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GATE EE · 2025

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GATE EE 2025 Solved Paper

INSIDE

- All 65 questions, in the order of the paper
- The official answer to every question
- A step-by-step solution to every question
- The answer key on one page

Question paper and official answer key: IIT Roorkee

Online, with the questions linked to their topic notes: **iaeacademy.in/study/papers/ee-2025**

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About this paper

65 questions, 100 marks. Question types: 42 MCQ · 4 MSQ · 19 NAT.

Official answer key: IIT Roorkee · https://gate2025.iitr.ac.in/doc/2025/2025_Key/EE_Keys.pdf

We placed each question under a subject and topic ourselves; the exam paper does not label them.

This paper online, with the questions linked to their topic notes:
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Answer key

Q	TYPE	MARKS	ANSWER
Q.1	MCQ	1	**D**
Q.2	MCQ	1	**A**
Q.3	MCQ	1	**A**
Q.4	MCQ	1	**B**
Q.5	MCQ	1	**C**
Q.6	MCQ	2	**C**
Q.7	MCQ	2	**C**
Q.8	MCQ	2	**B**
Q.9	MCQ	2	**C**
Q.10	MCQ	2	**A**
Q.11	MCQ	1	**B**
Q.12	MCQ	1	**B**
Q.13	MCQ	1	**B**
Q.14	MCQ	1	**A**
Q.15	MCQ	1	**B**
Q.16	MCQ	1	**C**
Q.17	MCQ	1	**D**
Q.18	MCQ	1	**B**
Q.19	MCQ	1	**B**
Q.20	MCQ	1	**A**
Q.21	MCQ	1	**A**
Q.22	MCQ	1	**B**
Q.23	MCQ	1	**D**
Q.24	MCQ	1	**A**
Q.25	MCQ	1	**C**
Q.26	MCQ	1	**A**
Q.27	MCQ	1	**A**
Q.28	MSQ	1	**A, C**
Q.29	MSQ	1	**A, C**
Q.30	NAT	1	**7**
Q.31	NAT	1	**5**
Q.32	NAT	1	**0.975 to 0.985**
Q.33	NAT	1	**0.380 to 0.400**
Q.34	NAT	1	**90**
Q.35	NAT	1	**66.25 to 66.75**
Q.36	MCQ	2	**A**
Q.37	MCQ	2	**A**
Q.38	MCQ	2	**C**
Q.39	MCQ	2	**C**
Q.40	MCQ	2	**A**
Q.41	MCQ	2	**B**
Q.42	MCQ	2	**B**
Q.43	MCQ	2	**A**
Q.44	MCQ	2	**A**
Q.45	MCQ	2	**C**
Q.46	MCQ	2	**A**
Q.47	MCQ	2	**A**
Q.48	MCQ	2	**A**
Q.49	MCQ	2	**B**
Q.50	MCQ	2	**C**
Q.51	MSQ	2	**B, D**
Q.52	MSQ	2	**A, D**
Q.53	NAT	2	**0 to 1**
Q.54	NAT	2	**2**
Q.55	NAT	2	**0**
Q.56	NAT	2	**2**
Q.57	NAT	2	**4.0 to 4.2**
Q.58	NAT	2	**78.0 to 81.0**
Q.59	NAT	2	**-2.00 to -1.94, or 1.94 to 2.00**
Q.60	NAT	2	**1.7 to 1.9**
Q.61	NAT	2	**200**
Q.62	NAT	2	**3**
Q.63	NAT	2	**8.5 to 10.0**
Q.64	NAT	2	**216**
Q.65	NAT	2	**0.1**

General Aptitude Q.1 to Q.10

Q.1 MCQ · 1 MARK

General Aptitude › Grammar & vocabulary

Kavya _____ go to work yesterday as she _____ feeling well.

Select the most appropriate option to complete the above sentence.

(A) didn't; isn't

(B) wouldn't; wasn't

(C) wasn't; wasn't

(D) couldn't; wasn't

Official answer: **D**

SOLUTION

1. The time marker "yesterday" puts the whole sentence in the past, so both blanks need past-tense forms.
2. Second blank: "as she _____ feeling well" gives the reason, and it holds at the same past time. "feeling" needs a form of "be" in front of it, and the past negative is "wasn't" (was + not + feeling, the past continuous). "isn't" is present tense and clashes with "yesterday", so (A) is out.
3. First blank: the word after it is the bare verb "go", so the blank must be a modal or "did". "wasn't go" is ungrammatical, because "was" would need "going" after it. So (C) is out.
4. That leaves (B) "wouldn't go" and (D) "couldn't go". In the past, "would not" signals refusal or unwillingness: she chose not to go. "could not" signals inability: she was not able to go.
5. The reason given is illness ("she wasn't feeling well"). Illness takes away your ability to do something; it is not a choice. So the modal of inability, "couldn't", fits the cause-and-effect link made by "as".
6. Completed sentence: "Kavya couldn't go to work yesterday as she wasn't feeling well." Answer: (D).

KEY IDEA

Fill the blanks in two passes. First use tense and grammar: "yesterday" needs past forms, and the bare verb "go" after the first blank needs a modal or "did". Then use meaning: illness stops you from being able to do something, so the modal of inability, "couldn't", fits better than the modal of refusal, "wouldn't".

WATCH OUT FOR

Picking (B) "wouldn't; wasn't" because it is also grammatical. "Wouldn't go" means Kavya refused to go, which is a choice. Being unwell is a reason she was unable to go, so "couldn't" is the right fit. Some students also pick (C) because it repeats "wasn't", but "wasn't go" is ungrammatical ("was" would need "going").

Q.2 MCQ · 1 MARK

General Aptitude › Reasoning

Good : Evil :: Genuine : _____

Select the most appropriate option to complete the analogy.

(A) Counterfeit

(B) Contraband

(C) Counterfoil

(D) Counterpart

Official answer: **A**

SOLUTION

1. Work out how the first pair is related. *Good* and *Evil* are opposites, so this is an **antonym** analogy.

2. The second pair must have the same relation, so we want the opposite of *Genuine* (real, authentic, not fake).
3. (A) *Counterfeit* means a fake made to pass as the real thing. That is the exact opposite of genuine.
4. (B) *Contraband* means goods that are illegal to import or export (smuggled goods). The word says the goods are illegal, not that they are fake, so it is not the opposite of genuine.
5. (C) *Counterfoil* is the part of a cheque, ticket or receipt that you keep as a record. Its meaning has nothing to do with being genuine or not.
6. (D) *Counterpart* is a person or thing that matches or corresponds to another. That means an equivalent, not an opposite.
7. Only *Counterfeit* keeps the antonym relation: Good : Evil :: Genuine : Counterfeit. The answer is (A).

KEY IDEA

Name the relation in the first pair before you look at the options. Here Good and Evil are antonyms, so you need the antonym of Genuine, and that is Counterfeit (a fake made to pass as real).

WATCH OUT FOR

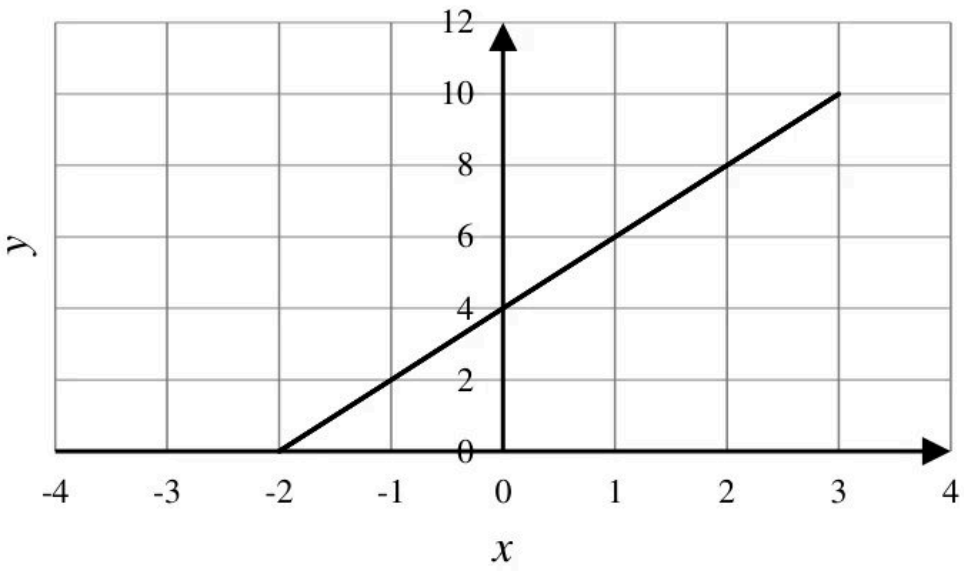
Three of the four options start with "Counter-", and that prefix can suggest "against" or "opposite". Choosing Counterpart or Counterfoil because of the prefix is the trap. Only Counterfeit actually means fake. Contraband also sounds wrong-doing and can tempt, but it means illegal goods, not false goods.

Q.3 MCQ · 1 MARK

General Aptitude › Geometry

Q.3 The relationship between two variables x and y is given by $x + py + q = 0$ and is shown in the figure. Find the values of p and q.

Note: The figure shown is representative.



(A) $p = -\frac{1}{2};\ q = 2$

(B) $p = 2\ ;\ q = -2$

(C) $p = \frac{1}{2}\ ;\ q = 4$

(D) $p = 2\ ;\ q = 4$

Official answer: **A**

SOLUTION

1. Read two points off the graph. The line meets the x-axis at $(-2, 0)$ and the y-axis at $(0, 4)$. It also passes through $(3, 10)$, which we use later as a check.
2. Slope from the two intercepts: $m = \dfrac{4 - 0}{0 - (-2)} = 2$. The y-intercept is $c = 4$.
3. Slope-intercept form $y = mx + c$ gives $y = 2x + 4$.

4. Rearrange into the given form, where the coefficient of x must be 1: $2x - y + 4 = 0$. Divide by 2 to get $x - \frac{1}{2}y + 2 = 0$.
5. Compare with $x + py + q = 0$: $p = -\frac{1}{2}$ and $q = 2$. The coefficient of x is fixed at 1, so these values are unique.
6. Check with $(-2, 0)$: $-2 + 0 + 2 = 0$. Check with $(3, 10)$: $3 - \frac{1}{2}(10) + 2 = 3 - 5 + 2 = 0$. Both points satisfy the equation, so the answer is (A).

KEY IDEA

Put the two intercepts straight into $x + py + q = 0$. At $(-2, 0)$ the equation becomes $-2 + q = 0$, so $q = 2$. At $(0, 4)$ it becomes $4p + q = 0$, so $p = -\frac{1}{2}$. The coefficient of x is fixed at 1, so the equation cannot be rescaled and p and q are unique.

WATCH OUT FOR

Seeing slope 2 and intercept 4 on the graph and choosing (D) $p = 2;\ q = 4$. Those numbers belong to $y = 2x + 4$, not to the form $x + py + q = 0$. A second trap is a sign slip when solving for x. From $y = 2x + 4$ you get $x = \frac{1}{2}y - 2$, which is $x - \frac{1}{2}y + 2 = 0$, not $x + \frac{1}{2}y + \ldots$. Getting that sign wrong points you to (C). Always check the answer by substituting one plotted point, such as $(-2, 0)$.

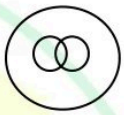
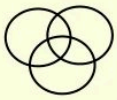
Q.4 MCQ · 1 MARK

General Aptitude › Reasoning

Q.4 Each row of Column-I has three items and each item is represented by a circle in Column-II. The arrangement of circles in Column-II represents the relationship among the items in Column-I.

Identify the option that has the most appropriate match between Column-I and Column-II.

Note: The figures shown are representative.

Column-I		Column-II	
(1)	Animals, Zebra, Giraffe	(P)	
(2)	Director, Producer, Actor	(Q)	
(3)	Word, Sentence, Novel	(R)	
(4)	Pianist, Guitarist, Instrumentalist	(S)	

(A) (1) – (Q); (2) – (P); (3) – (S); (4) – (R)

(B) (1) – (Q); (2) – (R); (3) – (S); (4) – (P)

(C) (1) – (S); (2) – (P); (3) – (R); (4) – (Q)

(D) (1) – (R); (2) – (S); (3) – (Q); (4) – (P)

Official answer: **B**

SOLUTION

1. Read the four diagrams first. (P): two sets that overlap each other, both inside a larger set. (Q): two separate (disjoint) sets, both inside a larger set. (R): three sets that each partly overlap the other two, with none inside another. (S): three nested sets, each fully inside the next.
2. (1) Animals, Zebra, Giraffe: zebras and giraffes are both animals, but no animal is both a zebra and a giraffe. That is two disjoint subsets inside a larger set, so (1) – (Q).

3. (2) Director, Producer, Actor: one person can hold any one, any two or all three of these roles (actor-directors, producer-directors, people who act, direct and produce), and no role includes another. That is three mutually overlapping sets, so (2) – (R).
4. (3) Word, Sentence, Novel: this is a part-of chain. Words make up a sentence, and sentences make up a novel, so each item sits wholly inside the next. That is drawn as nested circles, so (3) – (S). This is also the only diagram left once (1), (2) and (4) are matched.
5. (4) Pianist, Guitarist, Instrumentalist: every pianist and every guitarist is an instrumentalist, and some people play both piano and guitar. That is two overlapping subsets inside a larger set, so (4) – (P).
6. The matching is (1) – (Q); (2) – (R); (3) – (S); (4) – (P), which is option (B). Quick check: only (A) and (B) have (1) – (Q), and (A) gives (2) – (P) and (4) – (R), both wrong.

KEY IDEA

Each Venn diagram shows one relationship. Disjoint circles inside a big circle: two categories that never share a member, both belonging to a larger class. Overlapping circles inside a big circle: two categories that can share members, both inside a larger class. Three mutually overlapping circles: three roles a person can hold in any combination. Nested circles: a chain in which each item is contained in the next.

WATCH OUT FOR

Swapping (P) and (Q) for rows (1) and (4). Zebra and Giraffe can never overlap, so they need the diagram with separate inner circles (Q). Pianist and Guitarist can overlap, because one person can play both, so they need the diagram with overlapping inner circles (P). Another slip is to draw Director, Producer, Actor as nested circles. None of these roles is a subset of another, so the circles must overlap (R).

Q.5 MCQ · 1 MARK

General Aptitude › Numerical

What is the value of $\left(\frac{3^{81}}{27^4}\right)^{\frac{1}{3}}$?

(A) 3^{13}

(B) 3^{96}

(C) 3^{23}

(D) 3^{69}

Official answer: **C**

SOLUTION

1. Write everything in base 3. Since $27 = 3^3$, the rule $(a^m)^n = a^{mn}$ gives $27^4 = (3^3)^4 = 3^{12}$.
2. To divide powers of the same base, subtract the exponents ($a^m/a^n = a^{m-n}$): $\frac{3^{81}}{3^{12}} = 3^{81-12} = 3^{69}$.
3. To raise this to the power $\frac{1}{3}$, multiply the exponents: $\left(3^{69}\right)^{\frac{1}{3}} = 3^{69/3} = 3^{23}$.
4. Check by taking the cube root first: $\left(3^{81}\right)^{\frac{1}{3}} = 3^{27}$ and $\left(27^4\right)^{\frac{1}{3}} = \left(3^{12}\right)^{\frac{1}{3}} = 3^4$, so the value is $\frac{3^{27}}{3^4} = 3^{23}$ again.
5. The value is 3^{23}, which is option (C).

KEY IDEA

Write every term in the same base (here $27 = 3^3$) and then work only with exponents: multiply for a power of a power, and subtract for a quotient.

WATCH OUT FOR

Stopping at $3^{81-12} = 3^{69}$ and forgetting the outer power $\frac{1}{3}$, which gives option (D). The other slip is to leave 27^4 as it is, or to write it as 3^7 by adding $3 + 4$ when the exponents should be multiplied to give 3^{12}.

Q.6 MCQ · 2 MARKS

General Aptitude › Reading

Identify the option that has the most appropriate sequence such that a coherent paragraph is formed:

P. It is because deer, like most of the animals that tigers normally prey on, run much faster! It simply means, another day of empty stomach for the big cats.

Q. Tigers spend most of their life searching for food.

R. If they trace the scent of deer, tigers follow the trail, chase the deer for a mile or two in the dark, and yet may not catch them.

S. For several nights, they relentlessly prowl through the forest, hunting for a trail that may lead to their prey.

(A) S → P → R → Q

(B) R → P → S → Q

(C) Q → S → R → P

(D) P → Q → S → R

Official answer: **C**

SOLUTION

1. **Find the opening sentence.** An opener has to make sense with nothing before it and has to name its subject. Q, "Tigers spend most of their life searching for food.", is a general statement that names "Tigers" and depends on nothing earlier. S ("For several nights, they relentlessly prowl...") and R ("If they trace the scent of deer...") both use "they" before any animal has been named. P opens with "It is because", which has to explain something already said. So Q must come first, and only option (C) starts with Q.
2. **Pronoun reference: "they" must point back to a noun already mentioned.** In S, "For several nights, they relentlessly prowl through the forest, hunting for a trail that may lead to their prey.", "they" means the tigers from Q, and S gives the detail of the "searching for food". So Q → S.
3. **Chronological order: a hunt goes search, then trail, then chase.** S ends with the tigers "hunting for a trail that may lead to their prey". R picks up from there: "If they trace the scent of deer, tigers follow the trail, chase the deer for a mile or two in the dark, and yet may not catch them." So S → R.
4. **Cause and effect: "It is because" gives the reason for the statement just before it.** P, "It is because deer, like most of the animals that tigers normally prey on, run much faster!", explains R's "and yet may not catch them". Its last line, "It simply means, another day of empty stomach for the big cats.", ties back to Q's "searching for food" and closes the paragraph. So R → P.
5. **Check the options.** The order Q → S → R → P is option (C). Options (A), (B) and (D) start with S, R or P. Each of those sentences leans on something that has not been said yet: "they" with no noun before it, or "It is because" with nothing to explain. So none of them can open the paragraph.

KEY IDEA

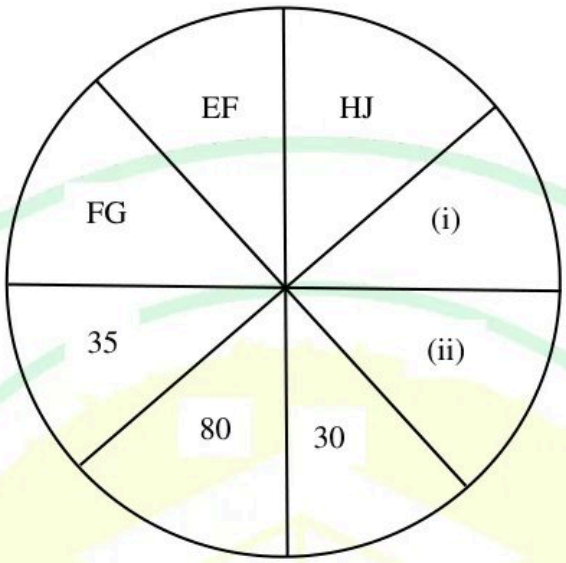
Start with the one sentence that makes sense on its own and names the subject (Q, "Tigers..."). Then join the rest using pronoun reference (S's "they" means the tigers), the order of events in a hunt (search, then trail, then chase: S → R) and cause and effect (P's "It is because" explains R's "may not catch them").

WATCH OUT FOR

Opening with S because "For several nights" sounds like a story's first line (option A), or with P because it holds the vivid punchline (option D). Neither works. S's "they" has no noun before it to refer to, and P's "It is because" needs an earlier statement to explain. The opener must be the sentence that names "Tigers", which is Q.

Q.7 MCQ · 2 MARKS

General Aptitude › Reasoning

Q.7	In the given figure, EF and HJ are coded as 30 and 80, respectively. Which one among the given options is most appropriate for the entries marked (i) and (ii) ?
	
(A)	(i) EH; (ii) 40
(B)	(i) JK; (ii) 36
(C)	(i) EG; (ii) 42
(D)	(i) PS; (ii) 14

Official answer: **C**

SOLUTION

1. Write the alphabet positions of the letters: $E = 5$, $F = 6$, $G = 7$, $H = 8$, $J = 10$.
2. Find the rule from the two given codes. Multiply the positions: $\mathrm{EF} \to 5 \times 6 = 30$ and $\mathrm{HJ} \to 8 \times 10 = 80$. Both match, so a letter pair is coded as the product of its letters' positions.
3. Find where the pairs sit in the figure. EF and its code 30 are in diametrically opposite sectors, and so are HJ and 80. So each code sits in the sector opposite its letter pair.
4. (i) is opposite 35, so (i) must be a letter pair whose positions multiply to 35. Since $35 = 5 \times 7$, the pair is E and G, so (i) is EG.
5. (ii) is opposite FG, so (ii) is the code of FG: $6 \times 7 = 42$.
6. Check the other options. (A): EH gives $5 \times 8 = 40$, not 35, and 40 is not 42. (B): JK gives $10 \times 11 = 110$, and 36 is not 42. (D): PS gives $16 \times 19 = 304$, and 14 is not 42. Only option (C), (i) EG

and (ii) 42, fits both entries.

KEY IDEA

Get the rule from both examples: EF = 30 and HJ = 80 are the products of alphabet positions (5×6 and 8×10). Then get the layout from the figure: each code sits in the sector diametrically opposite its letter pair. The layout says which pair (i) and (ii) belong to, and the rule gives their values.

WATCH OUT FOR

Assuming that (i) and (ii), which sit next to each other, are a letter pair and its code. On that reading option (A) looks right, because EH gives $5 \times 8 = 40$, matching its (ii) = 40. The figure rules this out: EF is not next to 30, and HJ is not next to 80. Each code is diametrically opposite its pair, so (ii) must be FG's code, 42. Other slips are adding the positions instead of multiplying them ($5 + 6 = 11$, not 30) and miscounting J as 9.

Q.8 MCQ · 2 MARKS

General Aptitude › Statistics & probability

Q.8 Scores obtained by two students P and Q in seven courses are given in the table below. Based on the information given in the table, which one of the following statements is INCORRECT?

P	22	89	50	45	78	60	39
Q	35	65	60	56	81	45	50

(A) Average score of P is less than the average score of Q.

(B) Median score of P is same as the median score of Q.

(C) Difference between the maximum and minimum scores of P is greater than the difference between the maximum and minimum scores of Q.

(D) Median score and the average score of Q are same.

Official answer: **B**

SOLUTION

1. Mean = sum of observations ÷ number of observations. Sum for P:
$22 + 89 + 50 + 45 + 78 + 60 + 39 = 383$, so $\bar{P} = \dfrac{383}{7} \approx 54.71$.

2. Sum for Q: $35+65+60+56+81+45+50=392$, so $\bar{Q}=\dfrac{392}{7}=56$. Since $54.71<56$, statement (A) is correct. (Both students have 7 scores, so comparing the sums $383<392$ gives the same result.)
3. Median of an odd number n of values: the $\left(\frac{n+1}{2}\right)$th value after sorting. With $n=7$, that is the 4th value.
4. P sorted: $22, 39, 45, \mathbf{50}, 60, 78, 89$, so the median of P is 50.
5. Q sorted: $35, 45, 50, \mathbf{56}, 60, 65, 81$, so the median of Q is 56. Since $50\neq 56$, statement (B) is INCORRECT.
6. Range = maximum $-$ minimum. P: $89-22=67$; Q: $81-35=46$. Since $67>46$, statement (C) is correct.
7. For Q, median $=56$ and mean $=56$, so statement (D) is correct.
8. The only incorrect statement is (B).

KEY IDEA

Sort each row before you read off the median. With 7 values, the median is the 4th value in sorted order. The mean is the sum divided by 7, and because both students have the same number of scores you can compare their means by comparing their sums. The difference between the maximum and minimum is the range.

WATCH OUT FOR

Reading the median from the unsorted table. The 4th column shows 45 for P and 56 for Q, and some students take those as the medians. The sorted median of P is actually 50. A second trap is marking the one statement you have checked and found true, even though the question asks for the INCORRECT one.

Q.9 MCQ · 2 MARKS

General Aptitude › Geometry

Spheres of unit diameter are centered at (l, m, n) where l, m, and n take every possible integer values.

The distance between two spheres is computed from the center of one sphere to the center of the other sphere. For a given sphere, x is the distance to its nearest sphere and y is the distance to its next nearest sphere. The value of $\frac{y}{x}$ is:

(A) $2\sqrt{2}$

(B) $\dfrac{1}{\sqrt{2}}$

(C) $\sqrt{2}$

(D) 2

Official answer: **C**

SOLUTION

1. The centres (l, m, n), with l, m, n all integers, form a cubic lattice of side 1. Every sphere sees the same pattern of neighbours, so we can take the sphere centred at the origin $(0, 0, 0)$.
2. By the 3D distance formula, the distance from the origin to the centre (l, m, n) is $\sqrt{l^2+m^2+n^2}$, where l, m, n are integers and not all zero.
3. The smallest possible value of $l^2+m^2+n^2$ is 1, for example at $(1, 0, 0)$, the next centre along one axis (one cube edge away). So $x=\sqrt{1}=1$. Because the diameter is 1, these spheres just touch.
4. The next possible value of $l^2+m^2+n^2$ is 2, for example at $(1, 1, 0)$, the opposite corner of a face of the unit cube (along the face diagonal). No integer point gives a value between 1 and 2, so $y=\sqrt{2}$.
5. $$\frac{y}{x}=\frac{\sqrt{2}}{1}=\sqrt{2}$$

   This is option (C).

KEY IDEA

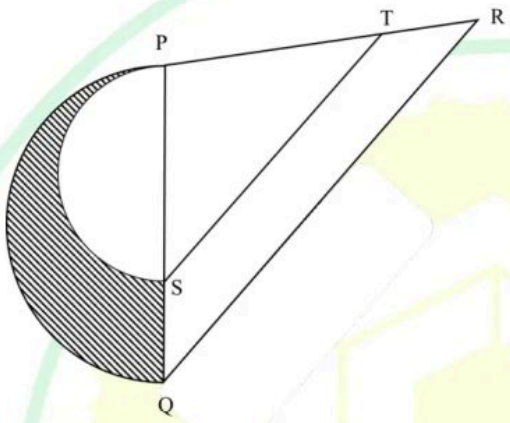
Integer centres form a unit cubic lattice, and the squared distance to any other centre is $l^2 + m^2 + n^2$. The nearest centre gives the smallest value of that sum (1, along an edge) and the next nearest gives the next value (2, along a face diagonal). So $x = 1$, $y = \sqrt{2}$, and $\frac{y}{x} = \sqrt{2}$.

WATCH OUT FOR

Taking the next nearest centre as the second one along the same axis, $(2, 0, 0)$, which gives $y = 2$ and option (D). The face-diagonal centre $(1, 1, 0)$ at $\sqrt{2}$ is closer. Other slips are jumping to the body diagonal $(1, 1, 1)$ at $\sqrt{3}$, which is the third-nearest, or inverting the ratio to get $\frac{1}{\sqrt{2}}$ (option B).

Q.10 MCQ · 2 MARKS

General Aptitude › Geometry

Q.10	In the triangle PQR, the lengths of PT and TR are in the ratio of 3:2. ST is parallel to QR. Two semicircles are drawn with PS and PQ as diameters, as shown in the figure. Which one of the following statements is true about the shaded area PQS? Note: The figure shown is representative.
	
(A)	The shaded area is $\frac{16}{9}$ times the area of the semicircle with the diameter PS.
(B)	The shaded area is equal to the area of the semicircle with the diameter PS.
(C)	The shaded area is $\frac{14}{9}$ times the area of the semicircle with the diameter PS.
(D)	The shaded area is $\frac{14}{25}$ times the area of the semicircle with the diameter PQ.

Official answer: **A**

SOLUTION

1. Basic proportionality theorem (Thales): ST is parallel to QR, so it divides the sides PQ and PR in the same ratio. That gives $PS : SQ = PT : TR = 3 : 2$.
2. Take $PS = 3k$ and $SQ = 2k$, so $PQ = PS + SQ = 5k$.

3. A semicircle with diameter d has area $\frac{1}{2}\pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{8}$. So the semicircle on PS has area $\frac{\pi(3k)^2}{8} = \frac{9\pi k^2}{8}$, and the semicircle on PQ has area $\frac{\pi(5k)^2}{8} = \frac{25\pi k^2}{8}$.
4. Both semicircles lie on the same side of PQ and share the end point P. The circle on PS touches the circle on PQ from inside at P, so the small semicircle lies entirely inside the large one. The shaded region is the large semicircle with the small one removed:

$$\text{Shaded} = \frac{25\pi k^2}{8} - \frac{9\pi k^2}{8} = \frac{16\pi k^2}{8} = 2\pi k^2$$

5. Compare this with the semicircle on PS:

$$\frac{\text{Shaded}}{\text{Semicircle on PS}} = \frac{16\pi k^2/8}{9\pi k^2/8} = \frac{16}{9}$$

   This matches option (A).
6. Check the other options. Against the semicircle on PQ the ratio is $\frac{16}{25}$, not $\frac{14}{25}$, so (D) is false. Since $\frac{16}{9} \neq 1$ and $\frac{16}{9} \neq \frac{14}{9}$, (B) and (C) are false as well.

KEY IDEA

Because ST is parallel to QR, the basic proportionality theorem gives $PS : SQ = PT : TR = 3 : 2$, so the diameters PS and PQ are in the ratio $3 : 5$. Semicircle areas go as the square of the diameter ($9 : 25$), and the shaded crescent is the difference, $25 - 9 = 16$ parts. Against the PS semicircle's 9 parts, that is $\frac{16}{9}$.

WATCH OUT FOR

Taking PQ as 3 + 2 but then comparing lengths instead of areas, which gives a ratio such as $\frac{5}{3}$ or $\frac{2}{3}$. Area scales with the square of the diameter. A second trap is reading $PS : SQ$ as $2 : 3$ because S looks close to Q in the figure. The figure is only representative, so the ratio has to come from the parallel line, not from how the drawing looks. Getting the ratio the wrong way round leads to $\frac{21}{4}$, which is not an option.

Electrical Engineering Q.11 to Q.65

Q.11 MCQ · 1 MARK

Engineering Mathematics › Calculus

Consider the set S of points $(x, y) \in \Re^2$ which minimize the real valued function

$$f(x, y) = (x + y - 1)^2 + (x + y)^2$$

Which of the following statements is true about the set S?

(A) The number of elements in the set S is finite and more than one.

(B) The number of elements in the set S is infinite.

(C) The set S is empty.

(D) The number of elements in the set S is exactly one.

Official answer: **B**

SOLUTION

1. Stationary points need $\nabla f = 0$: $\dfrac{\partial f}{\partial x} = 2(x + y - 1) + 2(x + y) = 0$ and $\dfrac{\partial f}{\partial y} = 2(x + y - 1) + 2(x + y) = 0$.
2. Both conditions are the same single equation, $4(x + y) - 2 = 0$. So the stationary points are every point on the line $x + y = \dfrac{1}{2}$, not one isolated point.
3. Check that these points are minima. The Hessian is $H = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$ and $\det H = 0$, so the second-derivative test cannot decide. Use the structure of f instead.
4. f depends only on $u = x + y$: $f = g(u) = (u - 1)^2 + u^2 = 2u^2 - 2u + 1 = 2\left(u - \dfrac{1}{2}\right)^2 + \dfrac{1}{2}$ (completing the square).
5. So $f(x, y) \geq \dfrac{1}{2}$ for every (x, y), with equality exactly when $u = x + y = \dfrac{1}{2}$. The global minimum value is $\dfrac{1}{2}$.
6. Hence $S = \left\{(x, y) \in \Re^2 : x + y = \dfrac{1}{2}\right\}$, a straight line (for example $(0, \frac{1}{2})$, $(\frac{1}{2}, 0)$, $(1, -\frac{1}{2})$, and so on). It has infinitely many elements, so option (B) is correct.

KEY IDEA

Here f depends on x and y only through $u = x + y$. Minimise the one-variable function $g(u) = (u - 1)^2 + u^2$ to get $u = \frac{1}{2}$. Every (x, y) with $x + y = \frac{1}{2}$ gives that same minimum, so the minimizers make up a whole line.

WATCH OUT FOR

Setting $\partial f/\partial x = 0$ and $\partial f/\partial y = 0$ and expecting two independent equations with one solution, which leads to (D). The two equations are identical, so they fix only $x + y$. Another trap is reading the singular Hessian ($\det H = 0$) as "no minimum exists" and picking (C). A zero determinant only means the test cannot decide, and completing the square shows $f \geq \frac{1}{2}$ with equality all along the line.

Q.12 MCQ · 1 MARK

Engineering Mathematics › Linear algebra

Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be the two eigenvectors corresponding to distinct eigenvalues of a 3×3 real symmetric matrix. Which one of the following statements is true?

(A) $\mathbf{v}_1^T \mathbf{v}_2 \neq 0$

(B) $\mathbf{v}_1^T \mathbf{v}_2 = 0$

(C) $\mathbf{v}_1 + \mathbf{v}_2 = \mathbf{0}$

(D) $\mathbf{v}_1 - \mathbf{v}_2 = \mathbf{0}$

Official answer: **B**

SOLUTION

1. Let A be the real symmetric matrix ($A^T = A$), with $A\mathbf{v}_1 = \lambda_1\mathbf{v}_1$, $A\mathbf{v}_2 = \lambda_2\mathbf{v}_2$ and $\lambda_1 \neq \lambda_2$. Eigenvectors are nonzero by definition.
2. Premultiply the second equation by $\mathbf{v}_1^T$: $\mathbf{v}_1^T A\mathbf{v}_2 = \lambda_2\, \mathbf{v}_1^T\mathbf{v}_2$.
3. Use symmetry: $\mathbf{v}_1^T A = (A^T\mathbf{v}_1)^T = (A\mathbf{v}_1)^T = \lambda_1\mathbf{v}_1^T$. So the same scalar is also $\mathbf{v}_1^T A\mathbf{v}_2 = \lambda_1\, \mathbf{v}_1^T\mathbf{v}_2$.
4. Subtract the two results:
$$(\lambda_1 - \lambda_2)\, \mathbf{v}_1^T\mathbf{v}_2 = 0.$$
Since $\lambda_1 \neq \lambda_2$, $\mathbf{v}_1^T\mathbf{v}_2 = 0$. The eigenvectors are orthogonal, so (B) is true and (A) is false.
5. (C) would give $\mathbf{v}_2 = -\mathbf{v}_1$ and (D) would give $\mathbf{v}_2 = \mathbf{v}_1$. In either case $A\mathbf{v}_2 = \lambda_1\mathbf{v}_2$ as well as $A\mathbf{v}_2 = \lambda_2\mathbf{v}_2$, so $(\lambda_1 - \lambda_2)\mathbf{v}_2 = \mathbf{0}$. That is impossible, because $\mathbf{v}_2 \neq \mathbf{0}$ and $\lambda_1 \neq \lambda_2$. (Also, $\mathbf{v}_1^T\mathbf{v}_2 = \mp\|\mathbf{v}_1\|^2 \neq 0$ would contradict the orthogonality just proved.) The answer is (B).

KEY IDEA

Write the scalar $\mathbf{v}_1^T A\mathbf{v}_2$ in two ways, once using $A\mathbf{v}_2 = \lambda_2\mathbf{v}_2$ and once using $A^T = A$ with $A\mathbf{v}_1 = \lambda_1\mathbf{v}_1$. Subtracting gives $(\lambda_1 - \lambda_2)\mathbf{v}_1^T\mathbf{v}_2 = 0$, so the eigenvectors are orthogonal whenever $\lambda_1 \neq \lambda_2$.

WATCH OUT FOR

Confusing linear independence with orthogonality. Eigenvectors for distinct eigenvalues are linearly independent for ANY matrix, but they are orthogonal ($\mathbf{v}_1^T\mathbf{v}_2 = 0$) only because this matrix is symmetric. A second mistake is choosing (C) or (D): eigenvectors of distinct eigenvalues can never be multiples of each other, so neither $\mathbf{v}_1 = -\mathbf{v}_2$ nor $\mathbf{v}_1 = \mathbf{v}_2$ can hold.

Q.13 MCQ · 1 MARK

Engineering Mathematics › Linear algebra

Let $\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$, and $\boldsymbol{b} = \begin{bmatrix} 1/3 \\ -1/3 \\ 0 \end{bmatrix}$. Then, the system of linear equations $\boldsymbol{Ax} = \boldsymbol{b}$ has

(A) a unique solution.

(B) infinitely many solutions.

(C) a finite number of solutions.

(D) no solution.

Official answer: **B**

SOLUTION

1. Write the augmented matrix:
$$[\boldsymbol{A} \mid \boldsymbol{b}] = \begin{bmatrix} 1 & 1 & 1 & 1/3 \\ -1 & -1 & -1 & -1/3 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$
2. Apply $R_2 \to R_2 + R_1$. Row 2 of the augmented matrix is exactly -1 times row 1, right-hand side included, so it becomes $\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$:
$$\begin{bmatrix} 1 & 1 & 1 & 1/3 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$
3. The two remaining coefficient rows $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 & -1 \end{bmatrix}$ are not multiples of each other, so $\rho(\boldsymbol{A}) = 2$. (Check: $\det \boldsymbol{A} = 0$ because rows 1 and 2 are proportional, so the rank must be less than 3.)
4. The zero row has a zero right-hand side, so no row reads $0 = \text{nonzero}$. Hence $\rho([\boldsymbol{A} \mid \boldsymbol{b}]) = 2$ as well.

5. Rank test (Rouché–Capelli): $\rho(\boldsymbol{A}) = \rho([\boldsymbol{A} \mid \boldsymbol{b}]) = 2 < n = 3$. The system is consistent and has $3 - 2 = 1$ free variable, so it has infinitely many solutions.
6. Explicitly, with $\boldsymbol{x} = (x, y, z)$: the third row gives $y - z = 0$, so $y = z = t$. The first row gives $x + 2t = \dfrac{1}{3}$, so $x = \dfrac{1}{3} - 2t$. Every real t gives a solution; for example $t = 0$ gives $\left(\frac{1}{3}, 0, 0\right)$. Check: $x + y + z = \frac{1}{3}$, $-(x + y + z) = -\frac{1}{3}$, $y - z = 0$. The answer is (B).

KEY IDEA

Compare $\rho(\boldsymbol{A})$ with $\rho([\boldsymbol{A} \mid \boldsymbol{b}])$. If they differ, there is no solution. If they are equal to n, the solution is unique. If they are equal but less than n, there are infinitely many solutions. In this question row 2 is -1 times row 1 and $\boldsymbol{b}$ follows the same relation ($-1/3 = -1 \times 1/3$), so the dependent equation adds no new information and no contradiction.

WATCH OUT FOR

Seeing $\det \boldsymbol{A} = 0$ and choosing 'no solution' (D) straight away. A singular matrix only rules out a unique solution. You still have to compare the rank of $\boldsymbol{A}$ with the rank of the augmented matrix. Option (C) is a trap: a linear system has exactly zero, one or infinitely many solutions, never a finite number greater than one.

Q.14 MCQ · 1 MARK

Engineering Mathematics › Linear algebra

Let $P = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and let I be the identity matrix. Then P^2 is equal to

(A) $2P - I$

(B) P

(C) I

(D) $P + I$

Official answer: **A**

SOLUTION

1. **Characteristic polynomial.** P is block-diagonal. Its upper 2×2 block $\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$ has trace 2 and determinant $2 \cdot 0 - (1)(-1) = 1$, so that block contributes $\lambda^2 - 2\lambda + 1 = (\lambda - 1)^2$. The 1×1 block $[1]$ contributes $(1 - \lambda)$. So $\det(P - \lambda I) = (\lambda - 1)^2(1 - \lambda) = -(\lambda - 1)^3$, and the only eigenvalue is $\lambda = 1$ (repeated three times).
2. By the Cayley-Hamilton theorem, $(P - I)^3 = 0$. That is a cubic, so check whether a lower power already vanishes.
3. $P - I = \begin{bmatrix} 1 & 1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Squaring it: the third row and third column are zero, so only the top-left 2×2 entries need checking. $(1,1)$: $1 \cdot 1 + 1 \cdot (-1) = 0$. $(1,2)$: $1 \cdot 1 + 1 \cdot (-1) = 0$. $(2,1)$: $(-1) \cdot 1 + (-1)(-1) = 0$. $(2,2)$: $(-1) \cdot 1 + (-1)(-1) = 0$. So $(P - I)^2 = 0$. Since $P - I \neq 0$, the **minimal polynomial** is $(\lambda - 1)^2$.
4. Expand, remembering that P and I commute: $(P - I)^2 = P^2 - 2P + I = 0 \;\Rightarrow\; P^2 = 2P - I$.
5. **Direct check:**

$$P^2 = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 0 \\ -2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and

$$2P - I = \begin{bmatrix} 4 - 1 & 2 & 0 \\ -2 & 0 - 1 & 0 \\ 0 & 0 & 2 - 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 0 \\ -2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The two agree.

6. Rule out the other options: $P^2 \neq P$ (the $(1,1)$ entry of P^2 is 3, not 2); $P^2 \neq I$ (its off-diagonal entries 2 and -2 are nonzero); and $P + I = \begin{bmatrix} 3 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ has $(2,2)$ entry 1, while P^2 has -1 there. So the answer is (A).

KEY IDEA

Every eigenvalue of P is 1, and $(P - I)^2 = 0$, so the minimal polynomial is $(\lambda - 1)^2$. Expanding $(P - I)^2 = 0$ gives $P^2 = 2P - I$ directly. For a 3×3 matrix like this one, simply multiplying P by itself and comparing with each option is just as quick, and it is a good check on the theory.

WATCH OUT FOR

Going only as far as Cayley-Hamilton, which gives the cubic $(P - I)^3 = 0$, and then trying to force a quadratic answer out of it without first checking that $(P - I)^2$ is already zero. The other slip is multiplying carelessly: a single sign error in the $(2,1)$ or $(2,2)$ entry of P^2 (true values -2 and -1) makes $P + I$ or P look like a match.

Q.15 MCQ · 1 MARK

Engineering Mathematics › Probability & statistics

Consider discrete random variables X and Y with probabilities as follows:

$$P(X = 0 \text{ and } Y = 0) = \frac{1}{4}$$
$$P(X = 1 \text{ and } Y = 0) = \frac{1}{8}$$
$$P(X = 0 \text{ and } Y = 1) = \frac{1}{2}$$
$$P(X = 1 \text{ and } Y = 1) = \frac{1}{8}$$

Given $X = 1$, the expected value of Y is

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{1}{8}$

(D) $\frac{1}{3}$

Official answer: **B**

SOLUTION

1. Check that the joint PMF is valid: $\frac{1}{4} + \frac{1}{8} + \frac{1}{2} + \frac{1}{8} = 1$.
2. Find the probability of the conditioning event by adding the two $X = 1$ entries over all values of Y: $P(X = 1) = P(X = 1, Y = 0) + P(X = 1, Y = 1) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$.
3. Conditional PMF $P(Y = y \mid X = 1) = \dfrac{P(X = 1, Y = y)}{P(X = 1)}$: $P(Y = 0 \mid X = 1) = \dfrac{1/8}{1/4} = \frac{1}{2}$ and $P(Y = 1 \mid X = 1) = \dfrac{1/8}{1/4} = \frac{1}{2}$. These add to 1, as they should.
4. Conditional expectation: $E[Y \mid X = 1] = \sum_y y\, P(Y = y \mid X = 1) = 0 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} = \frac{1}{2}$.
5. So the expected value of Y given $X = 1$ is $\frac{1}{2}$, which is option (B).

KEY IDEA

Conditioning on $X = 1$ keeps only the $X = 1$ entries of the joint PMF, rescaled by $P(X = 1)$ so they add to 1. Then $E[Y \mid X = 1] = \sum_y y\, P(Y = y \mid X = 1)$. Because Y takes only the values 0 and 1, this equals $P(Y = 1 \mid X = 1)$.

WATCH OUT FOR

Using the joint probability $P(X = 1, Y = 1) = \frac{1}{8}$ as the answer without dividing by $P(X = 1)$ gives option (C). Taking the unconditional mean $E[Y] = P(Y = 1) = \frac{1}{2} + \frac{1}{8} = \frac{5}{8}$ ignores the condition altogether, and $\frac{5}{8}$ is not among the options.

Q.16 MCQ · 1 MARK

Analog Electronics › Amplifiers

Which one of the following statements is true about the small signal voltage gain of a MOSFET based single stage amplifier?

(A) Common source and common gate amplifiers are both inverting amplifiers

(B) Common source and common gate amplifiers are both non-inverting amplifiers

(C) Common source amplifier is inverting and common gate amplifier is non-inverting amplifier

(D) Common source amplifier is non-inverting and common gate amplifier is inverting amplifier

Official answer: **C**

SOLUTION

1. Model: the MOSFET is biased in saturation. Its small-signal drain current is $i_d = g_m v_{gs}$, with r_o and the body effect neglected. The drain load is R_D and the output is taken at the drain in both configurations. Because the drain voltage is $V_{DD} - i_D R_D$, the small-signal output is $v_o = -i_d R_D$.
2. Common source: the input drives the gate and the source is at AC ground, so $v_{gs} = v_i$. Then
$$v_o = -g_m v_i R_D \;\Rightarrow\; A_v = \frac{v_o}{v_i} = -g_m R_D$$
The negative sign means the output is 180° out of phase with the input, so the amplifier is **inverting**.
3. Common gate: the gate is at AC ground and the input drives the source, so $v_{gs} = 0 - v_i = -v_i$. Then
$$v_o = -g_m(-v_i) R_D \;\Rightarrow\; A_v = \frac{v_o}{v_i} = +g_m R_D$$
The output is in phase with the input, so the amplifier is **non-inverting**. (With the body effect included the gain becomes $+(g_m + g_{mb})R_D$, and the sign is still positive.)
4. Physical check for CG: when the source voltage rises, v_{gs} falls, so the drain current falls. The drop across R_D then falls and the drain voltage rises. Input and output move together.
5. So CS is inverting and CG is non-inverting, which is option (C).

KEY IDEA

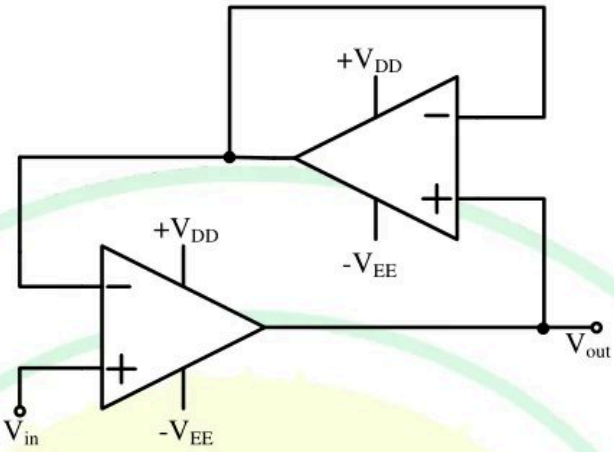
The sign of the gain depends on which terminal takes the input. At the gate (CS), $v_{gs} = +v_i$. At the source (CG), $v_{gs} = -v_i$. The output at the drain is always $-g_m v_{gs} R_D$, so the extra minus sign in CG cancels the inversion and gives a gain of the same size, $g_m R_D$, but positive.

WATCH OUT FOR

Students often assume every transistor amplifier with the output at the collector or drain is inverting, like the CE/CS stage, and choose (A). Driving the source makes $v_{gs} = -v_i$, and that second sign flip makes the common gate stage non-inverting. Another error is to confuse CG with the source follower (common drain). Both have positive gain, but the follower's gain is below 1 and its output is at the source, not the drain.

Q.17 MCQ · 1 MARK

Analog Electronics › Op-amps

Q.17	Assuming ideal op-amps, the circuit represents a
	
(A)	summing amplifier.
(B)	difference amplifier.
(C)	logarithmic amplifier.
(D)	buffer.

Official answer: **D**

SOLUTION

1. Assumptions: both op-amps are ideal (infinite open-loop gain, infinite input resistance, zero output resistance). The circuit has no resistors, capacitors or diodes, only wires.
2. Op-amp 2: its output is wired straight back to its own inverting input, which is 100% negative feedback. It is therefore a unity-gain voltage follower. By the virtual short $v_- = v_+$, its output is $v_{o2} = v_{+2} = V_{out}$.
3. Loop polarity: V_{out} passes through the follower (gain $+1$) and arrives at the inverting input of op-amp 1. The overall loop is therefore negative feedback, so the virtual-short rule applies to op-amp 1 as well.
4. Op-amp 1 inputs: $v_{+1} = V_{in}$ and $v_{-1} = v_{o2} = V_{out}$.
5. Virtual short at op-amp 1's inputs: $v_{-1} = v_{+1}$, so
$$V_{out} = V_{in} \quad \Rightarrow \quad \frac{V_{out}}{V_{in}} = 1$$
6. The gain is unity with no phase inversion. The input resistance is very high, because the input drives only the $+$ terminal of an ideal op-amp, which draws no current. The output resistance is zero (the ideal op-amp output). A unity-gain stage with these properties is a buffer. Answer (D).

KEY IDEA

With negative feedback, an ideal op-amp holds its two inputs at the same voltage (virtual short). Op-amp 2 is a follower that feeds V_{out} back unchanged to the inverting input of op-amp 1, so op-amp 1 forces $V_{out} = V_{in}$. Finite-gain check: with gains A_1 and A_2, $v_{o2} = \dfrac{A_2}{1+A_2}V_{out}$ and $V_{out} = \dfrac{A_1 V_{in}}{1 + A_1\dfrac{A_2}{1+A_2}}$, which tends to V_{in} as $A_1, A_2 \to \infty$. The loop is a two-op-amp voltage follower.

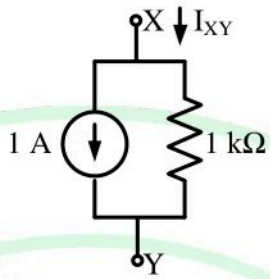
WATCH OUT FOR

Being thrown by the two op-amps and guessing a difference or summing amplifier. Summing and difference amplifiers need a resistor network to weight and combine their inputs, and a logarithmic amplifier needs a diode or BJT in its feedback path. This circuit has none of these and only one input, V_{in}. Another slip is reading op-amp 2's feedback as going to its $+$ terminal, which would make it positive feedback. Its output actually loops to its own $-$ input, and V_{out} goes to its $+$ input.

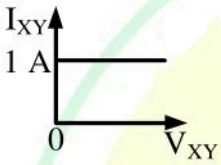
Q.18 MCQ · 1 MARK

Networks (Electric Circuits) › Laws & node/mesh

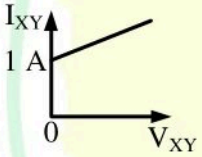
Q.18 The I-V characteristics of the element between the nodes X and Y is best depicted by



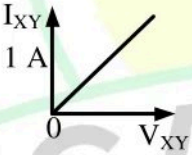
(A)



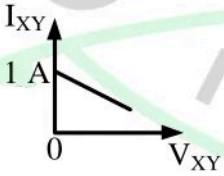
(B)



(C)



(D)



Official answer: **B**

SOLUTION

1. Take $V_{XY} = V_X - V_Y$. The current I_{XY} enters the element at X (passive sign convention), and at the top node it divides between the two parallel branches.
2. Current-source branch: the arrow points from the top node (X side) to the bottom node (Y side), so this branch carries exactly $1\,\text{A}$ from X towards Y whatever the voltage across it.
3. Resistor branch (Ohm's law): the current from X to Y through the $1\,\text{k}\Omega$ resistor is $\dfrac{V_{XY}}{1\,\text{k}\Omega}$.

4. KCL at the top node:
$$I_{XY} = 1\,\text{A} + \frac{V_{XY}}{1000\,\Omega}$$
5. This is a straight line. At $V_{XY} = 0$ it gives $I_{XY} = 1\,\text{A}$ (the intercept on the current axis), and its slope is $\frac{dI_{XY}}{dV_{XY}} = \frac{1}{1\,\text{k}\Omega} = 1\,\text{mS} > 0$.
6. Only option (B) starts at $1\,\text{A}$ and rises with V_{XY}. (A) has zero slope, which would need an open circuit in place of the resistor (an ideal current source alone). (C) has no source term, so it describes the resistor alone. (D) has a negative slope, which would need a negative resistance.

KEY IDEA

A current source in parallel with a resistor is a Norton source. KCL gives $I = I_s + V/R$, so the I-V characteristic is a straight line: the source sets the intercept on the current axis and the conductance $1/R$ sets the slope. The source's arrow direction relative to I_{XY} decides the sign of the intercept.

WATCH OUT FOR

Picking (A) because the current source seems to fix the terminal current at 1 A. That is only true for a source in series with the resistor. Here the resistor is in parallel, so it draws an extra V_{XY}/R and the line slopes upwards. A second slip is misreading the source direction as opposing I_{XY}, which gives $I_{XY} = V_{XY}/R - 1\,\text{A}$ (intercept $-1\,\text{A}$). That matches no option and should send you back to check the sign.

Q.19 MCQ · 1 MARK

Networks (Electric Circuits) › Laws & node/mesh

A nullator is defined as a circuit element where the voltage across the device and the current through the device are both zero. A series combination of a nullator and a resistor of value, R, will behave as a

(A) resistor of value R.

(B) nullator.

(C) open circuit.

(D) short circuit.

Official answer: **B**

SOLUTION

1. Nullator constraint (from the definition): $v_N = 0$ and $i_N = 0$ at the same time.
2. Series connection (KCL at the internal node): the same current i flows through both elements, so $i = i_N = 0$, and therefore $i_R = 0$.
3. Ohm's law on the resistor: $v_R = R\,i_R = R \times 0 = 0$.
4. KVL around the series branch: the terminal voltage is $v = v_N + v_R = 0 + 0 = 0$.
5. So the two-terminal combination has $v = 0$ **and** $i = 0$ together. That is exactly the definition of a nullator.
6. It is not an open circuit, which only forces $i = 0$ and leaves the voltage free. It is not a short circuit, which only forces $v = 0$ and leaves the current free. It is not a resistor of value R, because $v = Ri$ would allow any non-zero pair. Here both v and i are fixed at zero, so the answer is (B).

KEY IDEA

A nullator fixes BOTH $v = 0$ and $i = 0$. In series, its $i = 0$ passes to the resistor (KCL), which then has $v_R = 0$ (Ohm's law), so the terminal voltage is also zero (KVL). A nullator in series with any element that has finite impedance is still a nullator.

WATCH OUT FOR

Seeing only the zero current and picking open circuit (C), or seeing only the zero voltage and picking short circuit (D). An open circuit leaves the voltage free and a short circuit leaves the current free. Here both are pinned to zero, which is a nullator, not either of the ideal limits.

Q.20 MCQ · 1 MARK

Signals and Systems › LTI & convolution

Consider a discrete-time linear time-invariant (LTI) system $\boldsymbol{S}$, where

$$y[n] = \boldsymbol{S}\{x[n]\}$$

Let

$$\boldsymbol{S}\{\delta[n]\} = \begin{cases} 1, & n \in \{0, 1, 2\} \\ 0, & \text{otherwise} \end{cases}$$

where $\delta[n]$ is the discrete-time unit impulse function. For an input signal $x[n]$, the output $y[n]$ is

(A) $x[n] + x[n-1] + x[n-2]$

(B) $x[n-1] + x[n] + x[n+1]$

(C) $x[n] + x[n+1] + x[n+2]$

(D) $x[n+1] + x[n+2] + x[n+3]$

Official answer: **A**

SOLUTION

1. **Impulse response.** The response of the system to $\delta[n]$ is, by definition, its impulse response: $h[n] = 1$ for $n = 0, 1, 2$ and 0 otherwise. So
$$h[n] = \delta[n] + \delta[n-1] + \delta[n-2].$$
2. **Convolution sum.** The output of an LTI system is the convolution of the input with the impulse response:
$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k].$$
3. **Evaluate.** Only $k = 0, 1, 2$ contribute, and each has weight $h[k] = 1$:
$$y[n] = h[0]\, x[n] + h[1]\, x[n-1] + h[2]\, x[n-2] = x[n] + x[n-1] + x[n-2].$$
4. **Check with time invariance.** The input $\delta[n-1]$ must produce $h[n-1]$, which is nonzero at $n = 1, 2, 3$. Putting $x[n] = \delta[n-1]$ into $y[n] = x[n] + x[n-1] + x[n-2]$ gives $\delta[n-1] + \delta[n-2] + \delta[n-3]$, which is nonzero at exactly $n = 1, 2, 3$. It matches, so option (A) is correct.
5. **Why the other options fail.** Put $x[n] = \delta[n]$ into each one to get its impulse response. (B) gives $\delta[n-1] + \delta[n] + \delta[n+1]$, which is nonzero at $n \in \{-1, 0, 1\}$. (C) gives support $\{0, -1, -2\}$ and (D) gives $\{-1, -2, -3\}$. All three use a future sample such as $x[n+1]$, so their impulse responses reach into negative n. None of them has the given support $\{0, 1, 2\}$.

KEY IDEA

For an LTI system the response to $\delta[n]$ is the impulse response $h[n]$, and $y[n] = \sum_k h[k]\, x[n-k]$. Each nonzero sample $h[k]$ adds the term $h[k]\, x[n-k]$, a copy of the input delayed by k. Here $h[0] = h[1] = h[2] = 1$, so $y[n] = x[n] + x[n-1] + x[n-2]$.

WATCH OUT FOR

The most common slip is flipping the sign of the shift. $h[n]$ is nonzero at $n = 0, 1, 2$, and students then write $x[n] + x[n+1] + x[n+2]$, which is option (C). In the convolution sum, $h[k]$ multiplies $x[n-k]$, so a nonzero $h[k]$ at positive k means a delay ($x[n-k]$), not an advance. A quick check is to put $x[n] = \delta[n]$ into the option you picked and confirm that you get back the given $h[n]$.

Q.21 MCQ · 1 MARK

Signals and Systems › Signals basics

Consider a continuous-time signal

$$x(t) = -t^2\{u(t+4) - u(t-4)\}$$

where $u(t)$ is the continuous-time unit step function. Let $\delta(t)$ be the continuous-time unit impulse function. The value of

$$\int_{-\infty}^{\infty} x(t)\,\delta(t+3)\,dt$$

is

(A) -9

(B) 9

(C) 3

(D) -3

Official answer: **A**

SOLUTION

1. **Sifting property of the impulse:** $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$ whenever $x(t)$ is continuous at $t = t_0$. Here $\delta(t+3) = \delta\big(t-(-3)\big)$, so $t_0 = -3$.
2. **The window:** $u(t+4) - u(t-4)$ equals 1 for $-4 < t < 4$ and 0 for $|t| > 4$. Since $-4 < -3 < 4$, the window equals 1 at $t = -3$ and throughout a neighbourhood of it. So near $t = -3$ the signal is simply $x(t) = -t^2$, which is continuous there, and the sifting property applies. (The jumps at $t = \pm 4$ are far from $t = -3$.)
3. **Evaluate:**
$$x(-3) = -(-3)^2 \cdot 1 = -9$$
4. Therefore $\int_{-\infty}^{\infty} x(t)\,\delta(t+3)\,dt = -9$, which is option (A).

KEY IDEA

Sifting property: an impulse at t_0 inside an integral returns the value of the other factor at t_0. The impulse $\delta(t+3)$ sits at $t_0 = -3$, not $+3$. Check that t_0 lies inside the rectangular window $u(t+4) - u(t-4)$, which is 1 on $-4 < t < 4$.

WATCH OUT FOR

Three common slips. (1) Reading $\delta(t+3)$ as an impulse at $t = +3$. That happens to give the same value here because t^2 is even, so the wrong reasoning still lands on the right answer. (2) Dropping the leading minus sign and choosing 9 (B). (3) Forgetting to square and choosing ± 3 (C or D).

Q.22 MCQ · 1 MARK

Control Systems › Time response

Selected data points of the step response of a stable first-order linear time-invariant (LTI) system are given below. The closest value of the time-constant, in sec, of the system is

TIME (SEC)	0.6	1.6	2.6	10	∞
Output	0.78	1.65	2.18	2.98	3

(A) 1

(B) 2

(C) 3

(D) 4

Official answer: **B**

SOLUTION

1. A stable first-order system that starts from rest has the step response $y(t) = y_{ss}\left(1 - e^{-t/T}\right)$, where T is the time constant. The $t = \infty$ column gives the final value $y_{ss} = 3$.
2. Rearrange to get T from any single reading:
$$T = \frac{-t}{\ln\left(1 - \dfrac{y(t)}{y_{ss}}\right)}$$
3. At $t = 0.6$ s: $1 - \dfrac{0.78}{3} = 0.74$, so $T = \dfrac{0.6}{-\ln 0.74} = \dfrac{0.6}{0.301} \approx 1.99$ s.
4. At $t = 1.6$ s: $1 - \dfrac{1.65}{3} = 0.45$, so $T = \dfrac{1.6}{-\ln 0.45} = \dfrac{1.6}{0.799} \approx 2.00$ s.
5. At $t = 2.6$ s: $1 - \dfrac{2.18}{3} = 0.273$, so $T = \dfrac{2.6}{-\ln 0.273} = \dfrac{2.6}{1.297} \approx 2.00$ s. At $t = 10$ s: $1 - \dfrac{2.98}{3} = 0.00667$, so $T = \dfrac{10}{-\ln 0.00667} = \dfrac{10}{5.01} \approx 2.00$ s.
6. Quick check with the 63.2% rule. At $t = T$ the output is $1 - e^{-1} \approx 63.2\%$ of its final value: $0.632 \times 3 = 1.896$. That value falls between the readings at 1.6 s (1.65) and 2.6 s (2.18), so T lies between 1.6 s and 2.6 s. Only option (B) is in that range.
7. All four data points give the same value, $T \approx 2$ s. That agreement also confirms the model assumed in step 1 (first order, starting from zero output). Answer: option (B).

KEY IDEA

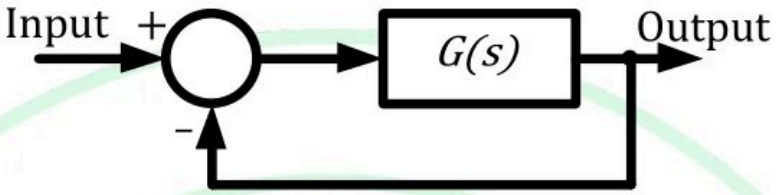
For a first-order step response $y(t) = y_{ss}(1 - e^{-t/T})$, normalise every reading by the final value y_{ss} (here 3). Then $T = -t/\ln(1 - y/y_{ss})$ at any point. Equivalently, T is the time at which the output reaches 63.2% of y_{ss}.

WATCH OUT FOR

Forgetting that the final value is 3, not 1. Looking for an output of 0.632 puts T between 0.6 s and 1.6 s and leads to option (A). The 63.2% mark is $0.632 \times 3 = 1.896$. A second trap is treating $t = 10$ s as the 2% settling time $4T$, which gives $T = 2.5$ s. In fact $2.98/3 = 99.3\%$, which is about $5T$.

Q.23 MCQ · 1 MARK

Control Systems › Nyquist

Q.23	The Nyquist plot of a strictly stable $G(s)$ having the numerator polynomial as $(s-3)$ encircles the critical point -1 once in the anti-clockwise direction. Which one of the following statements on the closed-loop system shown in figure, is correct?
	
(A)	The system stability cannot be ascertained.
(B)	The system is marginally stable.
(C)	The system is stable.
(D)	The system is unstable.

Official answer: **D**

SOLUTION

1. **Open-loop RHP poles.** $G(s)$ is strictly stable, so all its poles lie in the open left half-plane and none lie on the $j\omega$ axis. The poles of $1+G(s)$ are the poles of $G(s)$, so the number of open-loop RHP poles is
$$P = 0$$
2. **The zero at $s=3$ does not enter the count.** $(s-3)$ puts a zero of $G(s)$ in the RHP. The Nyquist criterion counts the zeros of $1+G(s)$. For this unity negative feedback loop those zeros are the closed-loop poles, because $T(s) = \dfrac{G(s)}{1+G(s)}$. A zero of $G(s)$ is neither a pole nor a zero of $1+G(s)$, so it plays no part in the count.
3. **Argument principle, with no sign convention assumed.** Let the Nyquist contour enclose the whole RHP. As s goes once round it, $G(s)$ traces the Nyquist plot. The net number of times that plot encircles -1, **counted in the same sense as the contour is traversed**, is
$$N = Z - P$$
Here Z is the number of zeros of $1+G(s)$ in the RHP, which is the number of closed-loop RHP poles.
4. **What $P=0$ implies.** With $P=0$ we get $N = Z \geq 0$. So the plot of a stable $G(s)$ can encircle -1 only in the direction the contour is traversed, and every net encirclement is one closed-loop RHP pole. Increasing ω (clockwise contour) gives clockwise encirclements. Decreasing ω (anticlockwise contour) gives anticlockwise ones. The question does not say which way the plot is traced, and its 'anticlockwise' fits the second reading. Either way, the number of encirclements is what decides stability.
5. **Put in the data.** The plot encircles -1 exactly once, so
$$Z = N + P = 1 + 0 = 1$$

The closed loop has one pole in the right half-plane.

6. **Ruling out marginal stability.** The point -1 is encircled, so it does not lie on the plot. That means $1 + G(j\omega) \neq 0$ for every ω, so no closed-loop pole sits on the $j\omega$ axis. The system is therefore not marginally stable. It has one RHP pole, so it is **unstable**: option (D).
7. **A concrete check.** Take $G(s) = \dfrac{s-3}{(s+1)(s+2)}$. It is strictly stable and its numerator is $(s-3)$. The closed-loop characteristic equation is

$$(s+1)(s+2) + (s-3) = s^2 + 4s - 1 = 0 \;\Rightarrow\; s = -2 \pm \sqrt{5}$$

so one root, $s \approx 0.236$, is in the RHP and $Z = 1$. A numerical trace of this plot, with $G(0) = -\frac{3}{2}$ to the left of -1, gives exactly one encirclement of -1. It is clockwise for ω from $-\infty$ to ∞ and anticlockwise when traced the other way. This is the situation the question describes, and the loop is unstable.

KEY IDEA

When the open loop is stable ($P = 0$), the closed loop is stable if and only if the Nyquist plot does not encircle -1. The number of net encirclements equals the number of closed-loop RHP poles, $Z = N$. The direction of an encirclement only shows which way the contour was traversed. RHP zeros of $G(s)$, such as the one at $s = 3$, are not counted in P or Z.

WATCH OUT FOR

A common error is to fix a clockwise contour, put the anticlockwise encirclement into $N = P - Z$ as $N = +1$, get $Z = -1$, and then call the data contradictory and pick (A). The question never says which way the plot is traced, and 'anticlockwise' is only meaningful relative to that direction. What does not depend on the direction is that -1 is encircled once, and with $P = 0$ that means one RHP closed-loop pole. Two other slips: counting the zero at $s = 3$ as an open-loop RHP pole, and choosing 'marginally stable' even though the plot does not pass through -1.

Q.24 MCQ · 1 MARK

Power Systems › Per unit & load flow

During a power failure, a domestic household uninterruptible power supply (UPS) supplies AC power to a limited number of lights and fans in various rooms. As per a Newton-Raphson load-flow formulation, the UPS would be represented as a

(A) Slack bus

(B) PV bus

(C) PQ bus

(D) PQV bus

Official answer: **A**

SOLUTION

1. Newton-Raphson load flow sorts buses by which two of the four bus quantities P, Q, $|V|$, δ are specified. Slack bus: $|V|$ and δ are specified, and P and Q are found by the solution. PV bus: P and $|V|$ are specified. PQ bus: P and Q are specified.
2. Every load-flow problem needs exactly one slack bus. It is the angle reference ($\delta = 0^\circ$), and it supplies whatever real and reactive power is left over once the loads and the network losses are met. The losses are not known until the solution has been found.
3. During the power failure the grid is disconnected. The household wiring, with its lights and fans, is an isolated network, and the UPS is its only source.
4. The lights and fans are loads, so their buses are PQ buses: their P and Q demands are specified.
5. The UPS has to supply the total load plus the losses in the house wiring. So its output P and Q cannot be fixed in advance; they come out of the solution. That rules out the PQ and PV types for the UPS bus, because both need P to be specified.

6. The UPS inverter holds its output voltage at a set magnitude (for example $230\ \mathrm{V}$) and sets the phase that every other bus voltage is measured against. So $|V|$ and δ are specified at the UPS bus. Those are exactly the quantities specified at a slack bus.
7. As the only source in an isolated network, the UPS is the reference bus and the balancing bus. That is the slack bus. 'PQV bus' is not a standard load-flow bus type. The answer is (A).

KEY IDEA

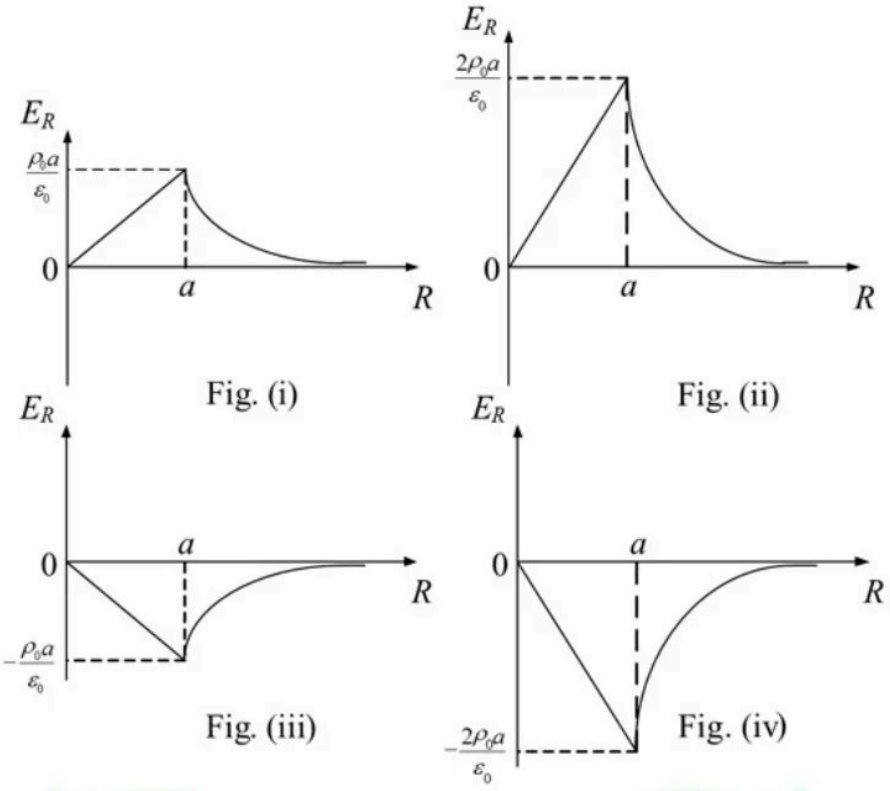
In an isolated network with one source, that source is the slack bus. Its output P and Q must cover the load plus unknown losses, so they cannot be specified, while its inverter fixes $|V|$ and supplies the reference angle $\delta = 0^\circ$.

WATCH OUT FOR

Calling the UPS a PQ bus because it 'delivers whatever the lights and fans demand'. That reasoning shows its P and Q are outputs of the solution, which is the definition of the slack bus, not a PQ bus. A PQ bus needs P and Q specified in advance, and a network made only of PQ buses has no angle reference and no bus to balance the losses.

Q.25 MCQ · 1 MARK

Electromagnetics › Electrostatics

Q.25	Which one of the following figures represents the radial electric field distribution E_R caused by a spherical cloud of electrons with a volume charge density, $\rho = -3\rho_0$ for $0 \le R \le a$ (both ρ_0, a are positive and R is the radial distance) and $\rho = 0$ for $R > a$?
	
(A)	Fig. (i)
(B)	Fig. (ii)
(C)	Fig. (iii)
(D)	Fig. (iv)

Official answer: **C**

SOLUTION

1. **Symmetry.** The charge is spherically symmetric, so $\mathbf{E} = E_R\,\hat{a}_R$ and E_R depends only on R. Gauss's law over a concentric sphere of radius R gives $E_R\,(4\pi R^2) = Q_{enc}/\varepsilon_0$, with $\varepsilon_0 \approx 8.854 \times 10^{-12}\ \mathrm{F/m}$.
2. **Inside ($0 \le R \le a$).** The enclosed charge is $Q_{enc} = (-3\rho_0)\cdot\frac{4}{3}\pi R^3 = -4\pi\rho_0 R^3$, so

$$E_R = \frac{-4\pi\rho_0 R^3}{4\pi\varepsilon_0 R^2} = -\frac{\rho_0 R}{\varepsilon_0}$$

This is a straight line through the origin with a negative slope.

3. **At the surface ($R = a$).** $E_R(a) = -\dfrac{\rho_0 a}{\varepsilon_0}$. This is the most negative value the field reaches.
4. **Outside ($R > a$).** Now the sphere encloses all the charge, $Q = -4\pi\rho_0 a^3$, so
$$E_R = \frac{-4\pi\rho_0 a^3}{4\pi\varepsilon_0 R^2} = -\frac{\rho_0 a^3}{\varepsilon_0 R^2}$$
This is negative and rises towards zero as $1/R^2$. At $R = a$ it equals $-\rho_0 a/\varepsilon_0$, the same as the inside value, so the field is continuous at the surface.
5. **Pick the figure.** The field is negative everywhere. It falls linearly to $-\rho_0 a/\varepsilon_0$ at $R = a$, then returns to zero as $1/R^2$. That is Fig. (iii), option (C). Figs. (i) and (ii) have the wrong sign: they would describe positive charge. Fig. (iv) has the right shape but twice the magnitude, which would need $\rho = -6\rho_0$.

KEY IDEA

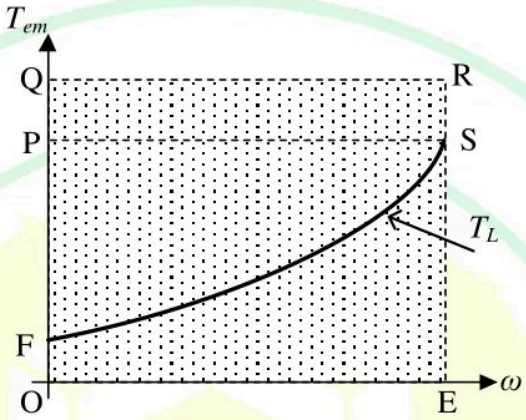
For a uniformly charged sphere, Gauss's law gives $E_R = \rho R/(3\varepsilon_0)$ inside, which grows linearly with R, and $E_R = Q/(4\pi\varepsilon_0 R^2)$ outside. The two agree at $R = a$. With $\rho = -3\rho_0$ the factor 3 cancels, leaving $E_R(a) = -\rho_0 a/\varepsilon_0$. The negative sign means the field points inward, towards the electron cloud.

WATCH OUT FOR

Dropping the minus sign because the cloud is described by positive constants ρ_0 and a. That gives the mirror-image graph, Fig. (i). A second slip is to forget the $\frac{4}{3}$ in the sphere's volume, or to use the cylinder result $\rho R/(2\varepsilon_0)$. Either one gives the wrong peak magnitude and can pull you towards Fig. (iv).

Q.26 MCQ · 1 MARK

Electrical Machines › Induction machines

Q.26	The operating region of the developed torque (T_{em}) and speed (ω) of an induction motor drive is given by the shaded region OQRE in the figure. The load torque (T_L) characteristic is also shown. The motor drive moves from the initial operating point O to the final operating point S. Which one of the following trajectories will take the shortest time?
	
(A)	$O - Q - R - S$
(B)	$O - P - S$
(C)	$O - E - S$
(D)	$O - F - S$

Official answer: **A**

SOLUTION

1. The drive obeys $J\frac{d\omega}{dt} = T_{em} - T_L$, so $dt = \frac{J\,d\omega}{T_{em}(\omega) - T_L(\omega)}$, and the total transit time from O to S is $t = \int_0^{\omega_E} \frac{J\,d\omega}{T_{em}(\omega) - T_L(\omega)}$.
2. The speed excursion is fixed (0 to ω_E, since both E and S sit at $\omega = \omega_E$), so t is minimized by making $T_{em}(\omega)$ as large as possible (i.e. by maximizing $T_{em} - T_L$) at every speed inside the feasible region

OQRE.

3. The upper boundary of the feasible region is the horizontal segment QR, at the constant torque level T_Q. This is the largest torque the drive can deliver at any speed between 0 and ω_E.
4. Along the path O–Q–R–S: torque rises from 0 to T_Q at $\omega = 0$ (negligible time, since speed is unchanged), then the drive accelerates at the constant maximum torque T_Q from $\omega = 0$ to $\omega = \omega_E$ (segment Q–R), and finally torque adjusts from T_Q down to the value needed at S, again at $\omega = \omega_E$ (negligible time).
5. Over the whole speed range, this path keeps T_{em} pinned at its maximum feasible value T_Q, giving the largest possible $(T_{em} - T_L)$ and hence the largest possible $d\omega/dt$ at every speed in $(0, \omega_E)$.
6. Any of the other paths (O–P–S, O–E–S, O–F–S) has $T_{em}(\omega) < T_Q$ over at least part of the range 0 to ω_E (P, the O–E–S diagonal, and F–S along/near T_L all lie below the QR level at intermediate speeds), so acceleration is lower there and the transit time is longer.
7. Hence O–Q–R–S, which rides the upper torque boundary of the feasible region for the entire speed excursion, takes the shortest time.
8. Therefore option (A), O–Q–R–S, is correct.

KEY IDEA

Since $J\,d\omega/dt = T_{em} - T_L$, the transit time over a fixed speed range is minimized by keeping the developed torque T_{em} at its largest feasible value at every speed, i.e., by riding the upper boundary of the allowed operating region.

WATCH OUT FOR

Assuming the shortest-time path is the one with the shortest-looking geometric route (e.g. the diagonal O–P–S or O–E–S) rather than reasoning about which path maximizes accelerating torque $(T_{em} - T_L)$ at every intermediate speed.

Q.27 MCQ · 1 MARK

Power Electronics › Harmonics & PF

The input voltage $v(t)$ and current $i(t)$ of a converter are given by, $v(t) = 300\sin(\omega t)$ V, $i(t) = 10\sin\left(\omega t - \frac{\pi}{6}\right) + 2\sin\left(3\omega t + \frac{\pi}{6}\right) + \sin\left(5\omega t + \frac{\pi}{2}\right)$ A, where $\omega = 2\pi \times 50$ rad/s. The input power factor of the converter is closest to

(A) 0.845

(B) 0.867

(C) 0.887

(D) 1.0

Official answer: **A**

SOLUTION

1. Average power comes only from terms of the same frequency in v and i. The voltage has only the fundamental, so the 3ω and 5ω current terms give zero average power. $P = V_{rms}\, I_{1,rms} \cos\phi_1$.
2. $V_{rms} = \dfrac{300}{\sqrt{2}} = 212.13$ V and $I_{1,rms} = \dfrac{10}{\sqrt{2}} = 7.071$ A. The fundamental current is $10\sin(\omega t - \pi/6)$ against $300\sin(\omega t)$, so the current lags by $\phi_1 = 30^\circ$.
3. $$P = 212.13 \times 7.071 \times \cos 30^\circ = 1500 \times 0.8660 = 1299.0 \text{ W}$$
4. The rms current includes every harmonic, because the rms of a sum of sinusoids of different frequencies is the root of the sum of the squared rms values.
$$I_{rms} = \sqrt{\frac{10^2 + 2^2 + 1^2}{2}} = \sqrt{52.5} = 7.246 \text{ A}$$
5. Apparent power: $S = V_{rms} I_{rms} = 212.13 \times 7.246 = 1537.0$ VA.
6. $$\text{PF} = \frac{P}{S} = \frac{1299.0}{1537.0} \approx 0.845$$

As a check, the distortion factor is $\frac{7.071}{7.246} = 0.976$ and the displacement factor is $\cos 30^\circ = 0.866$. Their product is 0.845. This is option (A).

KEY IDEA

Power factor = real power / apparent power. Only the fundamental current delivers real power, because the voltage is a pure sinusoid. The rms current counts every harmonic. So PF = distortion factor × displacement factor = 0.976 × 0.866.

WATCH OUT FOR

Using only the displacement factor, $\cos 30^\circ = 0.866$, which gives option (B) and ignores the harmonic current. Another slip is dividing by the fundamental rms current in the apparent power, or adding harmonic currents as if they also delivered power.

Q.28 MSQ · 1 MARK

Electrical Machines › Synchronous machines

Instrument(s) required to synchronize an alternator to the grid is/are

(A) Voltmeter

(B) Wattmeter

(C) Synchroscope

(D) Stroboscope

Official answer: **A, C**

SOLUTION

1. Synchronizing conditions: to connect an alternator to the grid without transients, four conditions must be satisfied at the moment the breaker closes: (i) equal RMS voltage magnitude, (ii) equal frequency, (iii) same phase sequence, and (iv) zero phase-angle difference between the incoming machine and the bus.
2. Voltmeter: connected across the incoming alternator and the bus (directly, or via the double-voltmeter/synchronizing-lamp arrangement), a voltmeter is used to confirm condition (i), equal voltage magnitude, before attempting synchronization.
3. Synchroscope: connected across the breaker (fed from the incoming machine's and the bus's potential transformers), a synchroscope is an instrument whose pointer position and direction of rotation directly indicate the phase-angle difference and whether the incoming machine is running fast or slow relative to the bus. The operator closes the breaker exactly when the pointer crosses the 'in-phase' (12 o'clock) position, which is the standard way of satisfying conditions (ii)-(iv) together.
4. Wattmeter: measures real power flow, a quantity that is only meaningful once the machine is already connected to the bus; it gives no information needed to decide when to close the breaker, so option (B) is not a synchronizing instrument.
5. Stroboscope: used to measure or observe mechanical rotational speed via a flashing light against a marked rotating part; it is not part of the standard electrical synchronizing (breaker-closing) procedure for an alternator, so option (D) is not a synchronizing instrument.
6. Hence the instruments required to synchronize an alternator to the grid are the Voltmeter and the Synchroscope, options (A) and (C).

KEY IDEA

Synchronizing an alternator to an infinite bus/grid requires matching (i) voltage magnitude, (ii) frequency, (iii) phase sequence, and (iv) phase angle (zero at the instant of closing the breaker). A voltmeter (or double-voltmeter method) checks voltage magnitude equality, and a synchroscope directly displays the phase-angle/slip condition, telling the operator exactly when to close the breaker.

WATCH OUT FOR

Picking the wattmeter because it seems like a natural "power system" instrument, but real/reactive power flow is only meaningful after paralleling, not during the synchronizing check itself. Similarly, confusing the stroboscope (used for measuring rotational/mechanical speed by a flashing-light effect) with the synchroscope (an electrical instrument that indicates phase difference/slip between two AC systems) is a common name-similarity trap.

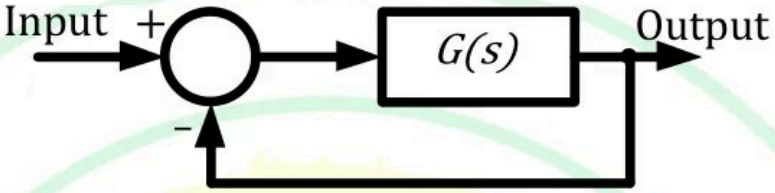
Q.29 MSQ · 1 MARK

Control Systems › Root locus

Q.29 The open-loop transfer function of the system shown in the figure, is

$$G(s) = \frac{Ks(s+2)}{(s+5)(s+7)}$$

For $K \geq 0$, which of the following real axis point(s) is/are on the root locus?



(A)	-1
(B)	-4
(C)	-6
(D)	-10

Official answer: **A, C**

SOLUTION

1. The loop has unity negative feedback (the take-off after $G(s)$ returns directly to the summing junction with a $-$ sign), so the characteristic equation is $1 + G(s) = 0$ and the standard root-locus rules for $K \geq 0$ apply to $G(s) = \dfrac{Ks(s+2)}{(s+5)(s+7)}$.
2. Open-loop zeros: $s = 0,\ -2$. Open-loop poles: $s = -5,\ -7$. All four are real, so they split the real axis into segments.
3. Real-axis rule for $K \geq 0$: a real point is on the locus if the total number of real open-loop poles and zeros to its right is odd.
4. $-2 < s < 0$: one point (0) to the right, which is odd, so this segment is on the locus. -1 lies here, so A is correct.

5. $-5 < s < -2$: two points $(0,\ -2)$ to the right, which is even, so this segment is not on the locus. -4 is not on it, so B is wrong.
6. $-7 < s < -5$: three points $(0,\ -2,\ -5)$ to the right, which is odd, so this segment is on the locus. -6 lies here, so C is correct.
7. $s < -7$: four points to the right, which is even, so this segment is not on the locus. -10 is not on it, so D is wrong.
8. Direct check with the magnitude/sign condition. A real point is on the locus for $K \geq 0$ only if $G(s) = -1$ gives a non-negative K:

$$s = -1: \quad G = \frac{K(-1)(1)}{(4)(6)} = -\frac{K}{24} \Rightarrow K = 24 \geq 0$$
$$s = -4: \quad G = \frac{K(-4)(-2)}{(1)(3)} = \frac{8K}{3} \Rightarrow K = -\frac{3}{8} < 0$$
$$s = -6: \quad G = \frac{K(-6)(-4)}{(-1)(1)} = -24K \Rightarrow K = \frac{1}{24} \geq 0$$
$$s = -10: \quad G = \frac{K(-10)(-8)}{(-5)(-3)} = \frac{16K}{3} \Rightarrow K = -\frac{3}{16} < 0$$

   Only -1 and -6 need a non-negative K, which confirms A and C.
9. Shape check: there are two poles and two zeros, so there are no asymptotes and both branches end at the zeros 0 and -2. The two branches that start at -7 and -5 meet in $(-7,\ -5)$ and break away into the complex plane as a conjugate pair; they cannot reach $(-2,\ 0)$ along the real axis because $(-5,\ -2)$ is not on the locus, so they re-enter the real axis in $(-2,\ 0)$ and end at the zeros. Setting $\frac{dK}{ds} = 0$ with $K = -\frac{(s+5)(s+7)}{s(s+2)}$ gives $s^2 + 7s + 7 = 0$, so $s = \frac{-7 \pm \sqrt{21}}{2} \approx -5.79,\ -1.21$: a breakaway point in $(-7,\ -5)$ and a break-in point in $(-2,\ 0)$, consistent with the segments found above.

KEY IDEA

For $K \geq 0$, a real-axis point lies on the root locus exactly when an odd number of real open-loop poles and zeros lie to its right. A quick cross-check is to put the point into $G(s) = -1$: it is on the locus only if the K that comes out is non-negative. Zeros count in the rule just as poles do, including the zero at the origin.

WATCH OUT FOR

Counting only the poles, or leaving out the zero at $s = 0$. If you count only poles, you shade $(-7,\ -5)$ correctly but mark every point to the right of -5 as off the locus, so you miss -1. If you drop the zero at the origin, the parity flips in every segment, and you pick -4 and -10 instead. Another slip is to assume that the branches from -5 and -7 run straight along the real axis to the zeros. The segment $(-5,\ -2)$ is not on the locus, so the branches leave the axis between the poles and come back between the zeros.

Q.30 NAT · 1 MARK

Signals and Systems › Fourier series

A continuous time periodic signal $x(t)$ is

$$x(t) = 1 + 2\cos 2\pi t + 2\cos 4\pi t + 2\cos 6\pi t.$$

If T is the period of $x(t)$, then $\frac{1}{T}\int_0^T |x(t)|^2\, dt =$ _____ (round off to the nearest integer).

Official answer: **7**

SOLUTION

1. **Period.** The cosines have frequencies $1, 2, 3$ Hz ($\omega = 2\pi, 4\pi, 6\pi$ rad/s). The fundamental frequency is their GCD, $f_0 = 1$ Hz, so $T = 1$ s and the three cosines are the 1st, 2nd and 3rd harmonics.

2. **What is asked.** $\frac{1}{T}\int_0^T |x(t)|^2\,dt$ is the average power P of the periodic signal. Since $x(t)$ is real, $|x(t)|^2 = x^2(t)$.
3. **Parseval's theorem (trigonometric form).** For $x(t) = a_0 + \sum_k A_k \cos(k\omega_0 t + \phi_k)$, the DC term and the harmonics are mutually orthogonal over one period, so every cross term integrates to zero and
$$P = a_0^2 + \sum_k \frac{A_k^2}{2}.$$
4. **DC term:** $a_0 = 1$, contributing $1^2 = 1$.
5. **Each harmonic:** $A_k = 2$, contributing $\frac{2^2}{2} = 2$; the three harmonics give $3 \times 2 = 6$.
6. **Total:**
$$P = 1 + 2 + 2 + 2 = 7.$$
7. **Check with exponential coefficients:** $2\cos k\omega_0 t = e^{jk\omega_0 t} + e^{-jk\omega_0 t}$, so $c_0 = 1$ and $c_{\pm 1} = c_{\pm 2} = c_{\pm 3} = 1$. Then $P = \sum_k |c_k|^2 = 1 + 6 \times 1 = 7$.

KEY IDEA

The quantity asked for is the average power of a periodic signal. By Parseval's theorem it equals the sum of the powers of the orthogonal Fourier components: a_0^2 for the DC term plus $A_k^2/2$ for each cosine of amplitude A_k. Equivalently it is $\sum |c_k|^2$ over the exponential coefficients. You never need to square $x(t)$ out and integrate.

WATCH OUT FOR

The usual slips: (1) squaring the amplitudes without halving, which gives $1 + 3 \times 4 = 13$; (2) halving the DC term as well, which gives $0.5 + 6 = 6.5$; (3) adding the amplitudes before squaring, which gives $(1 + 2 + 2 + 2)^2 = 49$, the peak value of $x^2(t)$ at $t = 0$ and not its average. The cross terms vanish because the harmonics are orthogonal, and the DC term contributes its full square.

Q.31 NAT · 1 MARK

Electrical and Electronic Measurements › Errors & standards

The maximum percentage error in the equivalent resistance of two parallel-connected resistors of $100\ \Omega$ and $900\ \Omega$ with each having a maximum 5% error is _____ % (round off to nearest integer value).

Official answer: **5**

SOLUTION

1. Nominal equivalent resistance: $R = \frac{R_1 R_2}{R_1 + R_2} = \frac{100 \times 900}{100 + 900} = 90\ \Omega$.
2. Find the limiting error from the total differential, treating R_1 and R_2 as independent variables:
$$\frac{\partial R}{\partial R_1} = \frac{R_2^2}{(R_1 + R_2)^2}, \qquad \frac{\partial R}{\partial R_2} = \frac{R_1^2}{(R_1 + R_2)^2}$$
3. Divide by $R = \frac{R_1 R_2}{R_1 + R_2}$ to get the relative form:
$$\frac{\delta R}{R} = \frac{R_2}{R_1 + R_2}\frac{\delta R_1}{R_1} + \frac{R_1}{R_1 + R_2}\frac{\delta R_2}{R_2}$$
Both weights are positive and add up to 1, so the worst case is when both resistors are off in the same direction.
4. Substitute the values:
$$\left|\frac{\delta R}{R}\right|_{\max} = \frac{900}{1000}(5\%) + \frac{100}{1000}(5\%) = 4.5\% + 0.5\% = 5\%$$
5. Check with the extreme values. At $+5\%$ the resistors are $105\ \Omega$ and $945\ \Omega$, so $R = \frac{105 \times 945}{1050} = 94.5\ \Omega$, which is $+5\%$ of $90\ \Omega$. At -5% they are $95\ \Omega$ and $855\ \Omega$, so $R = \frac{95 \times 855}{950} = 85.5\ \Omega$, which is -5%. If both resistors change by the same percentage, the parallel value changes by that same percentage.

6. Maximum percentage error $= 5\%$.

KEY IDEA

For resistors in parallel, the relative error in the result is a weighted average of the individual relative errors, with weights $R_2/(R_1 + R_2)$ and $R_1/(R_1 + R_2)$. The weights add up to 1, so when every resistor has the same maximum percentage error, the combination has that same maximum percentage error. Another way to see it: conductances add, so $G = G_1 + G_2$, and a sum of terms that each carry 5% error also carries at most 5%.

WATCH OUT FOR

Using the product/quotient rule on $R_1R_2/(R_1 + R_2)$ as if the numerator and denominator were independent: 5% + 5% for the product plus 5% for the sum gives 15%. The same resistors appear in both, so their errors partly cancel. Simply adding the two errors to get 10% is also wrong. A parallel combination never has a larger percentage error than the worst of its resistors.

Q.32 NAT · 1 MARK

Power Systems › Transmission & distribution

Consider a distribution feeder, with R/X ratio of 5. At the receiving end, a 350 kVA load is connected. The maximum voltage drop will occur from the sending end to the receiving end, when the power factor of the load is _____ (round off to three decimal places).

Official answer: **0.975 to 0.985**

SOLUTION

1. For a short feeder with series impedance $R + jX$ and a load current I at a lagging angle ϕ, the approximate voltage drop from the sending end to the receiving end is
$$\Delta V \approx I\,(R\cos\phi + X\sin\phi).$$
2. The load is fixed at 350 kVA, so with the receiving-end voltage held fixed the current magnitude I does not change. Only ϕ varies, so maximise $f(\phi) = R\cos\phi + X\sin\phi$.
3. Set $\dfrac{df}{d\phi} = -R\sin\phi + X\cos\phi = 0$, which gives $\tan\phi = \dfrac{X}{R}$. The second derivative is $-f(\phi) < 0$ at this point, so it is a maximum, and $f_{\max} = \sqrt{R^2 + X^2}$. The load angle equals the feeder impedance angle.
4. Given $R/X = 5$, so $\tan\phi = \dfrac{X}{R} = \dfrac{1}{5} = 0.2$.
5. Take $R = 5$ and $X = 1$ in any common unit. Then $\cos\phi = \dfrac{R}{\sqrt{R^2 + X^2}} = \dfrac{5}{\sqrt{26}} = \dfrac{5}{5.0990} = 0.98058$.
6. The power factor is $\cos\phi \approx 0.981$ (lagging).

KEY IDEA

With the load kVA fixed, the drop $I(R\cos\phi + X\sin\phi)$ is largest when the load's power-factor angle equals the feeder's impedance angle, so $\cos\phi = R/\sqrt{R^2 + X^2}$.

WATCH OUT FOR

Using $\tan\phi = R/X = 5$, which gives $\cos\phi = 1/\sqrt{26} = 0.196$, or assuming unity power factor gives the largest drop. The condition is $\tan\phi = X/R$, and the feeder's R/X ratio of 5 is the inverse of that.

Q.33 NAT · 1 MARK

Power Systems › Faults

The bus impedance matrix of a 3-bus system (in pu) is Z_bus = [[j0.059, j0.061, j0.038],[j0.061, j0.093, j0.066],[j0.038, j0.066, j0.110]]. A symmetrical fault (through a fault impedance of j0.007 pu) occurs at bus

2. Neglecting pre-fault loading conditions, the voltage at bus 1, during the fault is _____ pu (round off to three decimal places).

Official answer: **0.380 to 0.400**

SOLUTION

1. No pre-fault loading, so every bus starts at $V_i(0) = 1\angle 0^\circ$ pu. The fault is at bus 2 ($k = 2$) through $Z_f = j0.007$ pu.
2. Fault current from the bus impedance matrix: $I_f = \dfrac{V_2(0)}{Z_{22} + Z_f}$.
3. $$I_f = \frac{1}{j0.093 + j0.007} = \frac{1}{j0.100} = -j10 \text{ pu}$$
4. Fault current injected at bus 2 changes every bus voltage by $\Delta V_i = -Z_{i2} I_f$, so during the fault $V_i(F) = V_i(0) - Z_{i2} I_f$.
5. For bus 1 use $Z_{12} = j0.061$:
6. $$V_1(F) = 1 - (j0.061)(-j10) = 1 - 0.610$$
7. $$V_1(F) = 0.390 \text{ pu}$$
8. Check: $V_2(F) = 1 - (j0.093)(-j10) = 1 - 0.930 = 0.070$ pu, which equals $Z_f I_f$ in magnitude (0.007×10), as it should.

KEY IDEA

For a fault at bus k through impedance Zf, the fault current is I_f = V_k(0)/(Z_kk + Zf), and every bus voltage falls by Z_ik times I_f, so V_i(F) = 1 - Z_ik/(Z_kk + Zf).

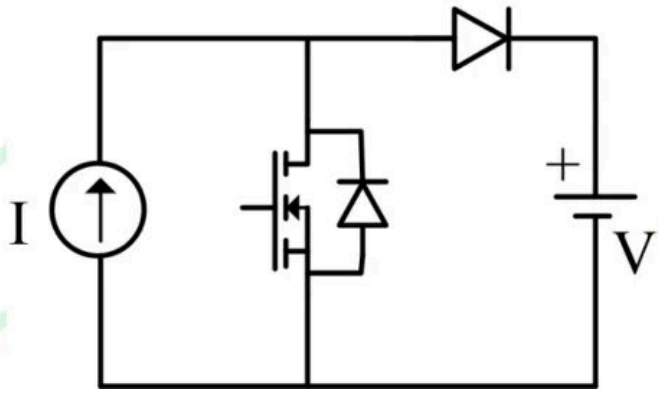
WATCH OUT FOR

Using Z_11 (j0.059) instead of the mutual term Z_12, or leaving out the fault impedance (dividing by 0.093 instead of 0.100), which gives 0.344 instead of 0.390.

Q.34 NAT · 1 MARK

Power Electronics › DC-DC converters

Q.34 In the circuit with ideal devices, the power MOSFET is operated with a duty cycle of 0.4 in a switching cycle with I =10 A and V =15 V. The power delivered by the current source, in W, is _____ *(round off to the nearest integer).*



Official answer: **90**

SOLUTION

1. The MOSFET is on for the duty ratio $D = 0.4$ and off for $1 - D = 0.6$ of each switching cycle. The source is an ideal current source, so its current is $I = 10$ A at every instant. Let v_s be the voltage across it, which is the voltage of the top node.
2. MOSFET ON: the switch connects the top node to the bottom rail, so $v_s = 0$. The series diode has its cathode at $V = 15$ V and its anode at 0 V, so it is reverse biased. No current reaches V, and the

source delivers $p = v_s I = 0$.

3. MOSFET OFF: the current I has no path through the switch, so it flows through the series diode into V. The diode conducts and, being ideal, drops no voltage, so $v_s = V = 15$ V. The source delivers $p = 15 \times 10 = 150$ W during this interval.
4. Average voltage across the current source over one cycle:
$$V_{s,\text{avg}} = D \cdot 0 + (1 - D)\, V = 0.6 \times 15 = 9 \text{ V}.$$
5. Since I is constant, the average power delivered by the source is
$$P = I\, V_{s,\text{avg}} = 10 \times 9 = 90 \text{ W}.$$
This agrees with $(1 - D) \times 150 = 0.6 \times 150 = 90$ W.
6. Check: the average current into V is $(1 - D)I = 0.6 \times 10 = 6$ A, so the power absorbed by V is $15 \times 6 = 90$ W. With ideal devices there is no loss, so this equals the power from the source.

KEY IDEA

A current source has constant current, so its average power is I times the average voltage across it. That voltage is 0 while the MOSFET is on and V while the MOSFET is off, so the average is (1-D)V.

WATCH OUT FOR

Using D instead of 1-D, which gives 0.4 x 15 x 10 = 60 W. The diode conducts, and the source sees V, only during the off time, 1-D = 0.6. Another slip is to take 15 V x 10 A = 150 W as the answer without averaging over the on interval, when the source is shorted.

Q.35 NAT · 1 MARK

Electrical Machines › Synchronous machines

The induced emf in a 3.3 kV, 4 pole, 3-phase star connected synchronous motor is considered to be equal and in phase with the terminal voltage under no load condition. On application of a mechanical load, the induced emf phasor is deflected by an angle of 2° mechanical with respect to the terminal voltage phasor. If the synchronous reactance is 2 Ω, and stator resistance is negligible, then the motor armature current magnitude, in ampere, during loaded condition is closest to, _____ (round off to two decimal places).

Official answer: **66.25 to 66.75**

SOLUTION

1. Phase voltage: $V = \dfrac{3300}{\sqrt{3}} = 1905.26$ V.
2. At no load, the induced emf equals the terminal voltage in both magnitude and phase: $|E| = |V|$.
3. Pole conversion: for a 4-pole machine, electrical angle $= \dfrac{P}{2} \times$ mechanical angle $= 2 \times 2^\circ = 4^\circ$. So the torque angle is $\delta = 4^\circ$ electrical.
4. Take $V = 1905.26 \angle 0^\circ$ V and $E = 1905.26 \angle (-4^\circ)$ V, and since stator resistance is negligible, $I_a = \dfrac{V - E}{jX_s}$.
5. For two equal-magnitude phasors separated by angle δ:
$$|V - E| = 2V \sin\left(\frac{\delta}{2}\right) = 2(1905.26)\sin(2^\circ) \approx 132.99 \text{ V}$$
6. Magnitude of current:
$$|I_a| = \frac{|V - E|}{X_s} = \frac{132.99}{2} \approx 66.49 \text{ A}$$

KEY IDEA

For a synchronous machine with P poles, a mechanical angle of rotor shift corresponds to an electrical (torque) angle of δ_elec = (P/2)·δ_mech. With |V| = |E| (no-load emf equals terminal voltage) and negligible armature resistance, the current is found from the phasor difference V − E = jXsIa, whose magnitude for two equal phasors separated by δ is 2V sin(δ/2).

WATCH OUT FOR

Treating the given 2° as the electrical torque angle directly, instead of converting it from mechanical to electrical degrees using δ_electrical = (P/2) × δ_mechanical for a 4-pole machine; this halves the actual current found.

Q.36 MCQ · 2 MARKS

Engineering Mathematics › Probability & statistics

Let X and Y be continuous random variables with probability density functions $P_X(x)$ and $P_Y(y)$, respectively. Further, let $Y = X^2$ and

$$P_X(x) = \begin{cases} 1, & x \in (0,1] \\ 0, & \text{otherwise} \end{cases}.$$

Which one of the following options is correct?

(A) $P_Y(y) = \begin{cases} \dfrac{1}{2\sqrt{y}}, & y \in (0,1] \\ 0, & \text{otherwise} \end{cases}$

(B) $P_Y(y) = \begin{cases} 1, & y \in (0,1] \\ 0, & \text{otherwise} \end{cases}$

(C) $P_Y(y) = \begin{cases} 1.5\sqrt{y}, & y \in (0,1] \\ 0, & \text{otherwise} \end{cases}$

(D) $P_Y(y) = \begin{cases} 2y, & y \in (0,1] \\ 0, & \text{otherwise} \end{cases}$

Official answer: **A**

SOLUTION

1. **Range of Y.** X takes values only in $(0,1]$, and $y = x^2$ is strictly increasing there, mapping $(0,1]$ onto $(0,1]$. So $Y \in (0,1]$: $F_Y(y) = 0$ for $y \le 0$ and $F_Y(y) = 1$ for $y > 1$, which gives $P_Y(y) = 0$ outside $(0,1]$.
2. **CDF method.** For $y \in (0,1]$, $F_Y(y) = P(X^2 \le y) = P(-\sqrt{y} \le X \le \sqrt{y}) = P(X \le \sqrt{y})$, because $X > 0$.
3. **Use the uniform density.** Since $\sqrt{y} \in (0,1]$, $F_Y(y) = \displaystyle\int_0^{\sqrt{y}} 1\,dx = \sqrt{y}$.
4. **Differentiate.** $P_Y(y) = \dfrac{d}{dy}\sqrt{y} = \dfrac{1}{2\sqrt{y}}$ for $y \in (0,1]$.
5. **Check with the change-of-variable formula.** For a monotone map, $P_Y(y) = P_X(x)\left|\dfrac{dx}{dy}\right|$. Here $x = \sqrt{y}$ and $\dfrac{dx}{dy} = \dfrac{1}{2\sqrt{y}}$, so $P_Y(y) = 1 \cdot \dfrac{1}{2\sqrt{y}}$. Same result.
6. **Why normalisation can't decide it.** Every option integrates to 1 over $(0,1]$: $\int_0^1 \dfrac{dy}{2\sqrt{y}} = 1$, $\int_0^1 1\,dy = 1$, $\int_0^1 1.5\sqrt{y}\,dy = 1.5 \cdot \dfrac{2}{3} = 1$ and $\int_0^1 2y\,dy = 1$. So checking that an option is a valid density doesn't pick the answer. The density derived above does, and it is option (A).

KEY IDEA

To find the density of a function of a random variable, work out the CDF first and then differentiate: $F_Y(y) = P(X^2 \le y) = P(X \le \sqrt{y}) = \sqrt{y}$ on $(0, 1]$, so $P_Y(y) = \dfrac{1}{2\sqrt{y}}$. For a monotone map, the shortcut $P_Y(y) = P_X(x)\left|\dfrac{dx}{dy}\right|$ gives the same answer.

WATCH OUT FOR

Assuming Y is uniform because X is uniform (option B). Squaring squeezes values near 0 closer together, so Y piles up near 0 and its density grows without bound as $y \to 0^+$. A second trap is checking only that an option integrates to 1. All four options do, so that test can't decide the answer. A third is multiplying by $\left|\dfrac{dy}{dx}\right|$ where the formula needs $\left|\dfrac{dx}{dy}\right|$.

Q.37 MCQ · 2 MARKS

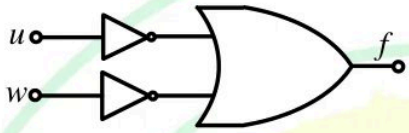
Digital Electronics › Boolean logic

Q.37 A Boolean function is given as

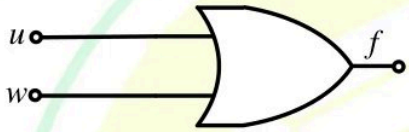
$$f = (\bar{u} + \bar{v} + \bar{w} + \bar{x}).(\bar{u} + \bar{v} + \bar{w} + x).(\bar{u} + v + \bar{w} + \bar{x}).(\bar{u} + v + \bar{w} + x)$$

The simplified form of this function is represented by

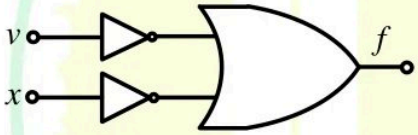
(A)



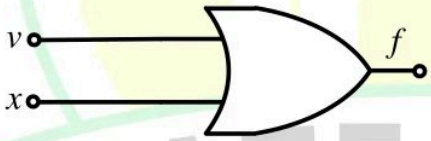
(B)



(C)



(D)



Official answer: **A**

SOLUTION

1. Notation: $\overline{a}$ is the complement of a. The dot between factors means AND. The function is a product of four sum terms (POS form):
$$f = (\overline{u} + \overline{v} + \overline{w} + \overline{x})(\overline{u} + \overline{v} + \overline{w} + x)(\overline{u} + v + \overline{w} + \overline{x})(\overline{u} + v + \overline{w} + x)$$
2. Take the first two factors together. They differ only in x. By the distributive law, $(A + \overline{x})(A + x) = A + x\overline{x} = A + 0 = A$. With $A = \overline{u} + \overline{v} + \overline{w}$:

$$(\overline{u}+\overline{v}+\overline{w}+\overline{x})(\overline{u}+\overline{v}+\overline{w}+x)=\overline{u}+\overline{v}+\overline{w}$$

3. The last two factors also differ only in x, so the same rule with $A=\overline{u}+v+\overline{w}$ gives
$$(\overline{u}+v+\overline{w}+\overline{x})(\overline{u}+v+\overline{w}+x)=\overline{u}+v+\overline{w}$$
4. Now $f=(\overline{u}+\overline{w}+\overline{v})(\overline{u}+\overline{w}+v)$. These two factors differ only in v. Apply the rule again with $B=\overline{u}+\overline{w}$:
$$f=(\overline{u}+\overline{w}+\overline{v})(\overline{u}+\overline{w}+v)=\overline{u}+\overline{w}$$
5. Check with maxterms. A sum term is 0 only when every literal in it is 0. So the four factors are 0 at $(u,v,w,x)=(1,1,1,1)$, $(1,1,1,0)$, $(1,0,1,1)$ and $(1,0,1,0)$ respectively, and 1 everywhere else. Between them these are exactly the four inputs with $u=1$ and $w=1$, taking every combination of v and x. So $f=0 \iff uw=1$, which means $f=\overline{uw}=\overline{u}+\overline{w}$ (De Morgan). The result does not depend on v or x.
6. To build $f=\overline{u}+\overline{w}$, pass u and w each through an inverter and feed both inverted signals into a 2-input OR gate. That is circuit (A). (This is the same as a 2-input NAND of u and w.) Circuit (B) gives $u+w$, (C) gives $\overline{v}+\overline{x}$ and (D) gives $v+x$, so none of them matches.

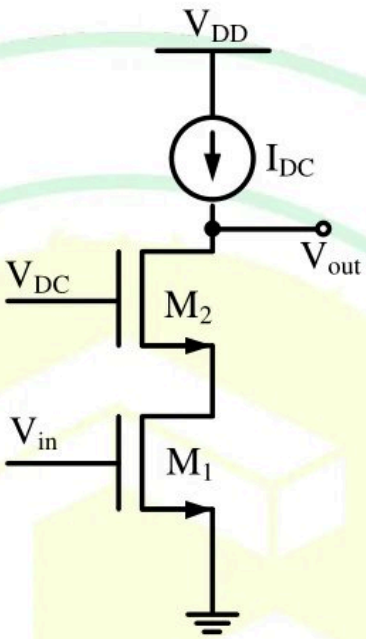
KEY IDEA

Adjacency, $(A+y)(A+\overline{y})=A$: when two sum terms differ only in one variable, that variable drops out. Use it twice here, first on x and then on v, to get $f=\overline{u}+\overline{w}=\overline{uw}$. As a check, the four maxterms are exactly the inputs where $u=w=1$.

WATCH OUT FOR

Two mistakes are common. One is to drop the complements and pick (B) $(u+w)$, forgetting that every factor has $\overline{u}$ and $\overline{w}$, never u or w. The other is to keep the wrong pair of variables and pick (C) or (D). Only v and x appear both complemented and uncomplemented across the factors, so those are the variables that cancel. The variables that are complemented in every factor, u and w, are the ones that stay.

Q.38 MCQ · 2 MARKS

Q.38	In the circuit, I_{DC} is an ideal current source. The transistors M_1 and M_2 are assumed to be biased in saturation, wherein V_{in} is the input signal and V_{DC} is fixed DC voltage. Both transistors have a small signal resistance of r_{ds} and trans-conductance of g_m. The small signal output impedance of this circuit is
	
(A)	$2r_{ds}$
(B)	$\frac{1}{g_m} + r_{ds}$
(C)	$g_m r_{ds}^2 + 2r_{ds}$
(D)	infinity

Official answer: **C**

SOLUTION

1. **Set up the small-signal circuit.** To find the output impedance, set the input to zero. V_{in} becomes AC ground. V_{DC} is a fixed DC voltage, so the gate of M_2 is also at AC ground. An ideal current source I_{DC} has infinite small-signal resistance, so it is an open circuit. What remains is the cascode M_2–M_1, seen from the drain of M_2.
2. **Model.** Both MOSFETs are in saturation. Each is modelled as a controlled source $g_m v_{gs}$ from drain to source, with r_{ds} in parallel with it. Body effect is neglected, because the question gives only g_m and r_{ds}.

3. **Apply a test source.** Drive a test voltage v_x into the V_{out} node. The current i_x it delivers flows down through M_2 and then through M_1 to ground, since the source of M_2 connects only to the drain of M_1.
4. **Lower device M_1.** Its gate and source are both at AC ground, so $v_{gs1} = 0$. Its controlled source is therefore dead and M_1 acts as a plain resistor r_{ds}. The whole of i_x flows through it, so the voltage at the source of M_2 is $v_s = i_x r_{ds}$.
5. **Upper device M_2.** With its gate at AC ground, $v_{gs2} = 0 - v_s = -i_x r_{ds}$. KCL at the drain of M_2: the current entering the drain splits between the controlled source and r_{ds}, so
$$i_x = g_m v_{gs2} + \frac{v_x - v_s}{r_{ds}} = -g_m r_{ds}\, i_x + \frac{v_x - i_x r_{ds}}{r_{ds}}$$
6. **Solve for v_x.** Multiply through by r_{ds} and collect terms:
$$i_x r_{ds} = -g_m r_{ds}^2\, i_x + v_x - i_x r_{ds}$$
$$v_x = \left(g_m r_{ds}^2 + 2 r_{ds}\right) i_x$$
The resistance looking into the drain of M_2 is therefore $g_m r_{ds}^2 + 2 r_{ds}$.
7. **Combine with the current source.** The ideal current source (infinite resistance) is in parallel with this at the output node, so
$$R_{out} = \infty \parallel \left(g_m r_{ds}^2 + 2 r_{ds}\right) = g_m r_{ds}^2 + 2 r_{ds}$$
This is option (C).

KEY IDEA

Looking into the drain of a MOSFET whose source has a resistance R_S to ground gives $R = r_{ds} + R_S + g_m r_{ds} R_S$. In a cascode, R_S is the r_{ds} of the lower transistor, so $R = 2r_{ds} + g_m r_{ds}^2$. The ideal current-source load is an open circuit in parallel with this, so it leaves the result unchanged.

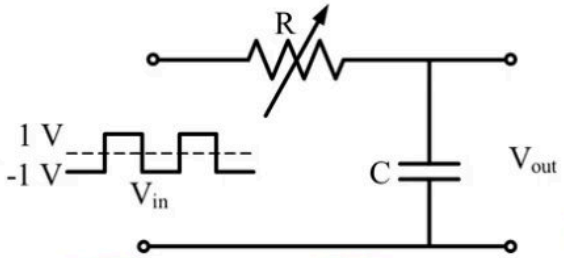
WATCH OUT FOR

Choosing (D) because the load is an ideal current source. The infinite resistance is in PARALLEL with the cascode at the output node, not in series, so it drops out and leaves the finite cascode resistance. Choosing (A) means treating M_2 as nothing more than r_{ds} and missing the $g_m r_{ds}^2$ boost that comes from its source being degenerated by M_1. Choosing (B) means treating M_2 as a source follower or diode ($1/g_m$), but here the output is taken at its drain.

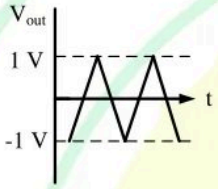
Q.39 MCQ · 2 MARKS

Networks (Electric Circuits) › Transients

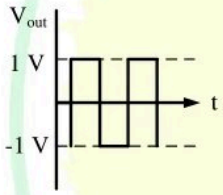
Q.39 In the circuit, shown below, if the values of R and C are very large, the form of the output voltage for a very high frequency square wave input, is best represented by



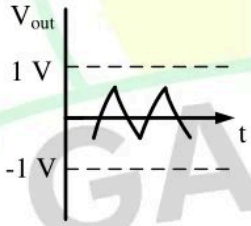
(A)



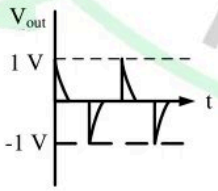
(B)



(C)



(D)



Official answer: **C**

SOLUTION

1. The output is taken across C, so the network is a first-order RC low-pass filter with time constant $\tau = RC$. A very high input frequency together with very large R and C means the half-period is tiny compared with the time constant: $\frac{T}{2} \ll RC$.
2. Average (DC) value: in periodic steady state the capacitor voltage returns to the same value every period, so the capacitor carries no average current. With zero average current there is zero average

voltage across R, so $\overline{V_{out}} = \overline{V_{in}} = \dfrac{(+1)+(-1)}{2} = 0$ V. The output is therefore centred on the time axis.

3. Because $T/2 \ll RC$, the capacitor voltage changes very little in one half-cycle. It stays small compared with 1 V (step 5 confirms this), so nearly the whole input appears across R. The charging current is then almost constant during each half-cycle: $i \approx \dfrac{\pm 1\ \text{V}}{R}$.
4. A constant current into a capacitor gives a linear ramp (the integrator approximation): $\dfrac{dV_{out}}{dt} = \dfrac{i}{C} \approx \pm\dfrac{1}{RC}$. So V_{out} ramps up while the input is $+1$ V and ramps down while it is -1 V, which gives a triangular wave.
5. Peak-to-peak swing over one half-period:
$$\Delta V_{out} \approx \frac{1}{RC}\cdot\frac{T}{2} = \frac{T}{2RC} \ll 1\ \text{V}$$
Exact check: in steady state V_{out} swings between $\pm V_p$. During a $+1$ V half-cycle it charges from $-V_p$ towards $+1$, so $V_p = 1-(1+V_p)e^{-T/(2RC)}$, which gives $V_p = \tanh\left(\dfrac{T}{4RC}\right) \approx \dfrac{T}{4RC}$ when $T \ll RC$. That agrees with the peak-to-peak value above.
6. So the output is a small-amplitude triangular wave centred on 0 V, far inside the ± 1 V lines. Option A has the right shape but a full ± 1 V swing: since $V_p = \tanh(T/4RC)$, that would need $RC \ll T$, and then the capacitor follows the input and the output is not triangular. Option B is what you would get across C when $RC \ll T$. Option D is the voltage across R (a differentiator) when $RC \ll T$. The answer is (C).

KEY IDEA

When the time constant is much longer than the half-period ($RC \gg T/2$), an RC network with the output across C acts as an integrator. The square wave in becomes a triangular wave out, with zero average and peak $\tanh(T/4RC) \approx T/(4RC)$, so the larger RC (or the higher the frequency), the smaller the triangle.

WATCH OUT FOR

Choosing (A) because 'integrator gives a triangle' and forgetting the amplitude. The integrator's output amplitude is about $T/(4RC)$, which is tiny here. A full ± 1 V swing would need $RC \ll T$, and in that case the output across C is square, not triangular. The other trap is picking (D): that is the differentiated waveform, taken across R with $RC \ll T$.

Q.40 MCQ · 2 MARKS

Signals and Systems › LTI & convolution

Let continuous-time signals $x_1(t)$ and $x_2(t)$ be

$$x_1(t) = \begin{cases} 1, & t \in [0,1] \\ 2-t, & t \in [1,2] \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad x_2(t) = \begin{cases} t, & t \in [0,1] \\ 2-t, & t \in [1,2] \\ 0, & \text{otherwise} \end{cases}$$

Consider the convolution $y(t) = x_1(t) * x_2(t)$. Then $\int_{-\infty}^{\infty} y(t)\,dt$ is

(A) 1.5

(B) 2.5

(C) 3.5

(D) 4

Official answer: **A**

SOLUTION

1. **Area property of convolution:** if $y(t) = x_1(t) * x_2(t)$, then
$$\int_{-\infty}^{\infty} y(t)\,dt = \left(\int_{-\infty}^{\infty} x_1(t)\,dt\right)\left(\int_{-\infty}^{\infty} x_2(t)\,dt\right)$$

This is $Y(0) = X_1(0)\,X_2(0)$, because $X(0) = \int_{-\infty}^{\infty} x(t)\,dt$. Directly: $\int y(t)\,dt = \int x_1(\tau)\left[\int x_2(t-\tau)\,dt\right]d\tau = A_2 \int x_1(\tau)\,d\tau = A_1 A_2$. Swapping the order of integration is valid here because both signals are bounded and last only a finite time.

2. **Area of $x_1(t)$:** a unit rectangle on $[0, 1]$ followed by a ramp falling from 1 to 0 on $[1, 2]$:

$$A_1 = \int_0^1 1\,dt + \int_1^2 (2 - t)\,dt = 1 + \tfrac{1}{2} = 1.5$$

3. **Area of $x_2(t)$:** a triangle with base $[0, 2]$ and peak 1 at $t = 1$:

$$A_2 = \int_0^1 t\,dt + \int_1^2 (2 - t)\,dt = \tfrac{1}{2} + \tfrac{1}{2} = 1$$

4. **Multiply the areas:**

$$\int_{-\infty}^{\infty} y(t)\,dt = A_1 A_2 = 1.5 \times 1 = 1.5$$

The answer is option (A).

KEY IDEA

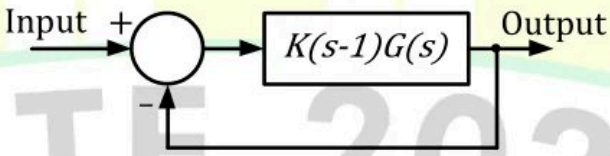
The area under a convolution is the product of the areas of the two signals, $\int y\,dt = \left(\int x_1\,dt\right)\left(\int x_2\,dt\right)$, which is the same statement as $Y(0) = X_1(0)X_2(0)$. So you never need to work out $y(t)$ itself.

WATCH OUT FOR

Adding the two areas instead of multiplying them: $1.5 + 1 = 2.5$ is exactly option (B). A second trap is counting only the rectangle part of $x_1(t)$ and taking its area as 1, which drops the triangle of area $\frac{1}{2}$ on $[1, 2]$. Working out the full piecewise convolution also reaches 1.5, but it takes much longer and leaves more room for algebra slips.

Q.41 MCQ · 2 MARKS

Control Systems › Routh–Hurwitz

Q.41	Let $G(s) = \frac{1}{(s+1)(s+2)}$. Then the closed-loop system shown in the figure below, is
	
(A)	stable for all $K > 2$.
(B)	unstable for all $K > 2$.
(C)	unstable for all $K > 1$.
(D)	stable for all $K > 1$.

Official answer: **B**

SOLUTION

1. With unity negative feedback, the open-loop transfer function is the forward block:
$$K(s-1)G(s) = \frac{K(s-1)}{(s+1)(s+2)}$$
2. Characteristic equation $1 + K(s-1)G(s) = 0$. Multiply through by $(s+1)(s+2)$:
$$(s+1)(s+2) + K(s-1) = s^2 + (3+K)s + (2-K) = 0$$
3. Routh array: row s^2: $1,\ 2-K$; row s^1: $3+K,\ 0$; row s^0: $2-K$.
4. For no sign change in the first column we need $3 + K > 0$ and $2 - K > 0$, so the closed loop is stable only for $-3 < K < 2$. (For a second-order polynomial this is the same as requiring every coefficient to be positive.)
5. At $K = 2$ the characteristic polynomial becomes $s^2 + 5s = s(s+5)$, so there is a closed-loop pole at $s = 0$ (marginally stable). For $K > 2$ the first column reads $1,\ 3+K > 0,\ 2-K < 0$: one sign change, so exactly one closed-loop pole lies in the right half-plane. You can see the same thing from the roots: their product is $2 - K < 0$, so they are real and of opposite sign.
6. Check the options. For every $K > 2$ the system is unstable, so (B) is true and (A) is false. At $K = 1.5$ (which is greater than 1) the system is stable, so (C) is false. At $K = 3$ (also greater than 1) it is unstable, so (D) is false. The answer is (B).

KEY IDEA

The forward path has a zero in the right half-plane at $s = +1$. As K increases, one root-locus branch leaves the pole at $s = -1$ and heads for that zero, crossing the origin at $K = 2$. For a second-order characteristic polynomial the Routh condition is simply that all coefficients are positive, here $3 + K > 0$ and $2 - K > 0$.

WATCH OUT FOR

Writing the characteristic equation as $(s+1)(s+2) + K(s+1)$ or $+K(1-s)$, which loses the sign of the right-half-plane zero. The constant term is $2 - K$, not $2 + K$. That sign slip makes the system look stable for every $K > 0$ and pushes students towards (A) or (D). The other trap is picking (C) because "higher gain destabilises": the loop is still stable for $1 < K < 2$, so the threshold is 2, not 1.

Q.42 MCQ · 2 MARKS

Signals and Systems › LTI & convolution

The continuous-time unit impulse signal is applied as an input to a continuous-time linear time-invariant system $\boldsymbol{S}$. The output is observed to be the continuous-time unit step signal $u(t)$. Which one of the following statements is true?

(A) Every bounded input signal applied to $\boldsymbol{S}$ results in a bounded output signal.

(B) It is possible to find a bounded input signal which when applied to $\boldsymbol{S}$ results in an unbounded output signal.

(C) On applying any input signal to $\boldsymbol{S}$, the output signal is always bounded.

(D) On applying any input signal to $\boldsymbol{S}$ the output signal is always unbounded.

Official answer: **B**

SOLUTION

1. **Impulse response.** The response of an LTI system to $\delta(t)$ is, by definition, its impulse response, so $h(t) = u(t)$. Convolution then gives
$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, u(t-\tau)\, d\tau = \int_{-\infty}^{t} x(\tau)\, d\tau$$
so $\boldsymbol{S}$ is an ideal integrator.
2. **BIBO stability test (CT LTI).** A continuous-time LTI system is BIBO stable if and only if $h(t)$ is absolutely integrable. Here
$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{0}^{\infty} 1\, dt = \infty$$
The integral diverges, so $\boldsymbol{S}$ is **not** BIBO stable. This rules out (A).
3. **A bounded input that gives an unbounded output (supports B).** Take $x(t) = u(t)$, which is bounded ($|x(t)| \le 1$). Then
$$y(t) = u(t) * u(t) = \int_{0}^{t} 1\, d\tau = t\, u(t)$$
This ramp grows without limit as $t \to \infty$, so (B) is true. The same example rules out (C), because here the output is not bounded.
4. **Rule out (D).** The input $x(t) = \delta(t)$ gives $y(t) = u(t)$, which is bounded. The zero input also gives zero output. So the output is not always unbounded.
5. Only (B) is true.

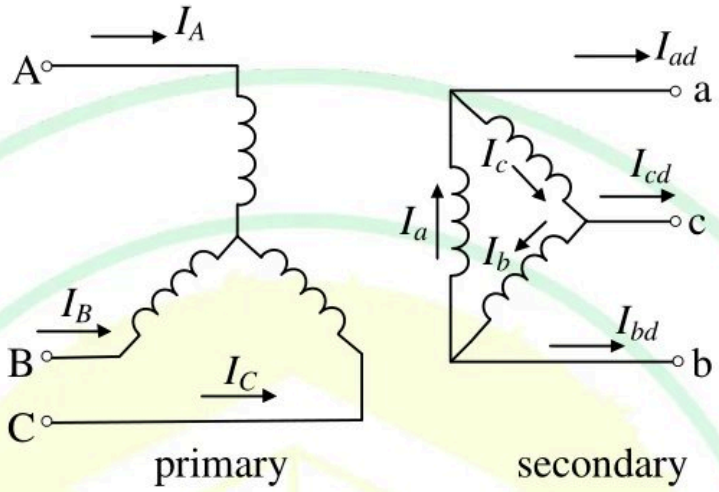
KEY IDEA

A continuous-time LTI system is BIBO stable if and only if $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$. An impulse response of $h(t) = u(t)$ makes the system an ideal integrator. Its integral diverges, so the system is unstable, and a bounded step input produces an unbounded ramp.

WATCH OUT FOR

Calling the system stable because its impulse response $u(t)$ is bounded (this leads to A). Stability needs $h(t)$ to be absolutely integrable, and being bounded is not enough. A second mistake is to pick (D) by reading 'unstable' as 'every output is unbounded'. Instability only means that at least one bounded input gives an unbounded output. Many inputs, such as $\delta(t)$ or a finite pulse, still give bounded outputs.

Q.43 MCQ · 2 MARKS

Q.43	The transformer connection given in the figure is part of a balanced 3-phase circuit where the phase sequence is "abc". The primary to secondary turns ratio is 2:1. If $(I_a + I_b + I_c = 0)$, then the relationship between I_A and I_{ad} will be
	
(A)	$\frac{\lvert I_A\rvert}{\lvert I_{ad}\rvert} = \frac{1}{2\sqrt{3}}$ and I_{ad} lags I_A by 30°.
(B)	$\frac{\lvert I_A\rvert}{\lvert I_{ad}\rvert} = \frac{1}{2\sqrt{3}}$ and I_{ad} leads I_A by 30°.
(C)	$\frac{\lvert I_A\rvert}{\lvert I_{ad}\rvert} = 2\sqrt{3}$ and I_{ad} lags I_A by 30°.
(D)	$\frac{\lvert I_A\rvert}{\lvert I_{ad}\rvert} = 2\sqrt{3}$ and I_{ad} leads I_A by 30°.

Official answer: **A**

SOLUTION

1. Each primary line terminal (A, B, C) is star-connected, so the primary line current equals that leg's primary winding current. By ampere-turn balance on each core leg (turns ratio N1:N2 = 2:1), the co-labelled secondary delta-winding current has magnitude $|I_a| = (N1/N2)|I_A| = 2|I_A|$, and is in phase with I_A (similarly $|I_b|$ with I_B, $|I_c|$ with I_C).
2. The secondary winding currents I_a, I_b, I_c satisfy $I_a+I_b+I_c=0$ and, with phase sequence abc, form a balanced set. Taking I_a as reference (in phase with I_A): $I_a = I\angle 0^\circ,\; I_b = I\angle{-120^\circ},\; I_c = I\angle{120^\circ}$, where $I = 2|I_A|$.

3. From the figure, at the delta corner feeding terminal a, the winding current I_a flows into that node and I_c flows out of it, alongside the external line current I_ad. KCL at that node gives I_{ad} = I_a - I_c.
4. Substituting the phasors: I_{ad} = I\angle 0^\circ - I\angle 120^\circ = I\left(1-\left(-\dfrac{1}{2}\right) - j\dfrac{\sqrt3}{2}\right) = I\left(\dfrac{3}{2} - j\dfrac{\sqrt3}{2}\right) = \sqrt3\,I\angle{-30^\circ}.
5. Therefore |I_{ad}| = \sqrt3\,I = \sqrt3\,(2|I_A|) = 2\sqrt3\,|I_A|, so \dfrac{|I_A|}{|I_{ad}|} = \dfrac{1}{2\sqrt3}.
6. Since I_{ad} is at -30^\circ relative to I_a, and I_a is in phase with I_A, I_{ad} lags I_A by 30^\circ. This matches option (A).

KEY IDEA

Match the primary star-winding current on each leg to the co-labelled secondary delta-winding current by ampere-turn balance (N1 I_primary = N2 I_secondary, in phase), then get the delta's external line current from the phasor difference of the two adjacent internal winding currents that meet at that terminal (I_ad = I_a - I_c), and finally compare magnitude and angle with the primary line current.

WATCH OUT FOR

Two common errors: (1) taking the delta line current as the sum (I_a + I_c) or as a single winding current (just I_a) instead of the correct phasor difference of the two windings meeting at that corner, which changes both the √3 factor and the ±30° shift; (2) getting the sign of the 30° shift backwards by mislabelling which winding is "incoming" vs "outgoing" at the corner, turning a lag into a lead (confusing options A and B, or C and D).

Q.44 MCQ · 2 MARKS

Electrical Machines › DC machines

A DC series motor with negligible series resistance is running at a certain speed driving a load, where the load torque varies as cube of the speed. The motor is fed from a 400 V DC source and draws 40 A armature current. Assume linear magnetic circuit. The external resistance, in Ω, that must be connected in series with the armature to reduce the speed of the motor by half, is closest to

(A) 23.28

(B) 4.82

(C) 46.7

(D) 0

Official answer: **A**

SOLUTION

1. Since the magnetic circuit is linear and this is a series motor, flux is proportional to armature current: φ ∝ Ia. Hence electromagnetic torque, $T = k\phi I_a \propto I_a^2$.
2. Initial condition (no external resistance, negligible series resistance): armature current $I_{a1} = 40\ \text{A}$, terminal voltage $V = 400\ \text{V}$, so back-EMF $E_1 = V = 400\ \text{V}$, at speed N_1.
3. The load torque varies as the cube of speed, $T_L \propto N^3$. At steady state the motor's electromagnetic torque equals the load torque at every operating speed. When the speed is halved, $N_2 = \dfrac{N_1}{2}$, so
$$\frac{T_2}{T_1} = \left(\frac{N_2}{N_1}\right)^3 = \frac{1}{8}$$
4. Since $T \propto I_a^2$:
$$\frac{I_{a2}^2}{I_{a1}^2} = \frac{1}{8} \implies I_{a2} = \frac{I_{a1}}{\sqrt{8}} = \frac{40}{2.828} = 14.142\ \text{A}$$
5. Back-EMF depends on both flux and speed, and here $\phi \propto I_a$, so $E \propto \phi N \propto I_a N$:
$$\frac{E_2}{E_1} = \left(\frac{I_{a2}}{I_{a1}}\right)\left(\frac{N_2}{N_1}\right) = \left(\frac{14.142}{40}\right)\left(\frac{1}{2}\right) = 0.1768$$
6. So
$$E_2 = 400 \times 0.1768 = 70.71\ \text{V}$$

7. With the external resistance R now inserted in the armature circuit (series/armature resistance itself still negligible):
$$V = E_2 + I_{a2}R \implies 400 = 70.71 + 14.142R$$
8. Solving:
$$R = \frac{400 - 70.71}{14.142} = \frac{329.29}{14.142} = 23.28\ \Omega$$
This matches option (A).

KEY IDEA
In a DC series motor with a linear magnetic circuit, flux is proportional to armature current (φ ∝ Ia), so electromagnetic torque T ∝ Ia², and back-EMF E ∝ φN ∝ Ia·N. Combining the load's T ∝ N³ law with the motor's T ∝ Ia² law, and then using the circuit equation V = E + Ia·R, pins down the external resistance needed for a given speed change.

WATCH OUT FOR
Assuming back-EMF stays proportional to speed alone (E ∝ N) as in a shunt/separately excited motor, forgetting that in a series motor with linear magnetic circuit, flux itself changes with armature current, so E ∝ φN ∝ Ia·N. Missing this collapses the whole ratio calculation.

Q.45 MCQ · 2 MARKS

Electrical Machines › Induction machines

A 3-phase, 400 V, 4 pole, 50 Hz star connected induction motor has the following parameters referred to the stator: $R_r' = 1\Omega$, $X_s = X_r' = 2\Omega$. Stator resistance, magnetizing reactance and core loss of the motor are neglected. The motor is run with constant V/f control from a drive. For maximum starting torque, the voltage and frequency output, respectively, from the drive, is closest to,

(A) 400 V and 50 Hz

(B) 200 V and 25 Hz

(C) 100 V and 12.5 Hz

(D) 300 V and 37.5 Hz

Official answer: **C**

SOLUTION

1. Neglecting stator resistance, the magnetizing branch and core loss, the per-phase equivalent circuit reduces to a single series loop: supply phasor in series with the total leakage reactance and the referred rotor resistance R_r'/s. The torque is
$$T = \frac{3}{\omega_s} \cdot \frac{V^2\,(R_r'/s)}{(R_r'/s)^2 + X_{20}^2}$$
where $X_{20} = X_s + X_r'$ is the total leakage reactance referred to the stator.
2. Maximizing T with respect to slip (for fixed V, f) gives the standard result
$$s_{maxT} = \frac{R_r'}{X_{20}}$$
3. At rated frequency (50 Hz): $X_{20} = X_s + X_r' = 2 + 2 = 4\ \Omega$.
4. Under constant V/f (scalar) control, the leakage reactance scales linearly with the drive's output frequency, while R_r' (a resistance) does not:
$$X_{20}(f) = 4 \times \frac{f}{50}\ \Omega$$
5. Starting always corresponds to slip $s = 1$. For maximum torque to occur exactly at starting, the peak-torque slip must equal 1:
$$s_{maxT} = 1 \ \Rightarrow\ R_r' = X_{20}(f)$$
$$1 = 4 \times \frac{f}{50}$$

6. Solving:
$$f = \frac{50}{4} = 12.5 \text{ Hz}$$
7. Constant V/f ratio, taken from the rated nameplate values (the ratio is the same whether line or phase quantities are used):
$$\frac{V}{f} = \frac{400}{50} = 8 \text{ V/Hz}$$
8. Therefore
$$V = 8 \times 12.5 = 100 \text{ V}$$
9. This matches option (C): 100 V and 12.5 Hz.

KEY IDEA
See keyIdea field above.

WATCH OUT FOR
Assuming $s_{maxT} = R'_r/X_{20}$ must be evaluated at the rated 50 Hz reactance (giving $s_{maxT} = 1/4$, not 1) and then either stopping there or wrongly forcing slip = 1 into the rated-frequency torque expression instead of solving for the frequency at which the reactance shrinks enough to make $s_{maxT} = 1$; a second common slip is treating R'_r as scaling with frequency along with the reactance, which cancels the frequency dependence and gives no solution.

Q.46 MCQ · 2 MARKS

Power Electronics › Inverters & PWM

The 3-phase modulating waveforms ($v_a(t), v_b(t)$ and $v_c(t)$), used in sinusoidal PWM in a Voltage Source Inverter (VSI) are $v_a(t) = 0.8\sin(\omega t)$ V, $v_b(t) = 0.8\sin(\omega t - \frac{2\pi}{3})$ V, $v_c(t) = 0.8\sin(\omega t + \frac{2\pi}{3})$ V, where $\omega = 2\pi \times 40$ rad/s is the fundamental frequency. The modulating waveforms are compared with a 10 kHz triangular carrier whose magnitude varies between +1 and -1. The VSI has a DC link voltage of 600 V and feeds a star connected motor. The per phase fundamental RMS motor voltage, in volts, is closest to

(A) 169.71

(B) 300.00

(C) 424.26

(D) 212.13

Official answer: **A**

SOLUTION

1. The carrier swings between -1 and $+1$, so its peak is 1. The modulating peak is 0.8, so the modulation index is $m_a = \frac{0.8}{1} = 0.8$. Since $m_a \leq 1$, the inverter works in the linear region of sinusoidal PWM, with no over-modulation.
2. In the linear region, each pole voltage measured from the DC mid-point contains a fundamental whose peak is $m_a \frac{V_{dc}}{2}$. This is because the leg voltage switches between $+\frac{V_{dc}}{2}$ and $-\frac{V_{dc}}{2}$, and its local average follows the modulating wave.
3. $$V_{ao,1(\text{peak})} = 0.8 \times \frac{600}{2} = 240 \text{ V}$$
4. The load is a balanced star. The three modulating waves have the same amplitude and are displaced by 120°, so the three pole voltages have the same fundamental amplitude and sum to zero. The star point therefore sits at the mid-point potential for the fundamental, and the fundamental phase voltage equals the fundamental pole voltage. Any common-mode or triplen component appears equally on all three poles and does not drop across the motor phase. So the peak fundamental phase voltage is 240 V.

5. $$V_{ph,1(\text{rms})} = \frac{240}{\sqrt{2}} \approx 169.71 \text{ V}$$

This is option (A).

KEY IDEA

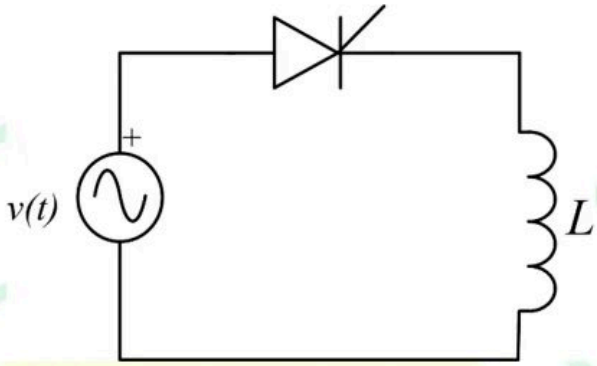
In the linear region of sinusoidal PWM ($m_a \le 1$), the fundamental phase voltage of a balanced star load has peak $m_a V_{dc}/2$. Divide by $\sqrt{2}$ for the RMS value.

WATCH OUT FOR

Using V_{dc} in place of $V_{dc}/2$ as the pole-voltage base gives a peak of 480 V. Another slip is to convert the RMS value of $m_a V_{dc}/2$ as if it were already RMS, or to multiply by $\sqrt{3}$ and quote the line voltage. Both lead to options (B), (C) or (D) instead of (A).

Q.47 MCQ · 2 MARKS

Power Electronics › Rectifiers

Q.47	An ideal sinusoidal voltage source $v(t) = 230\sqrt{2}\sin(2\pi \times 50t)\ V$ feeds an ideal inductor L through an ideal SCR with firing angle $\alpha = 0°$. If $L = 100\ mH$, then the peak of the inductor current, in ampere, is closest to
	
(A)	20.71
(B)	0
(C)	10.35
(D)	7.32

Official answer: **A**

SOLUTION

1. Source: $V_m = 230\sqrt{2} \approx 325.27$ V, $\omega = 2\pi \times 50 = 314.16$ rad/s, so $\omega L = 314.16 \times 0.1 = 31.416\ \Omega$.
2. The SCR is forward biased from $\omega t = 0$, when the source voltage turns positive. With firing angle $\alpha = 0°$ it is triggered at $\omega t = 0$ and conducts for as long as the current stays positive.
3. While the SCR conducts, the inductor obeys $L\dfrac{di}{dt} = V_m \sin \omega t$ with $i(0) = 0$.

4. Integrating:

$$i(\omega t) = \frac{V_m}{\omega L}\left(1 - \cos \omega t\right)$$

5. Since $1 - \cos \omega t \geq 0$, the current is never negative. It returns to zero only at $\omega t = 2\pi$, so the SCR conducts for the full cycle (conduction angle 360°). The negative half-cycle of the source only pulls the current back down to zero, not below it.
6. The current is largest where $\cos \omega t = -1$, that is at $\omega t = \pi$:

$$I_{peak} = \frac{2V_m}{\omega L}$$

7. $$I_{peak} = \frac{2 \times 325.27}{31.416} \approx 20.71 \text{ A}$$

   This is option (A).

KEY IDEA

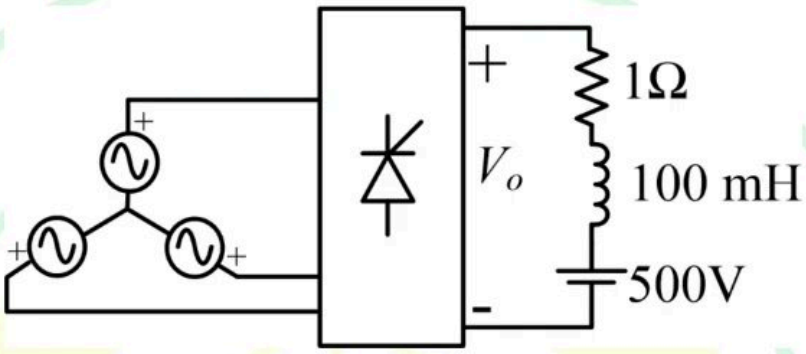
With a purely inductive load and $\alpha = 0^\circ$, the current is $\frac{V_m}{\omega L}(1 - \cos \omega t)$. It has a DC offset equal to its amplitude, so it never goes negative and the SCR never turns off. The peak is therefore twice $V_m/\omega L$.

WATCH OUT FOR

Treating the load like a resistor, or assuming the SCR turns off at $\omega t = \pi$ so that the current stops there. That gives $V_m/\omega L \approx 10.35$ A (option C). For an inductor the current lags the voltage and keeps flowing through the negative half-cycle, so the SCR does not turn off until $\omega t = 2\pi$.

Q.48 MCQ · 2 MARKS

Power Electronics › Rectifiers

Q.48	In the following circuit, the average voltage $V_o = 400\left(1+\frac{\cos\alpha}{3}\right)\text{V}$, where α is the firing angle. If the power dissipated in the resistor is 64 W, then the closest value of α in degrees is
	
(A)	35.9
(B)	46.4
(C)	41.4
(D)	0

Official answer: **A**

SOLUTION

1. The load is a resistor $R = 1\,\Omega$ in series with an inductor $L = 100\,\text{mH}$ and a DC source $E = 500\,\text{V}$, with its positive side toward the V_o+ terminal. So the converter must supply a voltage above E for current to flow.
2. The large inductance keeps the load current essentially constant, so $I_{rms} \approx I_o$ and the ripple is neglected.
3. The power in the resistor is $P = I_o^2 R = 64\,\text{W}$. With $R = 1\,\Omega$, $I_o = \sqrt{64/1} = 8\,\text{A}$.
4. The average voltage across the inductor is zero in steady state. Averaging KVL around the load gives $V_o = E + I_o R = 500 + 8 \times 1 = 508\,\text{V}$.
5. Set this equal to the given expression: $400\left(1+\frac{\cos\alpha}{3}\right) = 508$, so $1+\frac{\cos\alpha}{3} = 1.27$.
6. Then $\cos\alpha = 3 \times 0.27 = 0.81$, so $\alpha = \cos^{-1}(0.81) \approx 35.9^\circ$, which is option (A).

KEY IDEA

With a large inductor the load current is nearly constant, so the resistor power gives the current. Averaging KVL around the load, where the inductor drops no average voltage, then gives the converter's average output voltage as $V_o = E + I_o R$.

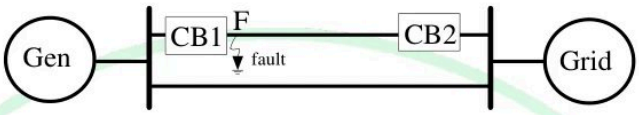
WATCH OUT FOR

Forgetting the 500 V source and writing $V_o = I_o R = 8\,\text{V}$, which gives an impossible $\cos\alpha$. Another slip is to treat the inductor as dropping average voltage.

Q.49 MCQ · 2 MARKS

Power Systems › Stability

Q.49	In the system shown below, the generator was initially supplying power to the grid. A temporary LLLG bolted fault occurs at F very close to circuit breaker 1. The circuit breakers open to isolate the line. The fault self-clears. The circuit breakers reclose and restore the line. Which one of the following diagrams best indicates the rotor accelerating and decelerating areas?



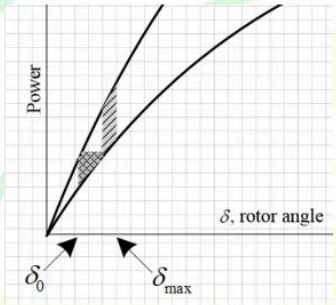


Fig. (i)

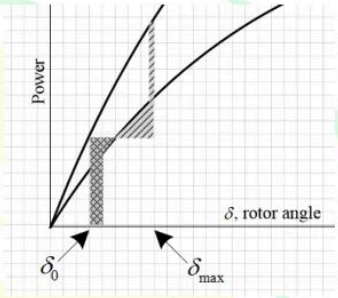


Fig. (ii)

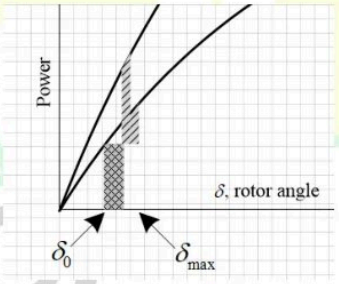


Fig. (iii)

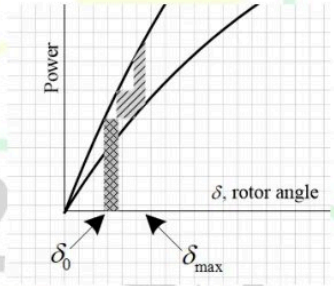


Fig. (iv)

(A)	Fig. (i)
(B)	Fig. (ii)
(C)	Fig. (iii)
(D)	Fig. (iv)

Official answer: **B**

SOLUTION

1. Before the fault both lines are in service and the generator delivers P_m. The operating point is δ_0 on the upper (two-line) power-angle curve, so $P_m = P_{max,1} \sin \delta_0$.
2. The bolted LLLG fault is at F, very close to circuit breaker 1, which sits at the generator end. The voltage at the generator terminal collapses to almost zero, so the electrical power delivered is $P_e \approx 0$ during the fault.

3. With $P_e \approx 0$ the accelerating power is the whole of P_m. The accelerating area is a rectangle, $A_1 = P_m(\delta_c - \delta_0)$, with its top at the horizontal P_m level and its base on the δ axis starting at δ_0.
4. The breakers open and isolate the faulted line. Only the other line remains, with higher reactance, so P_e follows the lower (one-line) curve. At the clearing angle this curve is still below P_m, so the net power is still accelerating. The rotor keeps speeding up and picks up a second accelerating area, a wedge between the P_m level and the lower curve.
5. The fault self-clears and the breakers reclose, restoring both lines. P_e jumps back to the upper curve, which is above P_m at that angle. The net power is now decelerating, and the decelerating area lies between the upper curve and the horizontal P_m level.
6. The required picture therefore shows, in order, a tall rectangle from δ_0 with its top at the P_m level, then a wedge between the P_m level and the lower curve, then a vertical jump to the upper curve with the decelerating area above the P_m level. This is Fig. (ii). Fig. (i) has only small patches and no full rectangle, so it does not show $P_e \approx 0$. Figs. (iii) and (iv) do not follow this sequence of areas. The answer is option B.

KEY IDEA

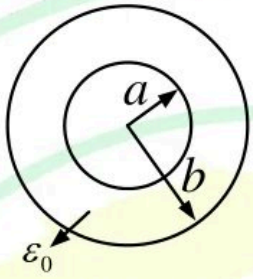
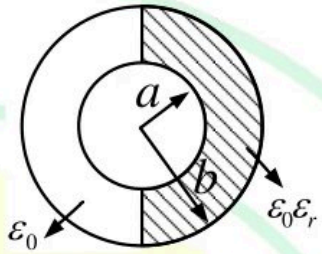
A bolted three-phase fault at the generator terminal makes $P_e \approx 0$, so the accelerating area is a full rectangle at the P_m level. With the faulted line out, P_e is on the lower curve, below P_m, which adds an accelerating wedge. Reclosing returns P_e to the upper curve, above P_m, which gives the decelerating area.

WATCH OUT FOR

Drawing the accelerating area between the pre-fault and during-fault curves as if the fault only reduced power partly. A bolted fault right at the generator end drops P_e to nearly zero. Another slip is forgetting the extra accelerating wedge while the single line is in service, or shading decelerating area before the line is restored.

Q.50 MCQ · 2 MARKS

Electromagnetics › Electrostatics

Q.50	An air filled cylindrical capacitor (capacitance C_0) of length L, with a and b as its inner and outer radii, respectively, consists of two coaxial conducting surfaces. Its cross-sectional view is shown in Fig. (i). In order to increase the capacitance, a dielectric material of relative permittivity ε_r is inserted inside 50% of the annular region as shown in Fig. (ii). The value of ε_r for which the capacitance of the capacitor in Fig. (ii), becomes $5C_0$ is
	 Fig. (i)  Fig. (ii)
(A)	4
(B)	5
(C)	9
(D)	10

Official answer: **C**

SOLUTION

1. **Air capacitor.** Put charge per unit length λ on the inner conductor. Gauss's law over a coaxial cylinder of radius ρ ($a < \rho < b$) gives $\mathbf{E} = \dfrac{\lambda}{2\pi\varepsilon_0\rho}\,\hat{a}_\rho$, where $\varepsilon_0 = 8.854 \times 10^{-12}\ \mathrm{F/m}$. So $V = \dfrac{\lambda}{2\pi\varepsilon_0}\ln\dfrac{b}{a}$ and
$$C_0 = \frac{\lambda L}{V} = \frac{2\pi\varepsilon_0 L}{\ln(b/a)}$$
2. **Geometry of the split.** The dielectric fills one angular half of the annulus: the dividing planes lie on a diameter, so they pass through the axis. Each half covers the full radial gap from a to b. The air-dielectric boundary is therefore a radial plane, and the radial field $\hat{a}_\rho$ lies in it.
3. **The field is the same in both halves.** Try $V(\rho) = V\,\dfrac{\ln(b/\rho)}{\ln(b/a)}$ in both regions. It satisfies Laplace's equation, equals V at $\rho = a$ and 0 at $\rho = b$, and gives a purely radial $\mathbf{E}$. On the interface, $\mathbf{E}$ is entirely tangential and the same on both sides, so tangential $\mathbf{E}$ is continuous. The normal component of $\mathbf{D}$ is zero on both sides, so that condition holds too. By uniqueness this is the field. The two halves have the

same V and the same $\mathbf{E}$; only $\mathbf{D} = \varepsilon\mathbf{E}$, and with it the surface charge on the conductors, is larger in the dielectric half. So the two halves act as **capacitors in parallel**.

4. **Each half.** Each half-annulus holds half the charge a full coaxial capacitor of the same material would hold at the same V. For example, in the air half $Q_{\text{air}} = \dfrac{\varepsilon_0 V}{\rho \ln(b/a)} \cdot \pi\rho L = \dfrac{C_0}{2}V$. Hence
$$C_{\text{air}} = \frac{C_0}{2}, \qquad C_{\text{diel}} = \frac{\varepsilon_r C_0}{2}$$
5. **Total capacitance.**
$$C = \frac{C_0}{2} + \frac{\varepsilon_r C_0}{2} = \frac{1+\varepsilon_r}{2}\,C_0$$
6. **Set $C = 5C_0$.**
$$\frac{1+\varepsilon_r}{2} = 5 \;\Rightarrow\; \varepsilon_r = 9$$
This is option (C).

KEY IDEA

Look at how the air-dielectric boundary lies relative to the field. Here the boundary is a radial plane, so the radial field $\mathbf{E}$ runs along it. Both regions then have the same voltage across them and the same $\mathbf{E}$, so they add as parallel capacitors, each scaled by its share of the angle: $C = \sum(\theta_i/2\pi)\,\varepsilon_{ri}\,C_0$. When the boundary is a cylinder $\rho = c$ instead, it is normal to $\mathbf{E}$, $\mathbf{D}$ is continuous across it, and the layers add in series.

WATCH OUT FOR

The common mistake is treating the two regions as series layers. That is right only when the dielectric forms a coaxial shell (a boundary at constant ρ), not when it fills an angular sector. The other trap is answering $\varepsilon_r = 5$ or 10: 5 assumes filling half the gap scales C_0 by ε_r, and 10 forgets that the air half still adds $C_0/2$. The correct equation is $\frac{1}{2}(1+\varepsilon_r) = 5$.

Q.51 MSQ · 2 MARKS

Electromagnetics › Electrostatics

Let $\mathbf{a}_R$ be the unit radial vector in the spherical co-ordinate system. For which of the following value(s) of n, the divergence of the radial vector field $\mathbf{f}(\mathbf{R}) = \mathbf{a}_R \dfrac{1}{R^n}$ is independent of R?

(A) -2

(B) -1

(C) 1

(D) 2

Official answer: **B, D**

SOLUTION

1. The field has only a radial component, $f_R = R^{-n}$, and it does not depend on θ or ϕ. So only the radial term of the spherical divergence survives:
$$\nabla \cdot \mathbf{f} = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2 f_R\right)$$
2. Substitute: $R^2 f_R = R^{2-n}$, so $\dfrac{\partial}{\partial R}\left(R^{2-n}\right) = (2-n)R^{1-n}$.
3. Divide by R^2:
$$\nabla \cdot \mathbf{f} = (2-n)\,R^{-(n+1)}$$
This is independent of R only when the power of R is zero or the coefficient is zero.
4. Case 1, the power of R is zero: $n+1 = 0 \Rightarrow n = -1$. Then $\mathbf{f} = R\,\mathbf{a}_R$ and $\nabla \cdot \mathbf{f} = (2-(-1))R^0 = 3$, a constant. So option (B) is correct.
5. Case 2, the coefficient is zero: $2 - n = 0 \Rightarrow n = 2$. Then $\mathbf{f} = \mathbf{a}_R/R^2$ (the inverse-square field of a point charge) and $\nabla \cdot \mathbf{f} = 0$ for every $R \neq 0$. Zero is also independent of R, so option (D) is correct.

6. Check the others: $n = -2$ gives $\nabla \cdot \mathbf{f} = 4R$, and $n = 1$ gives $\nabla \cdot \mathbf{f} = 1/R^2$. Both depend on R, so (A) and (C) are wrong. Answer: (B) and (D).

KEY IDEA

For a purely radial field in spherical coordinates, $\nabla \cdot \mathbf{f} = \frac{1}{R^2}\frac{d}{dR}(R^2 f_R)$. With $f_R = R^{-n}$ this gives $(2-n)R^{-(n+1)}$. That expression can lose its R-dependence in two ways: the power of R becomes zero ($n = -1$), or the coefficient becomes zero ($n = 2$, the inverse-square field, which has zero divergence).

WATCH OUT FOR

Setting only the power of R to zero and marking just (B). Students forget that a divergence of zero is also independent of R, so they miss $n = 2$, the inverse-square (Coulomb) field. A second trap is using the Cartesian-style derivative $\partial f_R/\partial R$ without the R^2 factor. That gives $-nR^{-n-1}$ and wrongly picks $n = -1$ and $n = 0$.

Q.52 MSQ · 2 MARKS

Networks (Electric Circuits) › Coupled circuits

Consider two coupled circuits, having self-inductances L_1 and L_2, that carry non-zero currents I_1 and I_2, respectively. The mutual inductance between the circuits is M with unity coupling coefficient. The stored magnetic energy of the coupled circuits is minimum at which of the following value(s) of $\frac{I_1}{I_2}$?

(A) $-\frac{M}{L_1}$

(B) $-\frac{M}{L_2}$

(C) $-\frac{L_1}{M}$

(D) $-\frac{L_2}{M}$

Official answer: **A, D**

SOLUTION

1. Energy stored in a coupled pair, with both currents referenced as entering the dotted terminals and $M > 0$ (a current flowing the other way just makes I_1 or I_2 negative, so this one expression covers both the aiding and the opposing case):
$$W = \tfrac{1}{2}L_1 I_1^2 + \tfrac{1}{2}L_2 I_2^2 + M I_1 I_2$$
2. Since $I_2 \neq 0$, write $W = I_2^2\, f(x)$ with $x = I_1/I_2$:
$$f(x) = \frac{W}{I_2^2} = \tfrac{1}{2}L_1 x^2 + Mx + \tfrac{1}{2}L_2$$
For a given I_2, $I_2^2 > 0$ is fixed, so minimising W is the same as minimising $f(x)$.
3. Minimise the quadratic:
$$\frac{df}{dx} = L_1 x + M = 0 \;\Rightarrow\; x = -\frac{M}{L_1}$$
The second derivative is $L_1 > 0$, so this is a minimum. Option (A) is correct.
4. Unity coupling: $k = \dfrac{M}{\sqrt{L_1 L_2}} = 1 \Rightarrow M^2 = L_1 L_2$. Hence
$$-\frac{M}{L_1} = -\frac{M^2}{L_1 M} = -\frac{L_1 L_2}{L_1 M} = -\frac{L_2}{M}$$
Option (D) is the same value, so it is also correct.
5. Check with a perfect square: with $M = \sqrt{L_1 L_2}$,
$$W = \tfrac{1}{2}\left(\sqrt{L_1}\, I_1 + \sqrt{L_2}\, I_2\right)^2 \geq 0$$
It reaches its lowest possible value, zero, when $\frac{I_1}{I_2} = -\sqrt{\frac{L_2}{L_1}}$. Both $-\frac{M}{L_1}$ and $-\frac{L_2}{M}$ equal $-\sqrt{\frac{L_2}{L_1}}$.

6. Options (B) and (C) both equal $-\frac{M}{L_2} = -\frac{L_1}{M} = -\sqrt{\frac{L_1}{L_2}}$. That is not $-\sqrt{L_2/L_1}$ unless $L_1 = L_2$, so neither is correct in general. Answer: (A) and (D).

KEY IDEA

$W = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M I_1 I_2$ is a quadratic in the ratio $x = I_1/I_2$, with its minimum at $x = -M/L_1$. When $k = 1$, $M^2 = L_1 L_2$ turns W into a perfect square, so the minimum energy is exactly zero at $x = -\sqrt{L_2/L_1}$. Both $-M/L_1$ and $-L_2/M$ reduce to that value.

WATCH OUT FOR

Stopping at $-M/L_1$ and marking only (A), without using $k = 1$ to see that $-L_2/M$ is the same number. Also common: minimising with respect to I_2 (which gives $I_2/I_1 = -M/L_2$) and then marking $-M/L_2$ as a value of I_1/I_2, when it is really the reciprocal ratio. A third slip is dropping the sign of the mutual term, which moves the minimum to a positive ratio.

Q.53 NAT · 2 MARKS

Engineering Mathematics › Vector calculus

Let $(x, y) \in \Re^2$. The rate of change of the real valued function,

$$V(x, y) = x^2 + x + y^2 + 1$$

at the origin in the direction of the point $(1, 2)$ is _____ *(round off to the nearest integer)*.

Official answer: **0 to 1**

SOLUTION

1. The rate of change of a scalar field in a given direction is the **directional derivative** $D_{\hat{u}}V = \nabla V \cdot \hat{u}$, where $\hat{u}$ is a **unit** vector along that direction.
2. Gradient: $\nabla V = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}\right) = (2x + 1,\ 2y)$.
3. At the origin: $\nabla V(0, 0) = (1,\ 0)$.
4. The direction from the origin $(0, 0)$ towards the point $(1, 2)$ is $\vec{d} = (1, 2) - (0, 0) = (1, 2)$, with $|\vec{d}| = \sqrt{1^2 + 2^2} = \sqrt{5}$, so $\hat{u} = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$.
5. Directional derivative:

$$D_{\hat{u}}V(0, 0) = (1)\left(\frac{1}{\sqrt{5}}\right) + (0)\left(\frac{2}{\sqrt{5}}\right) = \frac{1}{\sqrt{5}} \approx 0.447$$

6. Rounded to the nearest integer, as the question asks: $0.447 \to 0$.

KEY IDEA

"Rate of change in a direction" means the directional derivative: the gradient dotted with the unit vector pointing from the given point towards the target point. Here $\nabla V(0, 0) = (1, 0)$ and $\hat{u} = (1, 2)/\sqrt{5}$, which gives $1/\sqrt{5}$.

WATCH OUT FOR

Skipping the normalisation and computing $\nabla V \cdot (1, 2) = 1$, or answering with the maximum rate $|\nabla V(0, 0)| = 1$ (the rate along the gradient, not towards $(1, 2)$). A directional derivative is a rate per unit distance, so the direction vector has to be a unit vector. The correct value is $1/\sqrt{5} \approx 0.447$, which rounds to 0. The key's range of 0 to 1 happens to accept 1 as well, but only 0 follows from the definition.

Q.54 NAT · 2 MARKS

Engineering Mathematics › Differential equations

Consider ordinary differential equations given by

$\dot{x}_1(t) = 2x_2(t)$

$\dot{x}_2(t) = r(t)$

with initial conditions $x_1(0) = 1$ and $x_2(0) = 0$.

If $r(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$, then at $t = 1$, $x_1(t) =$ _____ (round off to the nearest integer).

Official answer: **2**

SOLUTION

1. The coupling runs one way only: x_2 depends on nothing but $r(t)$. So solve for x_2 first, then substitute it into the equation for x_1.
2. For $t \geq 0$ the input is $r(t) = 1$, so $\dot{x}_2 = 1$. Integrate from 0 to t using $x_2(0) = 0$:
$$x_2(t) = x_2(0) + \int_0^t 1 \, d\tau = t.$$
3. Substitute into the first equation: $\dot{x}_1(t) = 2x_2(t) = 2t$.
4. Integrate from 0 to t using $x_1(0) = 1$:
$$x_1(t) = x_1(0) + \int_0^t 2\tau \, d\tau = 1 + t^2.$$
5. Check with the Laplace transform, keeping the initial conditions: from $sX_2(s) - x_2(0) = \frac{1}{s}$ we get $X_2(s) = \frac{1}{s^2}$. Then $sX_1(s) - x_1(0) = 2X_2(s)$ gives $sX_1(s) - 1 = \frac{2}{s^2}$, so $X_1(s) = \frac{1}{s} + \frac{2}{s^3}$. Inverting gives $x_1(t) = 1 + t^2$, the same result.
6. At $t = 1$: $x_1(1) = 1 + 1^2 = 2$.

KEY IDEA

This is a chain of two integrators, $x_1 \leftarrow 2\int x_2$ and $x_2 \leftarrow \int r$. A unit step fed through it gives a ramp for x_2 and a parabola for x_1. Integrate twice and add the initial condition at each stage: $x_2 = t$, and $x_1 = 1 + t^2$.

WATCH OUT FOR

Dropping the initial condition $x_1(0) = 1$ gives $x_1(1) = 1$. This is the zero-state response only, and it misses the zero-input part $1/s$. Another common slip is forgetting the factor 2 in $\dot{x}_1 = 2x_2$, which gives $x_1 = 1 + t^2/2$, that is 1.5 at $t = 1$.

Q.55 NAT · 2 MARKS

Engineering Mathematics › Complex analysis

Let C be a clockwise oriented closed curve in the complex plane defined by $|z| = 1$. Further, let $f(z) = jz$ be a complex function, where $j = \sqrt{-1}$. Then, $\oint_C f(z)\, dz =$ _____ (round off to the nearest integer).

Official answer: **0**

SOLUTION

1. Start with the integrand. $f(z) = jz$ is a polynomial in z, so it is analytic at every point of the finite plane (an entire function). It has no singularity inside or on $|z| = 1$.
2. Apply **Cauchy's integral theorem**: if f is analytic on and inside a simple closed contour C, then $\oint_C f(z)\, dz = 0$. Traversing the contour clockwise instead of anticlockwise only changes the sign of the integral, and $-0 = 0$.
3. Check directly by parametrisation. The clockwise unit circle is $z = e^{-j\theta}$, $0 \leq \theta \leq 2\pi$, so $dz = -je^{-j\theta}\, d\theta$.
4. $$\oint_C jz\, dz = \int_0^{2\pi} je^{-j\theta}\left(-je^{-j\theta}\right) d\theta = \int_0^{2\pi} e^{-2j\theta}\, d\theta = \left[\frac{e^{-2j\theta}}{-2j}\right]_0^{2\pi} = \frac{1-1}{-2j} = 0$$
(using $-j \cdot j = -j^2 = 1$ and $e^{-4j\pi} = 1$).
5. A third check: jz has the antiderivative $\frac{jz^2}{2}$, which is single-valued in the whole plane, so its integral around any closed path is 0.
6. Therefore $\oint_C f(z)\, dz = 0$.

KEY IDEA

Before you compute anything, look for singularities inside the contour. $f(z) = jz$ is entire, so Cauchy's integral theorem gives $\oint_C f(z)\,dz = 0$. The orientation (clockwise) and the constant j only multiply that zero.

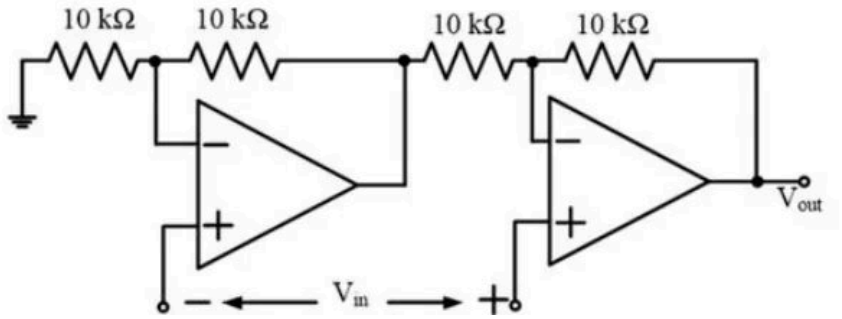
WATCH OUT FOR

Mixing up $\oint z\,dz$ with $\oint \frac{dz}{z}$ and writing $\pm 2\pi j$ (or $\pm 2\pi$ after multiplying by j). Only a pole inside the contour gives a nonzero result, and jz has none. A second slip is to think that the clockwise orientation must make the answer nonzero and negative. Reversing direction only flips the sign, and zero stays zero.

Q.56 NAT · 2 MARKS

Analog Electronics › Op-amps

Q.56 The op-amps in the following circuit are ideal. The voltage gain of the circuit is _____ *(round off to the nearest integer).*



Official answer: **2**

SOLUTION

1. Assume ideal op-amps: infinite open-loop gain and zero input current. Both op-amps have negative feedback, so each has a virtual short between its two inputs. Call the voltage at the input terminal marked '−' V_a and the voltage at the terminal marked '+' V_b, so that $V_{in} = V_b - V_a$.
2. Op-amp 1 is a non-inverting amplifier. V_a is on its + input, so its − input also sits at V_a. The two 10 kΩ resistors form a divider from its output V_1 to ground, with the − input at the midpoint. So
$$V_1 = \left(1 + \frac{10\,\text{k}\Omega}{10\,\text{k}\Omega}\right) V_a = 2V_a$$
3. Op-amp 2: by the virtual short, its − input sits at V_b. No current flows into the op-amp, so KCL at that node gives
$$\frac{V_1 - V_b}{10\,\text{k}\Omega} = \frac{V_b - V_{out}}{10\,\text{k}\Omega}$$
4. Solving for the output: $V_{out} = 2V_b - V_1$.
5. Substitute $V_1 = 2V_a$:
$$V_{out} = 2V_b - 2V_a = 2\,(V_b - V_a) = 2\,V_{in}$$
6. Voltage gain $= \dfrac{V_{out}}{V_{in}} = 2$.

KEY IDEA

This is the two-op-amp instrumentation amplifier. The first stage scales V_a by $(1 + R_2/R_1)$. The second stage outputs $(1 + R_4/R_3)V_b - (R_4/R_3)V_1$. When $R_4/R_3 = R_1/R_2$ (here all resistors are equal), the common-mode part cancels and $V_{out} = (1 + R_4/R_3)(V_b - V_a) = 2V_{in}$.

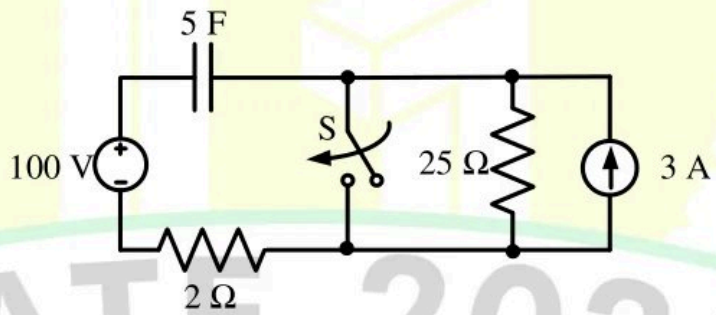
WATCH OUT FOR

Treating the second stage as a plain inverting amplifier, $V_{out} = -V_1 = -2V_a$, and forgetting that its + input carries V_b. That gives a gain of −2 or 1 instead of 2. A second mistake is taking op-amp 1's gain as 1 (a follower) when its − input actually sits at the midpoint of a 10 kΩ/10 kΩ divider.

Q.57 NAT · 2 MARKS

Networks (Electric Circuits) › Transients

Q.57 The switch (S) closes at $t = 0$ sec. The time, in sec, the capacitor takes to charge to 50 V is _____ *(round off to one decimal place).*



Official answer: **4.0 to 4.2**

SOLUTION

1. **Sign convention:** take v_C as the voltage of the capacitor plate on the source side minus the plate on node A's side.
2. **Steady state before $t = 0$ (S open):** at DC the capacitor is an open circuit, so no current flows in the series path through the 100 V source, the 5 F capacitor and the $2\,\Omega$ resistor. All of the 3 A therefore circulates through the current source and the $25\,\Omega$ resistor.
3. Voltage across the parallel group: $v_{AB} = 3 \times 25 = 75$ V, with the top node A positive (the source pushes current up into A, and it returns down through the $25\,\Omega$ resistor). The $2\,\Omega$ resistor carries no current, so it drops no voltage.
4. **KVL around the outer loop** (source, capacitor, A to B, then the $2\,\Omega$ resistor): $100 = v_C(0^-) + 75 + 0$, so $v_C(0^-) = 25$ V. The capacitor is already partly charged.
5. **Continuity of capacitor voltage:** $v_C(0^+) = v_C(0^-) = 25$ V.
6. **After S closes:** the switch shorts A to B, which removes the $25\,\Omega$ resistor and the 3 A source from the circuit. What remains is a series loop of the 100 V source, the 5 F capacitor and the $2\,\Omega$ resistor. So $v_C(\infty) = 100$ V with the same polarity, and $\tau = RC = 2 \times 5 = 10$ s.
7. **First-order response:** $v_C(t) = v_C(\infty) + [v_C(0^+) - v_C(\infty)]\, e^{-t/\tau} = 100 - 75\, e^{-t/10}$ V.
8. Set $v_C = 50$ V: $75\, e^{-t/10} = 50 \Rightarrow e^{-t/10} = \dfrac{2}{3}$.
9. $$t = 10 \ln \frac{3}{2} = 10 \times 0.4055 = 4.05 \text{ s} \approx 4.1 \text{ s}$$

KEY IDEA

The capacitor does not start uncharged. Before the switch closes, the 3 A source drives 75 V across the 25 Ω resistor, and that voltage opposes the 100 V source, so $v_C(0) = 100 - 75 = 25$ V. Closing S shorts out the whole parallel group. That leaves a plain series RC loop (100 V, 5 F, 2 Ω) with $v_C(\infty) = 100$ V and $\tau = 10$ s. Then use $v_C(t) = v_C(\infty) + [v_C(0^+) - v_C(\infty)]e^{-t/\tau}$.

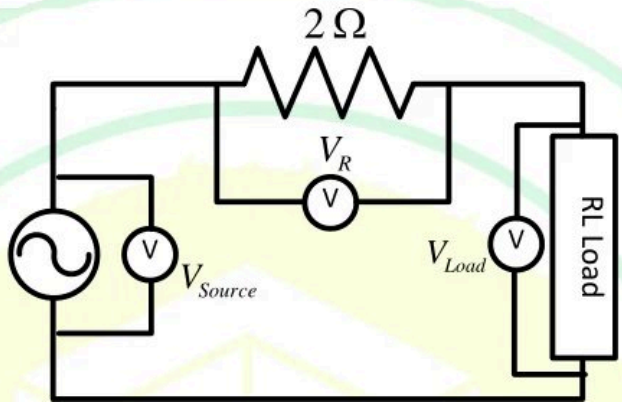
WATCH OUT FOR

Assuming the capacitor starts at 0 V gives $v_C = 100(1 - e^{-t/10})$ and $t = 10 \ln 2 \approx 6.9$ s, which is wrong. Another trap is taking $v_C(0)$ as the 75 V across the parallel group rather than 100 − 75 = 25 V. A third is leaving the 25 Ω resistor in the time constant after S closes, when the switch has shorted it out.

Q.58 NAT · 2 MARKS

Electrical and Electronic Measurements › V, I, power & energy

Q.58 In an experiment to measure the active power drawn by a single-phase RL Load connected to an AC source through a 2 Ω resistor, three voltmeters are connected as shown in the figure below. The voltmeter readings are as follows: $V_{Source} = 200\ V$, $V_R = 9\ V$, $V_{Load} = 199\ V$. Assuming perfect resistors and ideal voltmeters, the Load-active power measured in this experiment, in W, is _____ *(round off to one decimal place).*



Official answer: **78.0 to 81.0**

SOLUTION

1. This is the **three-voltmeter method**. A known series resistor $R = 2\ \Omega$ carries the load current I, and the three voltmeters read the supply voltage $V_S = 200$ V, the resistor voltage $V_R = 9$ V and the load voltage $V_L = 199$ V.
2. Take the current I as the reference phasor. The resistor voltage V_R is in phase with I, and the RL load voltage V_L leads I by the load power-factor angle ϕ. The supply voltage is the phasor sum $\vec{V}_S = \vec{V}_R + \vec{V}_L$, and the angle between $\vec{V}_R$ and $\vec{V}_L$ is ϕ, so by the **cosine rule**:
$$V_S^2 = V_R^2 + V_L^2 + 2V_R V_L \cos\phi \quad \Rightarrow \quad V_L \cos\phi = \frac{V_S^2 - V_R^2 - V_L^2}{2V_R}$$
3. The load active power is $P = V_L I \cos\phi$ with $I = \dfrac{V_R}{R}$ (ideal voltmeters draw no current, so the same I flows in R and the load). Substituting:
$$P = \frac{V_R}{R} \cdot \frac{V_S^2 - V_R^2 - V_L^2}{2V_R} = \frac{V_S^2 - V_R^2 - V_L^2}{2R}$$
4. Substitute the readings: $V_S^2 = 40000$, $V_R^2 = 81$, $V_L^2 = 39601$, so $V_S^2 - V_R^2 - V_L^2 = 40000 - 81 - 39601 = 318\ \text{V}^2$.
5. $$P = \frac{318}{2 \times 2} = 79.5\ \text{W}$$

6. Check: $I = 9/2 = 4.5$ A and $\cos\phi = \dfrac{318}{2 \times 9 \times 199} = \dfrac{318}{3582} = 0.0888$, so $P = 199 \times 4.5 \times 0.0888 = 79.5$ W. The power factor is about 0.089 lagging, a highly inductive load, which is consistent with an RL load.

KEY IDEA

Three-voltmeter method: the series resistor voltage is in phase with the current, so the supply voltage is the phasor sum of V_R and V_L with angle ϕ between them. The cosine rule gives $V_L \cos\phi$, and with $I = V_R/R$ the load power reduces to $P = (V_S^2 - V_R^2 - V_L^2)/(2R)$. Only the three readings and R are needed.

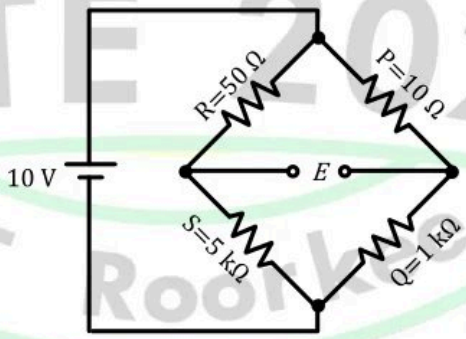
WATCH OUT FOR

Adding the voltages arithmetically ($9 + 199 = 208 \neq 200$) and concluding the data are inconsistent, when they are phasors at an angle. Other mistakes: dividing by $2V_R$ or by R instead of $2R$ (giving 17.7 W or 159 W), including the resistor's own loss $V_R^2/R = 40.5$ W in the load power, or using a minus sign in the cosine rule, which gives a negative $\cos\phi$.

Q.59 NAT · 2 MARKS

Electrical and Electronic Measurements › Bridges

Q.59 In the Wheatstone bridge shown below, the sensitivity of the bridge in terms of change in balancing voltage E for unit change in the resistance R, in mV/Ω, is _____ *(round off to two decimal places).*



Official answer: **-2.00 to -1.94, or 1.94 to 2.00**

SOLUTION

1. Check balance with the condition $RQ = PS$: $50 \times 1000 = 10 \times 5000 = 5 \times 10^4$. The bridge is balanced, so $E = 0$ at the operating point.
2. Take the bottom node as the reference. The battery's positive plate faces the top node, so the top node is at $V = 10$ V. The battery is connected across both dividers. The left node (R–S junction) is at $V_L = \dfrac{VS}{R+S}$ and the right node is at $V_R = \dfrac{VQ}{P+Q}$. V_R does not depend on R.
3. The output is $E = V_L - V_R$, so only the R–S divider responds to a change in R. Differentiating the divider gives
$$\frac{dE}{dR} = \frac{d}{dR}\left(\frac{VS}{R+S}\right) = -\frac{VS}{(R+S)^2}$$
4. Substitute $V = 10$ V, $S = 5000\ \Omega$ and $R = 50\ \Omega$:
$$\left|\frac{dE}{dR}\right| = \frac{10 \times 5000}{(5050)^2} = \frac{5 \times 10^4}{2.55025 \times 10^7} = 1.9606 \times 10^{-3}\ \text{V}/\Omega$$

5. The sensitivity is therefore $\approx 1.96\ \mathrm{mV/\Omega}$. The minus sign only shows that E falls as R rises (with E measured left node minus right node), which is why the official key accepts either sign.

KEY IDEA

Near balance, only the divider that contains the changed arm moves the detector voltage. With $E = \frac{VS}{R+S} - \frac{VQ}{P+Q}$, the sensitivity is $\left|\frac{dE}{dR}\right| = \frac{VS}{(R+S)^2}$. This is the same as the standard small-change result $\Delta E = \frac{VRS}{(R+S)^2} \cdot \frac{\Delta R}{R}$.

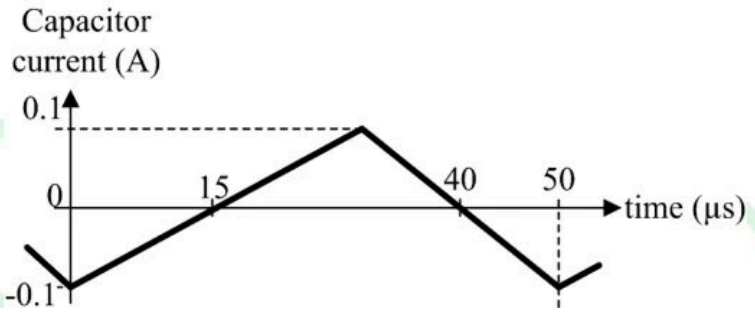
WATCH OUT FOR

There are three common slips. The first is using the equal-arm approximation $\Delta E \approx \frac{V}{4}\frac{\Delta R}{R}$, which gives $50\ \mathrm{mV/\Omega}$; it only holds when all four arms are equal, and here $R/S = 0.01$. The second is pairing R with the wrong arm, for example using $(R+P)$ instead of $(R+S)$; R and S are the two arms in series across the battery, meeting at the left detector terminal. The third is quoting the answer in $\mathrm{V/\Omega}$ (0.00196) when the question asks for $\mathrm{mV/\Omega}$.

Q.60 NAT · 2 MARKS

Power Electronics › DC-DC converters

Q.60 The steady state capacitor current of a conventional DC-DC buck converter, working in CCM, is shown in one switching cycle. If the input voltage is 30 V, the value of the inductor used, in mH, is _____ *(round off to one decimal place).*



Official answer: **1.7 to 1.9**

SOLUTION

1. In steady state the average capacitor current is zero, and in a buck converter the capacitor carries the ripple part of the inductor current. So the plotted triangle is the inductor ripple, and its peak-to-peak value is $\Delta I_L = 0.1 - (-0.1) = 0.2\ \mathrm{A}$.
2. From the figure, the minimum $-0.1\ \mathrm{A}$ occurs at $t = 0$ and again at $t = 50\ \mu\mathrm{s}$, so the switching period is $T = 50\ \mu\mathrm{s}$.
3. The inductor current rises while the switch is ON. The rising line goes from $-0.1\ \mathrm{A}$ at $t = 0$ to 0 at $t = 15\ \mu\mathrm{s}$, so its slope is $\frac{0.1}{15\ \mu\mathrm{s}}$. Reaching the peak, $+0.1\ \mathrm{A}$, means a rise of $0.2\ \mathrm{A}$, which takes $\frac{0.2}{0.1} \times 15 = 30\ \mu\mathrm{s}$. Check on the falling side: from the peak at $30\ \mu\mathrm{s}$ the current crosses zero at $40\ \mu\mathrm{s}$ ($0.1\ \mathrm{A}$ in $10\ \mu\mathrm{s}$) and reaches $-0.1\ \mathrm{A}$ at $50\ \mu\mathrm{s}$, as plotted.
4. So $T_{on} = 30\ \mu\mathrm{s}$, $T_{off} = 20\ \mu\mathrm{s}$, and the duty ratio is $D = \frac{30}{50} = 0.6$.
5. For the buck converter in CCM, $V_o = D\,V_s = 0.6 \times 30 = 18\ \mathrm{V}$.
6. During the ON interval the inductor voltage is $V_s - V_o = 12\ \mathrm{V}$, so $\Delta I_L = \frac{(V_s - V_o)\,DT}{L}$.
7. $$L = \frac{(V_s - V_o)\,DT}{\Delta I_L} = \frac{12 \times 30 \times 10^{-6}}{0.2} = 1.8 \times 10^{-3}\ \mathrm{H} = 1.8\ \mathrm{mH}$$

8. Check with the OFF interval, where the inductor voltage is $-V_o$: $L = \dfrac{V_o\,(1-D)\,T}{\Delta I_L} = \dfrac{18 \times 20 \times 10^{-6}}{0.2} = 1.8\ \text{mH}$, the same value.

KEY IDEA

In a buck converter the capacitor current is the inductor current minus its average, so the plotted triangle is the inductor ripple. The ON time read from the rising edge gives D. Then $L = (V_s - V_o)DT/\Delta I_L$.

WATCH OUT FOR

Taking the rise time as the zero crossing at $15\ \mu$s or the fall crossing at $40\ \mu$s instead of the peak at $30\ \mu$s, which gives the wrong duty ratio. Another slip is using $\Delta I_L = 0.1$ A (the peak) instead of the peak-to-peak 0.2 A.

Q.61 NAT · 2 MARKS

Signals and Systems › Fourier transform

An ideal low pass filter has frequency response given by

$$H(j\omega) = \begin{cases} 1, & |\omega| \le 200\pi \\ 0, & \text{otherwise} \end{cases}$$

Let $h(t)$ be its time domain representation. Then $h(0) =$ _____ (round off to the nearest integer).

Official answer: **200**

SOLUTION

1. Start from the inverse Fourier transform:
$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega)\, e^{j\omega t}\, d\omega$$
2. Set $t = 0$. Then $e^{j\omega t} = 1$, so $h(0)$ is the area under $H(j\omega)$ divided by 2π:
$$h(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega)\, d\omega$$
3. $H(j\omega) = 1$ only for $-200\pi \le \omega \le 200\pi$ and is 0 everywhere else. The area is therefore the width of the passband, 400π:
$$h(0) = \frac{1}{2\pi}\int_{-200\pi}^{200\pi} 1\, d\omega = \frac{400\pi}{2\pi} = 200$$
4. Check against the closed form.
$$h(t) = \frac{1}{2\pi}\int_{-200\pi}^{200\pi} e^{j\omega t}\, d\omega = \frac{1}{2\pi}\cdot\frac{e^{j200\pi t} - e^{-j200\pi t}}{jt} = \frac{\sin(200\pi t)}{\pi t}$$
Using $\sin x \approx x$ for small x:
$$h(0) = \lim_{t\to 0}\frac{\sin(200\pi t)}{\pi t} = \lim_{t\to 0}\frac{200\pi t}{\pi t} = 200$$
5. The result is already a whole number, so no rounding is needed: $h(0) = 200$.

KEY IDEA

The value of a signal at $t = 0$ is the total area under its Fourier transform divided by 2π: $h(0) = \frac{1}{2\pi}\int H(j\omega)\, d\omega$. For an ideal rectangular low-pass response of height 1 and cutoff ω_c, that area is $2\omega_c$, so $h(0) = \omega_c/\pi$. Here $\omega_c = 200\pi$ gives 200.

WATCH OUT FOR

Leaving out the $\frac{1}{2\pi}$ factor gives $400\pi \approx 1257$. Taking 200π as a frequency in hertz, or using only the positive half of the passband (width 200π instead of 400π), gives $h(0) = 100$. The passband runs from -200π to $+200\pi$ rad/s, and both halves count.

Q.62 NAT · 2 MARKS

Control Systems › State space

Consider the state-space model

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + Br(t),$$

$$y(t) = C\mathbf{x}(t)$$

where $\mathbf{x}(t)$, $r(t)$, $y(t)$ are the state, input and output, respectively. The matrices A, B, C are given below

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

The sum of the magnitudes of the poles is _____ *(round off to nearest integer)*.

Official answer: **3**

SOLUTION

1. The poles are the eigenvalues of A, which are the roots of the characteristic equation $\det(sI - A) = 0$.
2. $$sI - A = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix} \Rightarrow \det(sI - A) = s(s+3) + 2 = s^2 + 3s + 2$$
3. Factorise: $s^2 + 3s + 2 = (s+1)(s+2) = 0$, so $s = -1$ and $s = -2$.
4. Check for pole-zero cancellation. With $\operatorname{adj}(sI - A) = \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$, we get $G(s) = C(sI - A)^{-1}B = \dfrac{1}{s^2 + 3s + 2}$. The numerator is 1, so nothing cancels and both eigenvalues are poles of the transfer function too. (The matrix is in controllable canonical form, so you can also read the coefficients 2 and 3 straight from the last row.)
5. Sum of magnitudes: $|-1| + |-2| = 1 + 2 = 3$.

KEY IDEA

The poles of a state-space system are the eigenvalues of A, the roots of $\det(sI - A) = 0$. For a companion (controllable canonical) matrix, the last row $\begin{bmatrix} -a_0 & -a_1 \end{bmatrix}$ gives the characteristic polynomial $s^2 + a_1 s + a_0$ directly. Check $C(sI - A)^{-1}B$ for cancellation before treating every eigenvalue as a transfer-function pole.

WATCH OUT FOR

Adding the poles themselves, $(-1) + (-2) = -3$, which is the trace of A, instead of adding their magnitudes. Another common slip is a sign error in the last row: reading the polynomial as $s^2 - 3s - 2$ instead of $s^2 + 3s + 2$.

Q.63 NAT · 2 MARKS

Power Systems › Dispatch & control

Using shunt capacitors, the power factor of a 3-phase, 4 kV induction motor (drawing 390 kVA at 0.77 pf lag) is to be corrected to 0.85 pf lag. The line current of the capacitor bank, in A, is _____ (round off to one decimal place).

Official answer: **8.5 to 10.0**

SOLUTION

1. Real power of the motor is unchanged by the capacitors: $P = 390 \times 0.77 = 300.3\ \text{kW}$.
2. Initial reactive power: $Q_1 = 390 \sin(\cos^{-1} 0.77) = 390 \times 0.6381 = 248.85\ \text{kVAr}$ (lagging).
3. Reactive power needed at 0.85 lag: $Q_2 = P \tan(\cos^{-1} 0.85) = 300.3 \times 0.6197 = 186.1\ \text{kVAr}$.
4. Three-phase capacitor bank rating: $Q_c = Q_1 - Q_2 = 248.85 - 186.1 \approx 62.7\ \text{kVAr}$.
5. Line current of the bank: $I_c = \dfrac{Q_c}{\sqrt{3}\, V_L} = \dfrac{62.7 \times 10^3}{\sqrt{3} \times 4000} \approx 9.05\ \text{A} \approx 9.1\ \text{A}$, which lies in the key range 8.5 to 10.0.

KEY IDEA
Capacitors change only reactive power, so the bank supplies the difference between the motor's reactive power before and after correction. Then use the three-phase relation $Q_c = \sqrt{3} V_L I_c$ to get the line current.

WATCH OUT FOR
Using 390 kVA instead of the real power P in the $\tan$ formula for the final reactive power. Another is using the phase voltage in $Q_c = \sqrt{3} V_L I_c$, or treating the bank as single-phase and dividing by 4 kV alone.

Q.64 NAT · 2 MARKS

Power Systems › Dispatch & control

Two units, rated at 100 MW and 150 MW, are enabled for economic load dispatch. When the overall incremental cost is 10,000 Rs./MWh, the units are dispatched to 50 MW and 80 MW respectively. At an overall incremental cost of 10,600 Rs./MWh, the power output of the units are 80 MW and 92 MW, respectively. The total plant MW-output (without overloading any unit) at an overall incremental cost of 11,800 Rs./MWh is _____ (round off to the nearest integer).

Official answer: **216**

SOLUTION

1. Each unit has a linear incremental cost curve, so its output varies linearly with the system incremental cost λ (while it is within its limits): $P_i = P_i^0 + k_i(\lambda - \lambda_0)$, with $\lambda_0 = 10000$ and P_i^0 the output at that λ_0.
2. Unit 1 slope from the two given operating points: $k_1 = \dfrac{80 - 50}{10600 - 10000} = 0.05 \text{ MW per Rs./MWh}.$
3. Unit 2 slope: $k_2 = \dfrac{92 - 80}{10600 - 10000} = 0.02 \text{ MW per Rs./MWh}.$
4. At $\lambda = 11800$ the increase over the first point is $11800 - 10000 = 1800$.
5. Unit 1, unconstrained: $P_1 = 50 + 0.05 \times 1800 = 140 \text{ MW}$. This exceeds its 100 MW rating, and the question says no unit may be overloaded, so $P_1 = 100 \text{ MW}$.
6. Unit 2: $P_2 = 80 + 0.02 \times 1800 = 116 \text{ MW}$, which is within its 150 MW rating, so it stays as calculated.
7. Total plant output: $P = 100 + 116 = 216 \text{ MW}$.

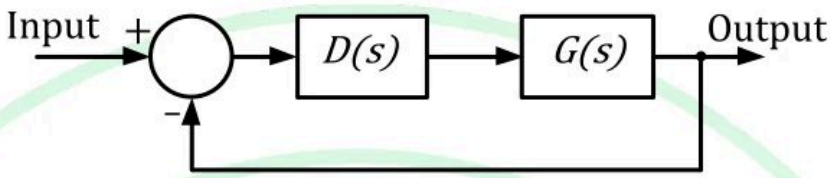
KEY IDEA
With linear incremental cost curves, each unit's output is a linear function of the common λ. Find each slope from the two given points, extrapolate to the new λ, then clamp any unit that passes its rating at that rating.

WATCH OUT FOR
Adding the unclamped outputs, $140 + 116 = 256$ MW, and so overloading the 100 MW unit. A unit that reaches its limit stays at the limit.

Q.65 NAT · 2 MARKS

Control Systems › Compensators & PID

Q.65 A controller $D(s)$ of the form $(1+K_Ds)$ is to be designed for the plant $G(s)=\frac{1000\sqrt{2}}{s(s+10)^2}$ as shown in the figure. The value of K_D that yields a phase margin of 45° at the gain cross-over frequency of 10 rad/sec is _____ *(round off to one decimal place).*



Official answer: **0.1**

SOLUTION

1. With unity negative feedback, the open-loop transfer function is
$$L(s)=D(s)G(s)=\frac{1000\sqrt{2}\,(1+K_Ds)}{s(s+10)^2}.$$
2. Plant magnitude at $\omega=10\ \text{rad/s}$: $|j10|=10$ and $|j10+10|^2=10^2+10^2=200$, so
$$|G(j10)|=\frac{1000\sqrt{2}}{10\times 200}=\frac{1000\sqrt{2}}{2000}=\frac{1}{\sqrt{2}}.$$
3. Plant phase at $\omega=10\ \text{rad/s}$: the integrator gives -90° and each of the two poles at $s=-10$ gives $-\tan^{-1}(10/10)=-45^\circ$, so
$$\angle G(j10)=-90^\circ-2(45^\circ)=-180^\circ.$$
4. Phase-margin condition at the gain crossover: $\text{PM}=180^\circ+\angle L(j10)=45^\circ$, so $\angle L(j10)=-135^\circ$. Since the plant already gives -180°, the controller has to add $+45^\circ$ at $\omega=10\ \text{rad/s}$.
5. Phase of the PD controller: $\angle D(j10)=\tan^{-1}(10K_D)=45^\circ$, so $10K_D=1$ and $K_D=0.1$.
6. Check that $10\ \text{rad/s}$ really is the gain crossover: $|D(j10)|=|1+j1|=\sqrt{2}$, so $|L(j10)|=\sqrt{2}\cdot\frac{1}{\sqrt{2}}=1$. The magnitude and phase conditions both hold at the same frequency, so $K_D=0.1$ is consistent.

KEY IDEA

At $\omega=10\ \text{rad/s}$ the plant has a phase of exactly -180° and a magnitude of $1/\sqrt{2}$. A 45° phase margin therefore needs the PD zero to add exactly 45° there, so $10K_D=1$. With that value, $|1+j|=\sqrt{2}$ cancels the plant's $1/\sqrt{2}$, so $10\ \text{rad/s}$ really is the gain crossover. The question is self-consistent, and the magnitude check confirms the answer.

WATCH OUT FOR

Using only one pole's phase (-45° in place of -90° for the squared factor $(s+10)^2$), which puts the plant phase at -135° and leads to the wrong conclusion that no controller phase is needed. Another mistake is computing $|(j10+10)^2|$ as $|j10+10|=10\sqrt{2}$ instead of 200, which spoils the magnitude check.

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