \frac{4.5 \times 10^6}{6 \times 10^6} = 0.75$$

4. In percent, $\eta = 0.75 \times 100 = 75\%$.

**KEY IDEA**
Quantum efficiency is electron-hole pairs created divided by photons incident. 'Maximum possible' means all created pairs count, so the ratio is taken directly.

**WATCH OUT FOR**
Using the ratio the wrong way round (photons divided by pairs) gives 133%, which is impossible. Another slip is leaving the answer as 0.75 instead of converting to percent.

## Q.26 MCQ · 2 MARKS

Engineering Mathematics › Vector calculus

Given $\vec{F} = (x^2 - 2y)\,\vec{i} - 4yz\,\vec{j} + 4xz^2\,\vec{k}$, the value of the line integral $\int_c \vec{F} \cdot d\vec{l}$ along the straight line $c$ from $(0, 0, 0)$ to $(1, 1, 1)$ is

**(A)** $3/16$

**(B)** $0$

**(C)** $-5/12$

**(D)** $-1$

Official answer: **D**

**SOLUTION**

1. Parametrise the straight line from $(0, 0, 0)$ to $(1, 1, 1)$: $x = t,\ y = t,\ z = t$ with $0 \le t \le 1$, so $dx = dy = dz = dt$.
2. Write the line integral in component form: $\int_c \vec{F} \cdot d\vec{l} = \int_c \left[(x^2 - 2y)\,dx - 4yz\,dy + 4xz^2\,dz\right]$.
3. Substitute the parametrisation into each component: $x^2 - 2y = t^2 - 2t,\ -4yz = -4t^2,\ 4xz^2 = 4t^3$.
4. Since $dx = dy = dz = dt$, add the three terms: $\vec{F} \cdot d\vec{l} = \left(t^2 - 2t - 4t^2 + 4t^3\right) dt = \left(4t^3 - 3t^2 - 2t\right) dt$.
5. Integrate over $0 \le t \le 1$:
$$\int_0^1 \left(4t^3 - 3t^2 - 2t\right) dt = \left[t^4 - t^3 - t^2\right]_0^1 = 1 - 1 - 1 = -1$$
6. The field is not conservative ($\nabla \times \vec{F} \neq \vec{0}$; for example the $\vec{k}$-component is $\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} = 0 - (-2) = 2$), so the value depends on the path and holds for this straight line only. The answer is $-1$, option (D).

**KEY IDEA**
To evaluate a line integral along a given path, parametrise the path, here $x = y = z = t$ for $0 \le t \le 1$. Write $\vec{F} \cdot d\vec{l} = F_x\,dx + F_y\,dy + F_z\,dz$ in terms of $t$ and integrate over the parameter range. Because $\nabla \times \vec{F} \neq \vec{0}$, no potential function exists, so the integral has to be taken along this particular path.

**WATCH OUT FOR**
Dropping the minus signs in $-2y$ or $-4yz$. Both give $t$-terms of the same sign, and losing either one moves the result to a distractor. A second mistake is to assume $\vec{F}$ is conservative and try to evaluate a potential at the endpoints: the curl is not zero, so the result depends on the path.

## Q.27 MCQ · 2 MARKS

Engineering Mathematics › Probability & statistics

Two bags $A$ and $B$ have equal number of balls. Bag $A$ has 20% red balls and 80% green balls. Bag $B$ has 30% red balls, 60% green balls and 10% yellow balls. Contents of Bags $A$ and $B$ are mixed thoroughly and a ball is randomly picked from the mixture. What is the chance that the ball picked is red?

**(A)** 20%

**(B)** 25%

**(C)** 30%

**(D)** 40%

Official answer: **B**

**SOLUTION**

1. Let each bag hold $N$ balls, so the mixture holds $2N$ balls.
2. Red balls from bag $A$: $0.2N$. Red balls from bag $B$: $0.3N$. Total red balls in the mixture: $0.2N + 0.3N = 0.5N$.
3. Every ball in the thoroughly mixed pile is equally likely to be picked, so by classical probability (favourable over total):
$$P(\text{red}) = \frac{0.5N}{2N} = 0.25 = 25\%$$
4. Check with the total probability theorem: since the bags contribute equal numbers of balls, a ball drawn from the mixture came from $A$ or from $B$ with probability $\frac{1}{2}$ each, so
$$P(\text{red}) = \tfrac{1}{2}(0.2) + \tfrac{1}{2}(0.3) = 0.25$$
5. The answer is 25%, option (B).

**KEY IDEA**

When equal-sized groups are pooled, the proportion of a colour in the mixture is the plain average of the group proportions: $\frac{1}{2}(20\% + 30\%) = 25\%$. This is the total probability theorem with $P(A) = P(B) = \frac{1}{2}$. The yellow balls in bag $B$ make no difference, because only the red fraction of each bag is used.

**WATCH OUT FOR**

Adding the two percentages ($20\% + 30\% = 50\%$) without dividing by the doubled total, which gives an option that is not even listed. Another mistake is letting the green or yellow split change the answer. Note too that the simple average works only because the bags hold equal numbers of balls. With unequal bags the average must be weighted by bag size.

## Q.28 MCQ · 2 MARKS

Engineering Mathematics › Linear algebra

Consider the following system of linear equations:

$$3x + 2ky = -2$$
$$kx + 6y = 2$$

Here $x$ and $y$ are the unknowns and $k$ is a real constant. The value of $k$ for which there are infinite number of solutions is

**(A)** $3$

**(B)** $1$

**(C)** $-3$

**(D)** $-6$

Official answer: **C**

**SOLUTION**

1. Write the system as $AX = B$ with $A = \begin{bmatrix} 3 & 2k \\ k & 6 \end{bmatrix}$ and $B = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$. For infinitely many solutions we need $\rho(A) = \rho([A \,|\, B]) < 2$, where 2 is the number of unknowns.
2. For the rank of $A$ to be below 2 we need $\det A = 0$:
$$3 \cdot 6 - (2k)(k) = 18 - 2k^2 = 0 \;\Rightarrow\; k^2 = 9 \;\Rightarrow\; k = \pm 3.$$
3. Check $k = 3$: the equations become $3x + 6y = -2$ and $3x + 6y = 2$. The left sides are the same but the right sides differ, so $\rho(A) = 1$ while $\rho([A \,|\, B]) = 2$. The system is inconsistent and has no solution.
4. Check $k = -3$: the equations become $3x - 6y = -2$ and $-3x + 6y = 2$. Multiplying the second by $-1$ gives $3x - 6y = -2$, which is the first equation again. So $\rho(A) = \rho([A \,|\, B]) = 1 < 2$.

5. Rank 1 with two unknowns leaves one free variable, so the solutions are $x = \dfrac{6y-2}{3}$ for every real $y$. That is infinitely many solutions.
6. Only $k = -3$ gives infinitely many solutions, which is option (C).

**KEY IDEA**

A $2 \times 2$ system has infinitely many solutions only when $\rho(A) = \rho([A\,|\,B]) < 2$. Put another way, the coefficient ratios must all be equal: $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$. Here that reads $\dfrac{3}{k} = \dfrac{2k}{6} = \dfrac{-2}{2} = -1$, and both equalities give $k = -3$.

**WATCH OUT FOR**

Stopping at $\det A = 0$ and choosing $k = 3$ (option A). A zero determinant only rules out a unique solution. At $k = 3$ the two lines are parallel and distinct ($3x + 6y = -2$ against $3x + 6y = 2$), so the system has no solution. You have to check that the augmented matrix has the same rank before calling the solutions infinite.

## Q.29 MCQ · 2 MARKS

Engineering Mathematics › Calculus

Consider the following equations

$$\frac{\partial V(x,y)}{\partial x} = px^2 + y^2 + 2xy$$
$$\frac{\partial V(x,y)}{\partial y} = x^2 + qy^2 + 2xy$$

where $p$ and $q$ are constants. $V(x,y)$ that satisfies the above equations is

**(A)** $p\dfrac{x^3}{3} + q\dfrac{y^3}{3} + 2xy + 6$

**(B)** $p\dfrac{x^3}{3} + q\dfrac{y^3}{3} + 5$

**(C)** $p\dfrac{x^3}{3} + q\dfrac{y^3}{3} + x^2y + xy^2 + xy$

**(D)** $p\dfrac{x^3}{3} + q\dfrac{y^3}{3} + x^2y + xy^2$

Official answer: **D**

**SOLUTION**

1. **Consistency check (equality of mixed partials).** A $V$ can exist only if $\dfrac{\partial}{\partial y}\left(\dfrac{\partial V}{\partial x}\right) = \dfrac{\partial}{\partial x}\left(\dfrac{\partial V}{\partial y}\right)$. Here $\dfrac{\partial}{\partial y}(px^2 + y^2 + 2xy) = 2y + 2x$ and $\dfrac{\partial}{\partial x}(x^2 + qy^2 + 2xy) = 2x + 2y$. They agree for every $p$ and $q$, and both right-hand sides are polynomials defined on the whole plane, so such a $V$ exists.
2. **Integrate the first equation in $x$, holding $y$ fixed:**
$$V = p\frac{x^3}{3} + xy^2 + x^2y + g(y)$$
where $g(y)$ is the 'constant' of integration, which may depend on $y$.
3. **Differentiate in $y$ and match the second equation:**
$$\frac{\partial V}{\partial y} = 2xy + x^2 + g'(y) = x^2 + qy^2 + 2xy \;\Rightarrow\; g'(y) = qy^2$$
4. **Integrate:** $g(y) = q\dfrac{y^3}{3} + C$, so
$$V(x,y) = p\frac{x^3}{3} + q\frac{y^3}{3} + x^2y + xy^2 + C$$
where $C$ is an arbitrary constant.
5. **Match the options.** With $C = 0$ this is option (D). Check: for (D), $\partial V/\partial x = px^2 + 2xy + y^2$ and $\partial V/\partial y = qy^2 + x^2 + 2xy$, both as given. The others fail the first equation: (A) gives $\partial V/\partial x = px^2 + 2y$, (B) gives $px^2$, and (C) gives $px^2 + 2xy + y^2 + y$, where the extra $xy$ term adds a stray $y$. The

constants $6$ and $5$ in (A) and (B) are not the problem, since a constant never changes a partial derivative. Those options fail because they are missing the $x^2y + xy^2$ terms.

**KEY IDEA**

To find $V$ from its two partial derivatives, integrate one of them, letting the 'constant' of integration be a function of the other variable. Then differentiate that result with respect to the other variable and match it to the second given partial to fix that function. For an MCQ, the quickest check is to differentiate each option and compare with both given partials.

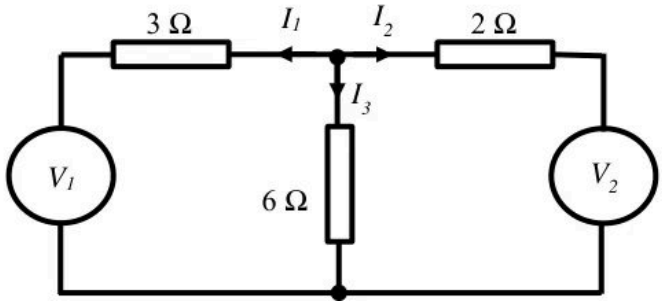
**WATCH OUT FOR**

One mistake is to integrate each equation on its own and add the results. That counts the shared cross terms twice, giving $2x^2y + 2xy^2$ instead of $x^2y + xy^2$. Another is to treat the $2xy$ term as if it came from an $xy$ (or $2xy$) term in $V$, which leads to (A) or (C). The right source is $x^2y + xy^2$: differentiating it in $x$ gives $2xy + y^2$, and in $y$ gives $x^2 + 2xy$. A third mistake is to use a true constant instead of $g(y)$ as the constant of integration after integrating in $x$.

## Q.30 NAT · 2 MARKS

Networks (Electric Circuits) › Theorems

Q.30 In the given circuit, superposition is applied. When $V_2$ is set to 0 V, the current $I_2$ is -6 A. When $V_1$ is set to 0 V, the current $I_1$ is +6 A. Current $I_3$ (in A) when both sources are applied will be (up to two decimal places) _____.




Official answer: **0.95 to 1.05**

**SOLUTION**

1. All three reference arrows point **away** from the top node A, so KCL at A gives $I_1 + I_2 + I_3 = 0$. Take the bottom wire as the reference, so $V_A$ is the voltage of node A. The source polarities are not needed, because the current given for each sub-circuit fixes that sub-circuit on its own.
2. **Only $V_1$ acting ($V_2$ shorted):** the $2\,\Omega$ resistor now runs from A to the reference through the short, so $I_2' = \dfrac{V_A'}{2} = -6 \;\Rightarrow\; V_A' = -12$ V.
3. Then $I_3' = \dfrac{V_A'}{6} = \dfrac{-12}{6} = -2$ A. As a check, KCL gives $I_1' = -(I_2' + I_3') = -(-6 - 2) = 8$ A.
4. **Only $V_2$ acting ($V_1$ shorted):** the $3\,\Omega$ resistor now runs from A to the reference through the short, so $I_1'' = \dfrac{V_A''}{3} = 6 \;\Rightarrow\; V_A'' = 18$ V.
5. Then $I_3'' = \dfrac{V_A''}{6} = \dfrac{18}{6} = 3$ A. As a check, $I_2'' = -(I_1'' + I_3'') = -(6 + 3) = -9$ A.
6. **Superposition:**

$$I_3 = I_3' + I_3'' = -2 + 3 = 1.00 \text{ A}$$

This agrees with $V_A = V_A' + V_A'' = -12 + 18 = 6$ V, which gives $I_3 = 6/6 = 1$ A.

**KEY IDEA**

When one source is set to zero, the resistor in that source's branch ends up connected between node A and the reference. The one branch current you are given then fixes the node voltage $V_A$ ($I_2' = V_A'/2$, $I_1'' = V_A''/3$). The $6\,\Omega$ current in each sub-circuit is $V_A/6$, and superposition adds the two. You never need the source values or their polarities.

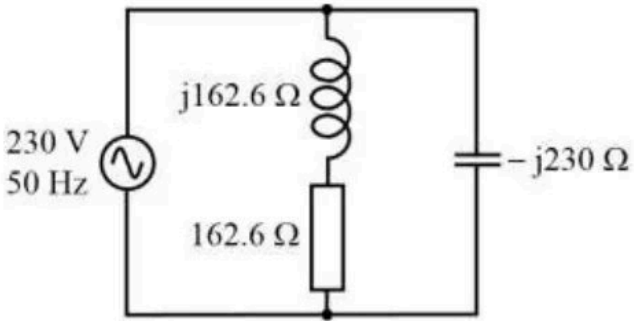
**WATCH OUT FOR**

One mistake is to read each given current as a current flowing into node A. All three reference arrows point away from A, so $I_2' = -6$ A makes $V_A' = -12$ V, not $+12$ V. Another is to try to find $V_1$ and $V_2$ first when their polarities are not shown. A third is to add the given currents $I_2'$ and $I_1''$ directly instead of first working out each sub-circuit's own $I_3$.

## Q.31 NAT · 2 MARKS

Networks (Electric Circuits) › AC, power & resonance

Q.31 In the figure, an RLC load is supplied by a 230 V, 50 Hz single phase source. The magnitude of the reactive power (in VAr) supplied by the source is _____.




Official answer: **66 to 68**

**SOLUTION**

1. Both branches are connected directly across the source, so each one sees the full $V = 230$ V (rms). Complex power is conserved, so the reactive power the source supplies equals the sum of the reactive powers taken by the two branches. Work out each branch separately and add.
2. R-L branch impedance: $Z_1 = 162.6 + j162.6 = 162.6\sqrt{2}\angle 45^\circ \approx 229.95\angle 45^\circ\ \Omega$.
3. Branch current: $|I_1| = \dfrac{230}{162.6\sqrt{2}} \approx 1.0002$ A (rms).
4. Reactive power absorbed by the inductor:
$$Q_L = |I_1|^2 X_L = \frac{230^2 \times 162.6}{2 \times 162.6^2} = \frac{52900}{325.2} \approx 162.67\ \text{VAr (absorbed, lagging)}$$
5. Capacitor branch: $Q_C = \dfrac{V^2}{X_C} = \dfrac{230^2}{230} = 230$ VAr. The capacitor supplies this reactive power (leading).
6. Net reactive power supplied by the source: $Q = Q_L - Q_C = 162.67 - 230 = -67.33$ VAr. The negative sign means the load as a whole is capacitive, so the source actually absorbs this reactive power instead of supplying it.
7. Admittance cross-check: $Y = \dfrac{1}{162.6(1+j)} + \dfrac{1}{-j230} = \dfrac{1}{325.2} + j\left(\dfrac{1}{230} - \dfrac{1}{325.2}\right)$ S. Then $S = V^2 Y^* = 162.67 - j67.33$ VA, which gives the same $Q = -67.33$ VAr.
8. The question asks for the magnitude: $|Q| \approx 67.33$ VAr.

**KEY IDEA**

In parallel branches across one voltage source, reactive power adds branch by branch (complex power is conserved). The inductor absorbs $|I_1|^2 X_L = V^2 X_L/|Z_1|^2$ and the capacitor supplies $V^2/X_C$. Their difference is what the source must provide or absorb.

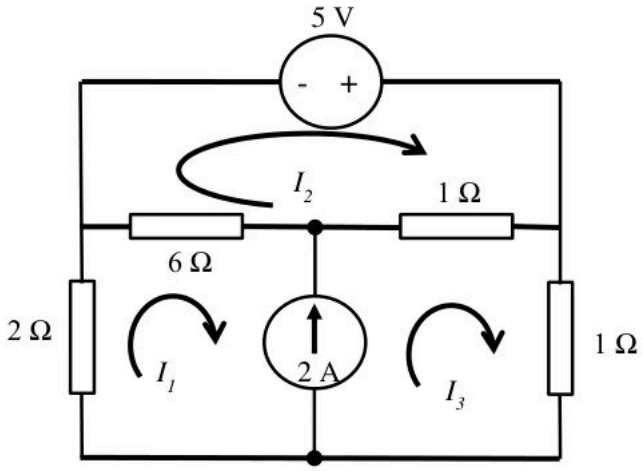
**WATCH OUT FOR**

Writing $Q_L = V^2/X_L = 230^2/162.6 \approx 325.4\ \text{VAr}$, as if the inductor had the full 230 V across it. It is in series with the resistor, so only part of the voltage appears across it, and you must use $|I_1|^2 X_L$ with $|I_1| = V/|Z_1|$. That shortcut gives $|Q| \approx 95.4\ \text{VAr}$, which is wrong. A second slip is to report $-67.33$ when the question asks for a magnitude.

## Q.32 MCQ · 2 MARKS

Networks (Electric Circuits) › Laws & node/mesh

Q.32 In the given circuit, the mesh currents $I_1$, $I_2$ and $I_3$ are




(A) $I_1 = 1$ A, $I_2 = 2$ A and $I_3 = 3$ A
(B) $I_1 = 2$ A, $I_2 = 3$ A and $I_3 = 4$ A
(C) $I_1 = 3$ A, $I_2 = 4$ A and $I_3 = 5$ A
(D) $I_1 = 4$ A, $I_2 = 5$ A and $I_3 = 6$ A

IN 8/14

Official answer: **A**

**SOLUTION**

1. **Current-source constraint.** The 2 A source is the only element in the branch shared by meshes 1 and 3. With both meshes clockwise, $I_3$ flows up this branch and $I_1$ flows down it. The source forces 2 A upward, so
$$I_3 - I_1 = 2 \quad (1)$$
2. **KVL in mesh 2 (clockwise).** Going left to right through the 5 V source takes us from − to +, which is a rise of 5 V. Along the bottom of mesh 2 we travel right to left. The leftward current in the 1 Ω is $(I_2 - I_3)$ and the leftward current in the 6 Ω is $(I_2 - I_1)$:
$$-5 + 1(I_2 - I_3) + 6(I_2 - I_1) = 0 \ \Rightarrow\ -6I_1 + 7I_2 - I_3 = 5 \quad (2)$$
3. **Supermesh of meshes 1 and 3.** KVL cannot be written for mesh 1 or mesh 3 on its own, because the voltage across the current source is unknown. Instead, go clockwise round the outer boundary of the two meshes and leave out the source: up the 2 Ω (current $I_1$), right through the 6 Ω (current $I_1 - I_2$), right through the 1 Ω (current $I_3 - I_2$), down the right 1 Ω (current $I_3$), then back along the bottom wire, which has no elements:
$$2I_1 + 6(I_1 - I_2) + 1(I_3 - I_2) + 1 \cdot I_3 = 0 \ \Rightarrow\ 8I_1 - 7I_2 + 2I_3 = 0 \quad (3)$$
4. **Substitute (1) into (2).** Put $I_3 = I_1 + 2$ into (2): $-6I_1 + 7I_2 - (I_1 + 2) = 5$, so $-7I_1 + 7I_2 = 7$ and $I_2 = I_1 + 1$.
5. **Substitute into (3).** $8I_1 - 7(I_1 + 1) + 2(I_1 + 2) = 0 \ \Rightarrow\ 3I_1 - 3 = 0 \ \Rightarrow\ I_1 = 1$ A.

6. **Back-substitute.** $I_2 = I_1 + 1 = 2$ A and $I_3 = I_1 + 2 = 3$ A. Check in (2): $-6 + 14 - 3 = 5$. Check in (3): $8 - 14 + 6 = 0$.
7. **Physical check with node voltages.** Take the bottom wire as $0$ V. The 2 Ω carries $I_1 = 1$ A upward, so $V_L = -2$ V. The right 1 Ω carries $I_3 = 3$ A downward, so $V_R = 3$ V. Then $V_R - V_L = 5$ V, which is exactly the source voltage, with + on the right. At node M, the 2 A from the source splits into 1 A leftward through the 6 Ω and 1 A rightward through the 1 Ω. Both branches give $V_M = 4$ V, so KCL and KVL hold everywhere. The answer is option (A).
8. **Why the supermesh equation decides it.** All four options satisfy $I_3 - I_1 = 2$ and $I_2 = I_1 + 1$, so equations (1) and (2) cannot separate them. Only (3) does. For example, option (B) gives $8(2) - 7(3) + 2(4) = 3 \neq 0$.

**KEY IDEA**

When a current source is the only element in a branch shared by two meshes, it gives a constraint between their mesh currents, here $I_3 - I_1 = 2$. The KVL equation that replaces those two meshes goes round their combined outer boundary (the supermesh) and leaves out the source, because the voltage across the source is unknown.

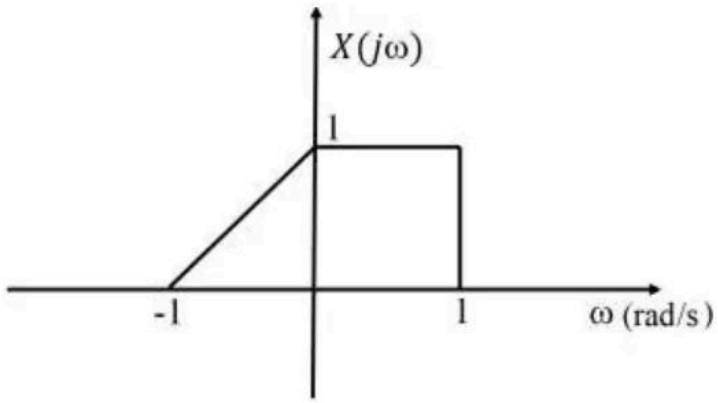
**WATCH OUT FOR**

There are two common mistakes. The first is to write KVL round mesh 1 or mesh 3 alone as if the current source had zero voltage across it. The second is to test the options only against the source constraint. Every option satisfies $I_3 - I_1 = 2$, and every option also satisfies the mesh-2 equation, so only the supermesh equation $8I_1 - 7I_2 + 2I_3 = 0$ picks out (A). Watch the sign of the 5 V source too: going clockwise in mesh 2 you meet − then +, which is a rise.

## Q.33 MCQ · 2 MARKS

Signals and Systems › Fourier transform

Q.33 The Fourier transform of a signal $x(t)$, denoted by $X(j\omega)$, is shown in the figure.




Let $y(t) = x(t) + e^{jt}x(t)$. The value of Fourier transform of $y(t)$ evaluated at the angular frequency $\omega = 0.5$ rad/s is

(A) 0.5 (B) 1 (C) 1.5 (D) 2.5

Official answer: **C**

**SOLUTION**

1. Read the spectrum from the figure: $X(j\omega) = 1 + \omega$ for $-1 \le \omega \le 0$ (the ramp from 0 at $\omega = -1$ to 1 at $\omega = 0$), $X(j\omega) = 1$ for $0 < \omega < 1$ (the flat part), and $X(j\omega) = 0$ elsewhere.
2. Frequency-shift property: multiplying a signal by $e^{j\omega_0 t}$ shifts its spectrum to the right by $\omega_0$, i.e. $e^{j\omega_0 t}x(t) \leftrightarrow X(j(\omega - \omega_0))$. Here $e^{jt} = e^{j(1)t}$, so $\omega_0 = 1$ rad/s.
3. By linearity,

$$Y(j\omega) = X(j\omega) + X(j(\omega - 1))$$

4. First term at $\omega = 0.5$: $\omega = 0.5$ lies on the flat part, so $X(j0.5) = 1$.
5. Second term at $\omega = 0.5$: $X(j(0.5-1)) = X(j(-0.5))$. The frequency $-0.5$ lies on the ramp, so $X(j(-0.5)) = 1 + (-0.5) = 0.5$.
6. Add the two:

$$Y(j0.5) = 1 + 0.5 = 1.5$$

which is option (C).

**KEY IDEA**

Multiplying by $e^{j\omega_0 t}$ in time shifts the spectrum to the right by $\omega_0$: $e^{j\omega_0 t}x(t) \leftrightarrow X(j(\omega - \omega_0))$. So $Y(j\omega) = X(j\omega) + X(j(\omega - 1))$. To evaluate the shifted copy at $\omega = 0.5$, read the original spectrum at $0.5 - 1 = -0.5$.

**WATCH OUT FOR**

Shifting the wrong way, i.e. reading $X(j(\omega + 1))$ at $\omega = 0.5$. That point, $1.5$, is outside the band, so the second term is taken as $0$ and the answer comes out as $1$ (option B). Another slip is reading the flat value $1$ at $\omega = -0.5$ as well, which gives $2$, not one of the options. The ramp gives $0.5$ at $\omega = -0.5$.

## Q.34 MCQ · 2 MARKS

Signals and Systems › z-transform

Let $y[n] = x[n] * h[n]$, where $*$ denotes convolution and $x[n]$ and $h[n]$ are two discrete time sequences. Given that the z-transform of $y[n]$ is $Y(z) = 2 + 3z^{-1} + z^{-2}$, the z-transform of $p[n] = x[n] * h[n-2]$ is

**(A)** $2 + 3z + z^{-2}$

**(B)** $3z + z^{-2}$

**(C)** $2z^2 + 3z + 1$

**(D)** $2z^{-2} + 3z^{-3} + z^{-4}$

Official answer: **D**

**SOLUTION**

1. Convolution property of the z-transform: $y[n] = x[n] * h[n] \;\Rightarrow\; Y(z) = X(z)\,H(z) = 2 + 3z^{-1} + z^{-2}$.
2. Time-shifting property: $h[n-2] \;\longleftrightarrow\; z^{-2}H(z)$.
3. Apply the convolution property to $p[n] = x[n] * h[n-2]$:

$$P(z) = X(z)\left[z^{-2}H(z)\right] = z^{-2}\,X(z)H(z) = z^{-2}\,Y(z)$$

4. Substitute $Y(z)$:

$$P(z) = z^{-2}\left(2 + 3z^{-1} + z^{-2}\right) = 2z^{-2} + 3z^{-3} + z^{-4}$$

5. Time-domain check: $Y(z)$ is a finite polynomial in $z^{-1}$, so $y[n] = 2\delta[n] + 3\delta[n-1] + \delta[n-2]$, i.e. $\{2, 3, 1\}$ for $n = 0, 1, 2$. Delaying one operand of a convolution delays the result by the same amount, so $p[n] = y[n-2] = \{2, 3, 1\}$ for $n = 2, 3, 4$, whose z-transform is $2z^{-2} + 3z^{-3} + z^{-4}$. This is option (D).

**KEY IDEA**

Convolution in time becomes multiplication in the z-domain, and a delay of $k$ samples multiplies by $z^{-k}$. Delaying either operand of a convolution by 2 samples delays the output by 2 samples: $x[n] * h[n-2] = y[n-2]$, so $P(z) = z^{-2}Y(z)$.

**WATCH OUT FOR**

Using $z^{+2}$ for the delay $h[n-2]$, which gives $z^2Y(z) = 2z^2 + 3z + 1$ (option C). A delay (shift right) multiplies by $z^{-2}$, not $z^2$. Another slip is to shift only part of the expression, which produces mixed-power options such as (A) or (B).

## Q.35 MCQ · 2 MARKS

Signals and Systems › DTFT & DFT

For the sequence $x[n] = \{1, -1, 1, -1\}$, with $n = 0, 1, 2, 3$, the DFT is computed as $X(k) = \sum_{n=0}^{3} x[n]e^{-j\frac{2\pi}{4}nk}$, for $k = 0, 1, 2, 3$. The value of $k$ for which $X(k)$ is not zero is

**(A)** 0

**(B)** 1

**(C)** 2

**(D)** 3

Official answer: **C**

**SOLUTION**

1. Recognise the sequence: $x[n] = \{1, -1, 1, -1\} = (-1)^n = e^{j\pi n} = e^{j\frac{2\pi}{4}\cdot 2n}$. It is a single complex exponential sitting exactly at DFT bin $k_0 = 2$.
2. Substitute into the 4-point DFT:
$$X(k) = \sum_{n=0}^{3} e^{j\frac{2\pi}{4}2n}\, e^{-j\frac{2\pi}{4}nk} = \sum_{n=0}^{3} e^{-j\frac{2\pi}{4}n(k-2)}$$
3. Orthogonality of the DFT basis (sum of the $N$-th roots of unity): $\sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}nm} = N$ if $m \equiv 0 \pmod N$, and 0 otherwise. With $m = k-2$ and $k \in \{0,1,2,3\}$, only $k = 2$ gives $m \equiv 0 \pmod 4$. So $X(k) = 4\,\delta[k-2]$, i.e. $X(k) = \{0, 0, 4, 0\}$.
4. Check directly: $X(0) = \sum x[n] = 1 - 1 + 1 - 1 = 0$; $X(2) = \sum x[n](-1)^n = 1 + 1 + 1 + 1 = 4$.
5. Check $k = 1$ using $e^{-j\frac{\pi}{2}n} = 1, -j, -1, j$ for $n = 0, 1, 2, 3$: $X(1) = (1)(1) + (-1)(-j) + (1)(-1) + (-1)(j) = 1 + j - 1 - j = 0$. Since $x[n]$ is real, $X(N-k) = X^*(k)$, so $X(3) = X^*(1) = 0$.
6. Only $X(2)$ is non-zero, so $k = 2$: option (C).

**KEY IDEA**

$x[n] = (-1)^n = e^{j\frac{2\pi}{N}\cdot\frac{N}{2}n}$ is one DFT basis vector, the one at the Nyquist bin $k = N/2$. Because the DFT basis vectors are orthogonal, all its energy falls in that one bin: $X(k) = N\,\delta[k - N/2]$. Here that is $4\,\delta[k-2]$.

**WATCH OUT FOR**

Picking $k = 0$ because it is the DC term, when in fact $X(0) = \sum x[n] = 0$ for this zero-mean sequence. A second mistake is getting the sign of $e^{-j\pi n/2}$ wrong at $n = 1, 3$, which makes $X(1)$ or $X(3)$ look non-zero. Remember $e^{-j\pi/2} = -j$ and $e^{-j3\pi/2} = +j$.

## Q.36 NAT · 2 MARKS

Control Systems › Nyquist

Consider the standard negative feedback configuration with $G(s) = \dfrac{s^2 + 0.2s + 100}{s^2 - 0.2s + 100}$ and $H(s) = \dfrac{1}{2}$. The number of clockwise encirclements of $(-1, 0)$ in the Nyquist plot of the Loop transfer-function $G(s)H(s)$ is _____

Official answer: **0**

**SOLUTION**

1. Loop transfer function:
$$G(s)H(s) = \frac{s^2 + 0.2s + 100}{2\,(s^2 - 0.2s + 100)}$$
2. Open-loop poles: $s^2 - 0.2s + 100 = 0 \Rightarrow s = 0.1 \pm j\sqrt{99.99} \approx 0.1 \pm j\,9.9995$. Both poles are in the right half-plane, so $P = 2$. No poles lie on the $j\omega$ axis, so the Nyquist contour needs no indentations.
3. Put $s = j\omega$:
$$G(j\omega)H(j\omega) = \frac{1}{2} \cdot \frac{(100 - \omega^2) + j0.2\omega}{(100 - \omega^2) - j0.2\omega}$$
The numerator and denominator are complex conjugates of each other, so they have equal magnitude.
4. Therefore $|G(j\omega)H(j\omega)| = \dfrac{1}{2}$ for every $\omega$. If $\theta(\omega)$ is the angle of the numerator $(100 - \omega^2) + j0.2\omega$, then $\angle G(j\omega)H(j\omega) = 2\theta(\omega)$. As $\omega$ goes from 0 to $\infty$, $\theta$ rises from 0 to $\pi$ (it passes $\pi/2$ at $\omega = 10$,

where $GH = -\frac{1}{2}$), so the plot goes once round the circle. On the infinite arc, $G(s)H(s) \to \frac{1}{2}$. The whole Nyquist plot therefore lies on the circle of radius $0.5$ centred at the origin.

5. The point $-1 + j0$ is at distance $1 > 0.5$ from the origin. It is outside that circle, so the plot never encircles it: $N = 0$.
6. Cross-check with the Nyquist criterion $N = Z - P$ (clockwise encirclements). The characteristic equation $1 + G(s)H(s) = 0$ gives $2(s^2 - 0.2s + 100) + (s^2 + 0.2s + 100) = 3s^2 - 0.2s + 300 = 0$. The roots are $s = \dfrac{0.2 \pm \sqrt{0.04 - 3600}}{6}$. Their real part is $+\dfrac{0.2}{6} > 0$, so both are in the RHP and $Z = 2$. Then $N = 2 - 2 = 0$, which agrees. (The closed loop is unstable, but that does not change the encirclement count.)

**KEY IDEA**

The zeros $-0.1 \pm j9.9995$ are the mirror images of the poles $0.1 \pm j9.9995$ across the $j\omega$ axis, so $G(s)$ is all-pass: $|G(j\omega)| = 1$ at every frequency. Multiplying by $H = \frac{1}{2}$ puts the entire Nyquist plot on a circle of radius $0.5$. The point $-1$ lies outside that circle and cannot be encircled.

**WATCH OUT FOR**

Students see two open-loop RHP poles ($P = 2$) and write $N = -2$ or $N = 2$, as if a stable closed loop were being assumed, or they take the count from $P$ alone. The encirclement count depends only on where the plot goes, and this plot stays on a circle of radius $0.5$, so $N = 0$. That gives $Z = N + P = 2$: the closed loop is unstable, but the answer asked for is still $0$. A second slip is writing the angle as $2\tan^{-1}\left(\frac{0.2\omega}{100-\omega^2}\right)$ with the principal-value arctangent. That formula goes wrong for $\omega > 10$ because it drops the quadrant.

## Q.37 MCQ · 2 MARKS

Control Systems › State space

Consider the linear system $\dot{x} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x$, with initial condition $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. The solution $x(t)$ for this system is

**(A)** $x(t) = \begin{bmatrix} e^{-t} & te^{-2t} \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

**(B)** $x(t) = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{2t} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

**(C)** $x(t) = \begin{bmatrix} e^{-t} & -t^2e^{-2t} \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

**(D)** $x(t) = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Official answer: **D**

**SOLUTION**

1. For the unforced system $\dot{x} = Ax$ the solution is $x(t) = e^{At}x(0)$, where $\Phi(t) = e^{At}$ is the state transition matrix.
2. Here $A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$ is diagonal, so every power of it is diagonal: $A^k = \begin{bmatrix} (-1)^k & 0 \\ 0 & (-2)^k \end{bmatrix}$. Summing the series term by term gives

$$e^{At} = \sum_{k=0}^{\infty} \frac{A^k t^k}{k!} = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix}$$

   This is the same as saying the two states are decoupled: $\dot{x}_1 = -x_1$ and $\dot{x}_2 = -2x_2$.
3. Check by Laplace transform: $sI - A = \begin{bmatrix} s+1 & 0 \\ 0 & s+2 \end{bmatrix}$, so $(sI - A)^{-1} = \begin{bmatrix} \dfrac{1}{s+1} & 0 \\ 0 & \dfrac{1}{s+2} \end{bmatrix}$ and

$$\Phi(t) = \mathcal{L}^{-1}\left[(sI - A)^{-1}\right] = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix}$$

4. So $x(t) = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} e^{-t} \\ e^{-2t} \end{bmatrix}$. This is option (D).
5. Why the others are wrong. (A) and (C) have a non-zero top-right entry. Step 2 shows that $e^{At}$ of a diagonal $A$ has no off-diagonal entries at all. A term like $te^{-2t}$ would need a Jordan block at $-2$, which means a repeated eigenvalue *and* a non-zero off-diagonal entry in $A$. This $A$ has neither. (B) has $e^{+2t}$, but the diagonal entries of $e^{At}$ are $e^{\lambda_i t}$ with $\lambda_1 = -1$ and $\lambda_2 = -2$, so it would need an eigenvalue at $+2$.
6. A quick check by substitution. All four matrices equal $I$ at $t = 0$, so $\Phi(0) = I$ cannot tell them apart. Putting each candidate into $\dot{x}_1 + x_1 = 0$ does. For (A), $x_1 = e^{-t} + te^{-2t}$ gives $\dot{x}_1 + x_1 = (1 - t)e^{-2t} \neq 0$. For (C), $x_1 = e^{-t} - t^2 e^{-2t}$ gives $\dot{x}_1 + x_1 = t(t - 2)e^{-2t} \neq 0$. For (B), $x_2 = e^{2t}$ gives $\dot{x}_2 + 2x_2 = 4e^{2t} \neq 0$. Only (D) satisfies both state equations and the initial condition. Because the solution is unique, the answer is (D).

**KEY IDEA**

For a diagonal $A$, $e^{At}$ is diagonal with entries $e^{\lambda_i t}$, because every power of a diagonal matrix is diagonal. Each state decays on its own, $x_i(t) = e^{\lambda_i t} x_i(0)$, and there are no cross terms.

**WATCH OUT FOR**

Using $\Phi(0) = I$ to eliminate options. Here every option equals $I$ at $t = 0$, including (C), whose $-t^2 e^{-2t}$ term vanishes at $t = 0$, so that check eliminates nothing. The other mistake is misreading $e^{-2t}$ as $e^{2t}$, which is option (B). The mode for an eigenvalue at $-2$ is $e^{-2t}$.

## Q.38 NAT · 2 MARKS

Control Systems › Routh–Hurwitz

Consider a standard negative feedback configuration with $G(s) = \dfrac{1}{(s+1)(s+2)}$ and $H(s) = \dfrac{s+\alpha}{s}$. For the closed loop system to have poles on the imaginary axis, the value of $\alpha$ should be equal to (up to one decimal place) _____.

Official answer: **8.9 to 9.1**

**SOLUTION**

1. Characteristic equation of the negative-feedback loop: $1 + G(s)H(s) = 0 \Rightarrow 1 + \dfrac{s+\alpha}{s(s+1)(s+2)} = 0 \Rightarrow s(s+1)(s+2) + (s+\alpha) = 0$.
2. Expand: $s^3 + 3s^2 + 2s + s + \alpha = 0 \Rightarrow s^3 + 3s^2 + 3s + \alpha = 0$.
3. Routh array:

$$\begin{array}{ll} s^3: & 1 \qquad 3 \\ s^2: & 3 \qquad \alpha \\ s^1: & \dfrac{3 \cdot 3 - 1 \cdot \alpha}{3} = \dfrac{9 - \alpha}{3} \\ s^0: & \alpha \end{array}$$

4. Stability needs every first-column entry positive: $\alpha > 0$ and $9 - \alpha > 0$, i.e. $0 < \alpha < 9$.
5. At the upper limit the $s^1$ row becomes entirely zero, which signals a pair of roots symmetric about the origin: $\dfrac{9 - \alpha}{3} = 0 \Rightarrow \alpha = 9$.
6. Auxiliary equation from the $s^2$ row with $\alpha = 9$: $3s^2 + 9 = 0 \Rightarrow s = \pm j\sqrt{3}$. Check by division: $s^3 + 3s^2 + 3s + 9 = (s^2 + 3)(s + 3)$, so the closed-loop poles are $s = \pm j\sqrt{3}$ and $s = -3$. The pair lies on the $j\omega$ axis and the loop oscillates at $\omega = \sqrt{3} \approx 1.732$ rad/s.
7. The other boundary, $\alpha = 0$, is not the intended answer: it makes $H(s) = s/s$, so the zero of $H(s)$ cancels its own pole and the characteristic equation becomes $s(s^2 + 3s + 3) = 0$. That gives a single

root at the origin (a cancelled mode), not the conjugate pair on the imaginary axis the question describes. Hence $\alpha = 9.0$.

**KEY IDEA**

Write the closed-loop characteristic polynomial $s(s+1)(s+2) + (s+\alpha)$, build its Routh array, and find the value of $\alpha$ that makes a whole row zero. The $s^1$ row vanishes at $\alpha = 9$. The auxiliary polynomial from the row above it ($3s^2 + 9$) then gives the imaginary-axis poles $\pm j\sqrt{3}$ directly.

**WATCH OUT FOR**

Dropping the $s$ in the denominator of $H(s)$ and writing $(s+1)(s+2) + (s+\alpha) = 0$, a second-order equation that can never put poles on the $j\omega$ axis. Another mistake is forgetting the extra $s$ contributed by $H(s)$'s numerator, which gives $s^3 + 3s^2 + 2s + \alpha$ and the wrong answer $\alpha = 6$. A third is giving $\alpha = 0$ just because it also zeroes a first-column entry: that only puts a cancelled root at the origin.

## Q.39 MCQ · 2 MARKS

Control Systems › Time response

Unit step response of a linear time invariant (LTI) system is given by $y(t) = (1 - e^{-2t})u(t)$. Assuming zero initial condition, the transfer function of the system is

**(A)** $\dfrac{1}{s+1}$

**(B)** $\dfrac{2}{(s+1)(s+2)}$

**(C)** $\dfrac{1}{s+2}$

**(D)** $\dfrac{2}{s+2}$

Official answer: **D**

**SOLUTION**

1. **Laplace transform of the step response** (zero initial conditions):
$$Y(s) = \frac{1}{s} - \frac{1}{s+2} = \frac{(s+2) - s}{s(s+2)} = \frac{2}{s(s+2)}$$
2. **Definition of the transfer function:** $G(s) = \dfrac{Y(s)}{R(s)}$ with zero initial conditions. The input is a unit step, so $R(s) = \dfrac{1}{s}$.
3. **Divide by the input:**
$$G(s) = \frac{2}{s(s+2)} \cdot s = \frac{2}{s+2}$$
This is option D.
4. **Check 1, final value theorem:** $y(\infty) = \lim_{s\to 0} s \cdot \dfrac{1}{s} \cdot G(s) = G(0)$. For D, $G(0) = \dfrac{2}{2} = 1$, which matches $1 - e^{-2t} \to 1$. Option C gives $G(0) = 0.5$ and fails. Options A and B also give $G(0) = 1$, so this check alone does not settle the answer.
5. **Check 2, initial slope:** the given response has $\dot{y}(t) = 2e^{-2t}$, so $\dot{y}(0^+) = 2$. By the initial value theorem, $\dot{y}(0^+) = \lim_{s\to\infty} s \cdot sY(s) = \lim_{s\to\infty} s\,G(s)$. This limit is 1 for A, 0 for B (a second-order system starts with zero slope), 1 for C and 2 for D. Only option D passes both checks.

**KEY IDEA**

With zero initial conditions, the transfer function is the Laplace transform of the output divided by the Laplace transform of the input. For a unit-step input that means $G(s) = s\,Y(s)$. The same result follows from differentiating the step response to get the impulse response, $h(t) = 2e^{-2t}u(t)$, whose transform is $\dfrac{2}{s+2}$.

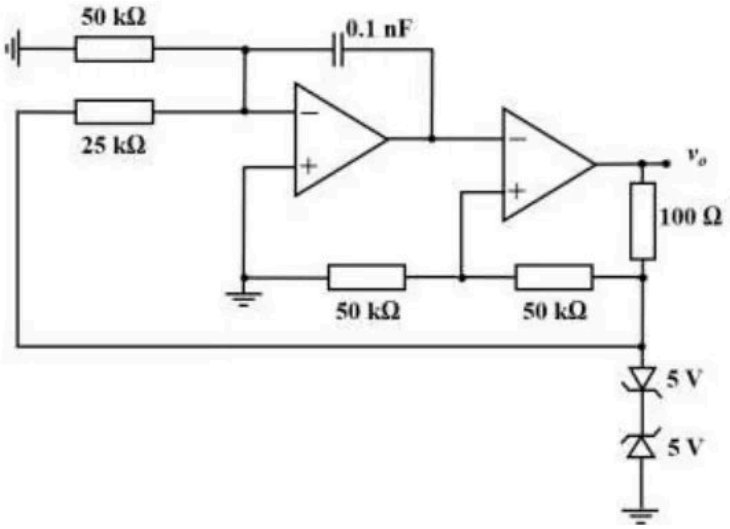
**WATCH OUT FOR**

Reading $1 - e^{-2t}$ as a pole at $s = -2$ and choosing $\dfrac{1}{s+2}$ (option C) without working out the gain. That system settles at $0.5$, not $1$. The numerator must be $2$ so that $G(0) = 1$. A second mistake is taking the transform of $y(t)$ itself as the transfer function, which forgets to divide by the step input's $\dfrac{1}{s}$.

## Q.40 NAT · 2 MARKS

Analog Electronics › Filters & waveforms

Q.40 In the given relaxation oscillator, the opamps and the zener diodes are ideal. The frequency (in kHz) of the square wave output $v_o$ is _____.




Official answer: **Marks to all**

## Q.41 NAT · 2 MARKS

Analog Electronics › Op-amps

An opamp that is powered from a $\pm 5$ V supply is used to build a non-inverting amplifier having a gain of 15. The slew rate of the opamp is $0.5 \times 10^6$ V/s. For a sinusoidal input with amplitude of $0.3$ V, the maximum frequency (in kHz) up to which it can be operated without any distortion is (up to one decimal place) ____.

Official answer: **17.5 to 17.9**

**SOLUTION**

1. Assume the op-amp is ideal except for its slew-rate limit. The non-inverting amplifier has a closed-loop gain of 15, so $v_o = 15\,v_i$.
2. Output peak: $V_m = 15 \times 0.3 = 4.5$ V. This is inside the $\pm 5$ V supply, so the output does not clip. The only thing that limits the frequency is the slew rate.
3. For $v_o = V_m \sin \omega t$, we get $\dfrac{dv_o}{dt} = \omega V_m \cos \omega t$. Its largest magnitude occurs at the zero crossings: $\left.\dfrac{dv_o}{dt}\right|_{\max} = \omega V_m = 2\pi f V_m$.
4. For no slew-rate distortion we need $2\pi f V_m \le SR$. This gives $f_{\max} = \dfrac{SR}{2\pi V_m}$, the full-power bandwidth for this output amplitude.
5. $$f_{\max} = \frac{0.5 \times 10^6}{2\pi \times 4.5} = \frac{0.5 \times 10^6}{28.274} \approx 17684 \text{ Hz}$$

6. $f_{\max} \approx 17.7\ \text{kHz}$

**KEY IDEA**

The slew rate limits the output of the op-amp, not its input. First find the output peak, $V_m = A_v V_{in}$, and check that it stays inside the supply rails so clipping is ruled out. Then set the sinusoid's largest slope, $2\pi f V_m$, equal to the slew rate: $f_{\max} = SR/(2\pi V_m)$.

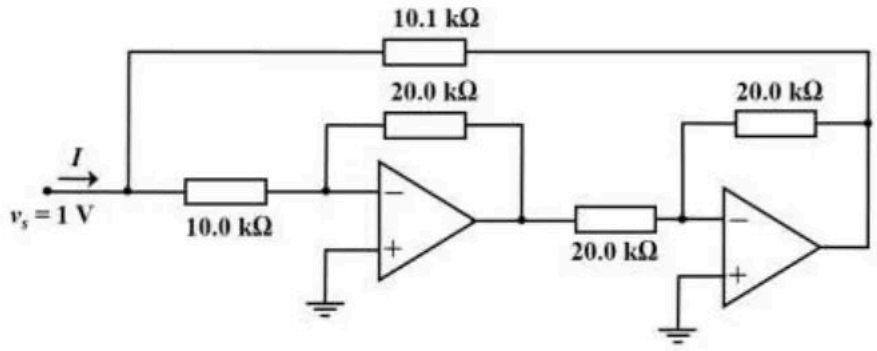
**WATCH OUT FOR**

Using the input amplitude of 0.3 V in place of the output peak of 4.5 V gives about 265 kHz, which is 15 times too high. A second mistake is to use the supply voltage of 5 V as $V_m$, which gives about 15.9 kHz. The supply only tells you whether the output clips. The slew-rate limit depends on the actual output swing.

## Q.42 NAT · 2 MARKS

Analog Electronics › Op-amps

Q.42 The circuit given uses ideal opamps. The current $I$ (in μA) drawn from the source $v_s$ is (up to two decimal places) ____.




IN 10/14

Official answer: **0.98 to 1.00**

**SOLUTION**

1. Assume ideal op-amps (infinite gain, infinite input resistance, zero output resistance). Each op-amp has negative feedback through its $20.0\ \text{k}\Omega$ resistor and its (+) input is grounded, so by the virtual short both inverting inputs sit at $0$ V. The source node is held at $v_s = 1$ V by the ideal source, so the $10.1\ \text{k}\Omega$ path does not feed anything back into the amplifiers.
2. Op-amp 1 is an inverting amplifier: $V_{o1} = -\dfrac{20.0}{10.0}\, v_s = -2 \times 1 = -2$ V.
3. Op-amp 2 is an inverting amplifier with gain $-1$: $V_{o2} = -\dfrac{20.0}{20.0}\, V_{o1} = +2$ V. Its output resistance is zero, so the $10.1\ \text{k}\Omega$ resistor connected to it does not change this value.
4. Current from the source node through the $10.0\ \text{k}\Omega$ resistor into the virtual ground: $I_1 = \dfrac{1-0}{10.0\ \text{k}\Omega} = 100\ \mu\text{A}$.
5. Current from the source node through the $10.1\ \text{k}\Omega$ resistor towards the output of op-amp 2: $I_2 = \dfrac{1-2}{10.1\ \text{k}\Omega} = -99.01\ \mu\text{A}$. The minus sign means this current actually flows from the op-amp 2 output into the source node.
6. KCL at the source node (the source must supply whatever leaves the node):
$$I = I_1 + I_2 = \left(\frac{1}{10} - \frac{1}{10.1}\right)\ \text{mA} = \frac{0.1}{101}\ \text{mA} \approx 0.990\ \mu\text{A}$$
7. To two decimal places, $I = 0.99\ \mu\text{A}$.

**KEY IDEA**

The two cascaded inverting stages give a net gain of $+2$, so the right end of the $10.1\ \text{k}\Omega$ resistor sits at $2v_s$. That resistor returns almost all of the current that the $10.0\ \text{k}\Omega$ resistor draws into the virtual ground, and the source supplies only the small difference $v_s\left(\frac{1}{10\ \text{k}\Omega} - \frac{1}{10.1\ \text{k}\Omega}\right)$. The circuit makes the source see a very large input resistance of about $1.01\ \text{M}\Omega$, and this is the principle behind bootstrapping and negative-impedance input stages.

**WATCH OUT FOR**

Stopping at $100\ \mu\text{A}$ (only the $10.0\ \text{k}\Omega$ branch), or taking $V_{o2} = -2\ \text{V}$ by forgetting that the second inverting stage flips the sign again. The second error gives $I = 1/10 + 3/10.1 \approx 397\ \mu\text{A}$. Another slip is rounding $1/10.1\ \text{mA}$ to $0.099\ \text{mA}$ too early. The answer is a small difference of two nearly equal currents, so keep at least five significant figures until the final subtraction.

## Q.43 MCQ · 2 MARKS

Digital Electronics › Boolean logic

The product of sum expression of a Boolean function $F(A, B, C)$ of three variables is given by

$$F(A, B, C) = (A + B + \overline{C}) \cdot (A + \overline{B} + \overline{C}) \cdot (\overline{A} + B + C) \cdot (\overline{A} + \overline{B} + \overline{C})$$

The canonical sum of product expression of $F(A, B, C)$ is given by

**(A)** $\overline{A}\,\overline{B}\,C + \overline{A}\,B\,C + A\,\overline{B}\,\overline{C} + A\,B\,C$

**(B)** $\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,B\,\overline{C} + A\,\overline{B}\,C + A\,B\,\overline{C}$

**(C)** $A\,B\,\overline{C} + A\,\overline{B}\,\overline{C} + \overline{A}\,B\,C + \overline{A}\,\overline{B}\,\overline{C}$

**(D)** $\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,B\,C + A\,B\,\overline{C} + A\,B\,C$

Official answer: **B**

**SOLUTION**

1. Complement is written with an overline, $\overline{X}$. Each sum term contains all three variables, so it is a **maxterm**. A maxterm is 0 for exactly one input: the one that makes every literal 0. An uncomplemented variable must be 0 and a complemented variable ($\overline{X}$) must be 1.
2. $(A + B + \overline{C}) = 0$ only at $ABC = 001$, so it is $M_1$. $(A + \overline{B} + \overline{C}) = 0$ only at $ABC = 011$, so it is $M_3$.
3. $(\overline{A} + B + C) = 0$ only at $ABC = 100$, so it is $M_4$. $(\overline{A} + \overline{B} + \overline{C}) = 0$ only at $ABC = 111$, so it is $M_7$.
4. So $F = \prod M(1, 3, 4, 7)$: $F$ is 0 at inputs 1, 3, 4 and 7 and 1 at every other input. The canonical SOP therefore uses the **complementary minterm set**: $F = \sum m(0, 2, 5, 6)$.
5. Write out each minterm. A 0 bit gives the complemented literal and a 1 bit gives the plain literal: $m_0 = \overline{A}\,\overline{B}\,\overline{C}$ (000), $m_2 = \overline{A}\,B\,\overline{C}$ (010), $m_5 = A\,\overline{B}\,C$ (101), $m_6 = A\,B\,\overline{C}$ (110).
6. $$F = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,B\,\overline{C} + A\,\overline{B}\,C + A\,B\,\overline{C}$$

   This is option (B). Check: at $ABC = 000$ the product is $(0 + 0 + 1)(0 + 1 + 1)(1 + 0 + 0)(1 + 1 + 1) = 1$, which agrees with $m_0$ being present. The distractors: (A) is $\sum m(1, 3, 4, 7)$, the maxterm indices wrongly used as minterms. (C) is $\sum m(0, 3, 4, 6)$ and (D) is $\sum m(0, 3, 6, 7)$. Both include inputs where $F = 0$.

**KEY IDEA**

In a canonical POS, each sum term is a maxterm and marks one input where $F = 0$. Read off its index by setting plain variables to 0 and complemented ones to 1. The canonical SOP is the sum of the minterms with every other index: $\prod M(S) = \sum m(\text{all indices not in } S)$.

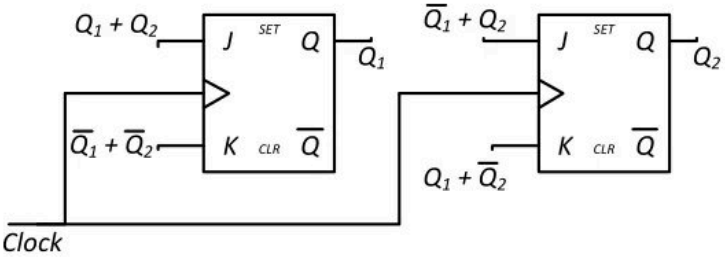
**WATCH OUT FOR**

Using the maxterm indices directly as minterms, which gives $\sum m(1, 3, 4, 7)$ = option (A). A second mistake is reading a maxterm's index the minterm way ($\overline{X} \to 0$). That turns $(A + B + \overline{C})$ into index 6 instead of 1, and the whole set comes out wrong.

## Q.44 MCQ · 2 MARKS

Digital Electronics › Sequential

Q.44 A 2-bit synchronous counter using two $J$-$K$ flip flops is shown. The expressions for the inputs to the $J$-$K$ flip flops are also shown in the figure. The output sequence of the counter starting from $Q_1Q_2 = 00$ is




(A) $00 \rightarrow 11 \rightarrow 10 \rightarrow 01 \rightarrow 00 \cdots$
(B) $00 \rightarrow 01 \rightarrow 10 \rightarrow 11 \rightarrow 00 \cdots$
(C) $00 \rightarrow 01 \rightarrow 11 \rightarrow 10 \rightarrow 00 \cdots$
(D) $00 \rightarrow 10 \rightarrow 11 \rightarrow 01 \rightarrow 00 \cdots$

Official answer: **C**

### SOLUTION

1. Read the inputs off the figure (overbar = complement): $J_1 = Q_1 + Q_2$, $K_1 = \overline{Q_1} + \overline{Q_2}$, $J_2 = \overline{Q_1} + Q_2$, $K_2 = Q_1 + \overline{Q_2}$. Both flip-flops share one clock, so all four inputs are worked out from the present state, and both outputs change on the same edge.
2. Use the J-K characteristic: $JK = 00$ holds, $01$ resets to 0, $10$ sets to 1, $11$ toggles.
3. Present state $Q_1Q_2 = 00$: $J_1K_1 = 0\,1 \Rightarrow Q_1 = 0$ (reset). $J_2K_2 = 1\,1 \Rightarrow Q_2$ toggles $0 \rightarrow 1$. Next state $01$.
4. Present state $01$: $J_1K_1 = 1\,1 \Rightarrow Q_1$ toggles $0 \rightarrow 1$. $J_2K_2 = 1\,0 \Rightarrow Q_2 = 1$ (set). Next state $11$.
5. Present state $11$: $J_1K_1 = 1\,0 \Rightarrow Q_1 = 1$ (set). $J_2K_2 = 1\,1 \Rightarrow Q_2$ toggles $1 \rightarrow 0$. Next state $10$.
6. Present state $10$: $J_1K_1 = 1\,1 \Rightarrow Q_1$ toggles $1 \rightarrow 0$. $J_2K_2 = 0\,1 \Rightarrow Q_2 = 0$ (reset). Next state $00$, which is where the counter started.
7. Cross-check with the characteristic equation $Q^+ = J\overline{Q} + \overline{K}Q$. Since $\overline{K_1} = Q_1Q_2$ and $\overline{K_2} = \overline{Q_1}Q_2$:
$$Q_1^+ = (Q_1 + Q_2)\overline{Q_1} + Q_1Q_2 = \overline{Q_1}Q_2 + Q_1Q_2 = Q_2$$
$$Q_2^+ = (\overline{Q_1} + Q_2)\overline{Q_2} + \overline{Q_1}Q_2 = \overline{Q_1}\,\overline{Q_2} + \overline{Q_1}Q_2 = \overline{Q_1}$$
So the next state is $Q_1^+Q_2^+ = Q_2\,\overline{Q_1}$: $00 \rightarrow 01 \rightarrow 11 \rightarrow 10 \rightarrow 00$, the same as the state-by-state table.
8. Sequence: $00 \rightarrow 01 \rightarrow 11 \rightarrow 10 \rightarrow 00 \cdots$. This is a 2-bit Gray-code (twisted-ring) counter, which is option (C).

**KEY IDEA**

In a synchronous counter, work out every J and K from the present state, then apply the J-K rules (00 hold, 01 reset, 10 set, 11 toggle) to both flip-flops at once. You can also check it in one line with $Q^+ = J\overline{Q} + \overline{K}Q$. Here the inputs simplify to $Q_1^+ = Q_2$ and $Q_2^+ = \overline{Q_1}$, a 2-bit twisted-ring counter, and that gives the Gray sequence 00, 01, 11, 10.

**WATCH OUT FOR**

Updating $Q_1$ first and then using the new $Q_1$ to work out $J_2$ and $K_2$ within the same clock edge. That is ripple-style thinking, and it is wrong for a synchronous counter, where every input comes from the old state. Other slips: mixing up $JK = 01$ (reset) with $10$ (set), and missing an overbar on the input labels. For example, $J_1 = Q_1 + Q_2$ has no bars, while $K_1 = \overline{Q_1} + \overline{Q_2}$ has two.

## Q.45 NAT · 2 MARKS

Digital Electronics › Processors & memory

A portion of an assembly language program written for an 8-bit microprocessor is given below along with explanations. The code is intended to introduce a software time delay. The processor is driven by a 5 MHz clock. The time delay (in $\mu$s) introduced by the program is _____.

```
      MVI B, 64H   ; Move immediate the given byte into register B. Takes 7 clock periods.
LOOP : DCR B       ; Decrement register B. Affects Flags. Takes 4 clock periods.
      JNZ LOOP     ; Jump to address with Label LOOP if zero flag is not set. Takes
                   ; 10 clock periods when jump is performed and 7 clock periods when
                   ; jump is not performed.
```

Official answer: **279 to 282**

### SOLUTION

1. Convert the loop count to decimal: $(64)_{16} = 6 \times 16 + 4 = (100)_{10}$. So $B$ starts at 100, and the loop body DCR B runs 100 times, once for each value from 100 down to 1.
2. JNZ LOOP jumps whenever $B \neq 0$ after the decrement. That happens 99 times, after $B$ becomes $99, 98, \ldots, 1$. On the 100th pass $B$ becomes 0, the zero flag is set ($Z = 1$), and the jump is not taken. This happens once.
3. Count the clock periods (T-states) for each instruction:

$$\begin{aligned} \text{MVI B, 64H} &: 1 \times 7 = 7 \\ \text{DCR B} &: 100 \times 4 = 400 \\ \text{JNZ (taken)} &: 99 \times 10 = 990 \\ \text{JNZ (not taken)} &: 1 \times 7 = 7 \end{aligned}$$

4. Total: $N_T = 7 + 400 + 990 + 7 = 1404$ clock periods.
5. Clock period: $T = \dfrac{1}{f} = \dfrac{1}{5\ \text{MHz}} = 0.2\ \mu\text{s}$. The question says each count is in clock periods of the 5 MHz clock that drives the processor.
6. Delay: $t_d = N_T \times T = 1404 \times 0.2\ \mu\text{s} = 280.8\ \mu\text{s}$.

**KEY IDEA**

For a delay loop with an initial count $N$, the loop body runs $N$ times, but the conditional jump is taken only $N - 1$ times and falls through once, at its shorter not-taken time. The total T-states are $N_T = T_{\text{init}} + N\,T_{\text{DCR}} + (N-1)\,T_{\text{JNZ,taken}} + T_{\text{JNZ,not taken}}$, and the delay is $N_T / f$. Convert a hexadecimal count such as $64\text{H}$ to decimal first.

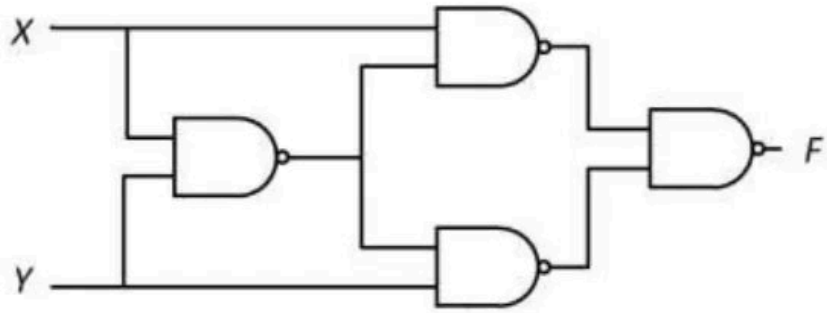
**WATCH OUT FOR**

Reading $64\text{H}$ as decimal 64 gives $7 + 256 + 630 + 7 = 900$ T-states, or $180\ \mu\text{s}$. Another error is to halve the 5 MHz clock, as if it were an 8085 crystal frequency, which doubles the answer to $561.6\ \mu\text{s}$. The question states that the processor itself is driven by a 5 MHz clock, so $T = 0.2\ \mu\text{s}$. Counting the jump as taken all 100 times is also wrong, even though it happens to land inside the key's range here.

## Q.46 MCQ · 2 MARKS

Digital Electronics › Boolean logic

Q.46 The Boolean function $F(X, Y)$ realized by the given circuit is




(A) $\bar{X}\,Y + X\,\bar{Y}$ (B) $\bar{X}\,\bar{Y} + X\,Y$ (C) $X + Y$ (D) $\bar{X} \cdot \bar{Y}$

Official answer: **A**

**SOLUTION**

1. Complement is written with - throughout. All four gates are 2-input NAND gates. Label them G1 (middle), G2 (top), G3 (bottom) and G4 (output).
2. G1 takes X and Y: $P = \overline{XY}$.
3. G2 takes X and P: $A = \overline{X\,\overline{XY}}$. G3 takes P and Y: $B = \overline{Y\,\overline{XY}}$.
4. G4 NANDs A and B. Write $A = \overline{u}$ with $u = X\,\overline{XY}$ and $B = \overline{v}$ with $v = Y\,\overline{XY}$. By De Morgan, $\overline{\overline{u} \cdot \overline{v}} = u + v$, so
$$F = \overline{A\,B} = X\,\overline{XY} + Y\,\overline{XY} = (X + Y)\,\overline{XY}$$
5. Apply De Morgan again, $\overline{XY} = \overline{X} + \overline{Y}$:
$$F = (X + Y)(\overline{X} + \overline{Y}) = X\overline{X} + X\overline{Y} + \overline{X}Y + Y\overline{Y} = \overline{X}\,Y + X\,\overline{Y}$$
since $X\overline{X} = Y\overline{Y} = 0$.
6. Check with a truth table: $(X, Y) = (0, 0)$ gives $P = 1$, $A = 1$, $B = 1$, $F = 0$. $(0, 1)$ gives $P = 1$, $A = 1$, $B = 0$, $F = 1$. $(1, 0)$ gives $P = 1$, $A = 0$, $B = 1$, $F = 1$. $(1, 1)$ gives $P = 0$, $A = 1$, $B = 1$, $F = 0$. The output is 1 exactly when the inputs differ, so the circuit is an XOR gate, $F = \overline{X}\,Y + X\,\overline{Y}$, which is option (A).

**KEY IDEA**

This is the standard four-NAND XOR. The first NAND makes $\overline{XY}$. Each input is then NANDed with it, and the last NAND turns the two terms into $X\,\overline{XY} + Y\,\overline{XY} = (X + Y)(\overline{X} + \overline{Y})$, which is $X \oplus Y$.

**WATCH OUT FOR**

Treating the output NAND as an AND or an OR of the two middle outputs and dropping a complement. Another slip is treating the gates as AND gates because the bubbles are small, which gives $XY$ or a wrong option such as (D). The quickest way to catch these is a four-row truth table with every gate treated as a NAND. It also separates XOR (A) from XNOR (B).

## Q.47 NAT · 2 MARKS

Electrical and Electronic Measurements › Errors & standards

The voltage and current drawn by a resistive load are measured with a 300 V voltmeter of accuracy $\pm 1\%$ of full scale and a 5 A ammeter of accuracy $\pm 0.5\%$ of full scale. The readings obtained are 200 V and 2.5 A.

The limiting error (in %) in computing the load resistance is (up to two decimal places) ____.

Official answer: **2.49 to 2.53**

**SOLUTION**

1. A full-scale accuracy fixes the same absolute error at every point on the scale. For the voltmeter: $\delta V = \pm 1\% \times 300\ \text{V} = \pm 3\ \text{V}$.
2. At the reading of $200\ \text{V}$, that is a relative error of $\dfrac{3}{200} \times 100 = \pm 1.5\%$.
3. For the ammeter: $\delta I = \pm 0.5\% \times 5\ \text{A} = \pm 0.025\ \text{A}$.
4. At the reading of $2.5\ \text{A}$, that is a relative error of $\dfrac{0.025}{2.5} \times 100 = \pm 1\%$.
5. The resistance is a quotient: $R = \dfrac{V}{I} = \dfrac{200}{2.5} = 80\ \Omega$. A limiting error is the worst case, so the relative errors of a quotient add; they do not subtract: $\dfrac{\delta R}{R} = \pm\left(\dfrac{\delta V}{V} + \dfrac{\delta I}{I}\right)$.
6. $$\frac{\delta R}{R} = \pm(1.5\% + 1\%) = \pm 2.50\%$$
7. So the limiting error in the computed load resistance is $2.50\%$, which is $\pm 2\ \Omega$ on $80\ \Omega$.

**KEY IDEA**

An instrument's accuracy stated as a percentage of full scale is a fixed absolute error, so convert it to a relative error at the actual reading. For a product or quotient, the limiting (worst-case) relative errors add.

**WATCH OUT FOR**

One mistake is to use the full-scale percentages directly ($1\% + 0.5\% = 1.5\%$) and forget that the error is a percentage of full scale, not of the reading. The other is to subtract the ammeter's relative error because $I$ is in the denominator. Worst-case errors of a quotient always add.

## Q.48 NAT · 2 MARKS

Electrical and Electronic Measurements › Bridges

A high Q coil having distributed (self) capacitance is tested with a Q-meter. First resonance at $\omega_1 = 10^6$ rad/s is obtained with a capacitance of 990 pF. The second resonance at $\omega_2 = 2 \times 10^6$ rad/s is obtained with a 240 pF capacitance. The value of the inductance (in mH) of the coil is (up to one decimal place) _____.

Official answer: **0.9 to 1.1**

**SOLUTION**

1. In a Q-meter the coil's distributed (self) capacitance $C_d$ is in parallel with the tuning capacitor $C$. Resonance therefore occurs when $\omega^2 L\,(C + C_d) = 1$.
2. First resonance, at $\omega_1 = 10^6\ \text{rad/s}$ with $C_1 = 990\ \text{pF}$:
$$\omega_1^2 L\,(C_1 + C_d) = 1$$
3. Second resonance, at $\omega_2 = 2 \times 10^6\ \text{rad/s}$ with $C_2 = 240\ \text{pF}$:
$$\omega_2^2 L\,(C_2 + C_d) = 1$$
4. Both products equal 1, so set them equal. $L$ cancels, and $\omega_2^2 = 4\,\omega_1^2$:
$$C_1 + C_d = 4\,(C_2 + C_d) \;\Rightarrow\; 990 + C_d = 960 + 4C_d \;\Rightarrow\; C_d = 10\ \text{pF}$$
This is the standard doubled-frequency result $C_d = \dfrac{C_1 - 4C_2}{3} = \dfrac{990 - 960}{3} = 10\ \text{pF}$.
5. Total resonating capacitance at the first resonance: $C_1 + C_d = 1000\ \text{pF} = 1000 \times 10^{-12}\ \text{F} = 10^{-9}\ \text{F}$.
6. Solve the first resonance condition for $L$:
$$L = \frac{1}{\omega_1^2 (C_1 + C_d)} = \frac{1}{(10^6)^2 \times 10^{-9}} = \frac{1}{10^{12} \times 10^{-9}} = \frac{1}{10^3} = 10^{-3}\ \text{H}$$

7. Check with the second resonance: $C_2 + C_d = 250\ \text{pF} = 2.5 \times 10^{-10}\ \text{F}$, so
$$\omega_2^2(C_2 + C_d) = 4 \times 10^{12} \times 2.5 \times 10^{-10} = 10^3 \ \Rightarrow\ L = 10^{-3}\ \text{H}$$
Both resonances give the same inductance.
8. Convert to the unit asked for: $L = 10^{-3}\ \text{H} = 1.0\ \text{mH}$.

**KEY IDEA**

A coil's self-capacitance $C_d$ adds to the Q-meter's tuning capacitance, so $\omega^2 L(C + C_d) = 1$ at every resonance. Two resonances give two equations. Equating them removes $L$ and gives $C_d$. For a frequency ratio of 2, $C_d = (C_1 - 4C_2)/3$. Put $C_d$ back into either equation to get $L$.

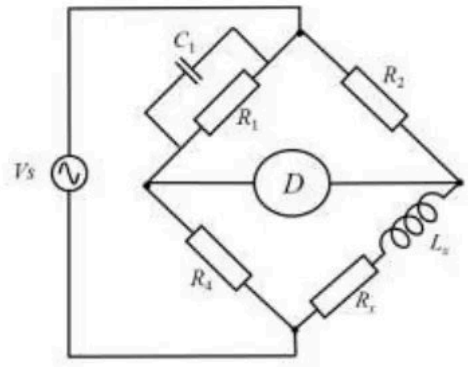
**WATCH OUT FOR**

The usual slip is the powers of ten in the last step. $1000\ \text{pF}$ is $10^{-9}\ \text{F}$, not $1\ \text{F}$, so $\omega_1^2(C_1 + C_d) = 10^{12} \times 10^{-9} = 10^3$. That gives $L = 1\ \text{mH}$, not $1\ \text{H}$ (1000 mH). Two other errors: leaving out $C_d$ altogether, which gives $L = 1/(10^{12} \times 990 \times 10^{-12}) \approx 1.01\ \text{mH}$ from the first resonance and a different value from the second, and subtracting $C_d$ from $C$ when it should be added.

## Q.49 NAT · 2 MARKS



Q.49 The inductance of a coil is measured using the bridge shown in the figure. Balance ($D = 0$) is obtained with $C_1 = 1$ nF, $R_1 = 2.2$ MΩ, $R_2 = 22.2$ kΩ, $R_4 = 10$ kΩ. The value of the inductance $L_x$ (in mH) is _____.




IN 12/14

Official answer: **221 to 223**

**SOLUTION**

1. Name the arms. The source $V_s$ is connected between the top and bottom nodes and the detector $D$ between the left and right nodes. $Z_1 = R_1 \parallel C_1$ (top to left), $Z_2 = R_2$ (top to right), $Z_4 = R_4$ (left to bottom), $Z_x = R_x + j\omega L_x$ (right to bottom). So $Z_1$ is opposite $Z_x$, and $Z_2$ is opposite $Z_4$.
2. Balance condition. $D = 0$ means the left and right nodes are at the same potential: $\dfrac{Z_4}{Z_1 + Z_4} = \dfrac{Z_x}{Z_2 + Z_x}$. This simplifies to $Z_1 Z_x = Z_2 Z_4$ (the products of opposite arms are equal). Therefore $Z_x = \dfrac{R_2 R_4}{Z_1} = R_2 R_4\, Y_1$.
3. Admittance of the parallel $R_1 C_1$ arm: $Y_1 = \dfrac{1}{R_1} + j\omega C_1$.
4. Substitute, then equate the real parts and the imaginary parts:
$$R_x + j\omega L_x = \frac{R_2 R_4}{R_1} + j\omega R_2 R_4 C_1 \ \Rightarrow\ L_x = R_2 R_4 C_1, \quad R_x = \frac{R_2 R_4}{R_1}$$
Neither result depends on the frequency. This is the Maxwell inductance-capacitance bridge.
5. Inductance:
$$L_x = (22.2 \times 10^3\ \Omega)(10 \times 10^3\ \Omega)(1 \times 10^{-9}\ \text{F}) = 0.222\ \text{H} = 222\ \text{mH}$$

6. Check: $R_x = \dfrac{(22.2\times10^3)(10\times10^3)}{2.2\times10^6}\ \Omega \approx 100.9\ \Omega$, which is a reasonable winding resistance. $R_1$ affects only $R_x$ and does not appear in $L_x$.

**KEY IDEA**

This is a Maxwell inductance-capacitance bridge. At balance, $Z_x = R_2R_4Y_1$. The parallel $R_1C_1$ arm contributes an admittance, so its $j\omega C_1$ term turns directly into $j\omega L_x$. That gives $L_x = R_2R_4C_1$, independent of frequency and of $R_1$.

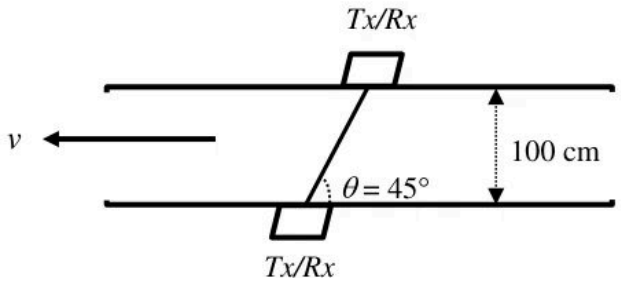
**WATCH OUT FOR**

One mistake is using the impedance of the $R_1C_1$ arm instead of its admittance. Another is pairing the wrong arms, for example writing $Z_1Z_4 = Z_2Z_x$ when the detector's position makes $Z_1$ opposite $Z_x$. A third is bringing $R_1$ into the inductance formula. Each of these gives a wrong or frequency-dependent $L_x$. The unit prefixes also cause slips: $22.2\ \text{k}\Omega \times 10\ \text{k}\Omega \times 1\ \text{nF} = 0.222\ \text{H}$, which is 222 mH, not 222 H or 22.2 mH.

## Q.50 NAT · 2 MARKS

Sensors and Industrial Instrumentation › Pressure & flow

Q.50 The average velocity $v$ of flow of clear water in a 100 cm (inner) diameter tube is measured using the ultrasonic flow meter as shown in the figure. The angle $\theta$ is 45°. The measured transit times are $t_1 = 0.9950$ ms and $t_2 = 1.0000$ ms. The velocity $v$ (in m/s) in the pipe is (up to one decimal place) ___.




Official answer: **3.5 to 5.7**

**SOLUTION**

1. Transit-time (ultrasonic) flowmeter. The sound path crosses the pipe at $\theta = 45^\circ$ to the axis, so its length is $L = \dfrac{D}{\sin\theta} = \dfrac{1\ \text{m}}{\sin 45^\circ} = \sqrt{2}\ \text{m}$. Here $D = 100\ \text{cm} = 1\ \text{m}$.
2. The flow adds $v\cos\theta$ to the sound speed $c$ on the downstream trip and subtracts it on the upstream trip. The shorter time $t_1 = 0.9950\ \text{ms}$ is therefore the downstream trip and $t_2 = 1.0000\ \text{ms}$ is the upstream trip: $t_1 = \dfrac{L}{c + v\cos\theta}$ and $t_2 = \dfrac{L}{c - v\cos\theta}$.
3. Subtract the reciprocals. This removes $c$, which is not given: $\dfrac{1}{t_1} - \dfrac{1}{t_2} = \dfrac{2v\cos\theta}{L}$.
4. Solve for $v$: $v = \dfrac{L}{2\cos\theta}\left(\dfrac{1}{t_1} - \dfrac{1}{t_2}\right) = \dfrac{D}{2\sin\theta\cos\theta}\left(\dfrac{1}{t_1} - \dfrac{1}{t_2}\right) = \dfrac{D}{\sin 2\theta}\left(\dfrac{1}{t_1} - \dfrac{1}{t_2}\right)$.
5. For $\theta = 45^\circ$, $\sin 2\theta = 1$ and $D = 1\ \text{m}$, so $v = \left(\dfrac{1}{t_1} - \dfrac{1}{t_2}\right) \times 1\ \text{m}$.
6. Numbers: $\dfrac{1}{0.9950\times10^{-3}\ \text{s}} = 1005.03\ \text{s}^{-1}$ and $\dfrac{1}{1.0000\times10^{-3}\ \text{s}} = 1000\ \text{s}^{-1}$. Therefore $v = 5.03\ \text{m/s} \approx 5.0\ \text{m/s}$.

**KEY IDEA**

In a transit-time ultrasonic flowmeter the difference of the reciprocal transit times, $\frac{1}{t_1} - \frac{1}{t_2} = \frac{2v\cos\theta}{L}$, gives the velocity without needing the sound speed. With $L = D/\sin\theta$ this becomes $v = \frac{D}{\sin 2\theta}\left(\frac{1}{t_1} - \frac{1}{t_2}\right)$.

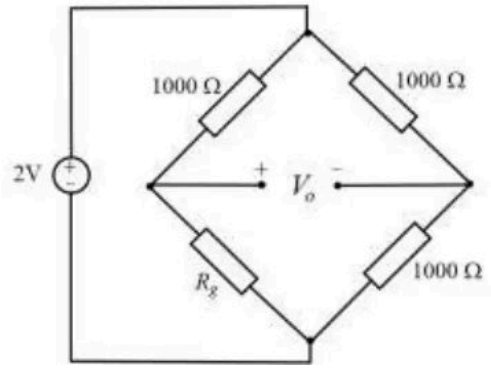
**WATCH OUT FOR**

Taking the path length as the pipe diameter $D$ instead of $D/\sin\theta$, or using $\Delta t/t^2$ with $L = D$ and so being off by the factor $\sin\theta\cos\theta$ or $\cos\theta$. Another slip is using $2D/\sin 2\theta$, or forgetting to convert ms to s.

## Q.51 NAT · 2 MARKS

Sensors and Industrial Instrumentation › Sensing principles

Q.51 A 1000 Ω strain gage ($R_g$) has a gage factor of 2.5. It is connected in the bridge as shown in figure. The strain gage is subjected with a positive strain of 400 μm/m. The output $V_o$ (in mV) of the bridge is (up to two decimal places) _____.




Official answer: **0.48 to 0.51**

**SOLUTION**

1. Gage factor: $G = \frac{\Delta R/R}{\varepsilon}$, so $\Delta R = G\,\varepsilon\,R = 2.5 \times 400 \times 10^{-6} \times 1000 = 1\ \Omega$.
2. The strain is positive, so the gage resistance rises: $R_g = 1000 + 1 = 1001\ \Omega$.
3. The gage $R_g$ is the bottom-left arm of the bridge; the other three arms are $1000\ \Omega$. The $2\ \text{V}$ source is across the top and bottom nodes, with its $+$ terminal at the top node. Take the bottom node as $0\ \text{V}$. $V_o$ is read from the left middle node $(+)$ to the right middle node $(-)$.
4. Left divider, measured from the bottom node: $V_L = 2 \times \frac{R_g}{1000 + R_g} = 2 \times \frac{1001}{2001}\ \text{V}$.
5. Right divider: $V_R = 2 \times \frac{1000}{1000 + 1000} = 1\ \text{V}$.
6. $$V_o = V_L - V_R = \frac{2002}{2001} - 1 = \frac{1}{2001}\ \text{V} \approx 0.4998\ \text{mV} \approx 0.50\ \text{mV}$$
7. Check with the quarter-bridge approximation: $V_o \approx \frac{V_s}{4}\frac{\Delta R}{R} = \frac{2}{4} \times 10^{-3}\ \text{V} = 0.5\ \text{mV}$, which agrees.

**KEY IDEA**

A strain gage changes resistance by $\Delta R/R = G\varepsilon$. In a quarter bridge with three equal fixed arms, the output is close to $\frac{V_s}{4}\frac{\Delta R}{R}$. Here the exact divider difference is $1/2001\ \text{V}$.

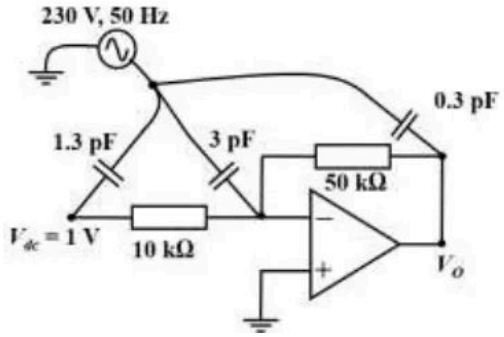
**WATCH OUT FOR**

Writing the strain $400\ \mu\text{m/m}$ as $400 \times 10^{-3}$ instead of $400 \times 10^{-6}$. Another slip is forgetting that the answer is asked in mV and giving $0.0005$ (volts), or reading the output with the wrong polarity.

## Q.52 NAT · 2 MARKS

Analog Electronics › Noise & shielding (IN)

Q.52 Assuming ideal opamp, the RMS voltage (in mV) in the output $V_o$ only due to the 230 V, 50 Hz interference is (up to one decimal place) _____.




Official answer: **10.6 to 11.0**

**SOLUTION**

1. **Assumptions:** the op-amp is ideal (infinite gain, no input current, zero output impedance), and $V_{dc} = 1\ \text{V}$ is an ideal dc source. The circuit is linear, so use superposition. Switch off $V_{dc}$, which only sets a dc output of $-\frac{50}{10} \times 1 = -5\ \text{V}$, and keep only the 50 Hz source, $V_n = 230\ \text{V}$ rms, $\omega = 2\pi(50)\ \text{rad/s}$.
2. **1.3 pF capacitor:** it ends on the node of the ideal voltage source $V_{dc}$. For ac that node is shorted to ground, so the current in this capacitor flows into the source and never reaches the amplifier. It adds nothing to $V_o$.
3. **0.3 pF capacitor:** it ends on the output of an ideal op-amp, which acts as a voltage source with zero output impedance. Current injected there cannot change $V_o$, so it adds nothing either.
4. **3 pF capacitor:** it ends on the inverting input. The (+) input is grounded, so the virtual short holds this node at exactly $0\ \text{V}$ and the whole source voltage appears across the 3 pF capacitor:
$$|I_n| = V_n\,\omega C = 230 \times 2\pi(50) \times 3 \times 10^{-12} = 2.168 \times 10^{-7}\ \text{A (rms)}$$
5. **KCL at the inverting node:** the ideal op-amp draws no input current. With $V_{dc}$ switched off, both ends of the $10\ \text{k}\Omega$ resistor are at $0\ \text{V}$, so it carries no current. All of $I_n$ therefore flows through the $50\ \text{k}\Omega$ feedback resistor:
$$I_n + \frac{V_o - 0}{R_f} = 0 \;\Rightarrow\; V_o = -I_n R_f = -j\omega C R_f\, V_n$$
6. **Output magnitude:**
$$|V_o| = |I_n|\,R_f = 2.168 \times 10^{-7} \times 50 \times 10^3 = 1.084 \times 10^{-2}\ \text{V} = 10.84\ \text{mV} \approx 10.8\ \text{mV (rms)}$$

**KEY IDEA**

Only a stray capacitance that ends on a high-impedance node lets interference through. The inverting input of an ideal op-amp is a virtual ground, so the 3 pF capacitor passes a current $V_n\omega C$ that must all flow through $R_f$. Capacitors that end on low-impedance nodes (the ideal dc source, or the op-amp output) send their current into those sources and have no effect on $V_o$.

**WATCH OUT FOR**

Adding all three capacitances (4.6 pF) and getting about 16.6 mV. The 1.3 pF current goes into the ideal dc source and the 0.3 pF current goes into the zero-impedance op-amp output, so neither reaches $V_o$. Other slips: treating the 1.3 pF current as an input through the 10 kΩ resistor with gain $-R_f/R_1$, or forgetting that the answer is asked for in mV.

## Q.53 NAT · 2 MARKS

Communications › Digital communication

The sampling rate for Compact Discs (CDs) is 44,000 samples/s. If the samples are quantized to 256 levels

and binary coded, the corresponding bit rate (in bits per second) is ____.

Official answer: **352000**

**SOLUTION**

1. Each sample is quantized to $256$ levels. With binary coding, the number of bits per sample is $n = \log_2 256 = 8$.
2. The bit rate is the sampling rate times the bits per sample: $R_b = f_s \cdot n$.
3. $R_b = 44{,}000 \times 8 = 352{,}000 \text{ bits/s}$.

**KEY IDEA**

Bit rate of a PCM stream equals the sampling rate times the number of bits per sample, where the bits per sample is $\log_2$ of the number of quantization levels.

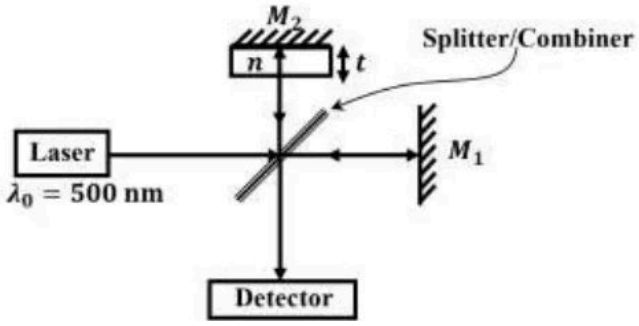
**WATCH OUT FOR**

Using the number of levels (256) in place of the number of bits (8), or multiplying $f_s$ by $\log_2 256$ and forgetting the 44,000 figure is the sampling rate itself, not a bandwidth that needs doubling.

## Q.54 NAT · 2 MARKS

Sensors and Industrial Instrumentation › Optical

Q.54 A Michelson Interferometer using a laser source of wavelength $\lambda_0 = 500$ nm, with both the mirrors ($M_1$ & $M_2$) fixed and positioned equidistant from the splitter/combiner is shown in the figure. When a dielectric plate of refractive index $n = 1.5$, of thickness $\boldsymbol{t}$ , is placed in front of the mirror $M_2$ , a dark fringe is observed on the detector. When the dielectric plate is removed without changing the position of the mirrors $M_1$ & $M_2$, a bright fringe is observed on the detector. The minimum thickness $\boldsymbol{t}$ (in nm) of the dielectric is _____.




Official answer: **249 to 251**

**SOLUTION**

1. The mirrors $M_1$ and $M_2$ are equidistant from the splitter/combiner. With the plate removed, the two arms therefore have zero path difference, so the detector sees a bright fringe, as the question states.
2. With the plate in front of $M_2$, the beam crosses it twice, once towards the mirror and once on the way back. In each pass the plate replaces a thickness $t$ of air by a thickness $nt$ of glass, so the optical path increases by $(n-1)t$ per pass. The total extra optical path is
$$\Delta = 2(n-1)t.$$
3. The detector sees a dark fringe, which needs destructive interference. Since the arms were otherwise equal, the extra path must be an odd number of half-wavelengths:
$$2(n-1)t = \left(m + \tfrac{1}{2}\right)\lambda_0, \quad m = 0, 1, 2, \ldots$$
4. The minimum thickness corresponds to $m = 0$:
$$t_{\min} = \frac{\lambda_0}{4(n-1)}$$
5. Substitute $\lambda_0 = 500$ nm and $n = 1.5$:

$$t_{\min} = \frac{500\ \text{nm}}{4 \times 0.5} = 250\ \text{nm}$$

**KEY IDEA**

A plate placed in one arm of a Michelson interferometer is crossed twice, so it adds an extra optical path of $2(n-1)t$. Equal arms give a bright fringe, so a dark fringe needs this extra path to equal an odd number of half-wavelengths.

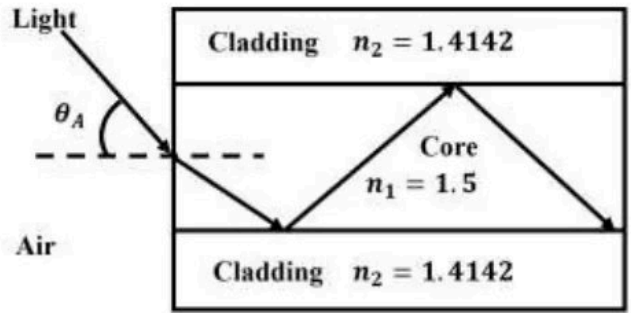
**WATCH OUT FOR**

Using a single pass, $(n-1)t = \lambda_0/2$, gives 500 nm. Another slip is using the full optical path $nt$ instead of the excess $(n-1)t$ over air. Both ignore that the beam goes through the plate twice and that only the excess over air matters.

## Q.55 NAT · 2 MARKS

Electromagnetics › Waveguides & fibres

Q.55 A multi-mode optical fiber with a large core diameter has a core refractive index $n_1 = 1.5$ and cladding refractive index $n_2 = 1.4142$. The maximum value of $\theta_A$ (in degrees) for which the incident light from air will be guided in the optical fiber is $\pm$_____.




Official answer: **29 to 31**

**SOLUTION**

1. **What the fibre needs:** the ray is guided only if it meets the core–cladding boundary at an angle (measured from the normal to that boundary) of at least the critical angle $\theta_c$, so that it is totally internally reflected. The largest launch angle $\theta_A$ in air that still meets this condition is the acceptance angle.
2. **Critical angle at the core–cladding boundary:** $\sin\theta_c = \dfrac{n_2}{n_1}$. The fibre axis runs parallel to the boundary, so at this limit the ray inside the core makes an angle $\theta_r = 90^\circ - \theta_c$ with the axis.
3. **Snell's law at the air–core end face** ($n_0 = 1$ for air):
$$n_0 \sin\theta_A = n_1 \sin\theta_r = n_1 \cos\theta_c = n_1\sqrt{1 - \frac{n_2^2}{n_1^2}} = \sqrt{n_1^2 - n_2^2}$$
This quantity is the numerical aperture, $\text{NA} = \sqrt{n_1^2 - n_2^2}$.
4. **Substitute the values:**
$$n_1^2 = 1.5^2 = 2.25, \qquad n_2^2 = 1.4142^2 = 1.99996$$
$$\text{NA} = \sqrt{2.25 - 1.99996} = \sqrt{0.25004} \approx 0.5000$$
(Note that $1.4142 \approx \sqrt{2}$, so $n_2^2 \approx 2$.)
5. **Acceptance angle:**
$$\theta_A = \sin^{-1}(\text{NA}) = \sin^{-1}(0.5000) \approx 30^\circ$$
(Keeping every digit gives $30.003^\circ$.)
6. **Result:** light from air is guided for launch angles up to $\pm 30^\circ$ from the fibre axis.

**KEY IDEA**

For a fibre launched from air, the acceptance angle satisfies $\sin\theta_A = \mathrm{NA} = \sqrt{n_1^2 - n_2^2}$. This comes from total internal reflection at the core–cladding boundary ( $\sin\theta_c = n_2/n_1$) together with Snell's law at the end face. Here $n_2 \approx \sqrt{2}$, so $\mathrm{NA} = \sqrt{2.25 - 2} = 0.5$.

**WATCH OUT FOR**

Answering with the critical angle $\theta_c = \sin^{-1}(n_2/n_1) = \sin^{-1}(0.9428) \approx 70.5^\circ$, or its complement inside the core, $90^\circ - \theta_c \approx 19.5^\circ$, instead of the angle in air. The core-side angle has to be carried through Snell's law at the end face, $\sin\theta_A = n_1\cos\theta_c$. Another slip is to take $\mathrm{NA} = n_1 - n_2$ instead of $\sqrt{n_1^2 - n_2^2}$.

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