IAE ACADEMY INSTITUTE FOR ENGINEERS

GATE EE · 2020

IAE Academy free study material · Updated 27 October 2026

GATE EE 2020 Solved Paper

INSIDE

- All 65 questions, in the order of the paper
- The official answer to every question
- Step-by-step worked solutions
- The answer key on one page

Question paper and official answer key: IIT Delhi

Online, with the questions linked to their topic notes: **iaeacademy.in/study/papers/ee-2020**

FREE COUNSELLING

Preparing for GATE EE? Talk to IAE Academy.

Or call **+91 84204 36524**

Every day, 10:30 am to 7:30 pm

Chat on WhatsApp

Free. Not for sale. Pass it on.

About this paper

65 questions, 100 marks. Question types: 43 MCQ · 22 NAT.

Official answer key: IIT Delhi

This paper numbers General Aptitude GA Q.1 to GA Q.10 and the subject questions Q.1 to Q.55: 65 questions in all.

We placed each question under a subject and topic ourselves; the exam paper does not label them.

This paper online, with the questions linked to their topic notes:
iaeacademy.in/study/papers/ee-2020

Answer key

Q	TYPE	MARKS	ANSWER
GA Q.1	MCQ	1	**C**
GA Q.2	MCQ	1	**B**
GA Q.3	MCQ	1	**C**
GA Q.4	MCQ	1	**B**
GA Q.5	MCQ	1	**C**
GA Q.6	MCQ	2	**B**
GA Q.7	MCQ	2	**C**
GA Q.8	MCQ	2	**C**
GA Q.9	MCQ	2	**A**
GA Q.10	MCQ	2	**C**
Q.1	MCQ	1	**A or D**
Q.2	MCQ	1	**C**
Q.3	MCQ	1	**B**
Q.4	MCQ	1	**D**
Q.5	MCQ	1	**C**
Q.6	MCQ	1	**A**
Q.7	MCQ	1	**A**
Q.8	MCQ	1	**D**
Q.9	MCQ	1	**A**
Q.10	MCQ	1	**A**
Q.11	MCQ	1	**C**
Q.12	MCQ	1	**Marks to all**
Q.13	MCQ	1	**C**
Q.14	MCQ	1	**A**
Q.15	MCQ	1	**B**
Q.16	NAT	1	**0.8774 to 0.8952**
Q.17	NAT	1	**4.10 to 4.40**
Q.18	NAT	1	**1.700 to 1.750**
Q.19	NAT	1	**13.80 to 14.20**
Q.20	NAT	1	**3**
Q.21	NAT	1	**162.41 to 162.59**
Q.22	NAT	1	**19.90 to 20.20**
Q.23	NAT	1	**123.00 to 135.00**
Q.24	NAT	1	**2**
Q.25	NAT	1	**0**
Q.26	MCQ	2	**A**
Q.27	MCQ	2	**C**
Q.28	MCQ	2	**C**
Q.29	MCQ	2	**C**
Q.30	MCQ	2	**D**
Q.31	MCQ	2	**B**
Q.32	MCQ	2	**C**
Q.33	MCQ	2	**D**
Q.34	MCQ	2	**B**
Q.35	MCQ	2	**C**
Q.36	MCQ	2	**B**
Q.37	MCQ	2	**C**
Q.38	MCQ	2	**B**
Q.39	MCQ	2	**C**
Q.40	MCQ	2	**D**
Q.41	MCQ	2	**A**
Q.42	MCQ	2	**Marks to all**
Q.43	MCQ	2	**A**
Q.44	NAT	2	**-3.05 to -2.95**
Q.45	NAT	2	**54.00 to 56.00**
Q.46	NAT	2	**123.00 to 127.00**
Q.47	NAT	2	**1.11 to 1.13**
Q.48	NAT	2	**Marks to all**
Q.49	NAT	2	**1.01 to 1.03**
Q.50	NAT	2	**4.95 to 5.05**
Q.51	NAT	2	**4.51 to 5.10**
Q.52	NAT	2	**11.50 to 12.00**
Q.53	NAT	2	**Marks to all**
Q.54	NAT	2	**210**
Q.55	NAT	2	**138.10 to 139.20**

General Aptitude GA Q.1 to Q.10

GA Q.1 MCQ · 1 MARK

General Aptitude › Grammar & vocabulary

This book, including all its chapters, _____ interesting. The students as well as the instructor _____ in agreement about it.

(A) is, was

(B) are, are

(C) is, are

(D) were, was

Official answer: **C**

SOLUTION

1. **Blank 1: find the real subject.** The subject is "This book", which is singular. The phrase "including all its chapters" sits between commas. It is a parenthetical add-on and does not change the number of the subject.
2. **Subject-verb agreement:** a singular subject takes a singular verb, so blank 1 is **is**. That rules out (B) "are" and (D) "were", which are both plural forms.
3. **Blank 2: the "as well as" rule.** In "A as well as B", the verb agrees with A, the noun before "as well as". "As well as the instructor" is an add-on phrase, like "along with" or "together with". It does not create a compound subject the way "and" does.
4. Here A is "The students", which is plural, so the verb must be plural: **are**. The present tense also matches the first sentence ("is interesting").
5. Check (A): "was" is singular, which does not agree with "The students", and it is past tense, which clashes with "is" in the first sentence. Rejected.
6. That leaves "is, are", which is option (C).

KEY IDEA

The verb agrees with the head subject. Phrases set off by commas ("including all its chapters") and add-on phrases ("as well as", "along with", "together with") do not change the subject's number. Only "and" makes a compound, plural subject.

WATCH OUT FOR

Matching the verb to the nearest noun. That gives "are" after "chapters" (plural) or "was" after "the instructor" (singular). Another mistake is treating "as well as" like "and". Here that happens to give the same plural verb, but with a singular first noun it would give the wrong one (e.g. "The instructor as well as the students IS ...").

GA Q.2 MCQ · 1 MARK

General Aptitude › Grammar & vocabulary

People were prohibited _____ their vehicles near the entrance of the main administrative building.

(A) to park

(B) from parking

(C) parking

(D) to have parked

Official answer: **B**

SOLUTION

1. The blank comes right after the verb **prohibit**, so the answer depends on which pattern *prohibit* takes. That is a matter of usage.

2. Verb pattern: *prohibit* takes an object, then **from**, then a gerund (the *-ing* form): *prohibit someone from doing something*. In the passive the object becomes the subject and *from* + gerund stays: *someone is prohibited from doing something*.
3. The sentence is passive (*People were prohibited* ...), so the blank needs **from** + gerund: *prohibited from parking*.
4. (A) *to park*: standard modern English does not put a *to*-infinitive after *prohibit*. That pattern belongs to *forbid* (*were forbidden to park*). (A) is wrong.
5. (C) *parking*: *prohibit* takes a bare gerund only when the activity itself is the object, as in the active *The rules prohibit parking near the entrance*. Here the people are the ones prohibited, so after the passive *People were prohibited* you cannot drop *from*. (C) is wrong.
6. (D) *to have parked*: a perfect infinitive places the parking before the prohibition, which makes no sense for a ban on future action. It is also the infinitive pattern that *prohibit* does not take. (D) is wrong.
7. The correct sentence is: *People were prohibited from parking their vehicles near the entrance of the main administrative building.* The answer is (B).

KEY IDEA

Prohibit takes **from** + gerund: *prohibit someone from doing something*, or in the passive *someone is prohibited from doing something*. Unlike *forbid*, it does not take a *to*-infinitive.

WATCH OUT FOR

Carrying the *forbid* pattern (*forbidden to park*) over to *prohibit* and picking (A). Another mistake is picking (C) because *prohibit parking* is correct in the active voice, without noticing that the passive *were prohibited*, with people as the subject, needs *from* before the gerund.

GA Q.3 MCQ · 1 MARK

General Aptitude › Reasoning

Select the word that fits the analogy:

Do : Undo :: Trust : _____

(A) Entrust

(B) Intrust

(C) Distrust

(D) Untrust

Official answer: **C**

SOLUTION

1. Find the relation in the first pair. "Undo" is the verb "Do" with the negating prefix un- added, and it means to reverse or cancel what was done. So the relation is: a word and its opposite, formed by adding the negating prefix that English normally uses with that word.
2. Apply the same relation to "Trust". We need the accepted English word that means the opposite of trust and is formed from "trust" with a negating prefix.
3. (A) "Entrust" means to give something or someone into another person's care. The prefix en- here means "put into"; it does not negate the word. Rejected.
4. (B) "Intrust" is an old, rarely used spelling of "entrust", with the same meaning. Rejected.
5. (D) "Untrust" is not used in standard modern English. The negating prefix that goes with "trust" is dis-, not un-. Rejected.
6. (C) "Distrust" means to lack trust in someone or something. It is the accepted opposite of "trust": dis- negates "trust" the same way un- negates "do".
7. So Do : Undo :: Trust : Distrust. The answer is (C).

KEY IDEA
In a word-formation analogy, first name exactly how the second word is made from the first. Here a negating prefix turns a word into its opposite (do → undo). Then use the prefix English actually uses with the new word. For "trust" that prefix is dis-, which gives "distrust".

WATCH OUT FOR
Copying the prefix instead of the relation. Students see un- in "Undo" and pick "Untrust", but English negates "trust" with dis-. Another trap is taking "Entrust" or "Intrust" because they look like prefixed forms of "trust". Neither one negates it: both mean to hand something over to someone's care.

GA Q.4 MCQ · 1 MARK

General Aptitude › Grammar & vocabulary

Stock markets _____ at the news of the coup.

(A) poised
(B) plunged
(C) plugged
(D) probed

Official answer: **B**

SOLUTION

1. **Work out what the blank needs.** The blank is the main verb of the sentence, in the past tense. Only the phrase "at the news of the coup" follows it and there is no object, so the verb has to work intransitively: it describes something the markets themselves did.
2. **Use the context.** A coup is a sudden, often violent seizure of power. It creates political uncertainty, investors sell, and prices fall sharply. So the verb should mean a sudden fall.
3. **(A) poised.** The word is almost always used as an adjective, meaning balanced or ready ("markets are poised for a rally"). As a verb, "to poise" means to balance something and takes an object. "Stock markets poised at the news" is not an idiomatic complete sentence, and it does not describe a reaction to bad news.
4. **(B) plunged.** Used intransitively, "to plunge" means to fall suddenly and steeply. "Stock markets plunged at the news of the coup" is grammatical, and it is the standard way to describe how markets react to a shock like a coup.
5. **(C) plugged.** "To plug" normally takes an object (you plug a hole, or plug a product). It has nothing to do with prices falling, and it leaves the sentence without an object.
6. **(D) probed.** "To probe" means to investigate. It takes an object ("probe the matter") or, used intransitively, needs "into" ("probe into the matter"). Markets do not investigate, and the sentence has no object for the verb to act on.
7. Only **plunged** fits both the grammar (a complete intransitive verb) and the meaning (a sharp fall after bad news). Answer: **(B)**.

KEY IDEA
The blank needs a verb with no object that describes how markets react to bad political news. "Plunge", used intransitively, means to fall suddenly and steeply, which is exactly what markets do after a coup.

WATCH OUT FOR
Being thrown by the look-alike words. Plunged, plugged and probed all start with "p" and differ by a letter or two, and "poised" is a common word in market reports ("markets are poised to rise"). Read each option in the sentence for meaning and check whether it needs an object. Don't pick the word that just sounds like finance.

GA Q.5 MCQ · 1 MARK

General Aptitude › Numerical

If P, Q, R, S are four individuals, how many teams of size exceeding one can be formed, with Q as a member?

(A) 5

(B) 6

(C) 7

(D) 8

Official answer: **C**

SOLUTION

1. Q is in every team, so a team is decided only by which of the other three people (P, R, S) join Q.
2. "Size exceeding one" means at least one of P, R, S must join Q.
3. Each of P, R, S is either in or out, independently, which gives $2^3 = 8$ subsets. Remove the empty subset, because that leaves Q alone in a team of size 1: $2^3 - 1 = 7$.
4. Check by team size, using combinations: size 2 gives $\binom{3}{1} = 3$ (QP, QR, QS), size 3 gives $\binom{3}{2} = 3$ (QPR, QPS, QRS), size 4 gives $\binom{3}{3} = 1$ ($PQRS$).
5. Total $= 3 + 3 + 1 = 7$, which is option (C).

KEY IDEA

Fix Q in the team, then count the subsets of the other three people. There are $2^3 = 8$ of them. Drop the empty one, because a team of Q alone has size 1. That leaves 7.

WATCH OUT FOR

Two slips are common. The first is keeping the empty subset, which counts Q alone as a team and gives 8 (option D). The second is leaving out the full team $PQRS$, which gives 6 (option B). A third error is counting every team of size 2 or more from all four people, which is $2^4 - 4 - 1 = 11$, and ignoring the condition that Q must be a member.

GA Q.6 MCQ · 2 MARKS

General Aptitude › Reading

Non-performing Assets (NPAs) of a bank in India is defined as an asset, which remains unpaid by a borrower for a certain period of time in terms of interest, principal, or both. Reserve Bank of India (RBI) has changed the definition of NPA thrice during 1993-2004, in terms of the holding period of loans. The holding period was reduced by one quarter each time. In 1993, the holding period was four quarters (360 days).

Based on the above paragraph, the holding period of loans in 2004 after the third revision was _____ days.

(A) 45

(B) 90

(C) 135

(D) 180

Official answer: **B**

SOLUTION

1. Read the unit from the passage. The holding period is counted in quarters, and "four quarters (360 days)" gives $1 \text{ quarter} = \frac{360}{4} = 90 \text{ days}$.
2. So "reduced by one quarter each time" means one quarter (90 days) comes off at each revision. It does not mean the period is cut by 25%.

3. The passage says RBI changed the definition "thrice", so there are three reductions between 1993 and 2004. The question asks for the period after the third revision.
4. Apply the reductions one at a time: $4 \to 3 \to 2 \to 1$ quarter, i.e. $360 \to 270 \to 180 \to 90$ days.
5. Or in one line: $360 - 3 \times 90 = 90$ days.
6. Check the other reading. Cutting by 25% each time gives $360 \times (0.75)^3 = 151.875$ days. That is not an option, which supports the one-quarter-of-time reading. The answer is 90 days, option (B).

KEY IDEA
The passage sets its own unit: "four quarters (360 days)" makes one quarter 90 days. Three revisions, each taking off one quarter, bring the period from 4 quarters down to 1 quarter, which is 90 days.

WATCH OUT FOR
Reading "reduced by one quarter" as a 25% cut compounded three times, which gives 151.875 days and matches no option. Another slip is stopping after two revisions (180 days, option D), or making four cuts in place of the three that "thrice" means (0 days), or halving the period each time (45 days, option A).

GA Q.7 MCQ · 2 MARKS

General Aptitude › Reasoning

Select the next element of the series: Z, WV, RQP, _

(A) LKJI
(B) JIHG
(C) KJIH
(D) NMLK

Official answer: **C**

SOLUTION

1. Write each letter's position in the alphabet (A = 1, ..., Z = 26): Z = 26; WV = 23, 22; RQP = 18, 17, 16.
2. Group size: the terms have 1, 2 and 3 letters, so the next term has 4 letters. Inside each term the letters run backwards, one step at a time.
3. Look at the letters skipped between one term and the next. Z to W skips Y, X (2 letters). V to R skips U, T, S (3 letters). The gap grows by 1 each time, so after P we skip 4 letters: O, N, M, L.
4. So the next term starts at K (position 11). Check with the first letters: $26 \to 23 \to 18$ has differences $3, 5$, so the next difference is 7, and $18 - 7 = 11$, which is K.
5. Four letters counting down from K: K (11), J (10), I (9), H (8), giving KJIH.
6. Check with the last letters: $26 \to 22 \to 16 \to 8$ has differences $4, 6, 8$, which is consistent. The other options break the rule: NMLK skips only O (1 letter), LKJI skips 3 letters (O, N, M) and JIHG skips 5. So the answer is (C) KJIH.

KEY IDEA
Two patterns run at once. Term length goes up by one (1, 2, 3, 4 letters, each term counting backwards), and the number of letters skipped between terms also goes up by one (2, 3, 4). So after P you skip O, N, M, L and start at K.

WATCH OUT FOR
Seeing only the four-letter length and picking the first backward run after P that looks right, such as NMLK (skips only O) or LKJI (skips O, N, M). Both miss the growing gap. Choosing JIHG means skipping one letter too many.

GA Q.8 MCQ · 2 MARKS

General Aptitude › Numerical

In four-digit integer numbers from 1001 to 9999, the digit group “37” (in the same sequence) appears _____ times.

(A) 270

(B) 279

(C) 280

(D) 299

Official answer: **C**

SOLUTION

1. A four-digit number $d_1d_2d_3d_4$ has three adjacent pairs of positions where "37" can sit: (d_1d_2), (d_2d_3) or (d_3d_4). Count the occurrences in each position separately, then add them.
2. Position $d_1d_2 = 37$: the numbers are $37__$. Both d_3 and d_4 are free, giving $10 \times 10 = 100$ numbers (3700 to 3799, all inside the range).
3. Position $d_2d_3 = 37$: the numbers are d_137d_4. The leading digit d_1 cannot be 0, so it has 9 choices, and d_4 has 10: $9 \times 10 = 90$ (1370 to 9379).
4. Position $d_3d_4 = 37$: the numbers are d_1d_237 with $d_1 \neq 0$: $9 \times 10 = 90$ (1037 to 9937). The smallest, 1037, is above 1001, so the lower limit of the range removes nothing.
5. Each occurrence is counted once, in its own position, so the total number of occurrences is
$$100 + 90 + 90 = 280$$
6. Check for overlaps: the first and second positions share d_2, which would have to be both 7 and 3, so they cannot both hold "37". The same is true of the second and third positions. Only the first and third positions can both hold "37", and that happens in exactly one number, 3737. That number contains the group **twice**, and both occurrences count.
7. So "37" appears 280 times, spread over $280 - 1 = 279$ different numbers. The question asks how many times the group appears, so the answer is 280, option (C).

KEY IDEA

Count the occurrences position by position. "37" can be the first two, middle two or last two digits, which gives 100 + 90 + 90. Only the first and third positions can both hold "37" at once, and only in 3737. That number has two occurrences, and both are counted.

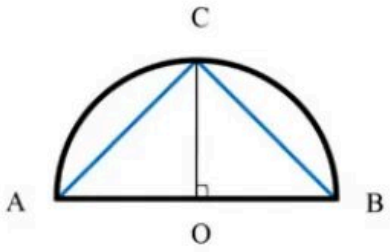
WATCH OUT FOR

Answering 279 (option B). That is the number of different numbers that contain "37": 3737 is counted once as a number even though it holds the group twice. The question asks how many times the group appears, so 3737 contributes 2 and the answer is 280. Another slip is allowing a leading 0 in the middle or last position (giving 100 instead of 90), which leads to answers such as 300 that are not among the options.

GA Q.9 MCQ · 2 MARKS

General Aptitude › Geometry

Q.No. 9 Given a semicircle with O as the centre, as shown in the figure, the ratio $\frac{\overline{AC}+\overline{CB}}{\overline{AB}}$ is _____, where $\overline{AC}$, $\overline{CB}$ and $\overline{AB}$ are chords.



(A) $\sqrt{2}$

(B) $\sqrt{3}$

(C) 2

(D) 3

Official answer: **A**

SOLUTION

1. Let the radius be r. Then $OA = OB = OC = r$, and the diameter is $\overline{AB} = 2r$.
2. The figure marks $OC \perp AB$. So C is the top of the arc, and triangles AOC and BOC both have a right angle at O.
3. Apply Pythagoras' theorem in $\triangle AOC$: $\overline{AC} = \sqrt{OA^2 + OC^2} = \sqrt{r^2 + r^2} = r\sqrt{2}$. The same working in $\triangle BOC$ gives $\overline{CB} = r\sqrt{2}$.
4. Check: the angle in a semicircle is 90°, so $\angle ACB = 90^\circ$ and $\overline{AC}^2 + \overline{CB}^2$ should equal $\overline{AB}^2$. Here $2r^2 + 2r^2 = 4r^2 = (2r)^2$, so the lengths agree.
5. Substitute into the ratio:

$$\frac{\overline{AC}+\overline{CB}}{\overline{AB}} = \frac{r\sqrt{2}+r\sqrt{2}}{2r} = \frac{2\sqrt{2}\,r}{2r} = \sqrt{2}$$

6. The radius cancels, so the ratio is $\sqrt{2}$ for any semicircle. That is option (A).

KEY IDEA

When C is the top of the semicircle ($OC \perp AB$), the triangles AOC and BOC are right-angled at O, with both legs equal to the radius r. So each chord is $r\sqrt{2}$ and the diameter is $2r$. The ratio does not depend on r and equals $\frac{2r\sqrt{2}}{2r} = \sqrt{2}$.

WATCH OUT FOR

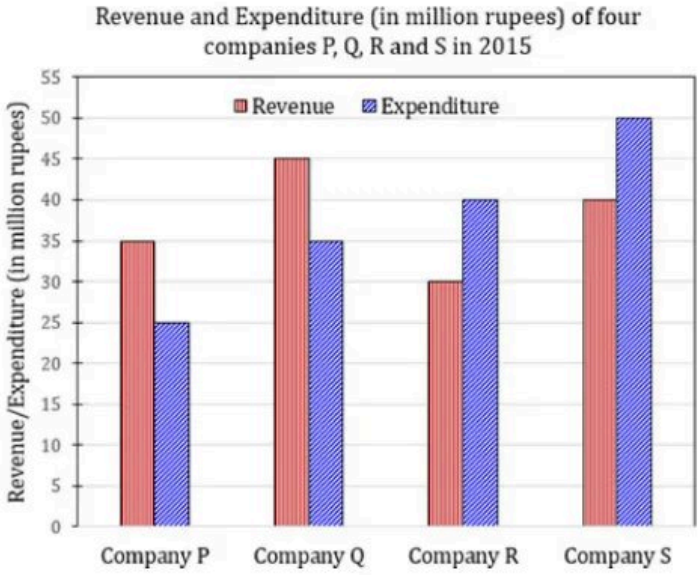
One mistake is to take each chord as equal to the radius, which gives $\frac{2r}{2r} = 1$. Another is to use $\overline{AB} = r$ in place of $2r$, which gives $2\sqrt{2}$. A third is to add the chords as squares, since $\overline{AC}^2 + \overline{CB}^2 = \overline{AB}^2$, and then pick option (C) 2. The question adds the lengths, not their squares.

GA Q.10 MCQ · 2 MARKS

General Aptitude › Data interpretation

Q.No. 10

The revenue and expenditure of four different companies P, Q, R and S in 2015 are shown in the figure. If the revenue of company Q in 2015 was 20% more than that in 2014, and company Q had earned a profit of 10% on expenditure in 2014, then its expenditure (in million rupees) in 2014 was _____.



(A) 32.7
(B) 33.7
(C) 34.1
(D) 35.1

Official answer: **C**

SOLUTION

1. Read the chart. Only company Q's 2015 revenue matters here: its revenue bar stands at 45 million rupees. The 2015 expenditure bar (35) is a distractor, because the question asks about 2014.
2. Revenue in 2015 was 20% more than in 2014, so $R_{2015} = 1.2\,R_{2014}$. That gives $R_{2014} = \dfrac{45}{1.2} = 37.5$ million rupees.
3. A profit of 10% on expenditure means profit $= 0.1\,E_{2014}$, so revenue $= E_{2014} + 0.1\,E_{2014} = 1.1\,E_{2014}$.
4. Set $1.1\,E_{2014} = 37.5$. Then $E_{2014} = \dfrac{37.5}{1.1} = 34.09\ldots \approx 34.1$ million rupees.
5. This value matches option (C).

KEY IDEA

Work backwards from the chart value in two steps. A 20% rise means dividing by 1.2 to get the 2014 revenue. A 10% profit on expenditure means revenue $= 1.1\times$ expenditure, so you then divide by 1.1. Together: $E_{2014} = \dfrac{45}{1.2 \times 1.1} = \dfrac{45}{1.32} \approx 34.1$.

WATCH OUT FOR

There are three usual slips. (1) Taking 20% off the 2015 revenue ($45 \times 0.8 = 36$) instead of dividing by 1.2. (2) Treating the profit as 10% of revenue, which gives $E = 0.9 \times 37.5 = 33.75$, close to the distractor 33.7. (3) Using Q's 2015 expenditure bar (35), which gives option (D) 35.1 or something near it. The 2015 expenditure is not needed at all.

Electrical Engineering Q.1 to Q.55

Q.1 MCQ · 1 MARK

Engineering Mathematics › Calculus

$ax^3 + bx^2 + cx + d$ is a polynomial on real x over real coefficients a, b, c, d wherein $a \neq 0$. Which of the following statements is true?

(A) d can be chosen to ensure that $x = 0$ is a root for any given set a, b, c.

(B) No choice of coefficients can make all roots identical.

(C) a, b, c, d can be chosen to ensure that all roots are complex.

(D) c alone cannot ensure that all roots are real.

Official answer: **A or D**

Q.2 MCQ · 1 MARK

Engineering Mathematics › Calculus

Which of the following is true for all possible non-zero choices of integers $m, n;\ m \neq n$, or all possible non-zero choices of real numbers $p, q;\ p \neq q$, as applicable?

(A) $\frac{1}{\pi}\int_0^{\pi} \sin m\theta \sin n\theta \, d\theta = 0$

(B) $\frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \sin p\theta \sin q\theta \, d\theta = 0$

(C) $\frac{1}{2\pi}\int_{-\pi}^{\pi} \sin p\theta \cos q\theta \, d\theta = 0$

(D) $\lim_{\alpha\to\infty} \frac{1}{2\alpha}\int_{-\alpha}^{\alpha} \sin p\theta \sin q\theta \, d\theta = 0$

Official answer: **C**

SOLUTION

1. The statement has to hold for **every** allowed choice, so one counter-example rules an option out. $m \neq n$ still allows $m = -n$, and $p \neq q$ still allows $p = -q$.
2. Product-to-sum identity: $\sin a\theta \sin b\theta = \frac{1}{2}\left[\cos(a-b)\theta - \cos(a+b)\theta\right]$.
3. (A): take $m = 1,\ n = -1$. Both are non-zero and unequal, so the choice is allowed. Then $\sin\theta \sin(-\theta) = -\sin^2\theta$, so
$$\frac{1}{\pi}\int_0^{\pi} -\sin^2\theta \, d\theta = \frac{1}{\pi}\left(-\frac{\pi}{2}\right) = -\frac{1}{2} \neq 0.$$
The orthogonality result also needs $m + n \neq 0$, so (A) is not always true.
4. (B): $[-\pi/2, \pi/2]$ is not a full period, so orthogonality does not apply. For $p = 1,\ q = 2$ the integrand is $\sin\theta \sin 2\theta = 2\sin^2\theta\cos\theta$, which is even, so
$$\frac{1}{2\pi}\cdot 2\int_0^{\pi/2} 2\sin^2\theta\cos\theta \, d\theta = \frac{1}{\pi}\left[\frac{2\sin^3\theta}{3}\right]_0^{\pi/2} = \frac{2}{3\pi} \neq 0.$$
(B) is false.
5. (C): odd-function property. $\sin p\theta$ is odd and $\cos q\theta$ is even, so $f(\theta) = \sin p\theta \cos q\theta$ satisfies $f(-\theta) = -f(\theta)$. The integral of an odd function over the symmetric interval $[-\pi, \pi]$ is 0, and this holds for **every** real p, q.
6. (D): for $p + q \neq 0$ the identity gives
$$\frac{1}{2\alpha}\int_{-\alpha}^{\alpha} \sin p\theta \sin q\theta \, d\theta = \frac{1}{2\alpha}\left[\frac{\sin(p-q)\alpha}{p-q} - \frac{\sin(p+q)\alpha}{p+q}\right] \to 0.$$
But $q = -p$ is allowed. Then the integrand is $-\sin^2 p\theta$ and
$$\frac{1}{2\alpha}\int_{-\alpha}^{\alpha} -\sin^2 p\theta \, d\theta = -\frac{1}{2} + \frac{\sin 2p\alpha}{4p\alpha} \to -\frac{1}{2} \neq 0.$$

(D) fails.

7. Only (C) is true for every allowed choice. Answer: **(C)**.

KEY IDEA

The only property that holds for every allowed choice is symmetry: $\sin p\theta \cos q\theta$ is odd, so its integral over $[-\pi, \pi]$ is zero whatever p and q are. The sine-sine orthogonality results in (A), (B) and (D) need $m + n \neq 0$ (or $p + q \neq 0$), and (B) also needs a full period, so each one has a counter-example.

WATCH OUT FOR

Choosing (A) or (D) from the memorised orthogonality relation "different frequencies integrate to zero". The question only requires $m \neq n$ (or $p \neq q$) and non-zero values, so negative values are allowed. With $n = -m$ or $q = -p$ the product becomes $-\sin^2$, and its integral (or long-run average) is $-\frac{1}{2}$, not 0.

Q.3 MCQ · 1 MARK

Signals and Systems › Laplace

Which of the following statements is true about the two sided Laplace transform?

(A) It exists for every signal that may or may not have a Fourier transform.

(B) It has no poles for any bounded signal that is non-zero only inside a finite time interval.

(C) The number of finite poles and finite zeroes must be equal.

(D) If a signal can be expressed as a weighted sum of shifted one sided exponentials, then its Laplace Transform will have no poles.

Official answer: **B**

SOLUTION

1. **(A) is false.** The two-sided transform $X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt$ exists only if some vertical strip of the s-plane is a region of convergence (ROC). Take $x(t) = 1$ for all t. The part for $t > 0$ converges only for $\sigma > 0$, and the part for $t < 0$ converges only for $\sigma < 0$. No σ satisfies both, so the ROC is empty and the transform does not exist.
2. **ROC of a finite-duration signal.** Let $x(t)$ be bounded, $|x(t)| \leq M$, and zero outside $[T_1, T_2]$. For every σ,
$$|X(s)| \leq \int_{T_1}^{T_2} Me^{-\sigma t}\,dt < \infty,$$
because a continuous function is integrated over a finite interval. So the ROC is the entire s-plane.
3. **(B) is true.** A pole is a point where $X(s)$ becomes unbounded, and a pole can never lie inside the ROC. Here the ROC is the whole s-plane, so $X(s)$ is finite everywhere and has no finite poles.
4. Check with a rectangular pulse: $x(t) = u(t) - u(t - T)$ gives $X(s) = \dfrac{1 - e^{-sT}}{s}$. The apparent pole at $s = 0$ is not a pole, because $\lim_{s\to 0} \dfrac{1 - e^{-sT}}{s} = T$ is finite. The singularity is removable.
5. **(C) is false.** For $u(t)$ the transform is $\dfrac{1}{s}$ with ROC $\mathrm{Re}(s) > 0$. It has one finite pole and no finite zero. In general, a strictly proper rational $X(s)$ has fewer finite zeros than finite poles.
6. **(D) is false.** Take one shifted one-sided exponential: $e^{-a(t-t_0)}u(t - t_0) \leftrightarrow \dfrac{e^{-st_0}}{s + a}$, with ROC $\mathrm{Re}(s) > -a$. It has a pole at $s = -a$. A weighted sum of such terms has poles at $s = -a_k$ unless the terms are specially arranged to cancel, as in the rectangular pulse of step 4.
7. Only statement (B) holds. The answer is (B).

KEY IDEA

A bounded, finite-duration signal gives a Laplace integral over a finite interval of a bounded integrand. That integral converges for every s, so the ROC is the whole s-plane. Poles cannot lie inside the ROC, so there are none. Any apparent pole, such as s = 0 in (1 - e^{-sT})/s, is a removable singularity.

WATCH OUT FOR

Seeing the factor 1/s in the rectangular-pulse transform (1 - e^{-sT})/s and concluding that finite-duration signals have a pole at s = 0. The numerator also vanishes there, so the limit is T and there is no pole. Two other errors: picking (A) because the Laplace transform 'generalises' the Fourier transform (a constant or two-sided signal can have an empty ROC), and picking (D) because only the delay factor e^{-st_0} is remembered and the 1/(s+a) factor is forgotten.

Q.4 MCQ · 1 MARK

Signals and Systems › z-transform

Consider a signal $x[n] = \left(\frac{1}{2}\right)^n 1[n]$, where $1[n] = 0$ if $n < 0$, and $1[n] = 1$ if $n \geq 0$. The z-transform of $x[n-k]$, $k > 0$ is $\frac{z^{-k}}{1 - \frac{1}{2}z^{-1}}$ with region of convergence being

(A) $|z| < 2$

(B) $|z| > 2$

(C) $|z| < 1/2$

(D) $|z| > 1/2$

Official answer: **D**

SOLUTION

1. **Write down the delayed signal.** With $y[n] = x[n-k]$,
$$y[n] = \left(\frac{1}{2}\right)^{n-k} 1[n-k],$$
which is zero for $n < k$ and equal to $\left(\frac{1}{2}\right)^{n-k}$ for $n \geq k$. Because $k > 0$, the sequence is right-sided and causal.

2. **Take the z-transform directly.**
$$Y(z) = \sum_{n=k}^{\infty} \left(\frac{1}{2}\right)^{n-k} z^{-n}.$$
Put $m = n - k$:
$$Y(z) = z^{-k} \sum_{m=0}^{\infty} \left(\frac{1}{2}z^{-1}\right)^m.$$

3. **Convergence condition.** The geometric series converges only when $\left|\frac{1}{2}z^{-1}\right| < 1$, that is, when $|z| > \frac{1}{2}$. Its sum there is $\frac{1}{1 - \frac{1}{2}z^{-1}}$, so
$$Y(z) = \frac{z^{-k}}{1 - \frac{1}{2}z^{-1}},$$
which is the expression given in the question. The factor z^{-k} is finite at every $z \neq 0$, and $z = 0$ is already outside $|z| > \frac{1}{2}$, so the delay does not shrink the region.

4. **Check with the poles.** Written in positive powers,
$$Y(z) = \frac{z^{1-k}}{z - \frac{1}{2}}.$$
It has a pole at $z = \frac{1}{2}$. For $k \geq 2$ it also has a pole of order $k - 1$ at $z = 0$; for $k = 1$ there is no pole at the origin. The outermost pole is at $|z| = \frac{1}{2}$. A right-sided sequence has its ROC outside the outermost

pole, and because this one is causal the ROC also includes $z=\infty$. So the ROC is $|z|>\frac{1}{2}$.

5. **Rule out the other options.** (A) $|z|<2$ and (C) $|z|<\frac{1}{2}$ are regions inside a circle, which belong to left-sided sequences. (B) $|z|>2$ lies inside the true ROC but is not all of it, because the series also converges for $\frac{1}{2}<|z|\le 2$.

$$x[n-k] \;\longleftrightarrow\; \frac{z^{-k}}{1-\frac{1}{2}z^{-1}}, \qquad \text{ROC: } |z|>\frac{1}{2}.$$

The answer is (D).

KEY IDEA

$\left(\frac{1}{2}\right)^n 1[n]$ is a right-sided exponential with a single pole at $z=\frac{1}{2}$, so its ROC is $|z|>\frac{1}{2}$. Delaying it by $k>0$ multiplies the transform by z^{-k}. That factor can only change whether $z=0$ or $z=\infty$ belongs to the ROC, and $z=0$ is already outside it, so the ROC stays $|z|>\frac{1}{2}$.

WATCH OUT FOR

Inverting the pole and writing $|z|>2$: the pole sits at $z=a=\frac{1}{2}$, not at $1/a$. A second mistake is to think the factor z^{-k}, or the shift itself, turns the sequence left-sided and gives $|z|<\frac{1}{2}$. A delay by $k>0$ keeps the sequence causal. Some students also say z^{-k} adds a pole of order k at the origin. In fact $z/(z-\frac{1}{2})$ already has a zero there, so the pole has order $k-1$ (none when $k=1$). Either way the annulus does not change.

Q.5 MCQ · 1 MARK

Engineering Mathematics › Complex analysis

The value of the following complex integral, with C representing the unit circle centered at origin in the counterclockwise sense, is:

$$\int_C \frac{z^2+1}{z^2-2z}\,dz$$

(A) $8\pi i$

(B) $-8\pi i$

(C) $-\pi i$

(D) πi

Official answer: **C**

SOLUTION

1. Factor the denominator: $z^2-2z=z(z-2)$, so $f(z)=\dfrac{z^2+1}{z(z-2)}$. The numerator is non-zero at $z=0$ (value 1) and at $z=2$ (value 5), so both are simple poles.
2. Locate the poles relative to C, the circle $|z|=1$: $z=0$ lies inside ($|0|<1$), while $z=2$ lies outside ($|2|=2>1$). Neither pole is on the contour, and only $z=0$ contributes.
3. Residue at the simple pole $z=0$:

$$\text{Res}_{z=0} f = \lim_{z\to 0} z\,f(z) = \lim_{z\to 0}\frac{z^2+1}{z-2} = \frac{0+1}{0-2} = -\frac{1}{2}$$

4. Cauchy's residue theorem for a closed contour traversed counterclockwise:

$$\int_C f(z)\,dz = 2\pi i \sum \text{Res}_{\text{inside}} = 2\pi i\left(-\frac{1}{2}\right) = -\pi i$$

5. Hence the integral equals $-\pi i$, which is option (C).

KEY IDEA

Residue theorem: $\oint_C f\,dz = 2\pi i$ times the sum of the residues at the poles inside C. Here only the simple pole $z = 0$ lies inside $|z| = 1$, and its residue is $\left.\frac{z^2+1}{z-2}\right|_{z=0} = -\frac{1}{2}$.

WATCH OUT FOR

Counting the pole at $z = 2$ as well. It lies outside the unit circle, and adding its residue $\frac{5}{2}$ gives $2\pi i \cdot 2 = 4\pi i$, which is not an option. The other common slip is a sign error in the residue, $\frac{1}{0-2} = -\frac{1}{2}$ and not $+\frac{1}{2}$. Getting that sign wrong, or taking the contour clockwise, gives πi, the distractor (D).

Q.6 MCQ · 1 MARK

Signals and Systems › Signals basics

x_R and x_A are, respectively, the rms and average values of $x(t) = x(t-T)$, and similarly, y_R and y_A are, respectively, the rms and average values of $y(t) = kx(t)$. k, T are independent of t. Which of the following is true?

(A) $y_A = kx_A;\ y_R = kx_R$

(B) $y_A = kx_A;\ y_R \neq kx_R$

(C) $y_A \neq kx_A;\ y_R = kx_R$

(D) $y_A \neq kx_A;\ y_R \neq kx_R$

Official answer: **A**

SOLUTION

1. **Periodicity.** $x(t) = x(t-T)$ says $x(t)$ is periodic with period T. Its average and rms values are therefore taken over one period. $y(t) = kx(t)$, with k a constant, has the same period T.
2. **Average value (linearity of the integral).**
$$y_A = \frac{1}{T}\int_0^T kx(t)\,dt = k \cdot \frac{1}{T}\int_0^T x(t)\,dt = kx_A$$
This holds for any constant k, positive, negative or zero. That rules out (C) and (D).
3. **RMS value (definition).**
$$y_R = \sqrt{\frac{1}{T}\int_0^T k^2x^2(t)\,dt} = \sqrt{k^2}\sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt} = |k|\,x_R$$
Squaring inside the rms does not stop it scaling with k. The constant k^2 comes out of the integral, and its square root comes out of the radical.
4. **Pick the option.** k is a constant scale factor (a gain). For $k \geq 0$, $|k| = k$, so $y_R = kx_R$. Both relations in (A) hold. Option (B) needs $y_R \neq kx_R$, which is false for every $k \geq 0$.
5. **A caveat on the sign.** Strictly, $y_R = |k|\,x_R$, because an rms value can never be negative. If k were negative (and $x_R \neq 0$), then $y_R = -kx_R \neq kx_R$, and the statement in (B) would be the true one. The question treats k as a plain non-negative scaling constant, and the official key (A) is the answer on that reading.

KEY IDEA

The average is a linear operation, so it scales by exactly k. The rms is the square root of a mean square: k^2 comes out of the integral and $\sqrt{k^2} = |k|$ comes out of the radical, so the rms scales by $|k|$. That equals k for a non-negative gain. The rms is not additive, $\mathrm{rms}(x_1 + x_2) \neq \mathrm{rms}(x_1) + \mathrm{rms}(x_2)$ in general, but it does scale with a constant multiplier.

WATCH OUT FOR

Reasoning that the rms is 'nonlinear because of the squaring' and so cannot scale by k, which leads to (B). The rms fails additivity, not scaling: multiplying the signal by k multiplies the rms by $|k|$. The opposite slip is forgetting that the rms is never negative, so $y_R = |k|x_R$ and not kx_R when $k < 0$. The question takes k as a non-negative gain.

Q.7 MCQ · 1 MARK

Electrical Machines › Synchronous machines

A three-phase cylindrical rotor synchronous generator has a synchronous reactance X_s and a negligible armature resistance. The magnitude of per phase terminal voltage is V_A and the magnitude of per phase induced emf is E_A. Considering the following two statements, P and Q,

P: For any three-phase balanced leading load connected across the terminals of this synchronous generator, V_A is always more than E_A

Q: For any three-phase balanced lagging load connected across the terminals of this synchronous generator, V_A is always less than E_A

which of the following options is correct?

(A) P is false and Q is true.

(B) P is true and Q is false.

(C) P is false and Q is false.

(D) P is true and Q is true.

Official answer: **A**

SOLUTION

1. Per-phase, neglecting armature resistance, the generator's phasor equation is $E_A = V_A + jIX_s$. Take V_A as the reference phasor and let the armature current be $I\angle\phi$, with $\phi > 0$ for a leading load and $\phi < 0$ for a lagging load.
2. Resolving the drop: $jIX_s\angle\phi = jIX_s\cos\phi - IX_s\sin\phi$, so
$$E_A = (V_A - IX_s\sin\phi) + jIX_s\cos\phi$$
3. Taking the magnitude squared:
$$|E_A|^2 - V_A^2 = (IX_s)^2 - 2V_AIX_s\sin\phi = IX_s\,(IX_s - 2V_A\sin\phi)$$
4. Lagging load: put $\phi = -\theta$ with $\theta > 0$. Then $\sin\phi = -\sin\theta$, so
$$|E_A|^2 - V_A^2 = IX_s\,(IX_s + 2V_A\sin\theta) > 0$$
for every lagging current, since both terms on the right are positive. Hence $E_A > V_A$, i.e. V_A is always less than E_A, for every lagging load: statement Q is TRUE.
5. Leading load: $\phi > 0$, so
$$|E_A|^2 - V_A^2 = IX_s\,(IX_s - 2V_A\sin\phi)$$
whose sign depends on whether IX_s is greater than or less than $2V_A\sin\phi$; it is not fixed.
6. In particular, whenever the leading current and angle are such that $IX_s > 2V_A\sin\phi$ (for instance, a small leading angle ϕ keeps $\sin\phi$ small, so even a moderate leading current satisfies this), the right side is positive, giving $E_A > V_A$, i.e. $V_A < E_A$: a direct counterexample to statement P.
7. Since P claims V_A is ALWAYS more than E_A for every leading load, and a leading-load case exists where $V_A < E_A$, statement P is FALSE.
8. Therefore P is false and Q is true: option (A).

KEY IDEA

For a cylindrical-rotor synchronous generator with negligible armature resistance, $E_A = V_A + jIX_s$. Writing this out for a load current at angle ϕ from V_A gives $|E_A|^2 - V_A^2 = IX_s(IX_s - 2V_A \sin\phi)$. For a lagging load ($\phi < 0$) this quantity is a sum of two positive terms and is always positive, so $E_A > V_A$ unconditionally. For a leading load ($\phi > 0$) the sign depends on the relative size of IX_s and $2V_A \sin\phi$, so no unconditional relation holds, and an "always" claim for leading loads must be false.

WATCH OUT FOR

Over-generalizing the familiar rule of thumb "lagging loads reduce terminal voltage, leading loads raise it" into a universal statement for BOTH directions. The rule is unconditionally true only for lagging loads (Q); for leading loads (P), whether V_A exceeds E_A depends on the magnitude of the leading current relative to $2V_A \sin\phi / X_s$, so treating P as always true (picking option D) is the typical error. A second common mistake is a sign slip when resolving $jI\angle\phi$, which can flip which of P/Q comes out true.

Q.8 MCQ · 1 MARK

Power Systems › Protection

A lossless transmission line with 0.2 pu reactance per phase uniformly distributed along the length of the line, connecting a generator bus to a load bus, is protected up to 80 % of its length by a distance relay placed at the generator bus. The generator terminal voltage is 1 pu. There is no generation at the load bus. The threshold pu current for operation of the distance relay for a solid three phase-to-ground fault on the transmission line is closest to:

(A) 1.00

(B) 3.61

(C) 5.00

(D) 6.25

Official answer: **D**

SOLUTION

1. Relay reach is 80 % of the line, so the reach impedance is $Z_r = 0.8 \times j0.2 = j0.16$ pu.
2. The relay is at the generator bus, and the generator terminal voltage is $1\angle 0^\circ$ pu. The relay therefore sees $V_r = 1\angle 0^\circ$ pu. The generator is treated as an ideal 1 pu source because the question gives no source impedance.
3. A distance relay operates when the apparent impedance V_r/I_r is at or below its reach. The threshold current is the current for which the apparent impedance equals the reach, which is a solid three-phase fault at the 80 % point.
4. For a solid fault at that point, the voltage at the relay equals the drop across the line section up to the fault: $V_r = I_f Z_r$. The line is lossless and uniformly distributed, so the impedance up to the fault is $j0.16$ pu.
5. $$I_{th} = \frac{V_r}{|Z_r|} = \frac{1}{0.16} = 6.25 \text{ pu}$$
6. A fault nearer than 80 % gives a smaller impedance and a larger current, so the relay operates. A fault beyond 80 % gives a smaller current, so the relay does not operate. The threshold is 6.25 pu, option (D).

KEY IDEA

A distance relay trips when the apparent impedance V/I is at or below its reach. At the reach point of a solid three-phase fault, the current is the relay voltage divided by the reach impedance. With a 1 pu source this is 1/(0.8 × 0.2).

WATCH OUT FOR

Using the full line reactance of 0.2 pu, which gives 5.00 pu (option C). The reach is only 80 % of the line, so the impedance is 0.16 pu. Another mistake is adding a source impedance that the question does not give.

Q.9 MCQ · 1 MARK

Power Systems › Per unit & load flow

Out of the following options, the most relevant information needed to specify the real power (P) at the PV buses in a load flow analysis is

(A) solution of economic load dispatch

(B) rated power output of the generator

(C) rated voltage of the generator

(D) base power of the generator

Official answer: **A**

SOLUTION

1. At a PV (generator) bus in a load flow study, the specified quantities are the real power P and the voltage magnitude $|V|$. The unknowns are the reactive power Q and the voltage angle δ.
2. The real power P_G at a PV bus is the generator's scheduled active output. This schedule is not fixed by the machine's nameplate. It is the operating point assigned to each unit by the economic load dispatch (ELD) solution, which shares the total demand among the units at least cost.
3. Check the other options. The rated power output is only a capacity limit, an upper bound on P_G, not the operating value. The rated voltage relates to the specified $|V|$, not to P. The base power only sets the per-unit scale and does not fix P.
4. So the most relevant information needed to specify P at the PV buses is the solution of economic load dispatch, which is option (A).

KEY IDEA

At a PV bus, P and $|V|$ are specified. P is the scheduled generation, which comes from the economic load dispatch. $|V|$ is the voltage set-point.

WATCH OUT FOR

Choosing the rated power output (B). The rating is only an upper limit on generation, while the scheduled P is the dispatched value. Likewise, the rated voltage (C) relates to $|V|$, not to P.

Q.10 MCQ · 1 MARK

Control Systems › Block diagrams & SFG

Consider a linear time-invariant system whose input $r(t)$ and output $y(t)$ are related by the following differential equation:

$$\frac{d^2y(t)}{dt^2} + 4y(t) = 6r(t)$$

The poles of this system are at

(A) $+2j, -2j$

(B) $+2, -2$

(C) $+4, -4$

(D) $+4j, -4j$

Official answer: **A**

SOLUTION

1. Take the Laplace transform with zero initial conditions, since a transfer function is defined that way: $s^2Y(s) + 4Y(s) = 6R(s)$.
2. Transfer function:

$$G(s) = \frac{Y(s)}{R(s)} = \frac{6}{s^2 + 4}$$

The gain of 6 in the numerator scales the response but has no effect on where the poles are.

3. The poles are the roots of the denominator, i.e. of the characteristic equation: $s^2 + 4 = 0 \Rightarrow s^2 = -4$.
4. $s = \pm\sqrt{-4} = \pm 2j$. Comparing with $s^2 + 2\zeta\omega_n s + \omega_n^2$ gives $\omega_n = 2\ \text{rad/s}$ and $\zeta = 0$. The system is undamped: it has a pair of simple conjugate poles on the imaginary axis, so it is marginally stable and oscillates steadily at $2\ \text{rad/s}$.
5. The poles are at $+2j$ and $-2j$, which is option (A).

KEY IDEA

Poles come only from the left-hand (output) side of the differential equation: its Laplace transform gives the characteristic polynomial $s^2 + 4$. The missing s term means $\zeta = 0$, so the poles sit on the imaginary axis at $\pm j\omega_n$ with $\omega_n = \sqrt{4} = 2\ \text{rad/s}$. The input-side gain of 6 plays no part.

WATCH OUT FOR

One slip is solving $s^2 = 4$ in place of $s^2 = -4$, which gives the real pair ± 2 (option B). Another is reading the coefficient 4 as the pole location, which gives ± 4 or $\pm 4j$ (options C and D), even though 4 is ω_n^2, not ω_n. A third is letting the right-hand-side gain of 6 affect the poles.

Q.11 MCQ · 1 MARK

Power Electronics › Harmonics & PF

A single-phase, full-bridge diode rectifier fed from a 230 V, 50 Hz sinusoidal source supplies a series combination of finite resistance, R, and a very large inductance, L. The two most dominant frequency components in the source current are:

(A) 50 Hz, 0 Hz

(B) 50 Hz, 100 Hz

(C) 50 Hz, 150 Hz

(D) 150 Hz, 250 Hz

Official answer: **C**

SOLUTION

1. Very large L: the dc-side current is essentially constant, I_o, with negligible ripple, because the inductor will not let the load current change appreciably within a cycle.
2. The bridge is built from uncontrolled diodes, so the firing angle is $\alpha = 0^\circ$ (no commutation overlap is assumed, since the source inductance is not given). The source current is $i_s = +I_o$ while D1, D2 conduct (positive half-cycle of v_s) and $i_s = -I_o$ while D3, D4 conduct (negative half-cycle). So i_s is a square wave of amplitude I_o at the supply frequency, in phase with v_s.
3. A square wave with half-wave symmetry, $i_s(t + T/2) = -i_s(t)$, has zero average value and no even harmonics. Only odd harmonics are present.
4. Fourier series:
$$i_s(t) = \frac{4I_o}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \cdots\right], \qquad \omega = 2\pi \times 50\ \text{rad/s}$$
so the peak of the nth harmonic is $I_n = \dfrac{4I_o}{n\pi}$ for odd n.
5. The largest component is the fundamental, $n = 1$, at $50\ \text{Hz}$. The next largest is $n = 3$, at $3 \times 50 = 150\ \text{Hz}$, with amplitude $\frac{1}{3}$ of the fundamental. The $n = 5$ component (250 Hz) is only $\frac{1}{5}$ of the fundamental, which is smaller.
6. 0 Hz is absent because the source current is alternating with zero average, and 100 Hz is an even harmonic and is absent. Hence the two most dominant components are $50\ \text{Hz}$ and $150\ \text{Hz}$: option (C).

KEY IDEA

With a very large load inductance, a single-phase diode bridge draws a square-wave source current. That wave has only odd harmonics, with amplitude $4I_o/(n\pi)$, so the two largest are the fundamental (50 Hz) and the third harmonic (150 Hz).

WATCH OUT FOR

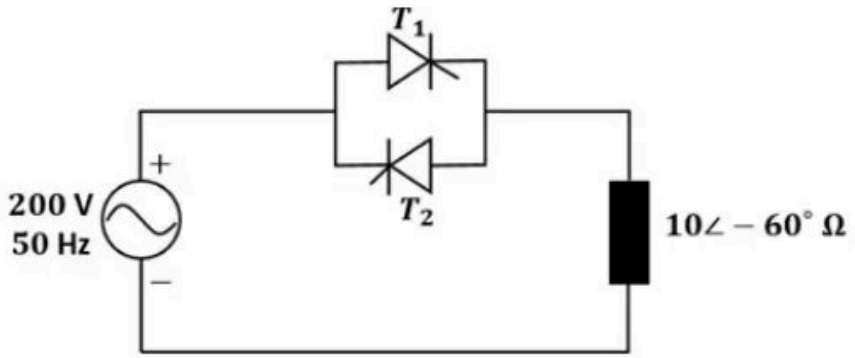
Treating the load as resistive and thinking of the output ripple at 100 Hz. The 100 Hz component belongs to the dc-side voltage and current, not to the source current. The source current has no dc term and no even harmonics because of its half-wave symmetry.

Q.12 MCQ · 1 MARK

Power Electronics › Rectifiers

Q.No. 12

Thyristor T_1 is triggered at an angle α (in degree), and T_2 at angle $180° + \alpha$, in each cycle of the sinusoidal input voltage. Assume both thyristors to be ideal. To control the load power over the range 0 to 2 kW, the minimum range of variation in α is:



(A) 0^o to 60^o

(B) 0^o to 120^o

(C) 60^o to 120^o

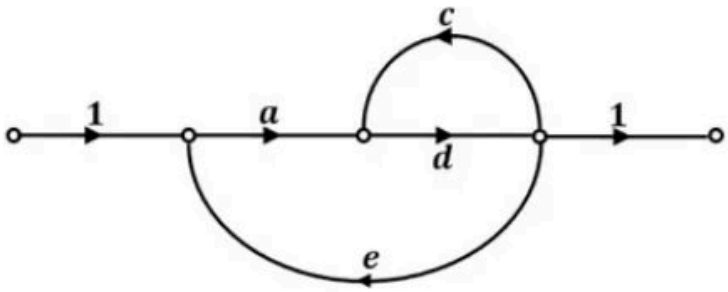
(D) 60^o to 180^o

Official answer: **Marks to all**

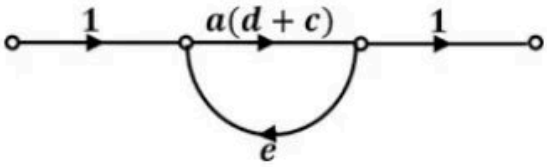
Q.13 MCQ · 1 MARK

Control Systems › Block diagrams & SFG

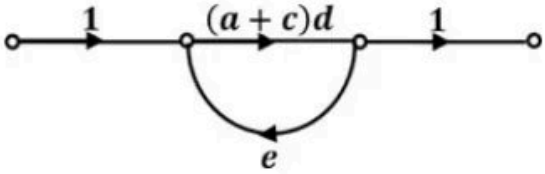
Q.No. 13 Which of the options is an equivalent representation of the signal flow graph shown here?



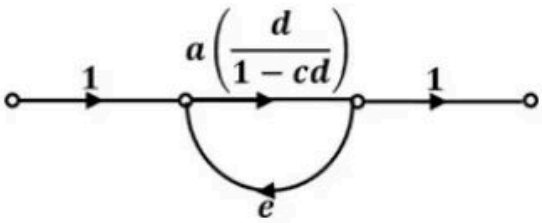
(A)



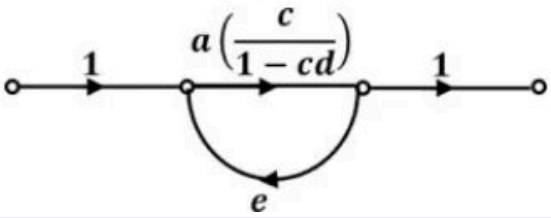
(B)



(C)



(D)



Official answer: **C**

SOLUTION

1. **Read the arrowheads.** Enlarged, the figure shows the arrowhead on arc c at its top pointing left, so c runs from x_3 back to x_2. The arrowhead on arc e at its bottom also points left, so e runs from x_3 back to x_1. Both are feedback branches, which means c is **not** in parallel with d.
2. **Remove node x_2 using its node equations.** Signals entering x_2 come from a and c, and its only outgoing branch is d:

$$x_2 = a\,x_1 + c\,x_3, \qquad x_3 = d\,x_2$$

Substituting gives $x_3 = ad\,x_1 + cd\,x_3$, so

$$x_3 = \frac{ad}{1 - cd}\,x_1$$

Seen as a rule, this is a forward branch d with feedback branch c around it. Their loop gain cd is positive (signal-flow-graph gains never change sign), so the pair reduces to one branch $\dfrac{d}{1 - cd}$ in series with a.

3. **Series rule.** The single branch from x_1 to x_3 therefore has gain
$$G = a\left(\frac{d}{1-cd}\right)$$
4. **What is left.** Nodes x_1 and x_3 both survive, and their equations are unchanged: $x_1 = x_0 + e\,x_3$ and $x_4 = x_3$. So branch e still runs from x_3 back to x_1 and wraps around G. That is exactly the graph drawn in option (C).
5. **Check with Mason's gain formula on the original graph.** There is one forward path, $P_1 = 1 \cdot a \cdot d \cdot 1 = ad$. There are two loops: $L_1 = cd$ (through $x_2 \to x_3 \to x_2$) and $L_2 = ade$ (through $x_1 \to x_2 \to x_3 \to x_1$). They share nodes x_2 and x_3, so there is no non-touching pair. That gives $\Delta = 1 - cd - ade$. The forward path touches both loops, so $\Delta_1 = 1$.
$$T = \frac{ad}{1-cd-ade}$$
6. **Option (C):**
$$T = \frac{G}{1-Ge} = \frac{\dfrac{ad}{1-cd}}{1-\dfrac{ade}{1-cd}} = \frac{ad}{1-cd-ade}$$
This matches. The other options do not: (A) gives $\dfrac{a(c+d)}{1-a(c+d)e}$, (B) gives $\dfrac{(a+c)d}{1-(a+c)de}$, and (D) gives $\dfrac{ac}{1-cd-ace}$. None of these equals T. The answer is **(C)**.

KEY IDEA

In a signal flow graph, a forward branch d with a feedback branch c around it forms a loop of gain cd. Branch gains simply multiply, with no minus sign, so the pair reduces to $\frac{d}{1-cd}$ and not $\frac{d}{1+cd}$. Removing a node such as x_2 never touches the branches between the nodes that remain, so e from x_3 to x_1 survives unchanged.

WATCH OUT FOR

Not checking which way arrow c points. If you read c as a second forward branch alongside d, you add them and get $a(d+c)$, which is option (A). There are two smaller slips. One is using the block-diagram negative-feedback form $\frac{d}{1+cd}$. The other is putting c in the numerator instead of d, which gives option (D). In a signal flow graph the numerator must be the forward branch, and the loop gain always goes in as $1 - (\text{loop gain})$.

Q.14 MCQ · 1 MARK

Analog Electronics › Amplifiers

A common-source amplifier with a drain resistance, $R_D = 4.7\ \text{k}\Omega$, is powered using a 10 V power supply. Assuming that the transconductance, g_m, is $520\ \mu\text{A/V}$, the voltage gain of the amplifier is closest to:

(A) -2.44

(B) -1.22

(C) 1.22

(D) 2.44

Official answer: **A**

SOLUTION

1. Assumptions: the MOSFET is biased in saturation, the output resistance r_o is neglected ($r_o \to \infty$), there is no load other than R_D, and the source is at AC ground (no unbypassed source resistance). The question gives no r_o, load or source resistor, so this is the intended model.
2. Small-signal model: replace the MOSFET with a voltage-controlled current source $g_m v_{gs}$ flowing from drain to source. The source is grounded, so $v_{gs} = v_{in}$.

3. The 10 V supply is a DC source, so it is AC ground and the top of R_D sits at 0 in the small-signal circuit. The current $g_m v_{gs}$ drawn into the drain has to come from that node through R_D. KCL at the drain gives $\frac{v_{out} - 0}{R_D} + g_m v_{gs} = 0$, so $v_{out} = -g_m R_D \, v_{gs}$.
4. Voltage gain:
$$A_v = \frac{v_{out}}{v_{in}} = -g_m R_D$$
The minus sign is the 180° phase inversion of a common-source stage.
5. Substitute the values:
$$A_v = -(520 \times 10^{-6}\ \text{A/V})(4.7 \times 10^3\ \Omega) = -2.444$$
6. The supply voltage (10 V) only sets the DC operating point, and its effect is already inside the given g_m. It does not appear in the small-signal gain. $A_v \approx -2.44$, which is option (A).

KEY IDEA

With r_o neglected and the source at AC ground, a common-source stage has small-signal gain $A_v = -g_m R_D$. The DC supply is AC ground, so it sets the bias but never appears in the gain.

WATCH OUT FOR

Dropping the minus sign and choosing +2.44 (D). A common-source amplifier inverts the signal. Another mistake is trying to use the 10 V supply in the gain, or halving the result to about 1.22 (options B and C). The supply only fixes the DC bias, and nothing in the circuit halves $g_m R_D$.

Q.15 MCQ · 1 MARK

Digital Electronics › Sequential

A sequence detector is designed to detect precisely 3 digital inputs, with overlapping sequences detectable. For the sequence (1,0,1) and input data (1,1,0,1,0,0,1,1,0,1,0,1,1,0), what is the output of this detector?

(A) 1,1,0,0,0,0,1,1,0,1,0,0

(B) 0,1,0,0,0,0,0,1,0,1,0,0

(C) 0,1,0,0,0,0,0,1,0,1,1,0

(D) 0,1,0,0,0,0,0,0,1,0,0,0

Official answer: **B**

SOLUTION

1. **Output count.** The detector looks at exactly 3 inputs at a time, so its first output appears once the 3rd bit has arrived. 14 input bits therefore give $14 - 3 + 1 = 12$ outputs. That matches the 12 entries in every option.
2. **Overlap rule.** Overlap is allowed, so the 3-bit window moves forward one bit at a time and a bit can belong to more than one detected sequence. At each clock the output is 1 exactly when the last three bits received are $1, 0, 1$.
3. Label the input $b_1 \ldots b_{14} = 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 1, 1, 0$.
4. **Slide the window** (bits $b_{i-2} b_{i-1} b_i$ for $i = 3$ to 14):
$110,\ \mathbf{101},\ 010,\ 100,\ 001,\ 011,\ 110,\ \mathbf{101},\ 010,\ \mathbf{101},\ 011,\ 110.$
5. The windows ending at b_4, b_{10} and b_{12} match 101. The windows ending at b_{10} and b_{12} share the bit $b_{10} = 1$, and only an overlapping detector counts both.
6. **Output sequence:** $0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0$. This is option (B).

KEY IDEA

An overlapping 3-bit sequence detector works like a sliding window. After each new bit, compare the last three bits received with the target (here 101) and output 1 on a match. With 14 inputs there are $14 - 3 + 1 = 12$ complete windows, so there are 12 outputs. A detected sequence may reuse bits from the previous detection.

WATCH OUT FOR

The usual mistake is to treat the detector as non-overlapping, resetting after the match that ends at b_{10}. That misses the 101 ending at b_{12}, because it reuses b_{10}, and gives $0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0$, which is not an option. Another mistake is to put each 1 at the first bit of the window instead of the last, or to expect 14 outputs rather than one per complete 3-bit window.

Q.16 NAT · 1 MARK

Engineering Mathematics › Differential equations

Consider the initial value problem below. The value of y at $x = \ln 2$, (rounded off to 3 decimal places) is _____.

$$\frac{dy}{dx} = 2x - y, \qquad y(0) = 1$$

Official answer: **0.8774 to 0.8952**

SOLUTION

1. Rewrite as a first-order linear ODE in standard form $\frac{dy}{dx} + P(x)\,y = Q(x)$: here $\frac{dy}{dx} + y = 2x$, so $P = 1$, $Q = 2x$.
2. Integrating factor: $\text{IF} = e^{\int P\,dx} = e^{x}$.
3. Solution rule $y \cdot \text{IF} = \int Q \cdot \text{IF}\,dx + C$:
$$ye^{x} = \int 2xe^{x}\,dx + C = 2(x-1)e^{x} + C$$
(integration by parts: $\int xe^{x}\,dx = xe^{x} - e^{x}$).
4. Divide by e^{x} to get the general solution: $y = 2x - 2 + Ce^{-x}$.
5. Apply the initial condition $y(0) = 1$: $1 = 0 - 2 + C \Rightarrow C = 3$, so $y = 2x - 2 + 3e^{-x}$. Check: $y' = 2 - 3e^{-x}$ and $2x - y = 2 - 3e^{-x}$, so the ODE holds.
6. At $x = \ln 2$: $e^{-\ln 2} = \frac{1}{2}$, so
$$y(\ln 2) = 2\ln 2 - 2 + \frac{3}{2} = 2\ln 2 - 0.5 = 1.386294 - 0.5 = 0.886294$$
7. Rounded to 3 decimal places: $y(\ln 2) \approx 0.886$.

KEY IDEA

$\frac{dy}{dx} + y = 2x$ is a first-order linear ODE. Use the integrating factor e^{x} to get $y = 2x - 2 + Ce^{-x}$. Then $y(0) = 1$ gives $C = 3$, and $e^{-\ln 2} = \frac{1}{2}$ makes the evaluation exact before you round.

WATCH OUT FOR

Using the initial condition before adding the constant C, or dropping C after integrating by parts. Getting the sign of the $-e^{x}$ term wrong in $\int xe^{x}\,dx = (x-1)e^{x}$. Writing $e^{-\ln 2} = 2$ instead of $\frac{1}{2}$ gives $2\ln 2 + 4 \approx 5.386$.

Q.17 NAT · 1 MARK

Electrical Machines › Induction machines

A three-phase, 50 Hz, 4-pole induction motor runs at no-load with a slip of 1 %. With full load, the slip increases to 5 %. The % speed regulation of the motor (rounded off to 2 decimal places) is _____.

Official answer: **4.10 to 4.40**

SOLUTION

1. Synchronous speed: $N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500$ rpm, fixed by the 50 Hz supply and 4 poles; it does not change with load.

2. Rotor speed from slip: $N = N_s(1 - s)$.
3. No-load speed: $N_{NL} = N_s(1 - 0.01) = 0.99\,N_s$.
4. Full-load speed: $N_{FL} = N_s(1 - 0.05) = 0.95\,N_s$.
5. % speed regulation, referenced to the full-load (rated) speed, exactly as voltage regulation is referenced to full-load voltage:

$$\%\ \text{Speed Regulation} = \frac{N_{NL} - N_{FL}}{N_{FL}} \times 100$$

6. Substitute:

$$= \frac{0.99N_s - 0.95N_s}{0.95N_s} \times 100 = \frac{0.04}{0.95} \times 100$$

7. Evaluate:

$$= 4.2105\ldots\% \approx 4.21\%$$

KEY IDEA

Synchronous speed $N_s = 120f/P$ is fixed by supply frequency and pole count and does not change with load; only the slip changes between no-load and full-load. Speed regulation is defined the same way as voltage regulation: as a percentage change measured relative to the full-load (rated operating) value, i.e. $(N_{NL} - N_{FL})/N_{FL} \times 100$.

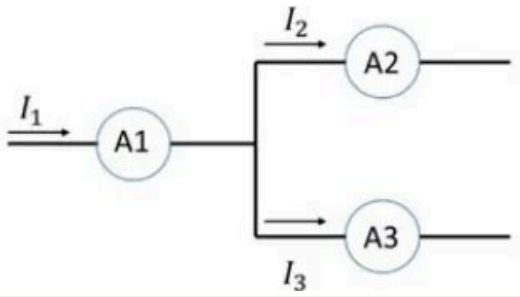
WATCH OUT FOR

Using the synchronous speed N_s in the denominator instead of the full-load speed N_{FL}: that gives $0.04/1 \times 100 = 4.00\%$, which is outside the accepted key range because it silently redefines "regulation" against the wrong reference speed.

Q.18 NAT · 1 MARK

Networks (Electric Circuits) › AC, power & resonance

Q.No. 18 Currents through ammeters A2 and A3 in the figure are $1\angle 10°$ and $1\angle 70°$, respectively. The reading of the ammeter A1 (rounded off to 3 decimal places) is _____ A.



Official answer: **1.700 to 1.750**

SOLUTION

1. KCL at the junction: the current through A1 is the phasor sum of the two branch currents, $I_1 = I_2 + I_3 = 1\angle 10° + 1\angle 70°$ A.
2. Magnitude of the sum of two phasors: $|I_2 + I_3|^2 = |I_2|^2 + |I_3|^2 + 2|I_2||I_3|\cos(70° - 10°) = 1 + 1 + 2\cos 60° = 3$, so $|I_1| = \sqrt{3}$. Equivalently, for two equal magnitudes, $|I_2 + I_3| = 2 \cdot 1 \cdot \cos\left(\frac{70° - 10°}{2}\right) = 2\cos 30° = \sqrt{3}$, and the resultant lies on the bisector, at $\frac{10° + 70°}{2} = 40°$.
3. Check in rectangular form: $1\angle 10° = 0.9848 + j0.1736$, $1\angle 70° = 0.3420 + j0.9397$, so $I_1 = 1.3268 + j1.1133$ A.
4. $$|I_1| = \sqrt{1.3268^2 + 1.1133^2} = \sqrt{3.000} = \sqrt{3} = 1.7321\ \text{A}, \qquad I_1 = 1.732\angle 40°\ \text{A}$$

5. An ammeter reads only the magnitude of the current, not its phase, so A1 reads $\sqrt{3} \approx 1.732\ \text{A}$.

KEY IDEA

In AC circuits KCL holds for phasors, not for meter readings. The current through A1 is the vector sum $1\angle 10^\circ + 1\angle 70^\circ$. The two currents are 60° apart, so the sum has magnitude $\sqrt{1+1+2\cos 60^\circ} = \sqrt{3}$.

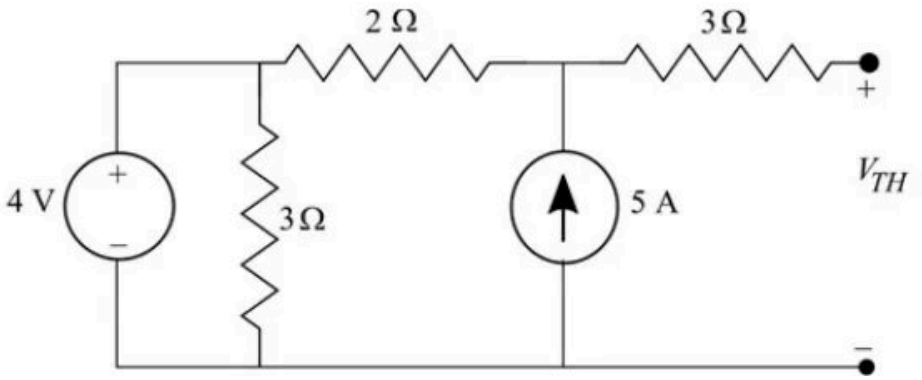
WATCH OUT FOR

Adding the ammeter readings as plain numbers ($1 + 1 = 2\ \text{A}$) ignores the 60° phase difference between the branch currents. Another slip is to use the 60° difference directly in $2\cos\theta$ instead of the half-angle, which gives $2\cos 60^\circ = 1\ \text{A}$. The correct result is $2\cos 30^\circ = \sqrt{3}\ \text{A}$.

Q.19 NAT · 1 MARK

Networks (Electric Circuits) › Theorems

Q.No. 19 The Thevenin equivalent voltage, V_{TH}, in V (rounded off to 2 decimal places) of the network shown below, is _____



Official answer: **13.80 to 14.20**

SOLUTION

1. **Open circuit at the output.** Nothing is connected across the output terminals, so no current flows in the right-hand $3\,\Omega$. With no current it drops no voltage, so $V_{TH} = V_B$, the voltage of the node where the $2\,\Omega$, the current source and the output $3\,\Omega$ meet, measured from the bottom rail.
2. **Voltage source in parallel with a resistor.** The left $3\,\Omega$ is connected directly across the ideal $4\,\text{V}$ source. It draws current from the source but cannot change the source's terminal voltage, so $V_A = 4\,\text{V}$ whatever that resistor's value is.
3. **KCL at node B.** The $5\,\text{A}$ source pushes current into B. The output branch is open, so the only way out of B is through the $2\,\Omega$ to node A:
$$\frac{V_B - 4}{2} = 5$$
4. **Solve.** $V_B = 4 + 5 \times 2 = 14\,\text{V}$.
5. **Result.** $V_{TH} = V_B = 14.00\,\text{V}$. The sign is positive because the + output terminal connects to B through the current-free $3\,\Omega$, and the – terminal is the bottom rail.

KEY IDEA

Only two facts are needed. An ideal voltage source fixes its node voltage no matter what resistor sits across it, and a resistor that carries no current (the open-circuited output $3\,\Omega$) drops no voltage. The whole $5\,\text{A}$ therefore flows through the $2\,\Omega$, so $V_{TH} = 4 + 5 \times 2 = 14\,\text{V}$.

WATCH OUT FOR

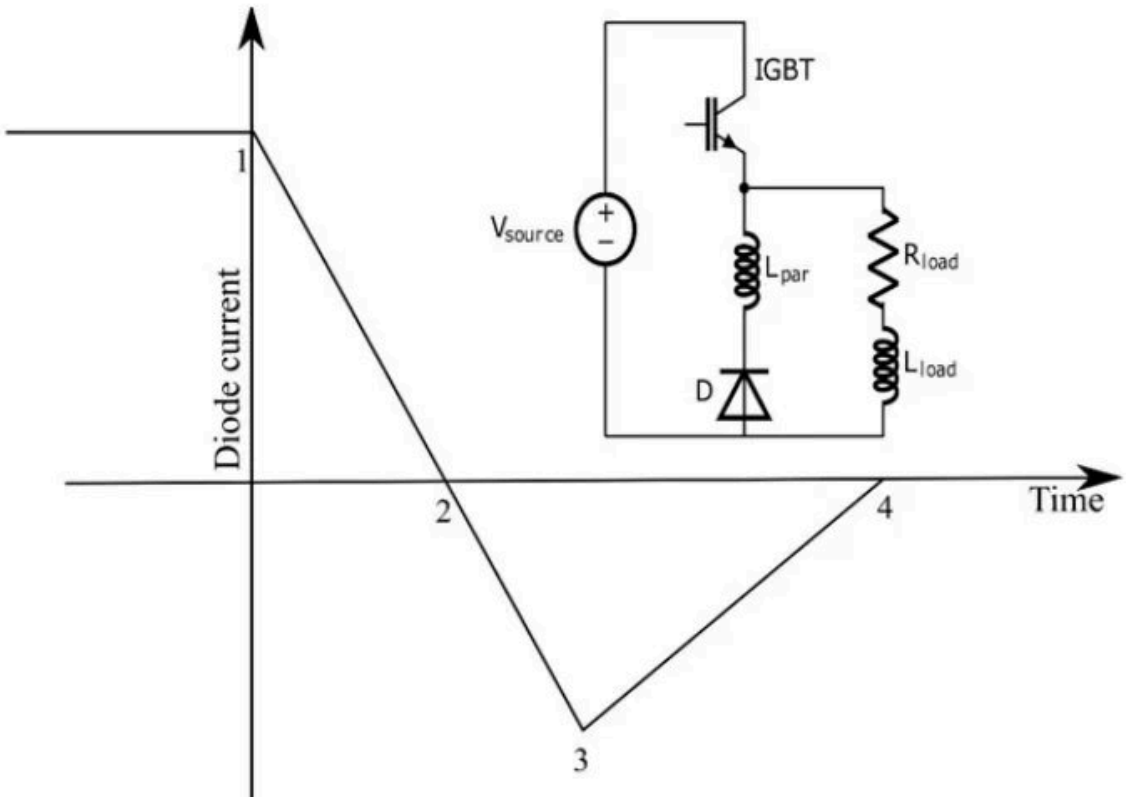
Letting the left $3\,\Omega$ carry part of the $5\,\text{A}$ (treating it as a divider with the $2\,\Omega$, or using source transformation on it). It is directly across an ideal voltage source, so it has no effect on V_A or V_B. Another slip is to count a voltage drop across the output $3\,\Omega$. With the terminals open it carries no current, so it drops nothing.

Q.20 NAT · 1 MARK

Power Electronics › Switches

Q.No. 20

A double pulse measurement for an inductively loaded circuit controlled by the IGBT switch is carried out to evaluate the reverse recovery characteristics of the diode, D, represented approximately as a piecewise linear plot of current vs time at diode turn-off. L_{par} is a parasitic inductance due to the wiring of the circuit, and is in series with the diode. The point on the plot (indicate your choice by entering 1, 2, 3 or 4) at which the IGBT experiences the highest current stress is _____.



Official answer: **3**

SOLUTION

1. Let I_{load} be the nearly constant current of the inductive load (the load time constant is much longer than the switching interval) and let i_D be the diode current, taken positive in the forward (freewheeling) direction. The IGBT, the diode branch and the load branch meet at one node, so by KCL
$$i_{IGBT} = I_{load} - i_D.$$
2. Before the IGBT turns on, the diode freewheels the full load current, so $i_D = I_{load}$ and $i_{IGBT} = 0$. When the IGBT turns on, the voltage across L_{par} is about V_{source}, so the diode current falls at $\frac{di_D}{dt} = -\frac{V_{source}}{L_{par}}$ (point 1 to point 2). The IGBT current rises as i_D falls.
3. At point 2 the diode current is zero, so $i_{IGBT} = I_{load}$. After point 2 the diode current is negative. Stored charge is being swept out of the diode, and the IGBT supplies this reverse current in addition to the load current: $i_{IGBT} = I_{load} + |i_D|$. It keeps increasing.

4. At point 3 the diode current is at its most negative value, the peak reverse recovery current $I_{rr,max}$. There
$$i_{IGBT} = I_{load} + I_{rr,max},$$
which is the largest value of i_{IGBT} over the whole interval.
5. After point 3 the diode current rises toward zero (point 4), so $|i_D|$ falls and i_{IGBT} falls back to I_{load}. The IGBT current stress is therefore highest at point 3, and the answer is 3.

KEY IDEA
At the IGBT turn-on, the IGBT current equals the load current minus the diode current. The IGBT current is largest where the diode current is most negative, which is the peak of the reverse recovery current.

WATCH OUT FOR
Picking point 2, where the diode current crosses zero, or point 1, where the diode current starts to fall. Neither is the peak of the reverse recovery current. The IGBT must also carry the diode's reverse current, so its maximum is at the most negative diode current.

Q.21 NAT · 1 MARK

Electrical Machines › Transformers

A single-phase, 4 kVA, 200 V/100 V, 50 Hz transformer with laminated CRGO steel core has rated no-load loss of 450 W. When the high-voltage winding is excited with 160 V, 40 Hz sinusoidal ac supply, the no-load losses are found to be 320 W. When the high-voltage winding of the same transformer is supplied from a 100 V, 25 Hz sinusoidal ac source, the no-load losses will be _____ W (rounded off to 2 decimal places).

Official answer: **162.41 to 162.59**

SOLUTION

1. Since $E = 4.44\, f\, N\, B_m\, A$ for a fixed winding, the peak flux density obeys $B_m \propto V/f$. Check the V/f ratio in all three given conditions: rated $200/50 = 4$; test $160/40 = 4$; target $100/25 = 4$. All three give the same ratio, so B_m is the SAME constant value in each case.
2. With B_m held constant, hysteresis loss $P_h \propto f$ and eddy current loss $P_e \propto f^2$. The B_m-dependence in each loss law contributes only an unchanging multiplicative factor, so only the frequency dependence survives when comparing conditions.
3. Let the rated-condition $(200\,\text{V}, 50\,\text{Hz})$ losses be P_h and P_e, so $P_h + P_e = 450\,\text{W}$.
4. At $160\,\text{V}, 40\,\text{Hz}$, the frequency ratio relative to rated is $40/50 = 0.8$, so
$$0.8P_h + (0.8)^2 P_e = 320 \;\Rightarrow\; 0.8P_h + 0.64P_e = 320.$$
5. Substituting $P_e = 450 - P_h$:
$$0.8P_h + 0.64(450 - P_h) = 320 \;\Rightarrow\; 0.16P_h = 32 \;\Rightarrow\; P_h = 200\,\text{W},\; P_e = 250\,\text{W}.$$
6. At $100\,\text{V}, 25\,\text{Hz}$ (same B_m), the frequency ratio relative to rated is $25/50 = 0.5$:
$$P_h' = 200(0.5) = 100\,\text{W}, \qquad P_e' = 250(0.5)^2 = 62.5\,\text{W}.$$
7. Total no-load loss $= 100 + 62.5 = 162.50\,\text{W}$.

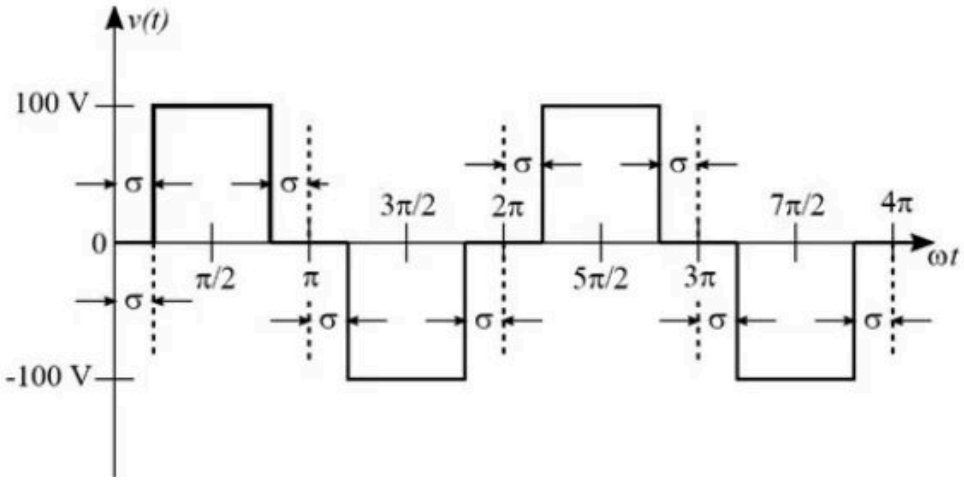
KEY IDEA
No-load (core) loss splits into hysteresis loss ($\propto$ f · B_m^n) and eddy current loss ($\propto$ f² · B_m²). Since E = 4.44 f N B_m A, the flux density B_m depends only on the ratio V/f. Whenever V/f is held at the same value across different test conditions, B_m stays constant, so the two loss components reduce to simple functions of frequency alone: P_h $\propto$ f and P_e $\propto$ f². This lets two known (V, f, loss) data points fix P_h and P_e at a reference condition, from which the loss at any other same-(V/f) condition follows by scaling.

WATCH OUT FOR
Forgetting to check that V/f is actually the same in all three conditions before dropping B_m from the proportionality: if V/f changed, B_m would change too and P_h $\propto$ f alone (or P_e $\propto$ f² alone) would no longer hold. A second common slip is mixing up which loss scales as f versus f² (swapping hysteresis and eddy current), or using absolute frequencies in the ratio instead of the ratio relative to the condition where P_h, P_e were computed.

Q.22 NAT · 1 MARK

Power Electronics › Inverters & PWM

Q.No. 22 A single-phase inverter is fed from a 100 V dc source and is controlled using a quasi-square wave modulation scheme to produce an output waveform, $v(t)$, as shown. The angle σ is adjusted to entirely eliminate the 3rd harmonic component from the output voltage. Under this condition, for $v(t)$, the magnitude of the 5th harmonic component as a percentage of the magnitude of the fundamental component is _____ (rounded off to 2 decimal places).



Official answer: **19.90 to 20.20**

SOLUTION

1. The output is $v = +V_{dc}$ for $\sigma < \omega t < \pi - \sigma$, $-V_{dc}$ for $\pi + \sigma < \omega t < 2\pi - \sigma$, and 0 in the intervals of width σ on either side of $\omega t = 0, \pi, 2\pi, \ldots$. Here $V_{dc} = 100$ V. The wave has odd symmetry and half-wave symmetry, and it is symmetric about $\omega t = \pi/2$ (quarter-wave symmetry). So only odd-order sine terms are present.
2. Fourier coefficient, using quarter-wave symmetry:
$$b_n = \frac{4}{\pi}\int_{\sigma}^{\pi/2} V_{dc}\sin n\omega t\, d(\omega t) = \frac{4V_{dc}}{n\pi}\cos n\sigma, \quad n = 1, 3, 5, \ldots$$
3. To remove the 3rd harmonic, set $\cos 3\sigma = 0$. The smallest non-trivial solution is $3\sigma = 90^\circ$, so $\sigma = 30^\circ$. The pulse width is then $\pi - 2\sigma = 120^\circ$.
4. Fundamental:
$$V_1 = \frac{4 \times 100}{\pi}\cos 30^\circ = \frac{400}{\pi}(0.8660) \approx 110.27 \text{ V}$$
5. 5th harmonic:
$$V_5 = \frac{4 \times 100}{5\pi}\cos 150^\circ, \qquad |V_5| = \frac{400}{5\pi}(0.8660) \approx 22.05 \text{ V}$$
6. Ratio:
$$\frac{|V_5|}{V_1} \times 100 = \frac{1}{5} \cdot \frac{|\cos 150^\circ|}{\cos 30^\circ} \times 100 = \frac{1}{5} \times 100 = 20.00\%$$

KEY IDEA

For a quasi-square wave with zero intervals of width σ at each end of the half-cycle, the nth harmonic is $\frac{4V_{dc}}{n\pi}\cos n\sigma$. Setting $\cos 3\sigma = 0$ gives $\sigma = 30^\circ$. Then $\cos 5\sigma = \cos 150^\circ = -\cos 30^\circ$ has the same magnitude as $\cos\sigma$, so $V_5/V_1 = 1/5$ exactly.

WATCH OUT FOR

Writing the harmonic as $\cos n\sigma$ with σ measured from the pulse centre, or taking the pulse width as σ instead of $\pi - 2\sigma$. Both give a wrong angle for eliminating the 3rd harmonic.

Q.23 NAT · 1 MARK

Power Systems › Dispatch & control

A single 50 Hz synchronous generator on droop control was delivering 100 MW power to a system. Due to increase in load, generator power had to be increased by 10 MW, as a result of which, system frequency dropped to 49.75 Hz. Further increase in load in the system resulted in a frequency of 49.25 Hz. At this condition, the power in MW supplied by the generator is _____ (rounded off to 2 decimal places).

Official answer: **123.00 to 135.00**

SOLUTION

1. Droop control gives a straight-line frequency-power characteristic, so the generator power rises in proportion to the frequency drop: $P = P_0 + K\,(f_0 - f)$, where $K = \dfrac{1}{R}$ is the slope in MW/Hz.
2. First operating point: $f_0 = 50\ \text{Hz}$ and $P_0 = 100\ \text{MW}$.
3. Second operating point: the power rises by $10\ \text{MW}$ to $110\ \text{MW}$ and the frequency falls to $49.75\ \text{Hz}$, a drop of $50 - 49.75 = 0.25\ \text{Hz}$.
4. Slope of the characteristic: $K = \dfrac{\Delta P}{\Delta f} = \dfrac{10\ \text{MW}}{0.25\ \text{Hz}} = 40\ \text{MW/Hz}$.
5. At $f = 49.25\ \text{Hz}$ the drop from the nominal $50\ \text{Hz}$ is $50 - 49.25 = 0.75\ \text{Hz}$.
6. Power supplied: $P = 100 + 40 \times 0.75 = 100 + 30 = 130.00\ \text{MW}$.
7. Cross-check from the second point: $110 + 40 \times (49.75 - 49.25) = 110 + 20 = 130.00\ \text{MW}$, which agrees.

KEY IDEA

A droop-controlled generator follows a straight line of power against frequency. Two points fix the slope in MW/Hz, and that slope then gives the power at any other frequency.

WATCH OUT FOR

Using the 10 MW increase as if it were measured from 49.75 Hz to 49.25 Hz, or working with the 0.5 Hz step alone and forgetting the 100 MW starting power. The slope must come from the first 0.25 Hz drop, and the final power is measured from the 100 MW starting point.

Q.24 NAT · 1 MARK

Control Systems › Nyquist

Consider a negative unity feedback system with forward path transfer function $G(s) = \dfrac{K}{(s+a)(s-b)(s+c)}$, where K, a, b, c are positive real numbers. For a Nyquist path enclosing the entire imaginary axis and right half of the s-plane in the clockwise direction, the Nyquist plot of $(1 + G(s))$, encircles the origin of $(1 + G(s))$-plane once in the clockwise direction and never passes through this origin

for a certain value of K. Then, the number of poles of $\dfrac{G(s)}{1+G(s)}$ lying in the open right half of the s-plane is _____.

Official answer: **2**

SOLUTION

1. Open-loop poles are $s=-a$, $s=+b$ and $s=-c$. Since $a,b,c>0$, exactly one of them ($s=b$) lies in the open right half-plane, and none lies on the imaginary axis. So $P=1$.
2. Write
$$1+G(s)=\frac{(s+a)(s-b)(s+c)+K}{(s+a)(s-b)(s+c)}.$$
At any root of the denominator the numerator equals $K\neq 0$, so nothing cancels: $1+G(s)$ has exactly the same poles as $G(s)$, and therefore also $P=1$ in the RHP.
3. The zeros of $1+G(s)$ are the roots of $(s+a)(s-b)(s+c)+K=0$, which is the closed-loop characteristic equation. Let Z be the number of these zeros inside the Nyquist contour. Because the plot never passes through the origin, $1+G(j\omega)\neq 0$ for every ω, so no zero sits on the imaginary axis and Z counts zeros in the open RHP only.
4. Nyquist criterion (argument principle): with the contour traversed clockwise and clockwise encirclements of the origin of the $(1+G)$-plane counted as positive,
$$N=Z-P.$$
5. One clockwise encirclement is given, so $N=+1$:
$$Z=N+P=1+1=2$$
6. The closed-loop transfer function is
$$\frac{G(s)}{1+G(s)}=\frac{K}{(s+a)(s-b)(s+c)+K}.$$
Its numerator is the non-zero constant K, so no pole-zero cancellation is possible. Its poles are exactly the zeros of $1+G(s)$, and 2 of them lie in the open right half of the s-plane.

KEY IDEA

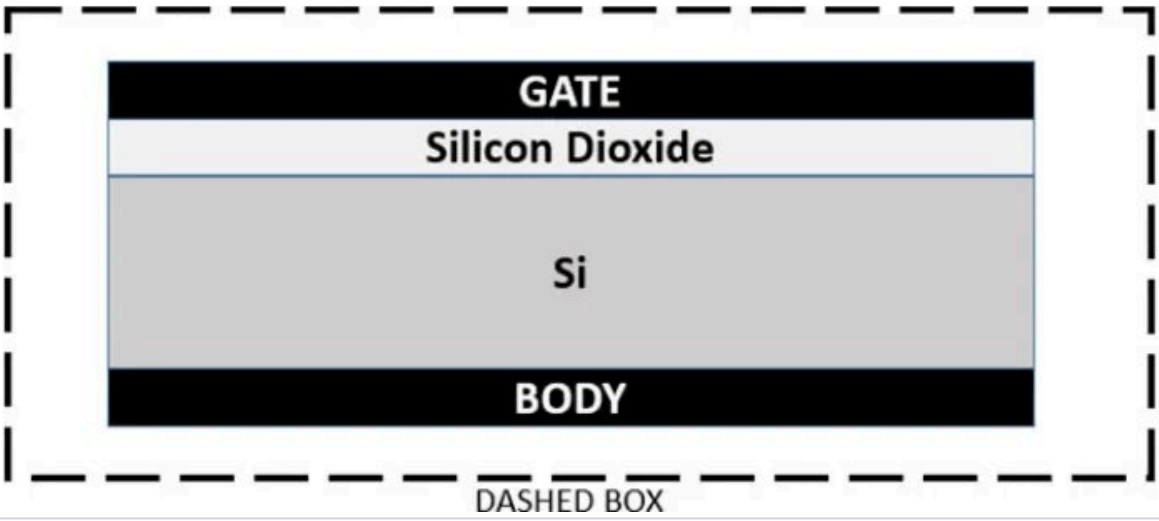
The plot is of $1+G(s)$, so encirclements are counted around the origin, not around -1. With a clockwise contour, $N=Z-P$: the open-loop pole at $s=+b$ gives $P=1$, the one clockwise encirclement gives $N=+1$, so $Z=2$. The zeros of $1+G(s)$ are the closed-loop poles, and here they cannot cancel against anything because the numerator of $G(s)$ is just the constant K.

WATCH OUT FOR

Using $N=P-Z$, which counts anticlockwise encirclements as positive, gives $Z=P-N=0$ and wrongly calls the system stable. The other common slip is to treat $G(s)$ as having no RHP poles and write $Z=N=1$, missing the pole at $s=+b$ that comes from the factor $(s-b)$.

Q.25 NAT · 1 MARK

Q.No. 25 The cross-section of a metal-oxide-semiconductor structure is shown schematically. Starting from an uncharged condition, a bias of $+3$ V is applied to the gate contact with respect to the body contact. The charge inside the silicon dioxide layer is then measured to be $+Q$. The total charge contained within the dashed box shown, upon application of bias, expressed as a multiple of Q (absolute value in Coulombs, rounded off to the nearest integer) is _____.



Official answer: **0**

Q.26 MCQ · 2 MARKS

Engineering Mathematics › Calculus

For real numbers, x and y, with $y = 3x^2 + 3x + 1$, the maximum and minimum value of y for $x \in [-2, 0]$ are respectively, _____.

(A) 7 and 1/4.

(B) 7 and 1.

(C) -2 and $-1/2$.

(D) 1 and 1/4.

Official answer: **A**

SOLUTION

1. On a closed interval, a continuous function reaches its absolute maximum and minimum either at a critical point inside the interval or at an endpoint (closed-interval method). So we need y at every critical point in $[-2, 0]$ and at $x = -2$ and $x = 0$.
2. Critical point: $\frac{dy}{dx} = 6x + 3 = 0 \Rightarrow x = -\frac{1}{2}$, and $-\frac{1}{2} \in [-2, 0]$. Since $\frac{d^2y}{dx^2} = 6 > 0$, this point is a local minimum.
3. Value at the critical point: $y\left(-\frac{1}{2}\right) = 3 \cdot \frac{1}{4} - \frac{3}{2} + 1 = \frac{3}{4} - \frac{6}{4} + \frac{4}{4} = \frac{1}{4}$.
4. Values at the endpoints: $y(-2) = 3(4) + 3(-2) + 1 = 12 - 6 + 1 = 7$, and $y(0) = 0 + 0 + 1 = 1$.
5. Comparing $\{7,\ 1,\ \frac{1}{4}\}$: the maximum is 7 (at $x = -2$) and the minimum is $\frac{1}{4}$ (at $x = -\frac{1}{2}$).

6. Check by completing the square: $y = 3\left(x^2 + x\right) + 1 = 3\left(x + \frac{1}{2}\right)^2 - \frac{3}{4} + 1 = 3\left(x + \frac{1}{2}\right)^2 + \frac{1}{4}$. The minimum is $\frac{1}{4}$ at the vertex $x = -\frac{1}{2}$, and y is largest at the point of the interval farthest from $x = -\frac{1}{2}$, which is $x = -2$ (distance $\frac{3}{2}$, against $\frac{1}{2}$ for $x = 0$). So the maximum and minimum are 7 and $\frac{1}{4}$: option (A).

KEY IDEA

For a continuous function on a closed interval, the absolute extrema can only occur at interior critical points or at the endpoints, so evaluate y at all of them and compare. Here $y = 3\left(x + \frac{1}{2}\right)^2 + \frac{1}{4}$ is an upward parabola whose vertex $x = -\frac{1}{2}$ lies inside $[-2, 0]$. The vertex gives the minimum, and the endpoint farther from the vertex gives the maximum.

WATCH OUT FOR

Checking only the endpoints gives $y(-2) = 7$ and $y(0) = 1$, which leads to option (B) "7 and 1". The interior critical point $x = -\frac{1}{2}$, where $y = \frac{1}{4}$, gets missed. Option (C) comes from confusing the x-values (the endpoint -2, the vertex $-\frac{1}{2}$) with the values of y.

Q.27 MCQ · 2 MARKS

Engineering Mathematics › Vector calculus

The vector function expressed by

$$\mathbf{F} = \mathbf{a}_x\left(5y - k_1 z\right) + \mathbf{a}_y\left(3z + k_2 x\right) + \mathbf{a}_z\left(k_3 y - 4x\right)$$

represents a conservative field, where $\mathbf{a}_x, \mathbf{a}_y, \mathbf{a}_z$ are unit vectors along x, y and z directions, respectively. The values of constants k_1, k_2, k_3 are given by:

(A) $k_1 = 3, k_2 = 3, k_3 = 7$

(B) $k_1 = 3, k_2 = 8, k_3 = 5$

(C) $k_1 = 4, k_2 = 5, k_3 = 3$

(D) $k_1 = 0, k_2 = 0, k_3 = 0$

Official answer: **C**

SOLUTION

1. Conservative field condition: on all of space, $\mathbf{F}$ is conservative exactly when $\nabla \times \mathbf{F} = \mathbf{0}$. Components: $F_x = 5y - k_1 z$, $F_y = 3z + k_2 x$, $F_z = k_3 y - 4x$.
2. x-component of the curl: $\dfrac{\partial F_z}{\partial y} - \dfrac{\partial F_y}{\partial z} = k_3 - 3 = 0 \Rightarrow k_3 = 3.$
3. y-component of the curl: $\dfrac{\partial F_x}{\partial z} - \dfrac{\partial F_z}{\partial x} = -k_1 - (-4) = 4 - k_1 = 0 \Rightarrow k_1 = 4.$
4. z-component of the curl: $\dfrac{\partial F_y}{\partial x} - \dfrac{\partial F_x}{\partial y} = k_2 - 5 = 0 \Rightarrow k_2 = 5.$
5. Check with a potential: $\phi = 5xy - 4xz + 3yz$ gives $\nabla\phi = (5y - 4z)\,\mathbf{a}_x + (5x + 3z)\,\mathbf{a}_y + (3y - 4x)\,\mathbf{a}_z$, which is $\mathbf{F}$ with $k_1 = 4, k_2 = 5, k_3 = 3$.
6. So $k_1 = 4,\ k_2 = 5,\ k_3 = 3$, which is option (C).

KEY IDEA

A field defined on all of space is conservative exactly when its curl is zero, so it is the gradient of a scalar potential. Set each of the three curl components to zero. Each one fixes one constant: the x-component gives k_3, the y-component gives k_1 and the z-component gives k_2. Then check by building the potential $\phi = 5xy - 4xz + 3yz$.

WATCH OUT FOR

The usual slip is the sign or order in the y-component of the curl. It is $\partial F_x/\partial z - \partial F_z/\partial x$, and reversing it gives $k_1 = -4$, which matches no option. The other mistake is testing divergence (solenoidal) instead of curl. Here $\nabla \cdot \mathbf{F} = 0$ for every choice of k_1, k_2, k_3, because no component depends on its own coordinate, so a divergence test says nothing. Option (D) is a trap: with every $k = 0$ the curl is $(-3, 4, -5) \neq \mathbf{0}$.

Q.28 MCQ · 2 MARKS

Electrical Machines › DC machines

A 250 V dc shunt motor has an armature resistance of $0.2\ \Omega$ and a field resistance of $100\ \Omega$. When the motor is operated on no-load at rated voltage, it draws an armature current of 5 A and runs at 1200 rpm. When a load is coupled to the motor, it draws total line current of 50 A at rated voltage, with a 5% reduction in the air-gap flux due to armature reaction. Voltage drop across the brushes can be taken as 1 V per brush under all operating conditions. The speed of the motor, in rpm, under this loaded condition, is closest to:

(A) 1200

(B) 1000

(C) 1220

(D) 900

Official answer: **C**

SOLUTION

1. Field current is fixed by the rated voltage and field resistance: $I_f = \dfrac{V}{R_f} = \dfrac{250}{100} = 2.5\ \text{A}$ (unchanged at no-load and under load, since the shunt field sees rated voltage throughout).
2. No-load condition: armature current $I_{a1} = 5\ \text{A}$ (given directly). Back EMF: $E_{b1} = V - I_{a1}R_a - V_{brush} = 250 - 5(0.2) - 2(1) = 250 - 1 - 2 = 247\ \text{V}$.
3. Loaded condition: total line current $I_L = 50\ \text{A}$, so armature current $I_{a2} = I_L - I_f = 50 - 2.5 = 47.5\ \text{A}$.
4. Back EMF under load: $E_{b2} = V - I_{a2}R_a - V_{brush} = 250 - 47.5(0.2) - 2(1) = 250 - 9.5 - 2 = 238.5\ \text{V}$.
5. Flux under load is reduced by armature reaction: $\phi_2 = 0.95\,\phi_1$.
6. Since $E_b \propto N\phi$: $\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \times \dfrac{\phi_1}{\phi_2} = \dfrac{238.5}{247} \times \dfrac{1}{0.95}$.
7. Computing: $\dfrac{238.5}{247} = 0.9656$, and $\dfrac{0.9656}{0.95} = 1.0164$.
8. $N_2 = 1200 \times 1.0164 \approx 1220\ \text{rpm}$, matching option (C).

KEY IDEA

For a dc shunt motor, back EMF $E_b = V - I_aR_a - V_{brush}$, and speed is proportional to E_b/ϕ (since $E_b \propto N\phi$). Field current stays fixed (rated voltage across a fixed field resistance), so armature current at each condition is found by subtracting field current from line current (no-load) or total line current (loaded).

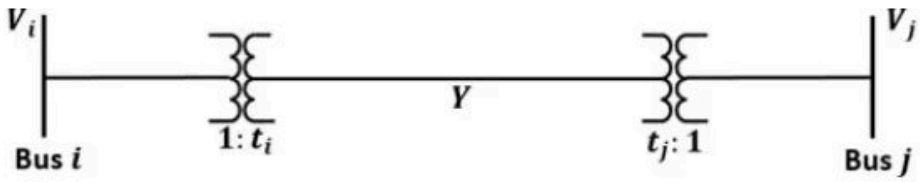
WATCH OUT FOR

Using the given no-load armature current directly (correct) but then forgetting to subtract the field current from the loaded total line current to get loaded armature current, using $I_a = 50\ \text{A}$ instead of 47.5 A. Also common: forgetting the 2 V total brush drop (1 V/brush × 2 brushes), or applying the flux reduction the wrong way (multiplying instead of dividing by 0.95 in the speed ratio).

Q.29 MCQ · 2 MARKS

Power Systems › Per unit & load flow

Q.No. 29 Two buses, i and j, are connected with a transmission line of admittance Y, at the two ends of which there are ideal transformers with turns ratios as shown. Bus admittance matrix for the system is:



(A) $\begin{bmatrix} -t_i t_j Y & t_j^2 Y \\ t_i^2 Y & -t_i t_j Y \end{bmatrix}$

(B) $\begin{bmatrix} t_i t_j Y & -t_j^2 Y \\ -t_i^2 Y & t_i t_j Y \end{bmatrix}$

(C) $\begin{bmatrix} t_i^2 Y & -t_i t_j Y \\ -t_i t_j Y & t_j^2 Y \end{bmatrix}$

(D) $\begin{bmatrix} t_i t_j Y & -(t_i - t_j)^2 Y \\ -(t_i - t_j)^2 Y & t_i t_j Y \end{bmatrix}$

Official answer: **C**

SOLUTION

1. Each ideal transformer scales voltage by its tap ratio and, with power conserved, scales current inversely. In the figure the bus sits on the 1 side of each transformer (ratios $1 : t_i$ and $t_j : 1$). The line-side voltages are therefore $V_i' = t_i V_i$ and $V_j' = t_j V_j$.
2. Current in the line from the i end to the j end: $I_L = Y\,(V_i' - V_j') = Y\,(t_i V_i - t_j V_j)$.
3. Power balance at the i transformer: $V_i I_i = V_i' I_L$, so $I_i = t_i I_L = t_i^2 Y\, V_i - t_i t_j Y\, V_j$.
4. Power balance at the j transformer: $V_j I_j = -V_j' I_L$, so $I_j = -t_j I_L = -t_i t_j Y\, V_i + t_j^2 Y\, V_j$.
5. Collect the currents as $\begin{bmatrix} I_i \\ I_j \end{bmatrix} = Y_{bus} \begin{bmatrix} V_i \\ V_j \end{bmatrix}$:
$$Y_{bus} = \begin{bmatrix} t_i^2 Y & -t_i t_j Y \\ -t_i t_j Y & t_j^2 Y \end{bmatrix}$$
6. Check: the matrix is symmetric, as it must be for a bilateral network with no phase shifter. The diagonals are positive, and the row sums are not zero unless $t_i = t_j$. Only option (C) has this form, so the answer is (C).

KEY IDEA

An ideal transformer with ratio $1 : t$ multiplies the voltage on the line side by t and the bus current by t. Each diagonal entry therefore picks up $t^2 Y$ and the off-diagonal entries pick up $t_i t_j Y$.

WATCH OUT FOR

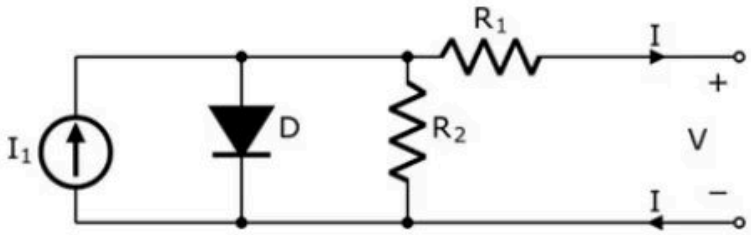
Scaling the voltage by the tap ratio but forgetting to scale the current as well, which gives tY terms instead of $t^2 Y$. Another slip is attaching the ratio to the wrong side of the transformer, so that the line-side voltage becomes V/t.

Q.30 MCQ · 2 MARKS

Networks (Electric Circuits) › Laws & node/mesh

Q.No. 30

Consider the diode circuit shown below. The diode, D, obeys the current-voltage characteristic $I_D = I_S\left(\exp\left(\frac{V_D}{nV_T}\right) - 1\right)$, where $n > 1$, $V_T > 0$, V_D is the voltage across the diode and I_D is the current through it. The circuit is biased so that voltage, $V > 0$ and current, $I < 0$. If you had to design this circuit to transfer maximum power from the current source (I_1) to a resistive load (not shown) at the output, what values of R_1 and R_2 would you choose?



(A) Large R_1 and large R_2.

(B) Small R_1 and small R_2.

(C) Large R_1 and small R_2.

(D) Small R_1 and large R_2.

Official answer: **D**

Q.31 MCQ · 2 MARKS

Analog Electronics › Diode circuits

A non-ideal diode is biased with a voltage of -0.03 V, and a diode current of I_1 is measured. The thermal voltage is 26 mV and the ideality factor for the diode is $15/13$. The voltage, in V, at which the measured current increases to $1.5I_1$ is closest to:

(A) -0.02

(B) -0.09

(C) -1.50

(D) -4.50

Official answer: **B**

SOLUTION

1. Use the Shockley diode equation with ideality factor η: $I = I_S\left(e^{V/(\eta V_T)} - 1\right)$. The bias is only about one ηV_T below zero, so the -1 term is not negligible and must be kept.
2. Effective thermal voltage: $\eta V_T = \frac{15}{13} \times 26\ \text{mV} = 30\ \text{mV} = 0.03\ \text{V}$.
3. At $V = -0.03$ V: $\frac{V}{\eta V_T} = \frac{-0.03}{0.03} = -1$, so $I_1 = I_S\left(e^{-1} - 1\right) = -0.6321\, I_S$ (a reverse current).
4. Required current: $1.5I_1 = 1.5 \times (-0.6321\, I_S) = -0.9482\, I_S$. Set $I_S\left(e^{V/0.03} - 1\right) = -0.9482\, I_S$.
5. Solve: $e^{V/0.03} = 1 - 0.9482 = 0.0518$, so $\frac{V}{0.03} = \ln(0.0518) \approx -2.96$.

6. Hence $V \approx -2.96 \times 0.03\ \text{V} \approx -0.089\ \text{V} \approx -0.09\ \text{V}$, which is option (B). Check that a solution exists: in reverse bias the current magnitude can only approach I_S, and $1.5 \times 0.632\, I_S = 0.948\, I_S$ is still less than I_S.

KEY IDEA

At a reverse bias of only about one ηV_T, keep the full Shockley equation $I = I_S\left(e^{V/(\eta V_T)} - 1\right)$. The reverse current saturates toward $-I_S$, so raising its magnitude by 50% (from $0.632\, I_S$ to $0.948\, I_S$) takes a lot more reverse voltage, and V ends up about three times the original bias.

WATCH OUT FOR

Dropping the -1 term and writing $I \approx I_S e^{V/(\eta V_T)}$. That approximation holds only for forward bias well above ηV_T. Using it here gives $V = -0.03 + 0.03 \ln 1.5 \approx -0.018\ \text{V} \approx -0.02\ \text{V}$, which is distractor (A). It moves the voltage toward zero even though the reverse current is supposed to grow. A second slip is using $V_T = 26\ \text{mV}$ without multiplying by the ideality factor.

Q.32 MCQ · 2 MARKS

Networks (Electric Circuits) › Laws & node/mesh

A benchtop dc power supply acts as an ideal 4 A current source as long as its terminal voltage is below 10 V. Beyond this point, it begins to behave as an ideal 10 V voltage source for all load currents going down to 0 A. When connected to an ideal rheostat, find the load resistance value at which maximum power is transferred, and the corresponding load voltage and current.

(A) Short, ∞ A, 10 V

(B) Open, 4 A, 0 V

(C) 2.5 Ω, 4 A, 10 V

(D) 2.5 Ω, 4 A, 5 V

Official answer: **C**

SOLUTION

1. **Supply characteristic.** For $0 \le V < 10\ \text{V}$ the supply is an ideal current source, $I = 4\ \text{A}$. For $0 \le I \le 4\ \text{A}$ it is an ideal voltage source, $V = 10\ \text{V}$. The two pieces meet at the corner $(V, I) = (10\ \text{V},\ 4\ \text{A})$.
2. **Corner load (Ohm's law).** The load line $V = IR$ passes through that corner when
$$R = \frac{10\ \text{V}}{4\ \text{A}} = 2.5\ \Omega$$
3. **Current-source region, $R \le 2.5\ \Omega$.** Here $V = 4R \le 10\ \text{V}$, so the supply holds $I = 4\ \text{A}$ and
$$P = I^2 R = 16R$$
Power rises with R and reaches $16 \times 2.5 = 40\ \text{W}$ at $R = 2.5\ \Omega$.
4. **Voltage-source region, $R \ge 2.5\ \Omega$.** Here $I = \dfrac{10}{R} \le 4\ \text{A}$, so the supply holds $V = 10\ \text{V}$ and
$$P = \frac{V^2}{R} = \frac{100}{R}$$
Power falls as R increases, starting from $\dfrac{100}{2.5} = 40\ \text{W}$ at $R = 2.5\ \Omega$.
5. **Maximum.** P rises up to $R = 2.5\ \Omega$ and falls beyond it, and both expressions give $40\ \text{W}$ at the corner. So maximum power is delivered at $R = 2.5\ \Omega$, where $I = 4\ \text{A}$, $V = 10\ \text{V}$ and $P_{max} = 40\ \text{W}$.
6. **Checking the other options.** A short makes $V = 0$ and an open makes $I = 0$, so both deliver zero power. Option (A) is also impossible as printed, because the supply limits its current to $4\ \text{A}$ and a short cannot hold $10\ \text{V}$. Option (D) pairs $2.5\ \Omega$ and $4\ \text{A}$ with $5\ \text{V}$, which breaks Ohm's law, since $4 \times 2.5 = 10\ \text{V}$. The answer is (C).

KEY IDEA

This supply has no linear Thevenin equivalent. It is current-limited below 10 V and voltage-limited up to 4 A, so the rule $R_L = R_{th}$ does not apply. Delivered power $P = VI$ rises along the current-limited branch ($P = 16R$) and falls along the voltage-limited branch ($P = 100/R$), so the maximum is at the knee of the V–I characteristic, $(10\ \text{V}, 4\ \text{A})$, which the load reaches when $R = 10/4 = 2.5\ \Omega$.

WATCH OUT FOR

Applying the linear maximum-power-transfer rule $R_L = R_{th}$. An ideal current source has $R_{th} = \infty$, which suggests an open load, and an ideal voltage source has $R_{th} = 0$, which suggests a short. Both loads deliver zero power here. A second trap is the matched-load habit of taking half the source voltage (5 V), which gives option (D). But 4 A through 2.5 Ω is 10 V, not 5 V, so (D) breaks Ohm's law.

Q.33 MCQ · 2 MARKS

Electromagnetics › Electrostatics

The static electric field inside a dielectric medium with relative permittivity, $\varepsilon_r = 2.25$, expressed in cylindrical coordinate system is given by the following expression

$$\mathbf{E} = \hat{a}_r\, 2r + \hat{a}_\varphi \left(\frac{3}{r}\right) + \hat{a}_z\, 6$$

where $\hat{a}_r, \hat{a}_\varphi, \hat{a}_z$ are unit vectors along r, φ and z directions, respectively. If the above expression represents a valid electrostatic field inside the medium, then the volume charge density associated with this field in terms of free space permittivity, ε_0, in SI units is given by:

(A) $3\varepsilon_0$

(B) $4\varepsilon_0$

(C) $5\varepsilon_0$

(D) $9\varepsilon_0$

Official answer: **D**

SOLUTION

1. Use Gauss's law in point form. In a linear dielectric $\mathbf{D} = \varepsilon_0\varepsilon_r\mathbf{E}$, so $\rho_v = \nabla \cdot \mathbf{D} = \varepsilon_0\varepsilon_r\, \nabla \cdot \mathbf{E}$. The medium is uniform, so ε_r comes out of the divergence. Here $\varepsilon_0 \approx 8.854 \times 10^{-12}\ \text{F/m}$, but the answer is asked for in terms of ε_0.
2. Divergence in cylindrical coordinates:
$$\nabla \cdot \mathbf{E} = \frac{1}{r}\frac{\partial (rE_r)}{\partial r} + \frac{1}{r}\frac{\partial E_\varphi}{\partial \varphi} + \frac{\partial E_z}{\partial z}$$
with $E_r = 2r$, $E_\varphi = 3/r$, $E_z = 6$.
3. Radial term: $rE_r = 2r^2$, so $\dfrac{1}{r}\dfrac{\partial (2r^2)}{\partial r} = \dfrac{4r}{r} = 4$.
4. Azimuthal term: $E_\varphi = 3/r$ does not depend on φ, so it contributes 0. Axial term: $E_z = 6$ is constant, so it contributes 0. Hence $\nabla \cdot \mathbf{E} = 4\ \text{V/m}^2$.
5. Check that the field is a valid electrostatic field, i.e. $\nabla \times \mathbf{E} = 0$. In cylindrical coordinates: r-component $\dfrac{1}{r}\dfrac{\partial E_z}{\partial \varphi} - \dfrac{\partial E_\varphi}{\partial z} = 0 - 0 = 0$; φ-component $\dfrac{\partial E_r}{\partial z} - \dfrac{\partial E_z}{\partial r} = 0 - 0 = 0$; z-component $\dfrac{1}{r}\left[\dfrac{\partial (rE_\varphi)}{\partial r} - \dfrac{\partial E_r}{\partial \varphi}\right] = \dfrac{1}{r}\left[\dfrac{\partial (3)}{\partial r} - 0\right] = 0$. So the field is curl-free (for $r \neq 0$), as the question states.
6. Volume charge density:
$$\rho_v = \varepsilon_0\varepsilon_r\, \nabla \cdot \mathbf{E} = 2.25 \times 4\, \varepsilon_0 = 9\varepsilon_0\ \text{C/m}^3$$
which is option (D).

KEY IDEA

Free charge comes from the divergence of $\mathbf{D}$, not of $\mathbf{E}$: in a uniform linear dielectric, $\rho_v = \nabla \cdot \mathbf{D} = \varepsilon_0 \varepsilon_r \nabla \cdot \mathbf{E}$. In cylindrical coordinates the radial term is $\frac{1}{r}\frac{\partial (r E_r)}{\partial r}$, and a φ-component that depends only on r adds nothing to the divergence.

WATCH OUT FOR

Using $\rho_v = \varepsilon_0 \nabla \cdot \mathbf{E}$ and leaving out ε_r gives $4\varepsilon_0$ (option B), which is the total (free plus bound) charge density, not the free charge. Another slip is to take the radial term as $\partial E_r / \partial r = 2$ and forget the $\frac{1}{r}\frac{\partial (r\,\cdot)}{\partial r}$ form, which gives $2.25 \times 2 = 4.5\varepsilon_0$, an answer that is not among the options.

Q.34 MCQ · 2 MARKS

Control Systems › Time response

Consider a permanent magnet dc (PMDC) motor which is initially at rest. At $t = 0$, a dc voltage of $5\ \text{V}$ is applied to the motor. Its speed monotonically increases from $0\ \text{rad/s}$ to $6.32\ \text{rad/s}$ in $0.5\ \text{s}$ and finally settles to $10\ \text{rad/s}$. Assuming that the armature inductance of the motor is negligible, the transfer function for the motor is

(A) $\dfrac{10}{0.5s + 1}$

(B) $\dfrac{2}{0.5s + 1}$

(C) $\dfrac{2}{s + 0.5}$

(D) $\dfrac{10}{s + 0.5}$

Official answer: **B**

SOLUTION

1. With armature inductance neglected, the electrical time constant drops out and the PMDC motor's speed-to-voltage transfer function is first order: $\dfrac{\Omega(s)}{V(s)} = \dfrac{K}{\tau s + 1}$. A monotonic rise with no overshoot is what a first-order response looks like.
2. For a step input of $5\ \text{V}$, $V(s) = \dfrac{5}{s}$, and the first-order step response is $\omega(t) = 5K\left(1 - e^{-t/\tau}\right)$.
3. Steady state, using the final value theorem: $\omega(\infty) = \lim_{s \to 0} s \cdot \dfrac{K}{\tau s + 1} \cdot \dfrac{5}{s} = 5K$. Setting this equal to $10\ \text{rad/s}$ gives $K = \dfrac{10}{5} = 2\ \text{rad/s per V}$.
4. Time constant: $\dfrac{\omega(0.5)}{\omega(\infty)} = \dfrac{6.32}{10} = 0.632 = 1 - e^{-1}$. So $e^{-0.5/\tau} = e^{-1}$, which means $t = 0.5\ \text{s}$ is exactly one time constant: $\tau = 0.5\ \text{s}$.
5. Therefore
$$\frac{\Omega(s)}{V(s)} = \frac{2}{0.5s + 1}$$
which is option (B).
6. Checking the other options: (C) $\dfrac{2}{s + 0.5} = \dfrac{4}{2s + 1}$ has DC gain 4 and $\tau = 2\ \text{s}$. (D) $\dfrac{10}{s + 0.5} = \dfrac{20}{2s + 1}$ has DC gain 20 and $\tau = 2\ \text{s}$. (A) has the right time constant, but its DC gain is 10, so a $5\ \text{V}$ step would settle at $50\ \text{rad/s}$, not $10\ \text{rad/s}$.

KEY IDEA

If armature inductance is negligible, a PMDC motor behaves as a first-order system $\frac{K}{\tau s+1}$. Its DC gain is the final speed divided by the step voltage ($10/5=2$). Its time constant is the time taken to reach 63.2% of the final value (0.5 s).

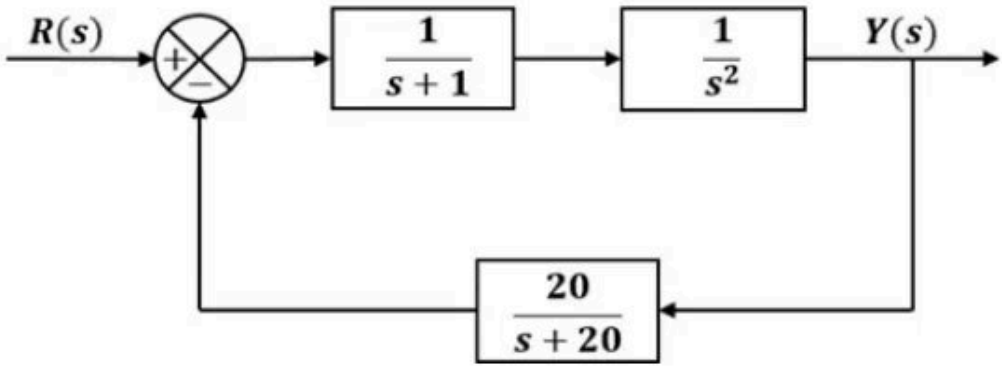
WATCH OUT FOR

Using the final speed (10 rad/s) as the gain and forgetting to divide by the 5 V input, which gives option (A). Another mistake is reading the pole form $\frac{K}{s+a}$ as if a were the time constant. In fact $\tau=1/a$ and the DC gain is K/a, so $\frac{2}{s+0.5}$ has $\tau=2$ s and DC gain 4.

Q.35 MCQ · 2 MARKS

Control Systems › Routh–Hurwitz

Q.No. 35 Which of the following options is correct for the system shown below?



(A) 4^{th} order and stable

(B) 3^{rd} order and stable

(C) 4^{th} order and unstable

(D) 3^{rd} order and unstable

Official answer: **C**

SOLUTION

1. Forward path (two blocks in cascade): $G(s)=\frac{1}{s+1}\cdot\frac{1}{s^2}=\frac{1}{s^2(s+1)}$. Feedback path: $H(s)=\frac{20}{s+20}$, with negative feedback.
2. Closed-loop transfer function:
$$\frac{Y(s)}{R(s)}=\frac{G(s)}{1+G(s)H(s)}=\frac{s+20}{s^2(s+1)(s+20)+20}$$
3. Characteristic equation: $s^2(s^2+21s+20)+20=0 \;\Rightarrow\; s^4+21s^3+20s^2+0\cdot s+20=0$. The highest power of s is 4, so the system is **4th order**. No pole-zero cancellation reduces this: the only closed-loop zero is $s=-20$, and substituting it into the characteristic polynomial gives $(-20)^2(-19)(0)+20=20\neq 0$, so $s=-20$ is not a closed-loop pole.
4. Necessary condition for stability: every coefficient of the characteristic polynomial must be present and positive. Here the s^1 coefficient is zero, so the system cannot be stable. The Routh array tells us which kind of instability this is.
5. Routh array:

$$
\begin{aligned}
s^4 &: \ 1 \quad 20 \quad 20 \\
s^3 &: \ 21 \quad 0 \\
s^2 &: \ \frac{21\cdot 20 - 1\cdot 0}{21} = 20 \quad \frac{21\cdot 20 - 1\cdot 0}{21} = 20 \\
s^1 &: \ \frac{20\cdot 0 - 21\cdot 20}{20} = -21 \\
s^0 &: \ 20
\end{aligned}
$$

6. The first column is $1, 21, 20, -21, 20$. The sign changes twice ($20 \to -21$ and $-21 \to 20$), so two closed-loop poles lie in the right half of the s-plane. The system is **unstable**, not merely marginally stable.
7. The system is 4th order and unstable, so the answer is option (C).

KEY IDEA

The order of the closed-loop system is the degree of $1 + G(s)H(s) = 0$ after clearing fractions, and that degree includes the pole in the feedback block. If any coefficient of the characteristic polynomial is missing (here the s^1 term), the system cannot be stable. A Routh array then counts the right-half-plane poles: two sign changes here.

WATCH OUT FOR

Counting only the forward-path poles ($s^2(s+1)$, which gives 3rd order) and ignoring the pole of the feedback block at $s = -20$. A second mistake is to treat the missing s^1 coefficient as a sign of marginal stability. The Routh array shows two sign changes, which means two poles in the right half-plane, so the system is truly unstable.

Q.36 MCQ · 2 MARKS

Control Systems › Routh–Hurwitz

Consider a negative unity feedback system with the forward path transfer function $\dfrac{s^2+s+1}{s^3+2s^2+2s+K}$, where K is a positive real number. The value of K for which the system will have some of its poles on the imaginary axis is _____.

(A) 9

(B) 8

(C) 7

(D) 6

Official answer: **B**

SOLUTION

1. Closed-loop characteristic equation (unity negative feedback): $1 + G(s) = 0$. Multiplying through by the denominator of $G(s)$ gives $(s^3 + 2s^2 + 2s + K) + (s^2 + s + 1) = 0$.
2. Combine terms:
$$s^3 + 3s^2 + 3s + (K+1) = 0$$
3. Routh array:
$$
\begin{aligned}
s^3 &: \ 1 \quad 3 \\
s^2 &: \ 3 \quad K+1 \\
s^1 &: \ \frac{3\cdot 3 - 1\cdot(K+1)}{3} = \frac{8-K}{3} \\
s^0 &: \ K+1
\end{aligned}
$$
4. Poles on the imaginary axis appear when a complete row of the Routh array becomes zero. The s^0 entry $K+1$ cannot be zero for $K > 0$, so the only possibility is the s^1 row: $\dfrac{8-K}{3} = 0 \Rightarrow K = 8$.
5. Auxiliary equation from the s^2 row at $K = 8$: $3s^2 + 9 = 0 \Rightarrow s = \pm j\sqrt{3}$, so two closed-loop poles sit on the $j\omega$ axis at $\omega = \sqrt{3}\ \text{rad/s}$.

6. Check: at $K = 8$ the characteristic polynomial is $s^3 + 3s^2 + 3s + 9 = (s+3)(s^2+3)$, with roots -3 and $\pm j\sqrt{3}$. Hence $K = 8$, option (B).

KEY IDEA

For a unity negative feedback loop, the closed-loop characteristic polynomial is the open-loop denominator plus the open-loop numerator. Build the Routh array for it. A full row of zeros marks poles on the $j\omega$ axis, and the auxiliary polynomial from the row above gives their location.

WATCH OUT FOR

Running Routh on the open-loop denominator $s^3 + 2s^2 + 2s + K$ alone. Its s^1 entry is $(4 - K)/2$, which gives $K = 4$, and that is not among the options. The numerator $s^2 + s + 1$ has to be added to the denominator first. A second slip is getting the s^1 entry as $(9 + K + 1)/3$ by using the wrong sign in the cross-multiplication.

Q.37 MCQ · 2 MARKS

Signals and Systems › LTI & convolution

Suppose for input $x(t)$ a linear time-invariant system with impulse response $h(t)$ produces output $y(t)$, so that $x(t) * h(t) = y(t)$. Further, if $|x(t)| * |h(t)| = z(t)$, which of the following statements is true?

(A) For all $t \in (-\infty, \infty)$, $z(t) \le y(t)$

(B) For some but not all $t \in (-\infty, \infty)$, $z(t) \le y(t)$

(C) For all $t \in (-\infty, \infty)$, $z(t) \ge y(t)$

(D) For some but not all $t \in (-\infty, \infty)$, $z(t) \ge y(t)$

Official answer: **C**

SOLUTION

1. The question compares $y(t)$ and $z(t)$ with $\le$ and $\ge$, so the signals are real-valued. Write both outputs as convolution integrals:
$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau, \qquad z(t) = \int_{-\infty}^{\infty} |x(\tau)|\, |h(t-\tau)|\, d\tau$$
2. For every τ, $|x(\tau)\, h(t-\tau)| = |x(\tau)|\, |h(t-\tau)|$. The magnitude of an integral is at most the integral of the magnitude (the triangle inequality for integrals), so
$$|y(t)| = \left| \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \right| \le \int_{-\infty}^{\infty} |x(\tau)|\, |h(t-\tau)|\, d\tau = z(t)$$
3. A real number is never larger than its magnitude, so $y(t) \le |y(t)|$. Combining this with step 2:
$$y(t) \le |y(t)| \le z(t) \quad \text{for every } t$$
No condition on x or h was used, so $z(t) \ge y(t)$ for all $t \in (-\infty, \infty)$, for every input and every impulse response. This is option (C).
4. (D) says $z(t) \ge y(t)$ holds for some but NOT all t. Step 3 shows it holds at every t, so the 'not all' part is never true and (D) is always false.
5. (A) is not always true. Take $x(t) = u(t)$ and $h(t) = -u(t)$. For $t \ge 0$,
$$y(t) = \int_0^t (1)(-1)\, d\tau = -t, \qquad z(t) = \int_0^t (1)(1)\, d\tau = t$$
and both are zero for $t < 0$. So $y(t) = -t\, u(t)$ and $z(t) = t\, u(t)$. At $t = 1$, $z = 1 > y = -1$, so $z(t) \le y(t)$ fails and (A) is false.
6. (B) is not always true either. Take $x(t) = h(t) = u(t)$. Both signals are non-negative, so $|x| = x$ and $|h| = h$, giving $z(t) = y(t) = t\, u(t)$. Then $z(t) \le y(t)$ holds at EVERY t (with equality), so 'some but not all' is false. (B) does happen to hold for the pair in step 5, which is why one example cannot settle this. The question asks which statement is true for every x and h, and only (C) is. Answer: (C).

KEY IDEA

The magnitude of a convolution is at most the convolution of the magnitudes, $|x * h|(t) \le (|x| * |h|)(t)$, at every t. This is the triangle inequality applied to the convolution integral. Since a real number is at most its magnitude, $y(t) \le |y(t)| \le z(t)$ everywhere, for any x and h.

WATCH OUT FOR

Testing a single pair of signals and reading the options as claims about that pair. With $x = u(t)$ and $h = -u(t)$, $z(t) \le y(t)$ holds for $t < 0$ and fails for $t > 0$, which tempts students into (B). The question asks what is true for EVERY x and h, and one pair can never establish that. Some students also pick (D) without seeing that its 'but not all' clause makes it false, because the inequality holds at every instant.

Q.38 MCQ · 2 MARKS

Signals and Systems › Laplace

The causal realization of a system transfer function $H(s)$ having poles at $(2, -1)$, $(-2, 1)$ and zeroes at $(2, 1)$, $(-2, -1)$ will be

(A) stable, real, allpass

(B) unstable, complex, allpass

(C) unstable, real, highpass

(D) stable, complex, lowpass

Official answer: **B**

SOLUTION

1. Read each coordinate pair as a point $s = \sigma + j\omega$ in the s-plane. The poles are $p_1 = 2 - j$ and $p_2 = -2 + j$. The zeros are $z_1 = 2 + j$ and $z_2 = -2 - j$.
2. Build the transfer function (unity gain). Each pair of factors is a difference of squares:
$$H(s) = \frac{(s-2-j)(s+2+j)}{(s-2+j)(s+2-j)} = \frac{s^2 - (2+j)^2}{s^2 - (2-j)^2} = \frac{s^2 - 3 - 4j}{s^2 - 3 + 4j}$$
3. Real or complex: a system with a real impulse response has a real-coefficient $H(s)$, so its complex poles come in conjugate pairs. The conjugate of the pole $2 - j$ is $2 + j$, which is a zero, not a pole. The poles are not a conjugate pair, the coefficients $-3 \mp 4j$ are complex, and so $h(t)$ is complex-valued. The system is **complex**.
4. All-pass test: each zero must be the mirror image of a pole about the $j\omega$ axis, $z = -p^*$. Here $-(2 - j)^* = -(2 + j) = -2 - j = z_2$ and $-(-2 + j)^* = -(-2 - j) = 2 + j = z_1$. Both zeros pass.
5. Confirm on the $j\omega$ axis. Put $s = j\omega$, so $s^2 = -\omega^2$:
$$|H(j\omega)|^2 = \frac{|-(\omega^2+3) - 4j|^2}{|-(\omega^2+3) + 4j|^2} = \frac{(\omega^2+3)^2 + 16}{(\omega^2+3)^2 + 16} = 1 \quad \text{for all } \omega$$
The magnitude is flat, so the system is **all-pass**. Any other constant gain K gives $|H(j\omega)| = |K|$, which is still flat.
6. Stability from the ROC: the ROC of a causal system lies to the right of its rightmost pole. The pole $2 - j$ is in the right half-plane, so the ROC is $\mathrm{Re}(s) > 2$. That region does not contain the $j\omega$ axis, so the causal realization is **unstable**. The residue of $H(s)$ at $s = 2 - j$ is $\frac{(2-j)^2 - 3 - 4j}{(2-j) + 2 - j} = \frac{-8j}{4 - 2j} \neq 0$, so $h(t)$ really does contain a term in $e^{(2-j)t}u(t)$, whose magnitude e^{2t} grows without bound.
7. The system is unstable, complex and all-pass, which is option (B). Every other option fails at least one test: (A) and (D) say stable, (A) and (C) say real, (C) says highpass and (D) says lowpass.

KEY IDEA

Read each property straight from the pole-zero pattern. Zeros that mirror the poles about the $j\omega$ axis ($z = -p^*$) give an all-pass. Complex poles without their conjugates give complex coefficients, so the system is complex. A causal system with a right-half-plane pole has ROC $\mathrm{Re}(s) > 2$, which misses the $j\omega$ axis, so it is unstable.

WATCH OUT FOR

Taking the symmetric-looking pattern as proof the system is real. The poles $2 - j$ and $-2 + j$ are symmetric about the origin (p and $-p$), not about the real axis. Symmetry about the origin only makes H a function of s^2; it does not make the coefficients real. A second trap is assuming an all-pass is automatically stable. All-pass describes only $|H(j\omega)|$. Causality combined with the right-half-plane pole at $2 - j$ makes the system unstable, which rules out (A).

Q.39 MCQ · 2 MARKS

Signals and Systems › LTI & convolution

Which of the following options is true for a linear time-invariant discrete time system that obeys the difference equation:

$$y[n] - ay[n-1] = b_0 x[n] - b_1 x[n-1]$$

(A) $y[n]$ is unaffected by the values of $x[n-k];\ k > 2$.

(B) The system is necessarily causal.

(C) The system impulse response is non-zero at infinitely many instants.

(D) When $x[n] = 0,\ n < 0$, the function $y[n];\ n > 0$ is solely determined by the function $x[n]$.

Official answer: **C**

Q.40 MCQ · 2 MARKS

Electromagnetics › Electrostatics

Let $\mathbf{a}_r$, $\mathbf{a}_\phi$ and $\mathbf{a}_z$ be unit vectors along r, ϕ and z directions, respectively in the cylindrical coordinate system. For the electric flux density given by $\mathbf{D} = (\mathbf{a}_r\, 15 + \mathbf{a}_\phi\, 2r - \mathbf{a}_z\, 3rz)\ \mathrm{Coulomb/m^2}$, the total electric flux, in Coulomb, emanating from the volume enclosed by a solid cylinder of radius 3 m and height 5 m oriented along the z-axis with its base at the origin is:

(A) 54π

(B) 90π

(C) 108π

(D) 180π

Official answer: **D**

SOLUTION

1. The total flux leaving a closed surface is $\Psi = \oint_S \mathbf{D} \cdot d\mathbf{S} = \int_V (\nabla \cdot \mathbf{D})\, dV$ (divergence theorem). The volume is $0 \le r \le 3$, $0 \le \phi \le 2\pi$, $0 \le z \le 5$.
2. Here $D_r = 15$, $D_\phi = 2r$, $D_z = -3rz$. The divergence in cylindrical coordinates is
$$\nabla \cdot \mathbf{D} = \frac{1}{r}\frac{\partial (rD_r)}{\partial r} + \frac{1}{r}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z} = \frac{1}{r}\frac{\partial (15r)}{\partial r} + 0 + \frac{\partial (-3rz)}{\partial z} = \frac{15}{r} - 3r$$
The ϕ term is zero because $D_\phi = 2r$ does not depend on ϕ.
3. The volume element is $dV = r\, dr\, d\phi\, dz$, so the integrand becomes $\left(\frac{15}{r} - 3r\right) r = 15 - 3r^2$.
4. $$\Psi = \int_0^5 \int_0^{2\pi} \int_0^3 (15 - 3r^2)\, dr\, d\phi\, dz = \left[15r - r^3\right]_0^3 \times 2\pi \times 5 = (45 - 27)(10\pi) = 180\pi\ \mathrm{C}$$
5. Check by integrating over the three faces. Curved side ($r = 3$, outward normal $\hat{a}_r$, $D_r = 15$): $15 \times (2\pi \cdot 3 \cdot 5) = 450\pi$. Top ($z = 5$, outward normal $\hat{a}_z$, $D_z = -15r$): $\int_0^{2\pi}\int_0^3 (-15r)\, r\, dr\, d\phi = -15 \cdot \frac{27}{3} \cdot 2\pi =$

-270π. Bottom ($z = 0$): $D_z = 0$, so the flux is 0. The $\hat{a}_\phi$ component is tangential to every face and contributes nothing.

6. Total: $450\pi - 270\pi + 0 = 180\pi$ C, which agrees with the volume integral. The answer is option (D).

KEY IDEA

The flux out of a closed surface is the volume integral of $\nabla \cdot \mathbf{D}$ (divergence theorem). In cylindrical coordinates the radial term is $\frac{1}{r}\frac{\partial(rD_r)}{\partial r}$, and the extra r in $dV = r\,dr\,d\phi\,dz$ cancels the $\frac{1}{r}$. The ϕ component carries no flux through any face of a cylinder centred on the z-axis.

WATCH OUT FOR

Taking $\frac{\partial D_r}{\partial r} = 0$ (as if in Cartesian coordinates), which drops the radial term and gives -270π, which is not an option. Leaving out the Jacobian r in dV is another route to a wrong answer. Other slips are giving the top face the wrong sign (its outward normal is $+\hat{a}_z$ and $D_z = -15r$ there, so its flux is negative) and counting flux from D_ϕ, which is tangential to every face.

Q.41 MCQ · 2 MARKS

Control Systems › Bode & margins

A stable real linear time-invariant system with single pole at p, has a transfer function $H(s) = \dfrac{s^2 + 100}{s - p}$ with a dc gain of 5. The smallest positive frequency, in rad/s, at unity gain is closest to:

(A) 8.84

(B) 11.08

(C) 78.13

(D) 122.87

Official answer: **A**

SOLUTION

1. **DC gain fixes the pole.** Put $s = 0$: $H(0) = \dfrac{100}{-p} = 5 \Rightarrow p = -20$. The pole is in the left half-plane, which agrees with the system being stable. So $H(s) = \dfrac{s^2 + 100}{s + 20}$.
2. **Magnitude on the $j\omega$ axis.** With $s = j\omega$, the numerator is $100 - \omega^2$ (real) and the denominator is $20 + j\omega$, so
$$|H(j\omega)| = \frac{|100 - \omega^2|}{\sqrt{\omega^2 + 400}}$$
3. **Unity-gain condition** $|H(j\omega)| = 1$. Square both sides:
$$(100 - \omega^2)^2 = \omega^2 + 400$$
4. **Substitute $x = \omega^2$:** $x^2 - 200x + 10000 = x + 400 \Rightarrow x^2 - 201x + 9600 = 0$.
5. **Quadratic formula:**
$$x = \frac{201 \pm \sqrt{201^2 - 4(9600)}}{2} = \frac{201 \pm \sqrt{40401 - 38400}}{2} = \frac{201 \pm \sqrt{2001}}{2} = \frac{201 \pm 44.73}{2}$$
which gives $x = 78.13$ or $x = 122.87$.
6. **Take square roots, since these are values of ω^2:** $\omega = \sqrt{78.13} = 8.84$ rad/s or $\omega = \sqrt{122.87} = 11.08$ rad/s. Check at $\omega = 8.84$: $|100 - 78.13| = 21.87$ and $\sqrt{78.13 + 400} = 21.87$, so the gain is indeed 1.
7. **Pick the smaller root.** The gain starts at 5 at $\omega = 0$, falls to zero at the notch $\omega = 10$ rad/s (the zeros $s = \pm j10$), then rises without bound. It therefore crosses unity once below the notch and once above it. The smallest positive unity-gain frequency is 8.84 rad/s, option (A).

KEY IDEA

Use the dc gain $H(0)$ to find the unknown pole. Then set $|H(j\omega)|^2 = 1$, which is a quadratic in ω^2. The zeros at $\pm j10$ make a notch at $\omega = 10$, so the gain falls from 5 to 0 and then rises again, and it crosses unity once on each side of the notch. The smaller crossing is the answer.

WATCH OUT FOR

Picking 78.13 (option C) or 122.87 (option D). Those are the roots for ω^2, not ω, and the options are set up to catch anyone who forgets the square root. Another trap is a sign slip in $H(0) = 100/(-p)$ that gives $p = +20$. That pole is unstable and contradicts the problem statement, though it gives the same magnitude, so check the sign against stability.

Q.42 MCQ · 2 MARKS

Engineering Mathematics › Linear algebra

The number of purely real elements in a lower triangular representation of the given 3×3 matrix, obtained through the given decomposition is _____.

$$\begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 7 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} a_{11} & 0 & 0 \\ a_{12} & a_{22} & 0 \\ a_{13} & a_{23} & a_{33} \end{bmatrix}^T$$

(A) 5

(B) 6

(C) 8

(D) 9

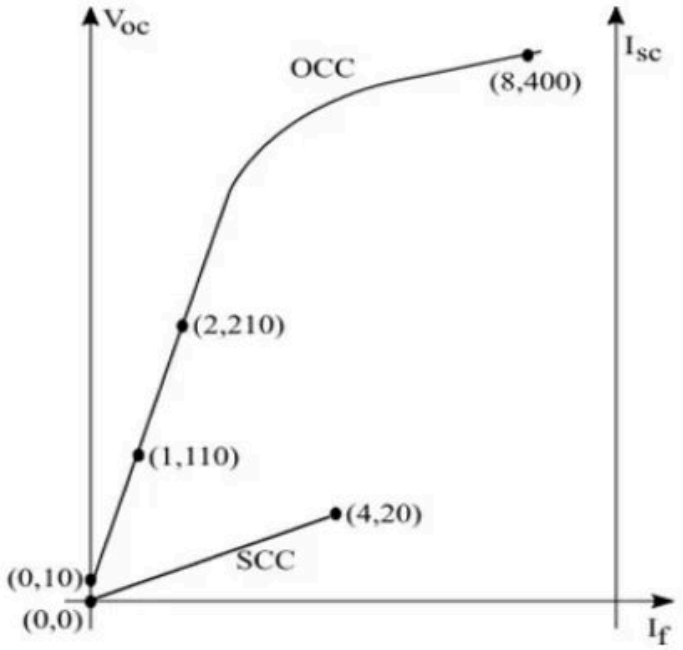
Official answer: **Marks to all**

Q.43 MCQ · 2 MARKS

Electrical Machines › Synchronous machines

Q.No. 43

The figure below shows the per-phase Open Circuit Characteristics (measured in V) and Short Circuit Characteristics (measured in A) of a 14 kVA, 400 V, 50 Hz, 4-pole, 3-phase, delta connected alternator, driven at 1500 rpm. The field current, I_f is measured in A. Readings taken are marked as respective (x, y) coordinates in the figure. Ratio of the unsaturated and saturated synchronous impedances $(Z_{s(unsat)}/Z_{s(sat)})$ of the alternator is closest to:



(A) 2.100
(B) 2.025
(C) 2.000
(D) 1.000

Official answer: **A**

Q.44 NAT · 2 MARKS

Engineering Mathematics › Vector calculus

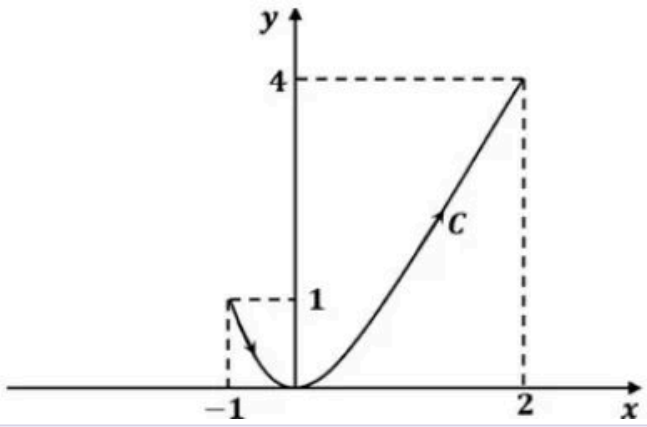
Q.No. 44 Let $\mathbf{a}_x$ and $\mathbf{a}_y$ be unit vectors along x and y directions, respectively. A vector function is given by

$$\mathbf{F} = \mathbf{a}_x\, y - \mathbf{a}_y\, x$$

The line integral of the above function

$$\int_C \mathbf{F} \cdot d\boldsymbol{l}$$

along the curve C, which follows the parabola $y = x^2$ as shown below is _____ (rounded off to 2 decimal places).



Official answer: **-3.05 to -2.95**

SOLUTION

1. Read the path from the figure: C is the arc of $y = x^2$ from $(-1, 1)$ through the origin to $(2, 4)$. The arrowheads show that direction, so x runs from -1 to 2.
2. Write out the integrand using $d\mathbf{l} = \mathbf{a}_x\, dx + \mathbf{a}_y\, dy$:
$$\mathbf{F} \cdot d\mathbf{l} = y\, dx - x\, dy$$
3. Parametrise by x along the curve: $y = x^2$, so $dy = 2x\, dx$.
4. Substitute:
$$y\, dx - x\, dy = x^2\, dx - x(2x\, dx) = -x^2\, dx$$
5. Integrate along the path:
$$\int_C \mathbf{F} \cdot d\mathbf{l} = \int_{-1}^{2} -x^2\, dx = -\left[\frac{x^3}{3}\right]_{-1}^{2} = -\left(\frac{8}{3} + \frac{1}{3}\right) = -3$$
6. Check: $\nabla \times \mathbf{F} = \left(\frac{\partial(-x)}{\partial x} - \frac{\partial y}{\partial y}\right)\mathbf{a}_z = -2\,\mathbf{a}_z \neq 0$. The field is not conservative, so the integral depends on the path and has to be taken along the parabola itself. The answer is -3.00.

KEY IDEA

For a line integral along $y = x^2$, write $\mathbf{F} \cdot d\mathbf{l} = F_x\,dx + F_y\,dy$. Then replace y and dy with x^2 and $2x\,dx$ and integrate in x from the start point to the end point shown by the arrows. Here $\nabla \times \mathbf{F} = -2\,\mathbf{a}_z \neq 0$, so no potential function exists. The integral cannot be found from the endpoints alone and must be taken along the given curve.

WATCH OUT FOR

Integrating only over the right branch (x from 0 to 2), which gives $-8/3 \approx -2.67$. Other common slips: reversing the direction of travel, which gives $+3$; dropping the minus sign on the $\mathbf{a}_y\,x$ term, so the integrand becomes $3x^2\,dx$ and the answer 9; and treating the field as conservative by taking a difference between the endpoints.

Q.45 NAT · 2 MARKS

Networks (Electric Circuits) › Transients

A resistor and a capacitor are connected in series to a 10 V dc supply through a switch. The switch is closed at $t = 0$, and the capacitor voltage is found to cross 0 V at $t = 0.4\tau$, where τ is the circuit time constant. The absolute value of percentage change required in the initial capacitor voltage if the zero crossing has to happen at $t = 0.2\tau$ is _____ (rounded off to 2 decimal places).

Official answer: **54.00 to 56.00**

SOLUTION

1. Write the first-order step response of a series RC circuit: $v_C(t) = v_C(\infty) + [v_C(0^+) - v_C(\infty)]\,e^{-t/\tau}$, with $v_C(\infty) = 10\,\text{V}$ because the capacitor charges to the supply voltage. Let $V_0 = v_C(0^+) = v_C(0^-)$, since capacitor voltage cannot jump.
2. For $t > 0$ the capacitor voltage moves steadily towards $+10\,\text{V}$, so it can only cross zero if it starts negative, i.e. $V_0 < 0$. Setting $v_C(t_0) = 0$: $10 + (V_0 - 10)e^{-t_0/\tau} = 0 \Rightarrow V_0 = 10\left(1 - e^{t_0/\tau}\right)$.
3. Present case, $t_0 = 0.4\tau$: $V_0 = 10\left(1 - e^{0.4}\right) = 10(1 - 1.49182) = -4.9182\,\text{V}$.
4. Required case, $t_0 = 0.2\tau$: $V_0' = 10\left(1 - e^{0.2}\right) = 10(1 - 1.22140) = -2.2140\,\text{V}$.
5. Percentage change relative to the original initial voltage:
$$\frac{V_0' - V_0}{V_0} \times 100 = \frac{-2.2140 - (-4.9182)}{-4.9182} \times 100 = \frac{2.7042}{-4.9182} \times 100 = -54.98\%$$
The initial voltage has to be reduced in magnitude by about 55%.
6. Check in closed form: $\left|\dfrac{e^{0.4} - e^{0.2}}{e^{0.4} - 1}\right| = \dfrac{e^{0.2}\left(e^{0.2} - 1\right)}{\left(e^{0.2} - 1\right)\left(e^{0.2} + 1\right)} = \dfrac{e^{0.2}}{e^{0.2} + 1} = \dfrac{1.22140}{2.22140} = 0.54983$, so the absolute percentage change is 54.98%.

KEY IDEA

A capacitor charging towards $+10\,\text{V}$ can only pass through zero if its initial voltage is negative. Setting $v_C(t_0) = 0$ in $v_C(t) = 10 + (V_0 - 10)e^{-t/\tau}$ gives $V_0 = 10(1 - e^{t_0/\tau})$. Work out V_0 for both crossing times and compare them. The ratio simplifies to $e^{0.2}/(1 + e^{0.2})$.

WATCH OUT FOR

Assuming the capacitor starts uncharged (then it never crosses 0 V), or putting $e^{-t_0/\tau}$ in place of $e^{t_0/\tau}$ when solving for V_0. Another mistake is dividing the change by the new voltage V_0' instead of the original V_0, which gives about 122% instead of 54.98%.

Q.46 NAT · 2 MARKS

Networks (Electric Circuits) › AC, power & resonance

A cylindrical rotor synchronous generator with constant real power output and constant terminal voltage is supplying 100 A current to a 0.9 lagging power factor load. An ideal reactor is now connected in parallel with the load, as a result of which the total lagging reactive power requirement of the load is twice the

previous value while the real power remains unchanged. The armature current is now _____ A (rounded off to 2 decimal places).

Official answer: **123.00 to 127.00**

SOLUTION

1. Split the original armature current into its active and reactive parts. The terminal voltage V is constant, so $P = VI\cos\phi \propto I\cos\phi$ and $Q = VI\sin\phi \propto I\sin\phi$:
$$I_p = 100 \times 0.9 = 90\ \text{A}, \qquad I_q = 100\sqrt{1 - 0.9^2} = 100\sqrt{0.19} = 43.589\ \text{A (lagging)}$$
2. An ideal reactor absorbs only lagging reactive power, so the real power stays the same. With V and P both constant, the active component does not change: $I'_p = 90\ \text{A}$.
3. The question says the total lagging reactive power (load plus reactor) is now twice what it was. At the same V, the reactive component of current doubles too:
$$I'_q = 2 \times 43.589 = 87.178\ \text{A}$$
4. Add the two components in quadrature. Since $I_q^2 = 100^2 \times 0.19 = 1900$, we have $I_q'^2 = 4 \times 1900 = 7600$:
$$I' = \sqrt{I_p'^2 + I_q'^2} = \sqrt{90^2 + 7600} = \sqrt{8100 + 7600} = \sqrt{15700}$$
5. $$I' = 125.30\ \text{A}$$
The new power factor is $90/125.30 = 0.718$ lagging. You do not need the machine's reactance or excitation: at a fixed terminal voltage, the generator supplies whatever P and Q the load draws, and the excitation only settles the internal EMF needed to do that.

KEY IDEA

The terminal voltage is fixed, so the active current tracks P and the reactive current tracks Q. The reactor leaves the active current at $90\ \text{A}$ and doubles the lagging reactive current from $43.59\ \text{A}$ to $87.18\ \text{A}$. The new armature current is these two added in quadrature.

WATCH OUT FOR

Doubling the whole current to 200 A, or doubling the angle ϕ instead of Q (which gives $100 \times 0.9/\cos(2\phi) = 146.9\ \text{A}$). Another slip is adding the reactor's current to the load current by simple arithmetic instead of in quadrature. Only the reactive component doubles, and the active component stays at $90\ \text{A}$.

Q.47 NAT · 2 MARKS

Power Systems › Transmission & distribution

Q.No. 47 Bus 1 with voltage magnitude $V_1 = 1.1$ pu is sending reactive power Q_{12} towards bus 2 with voltage magnitude $V_2 = 1$ pu through a lossless transmission line of reactance X. Keeping the voltage at bus 2 fixed at 1 pu, magnitude of voltage at bus 1 is changed, so that the reactive power Q_{12} sent from bus 1 is increased by 20%. Real power flow through the line under both the conditions is zero. The new value of the voltage magnitude, V_1, in pu (rounded off to 2 decimal places), at bus 1 is _____.



Official answer: **1.11 to 1.13**

SOLUTION

1. The line is lossless, so $P_{12} = \dfrac{V_1 V_2}{X}\sin\delta$. Real power flow is zero in both conditions, so $\sin\delta = 0$ and $\delta = 0$ in both.
2. The reactive power sent from bus 1 is $Q_{12} = \dfrac{V_1^2 - V_1 V_2 \cos\delta}{X}$. With $\delta = 0$ and $V_2 = 1$ pu this becomes $Q_{12} = \dfrac{V_1(V_1 - 1)}{X}$.
3. Initial case, $V_1 = 1.1$ pu: $Q_{12} = \dfrac{1.1(1.1 - 1)}{X} = \dfrac{0.11}{X}$.
4. Q_{12} is increased by 20%, so the new value is $Q'_{12} = 1.2 \times \dfrac{0.11}{X} = \dfrac{0.132}{X}$. The reactance X and V_2 are unchanged.
5. Equate the new Q_{12} to the formula: $V_1(V_1 - 1) = 0.132$, which gives $V_1^2 - V_1 - 0.132 = 0$.
6. Solve the quadratic: $V_1 = \dfrac{1 \pm \sqrt{1 + 4(0.132)}}{2} = \dfrac{1 \pm \sqrt{1.528}}{2}$, with $\sqrt{1.528} \approx 1.2361$.
7. The negative root gives a negative voltage magnitude, which is not physical, so reject it. Then $V_1 = \dfrac{1 + 1.2361}{2} \approx 1.118$ pu, which is 1.12 pu to 2 decimal places.

KEY IDEA

With zero real power flow the angle is zero, so reactive power is $Q_{12} = V_1(V_1 - V_2)/X$. A 20% increase in Q_{12} gives a quadratic in V_1. Take its positive root.

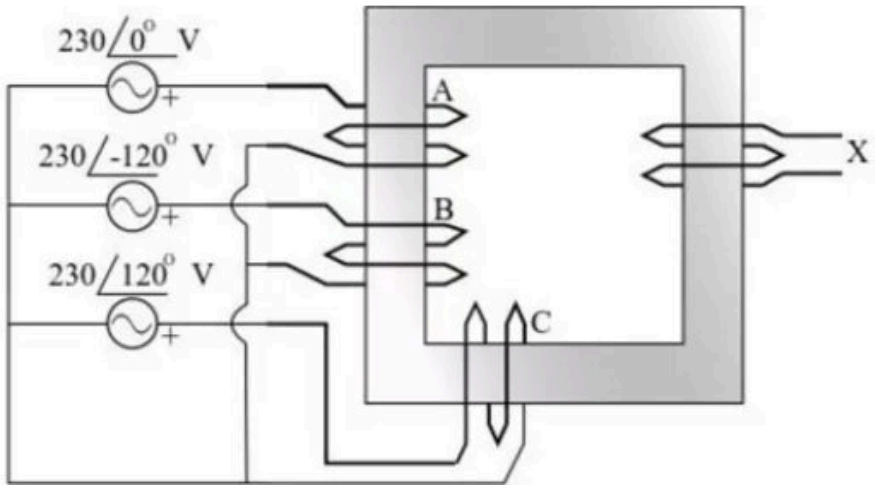
WATCH OUT FOR

Using a 20% rise in voltage instead of a 20% rise in reactive power. Another is treating Q as linear in V_1. Q depends on V_1^2, so V_1 rises by only about 1.6%, not 20%.

Q.48 NAT · 2 MARKS

Electrical Machines › Transformers

Q.No. 48 Windings 'A', 'B' and 'C' have 20 turns each and are wound on the same iron core as shown, along with winding 'X' which has 2 turns. The figure shows the sense (clockwise/anti-clockwise) of each of the windings only and does not reflect the exact number of turns. If windings 'A', 'B' and 'C' are supplied with balanced 3-phase voltages at 50 Hz and there is no core saturation, the no-load RMS voltage (in V, rounded off to 2 decimal places) across winding 'X' is _____.



Official answer: **Marks to all**

Q.49 NAT · 2 MARKS

Electrical Machines › Synchronous machines

A cylindrical rotor synchronous generator has steady state synchronous reactance of 0.7 pu and subtransient reactance of 0.2 pu. It is operating at $(1 + j0)$ pu terminal voltage with an internal emf of $(1 + j0.7)$ pu. Following a three-phase solid short circuit fault at the terminal of the generator, the magnitude of the subtransient internal emf (rounded off to 2 decimal places) is _____ pu.

Official answer: **1.01 to 1.03**

SOLUTION

1. Because rotor flux linkages cannot change instantaneously at the instant of a fault, the emf behind the subtransient reactance must be evaluated using the prefault load current, first found from the steady-state circuit.
2. Prefault steady-state relation: $E = V + jX_dI$, with $X_d = 0.7$ pu, $V = 1\angle 0^\circ$ pu, $E = 1 + j0.7$ pu.
3. Solve for the prefault current:
$$I = \frac{E - V}{jX_d} = \frac{(1 + j0.7) - (1 + j0)}{j0.7} = \frac{j0.7}{j0.7} = 1\angle 0^\circ \text{ pu}$$
4. The subtransient internal emf is the emf behind the subtransient reactance, evaluated at this same prefault current: $E'' = V + jX''_dI$, with $X''_d = 0.2$ pu.
5. Compute:
$$E'' = (1 + j0) + j0.2(1) = 1 + j0.2 \text{ pu}$$
6. Magnitude:

$$|E''| = \sqrt{1^2 + 0.2^2} = \sqrt{1.04} \approx 1.02 \text{ pu}$$

KEY IDEA

Flux linkages (and hence the emf behind the subtransient reactance) cannot change instantaneously at the instant of fault, so E'' is computed using the prefault load current together with the subtransient reactance, evaluated at the prefault terminal voltage.

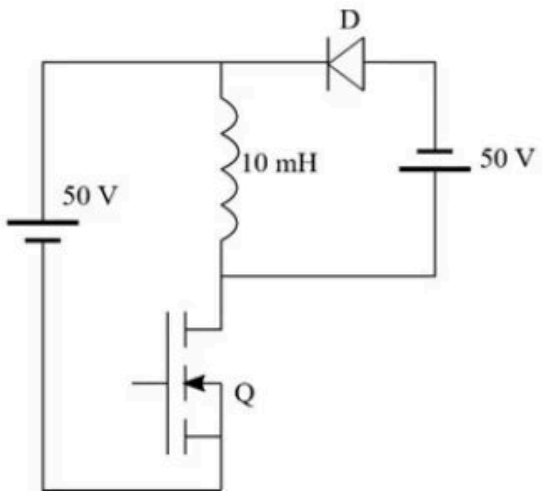
WATCH OUT FOR

Using the subtransient reactance to recompute a "prefault" current, or trying to use the given internal emf $E = 1 + j0.7$ directly as E'', instead of first finding the actual prefault line current from the steady-state relation $E = V + jX_dI$ and then applying that same current with X_d''.

Q.50 NAT · 2 MARKS

Power Electronics › DC-DC converters

Q.No. 50 In the dc-dc converter circuit shown, switch Q is switched at a frequency of 10 kHz with a duty ratio of 0.6. All components of the circuit are ideal, and the initial current in the inductor is zero. Energy stored in the inductor in mJ (rounded off to 2 decimal places) at the end of 10 complete switching cycles is _____.



Official answer: **4.95 to 5.05**

SOLUTION

1. Data: $V_s = 50$ V, $E_b = 50$ V, $L = 10$ mH, $f = 10$ kHz, so $T = 100$ μs. The duty ratio is $D = 0.6$, so $T_{on} = DT = 60$ μs and $T_{off} = (1 - D)T = 40$ μs. Call the inductor's top end node A (the source's positive rail) and its bottom end node B (the drain of Q). Take the inductor current i_L as positive from A to B.
2. Q ON: node B is connected to the source's negative terminal, so $v_L = v_A - v_B = +50$ V. The battery's positive terminal is at B (0 V), so its negative terminal, which is the diode's anode, sits at -50 V. The cathode is at $+50$ V, so D is reverse biased and blocks. The current rises:
$$\Delta i_{on} = \frac{V_s}{L} T_{on} = \frac{50}{10 \times 10^{-3}} \times 60 \times 10^{-6} = 0.3 \text{ A}$$
3. Q OFF: i_L cannot change instantly. It flows from B into the battery's positive terminal, out of its negative terminal, through D (anode to cathode) and back to A. Around this loop, $v_A = v_B - E_b$, so $v_L = -50$ V: the battery absorbs energy and the current falls:
$$\Delta i_{off} = -\frac{E_b}{L} T_{off} = -\frac{50}{10 \times 10^{-3}} \times 40 \times 10^{-6} = -0.2 \text{ A}$$
4. Check that the current never reaches zero. In cycle 1 it rises from 0 to 0.3 A and falls to 0.1 A, so it stays positive and D conducts for the whole off interval. Each later cycle starts higher, so the same

holds every cycle and the conduction is continuous.

5. Net change per cycle: $\Delta i = 0.3 - 0.2 = 0.1$ A. Equivalently, the volt-seconds do not balance: $(50)(60\ \mu\text{s}) - (50)(40\ \mu\text{s}) = 1\ \text{mV·s}$ per cycle, and $1\ \text{mV·s}/10\ \text{mH} = 0.1$ A.
6. At the end of 10 complete cycles (each cycle ends at the end of its off interval): $I = 10 \times 0.1 = 1$ A.
7. Energy stored:

$$W = \tfrac{1}{2}LI^2 = \tfrac{1}{2} \times 10 \times 10^{-3} \times 1^2 = 5 \times 10^{-3}\ \text{J} = 5.00\ \text{mJ}$$

KEY IDEA

The inductor sees +50 V for DT and −50 V (the battery) for $(1-D)T$. With $D = 0.6$ the volt-seconds do not balance, so the current climbs by $\frac{50}{L}(2D-1)T = 0.1$ A in every cycle. After 10 cycles it is 1 A, and $W = \frac{1}{2}LI^2$.

WATCH OUT FOR

Reading the right-hand battery the wrong way up. If you do, the inductor seems to see +50 V all the time and you get 125 mJ. Read the polarity from the plate lengths: the long plate is positive. Another slip is to assume steady state (zero net change per cycle). This circuit starts from zero current and is not in steady state, because the volt-seconds per cycle are not zero when $D \neq 0.5$.

Q.51 NAT · 2 MARKS

Power Electronics › Rectifiers

A single-phase, full-bridge, fully controlled thyristor rectifier feeds a load comprising a 10 Ω resistance in series with a very large inductance. The rectifier is fed from an ideal 230 V, 50 Hz sinusoidal source through cables which have negligible internal resistance and a total inductance of 2.28 mH. If the thyristors are triggered at an angle α = 45°, the commutation overlap angle in degree (rounded off to 2 decimal places) is _____.

Official answer: **4.51 to 5.10**

SOLUTION

1. Given: $V_s = 230$ V rms, so $V_m = 230\sqrt{2} = 325.27$ V; $R = 10\ \Omega$; $\omega L_s = 2\pi(50)(2.28 \times 10^{-3}) = 0.7163\ \Omega$; firing angle $\alpha = 45^\circ$. The load inductance is very large, so the load current I_{dc} is constant (ripple-free).
2. With source inductance, the average output voltage of a single-phase fully controlled bridge is

$$V_{dc} = \frac{2V_m}{\pi}\cos\alpha - \frac{2\omega L_s}{\pi}I_{dc}.$$

   The second term is the drop caused by commutation overlap.
3. Ideal term: $\frac{2V_m}{\pi}\cos 45^\circ = 207.07 \times 0.7071 = 146.42$ V. Drop coefficient: $\frac{2\omega L_s}{\pi} = 0.4560\ \Omega$.
4. The load is R in series with a very large inductance, so $I_{dc} = V_{dc}/R = V_{dc}/10$. Then $V_{dc} = 146.42 - 0.0456\,V_{dc}$, so

$$V_{dc} = \frac{146.42}{1.0456} = 140.04\ \text{V}, \qquad I_{dc} = 14.004\ \text{A}.$$

5. During overlap the output voltage is the average of the two source voltages. Over a full half-cycle this gives $V_{dc} = \frac{V_m}{\pi}[\cos\alpha + \cos(\alpha+\mu)]$. Equating this with the expression in step 2 gives

$$\cos\alpha - \cos(\alpha+\mu) = \frac{2\omega L_s I_{dc}}{V_m}.$$

6. Right side: $\frac{2(0.7163)(14.004)}{325.27} = 0.06168$.
7. $\cos(\alpha+\mu) = 0.70711 - 0.06168 = 0.64543$, so $\alpha + \mu = \cos^{-1}(0.64543) = 49.80^\circ$.
8. Overlap angle: $\mu = 49.80^\circ - 45^\circ \approx 4.80^\circ$.

KEY IDEA

With source inductance and a constant load current, the overlap angle follows from $\cos\alpha - \cos(\alpha+\mu) = 2\omega L_s I_{dc}/V_m$. Here I_{dc} is not given, so first find it from the resistive load together with the voltage-drop equation $V_{dc} = \frac{2V_m}{\pi}\cos\alpha - \frac{2\omega L_s}{\pi} I_{dc}$.

WATCH OUT FOR

Using the ideal output $V_{dc} = \frac{2V_m}{\pi}\cos\alpha$ to get I_{dc}, which ignores the overlap drop. That gives $I_{dc} = 14.64$ A and an overlap of about 5.0 degrees, which is slightly wrong. Another slip is writing the drop as $\frac{\omega L_s}{\pi} I_{dc}$, which holds for a half-wave circuit, not the full bridge with its factor of 2.

Q.52 NAT · 2 MARKS

Electrical and Electronic Measurements › Errors & standards

A non-ideal Si-based pn junction diode is tested by sweeping the bias applied across its terminals from -5 V to $+5$ V. The effective thermal voltage, V_T, for the diode is measured to be (29 ± 2) mV. The resolution of the voltage source in the measurement range is 1 mV. The percentage uncertainty (rounded off to 2 decimal places) in the measured current at a bias voltage of 0.02 V is _____.

Official answer: **11.50 to 12.00**

SOLUTION

1. Use the full diode equation. The effective thermal voltage already includes the ideality factor, so
$$I = I_s\left(e^{V/V_T} - 1\right)$$
The saturation current I_s is not given with an uncertainty, so it is treated as exact. The only uncertain quantities are the bias V and the thermal voltage V_T.
2. Data: $V = 0.02\ \text{V} = 20\ \text{mV}$, $\delta V = 1\ \text{mV}$ (the resolution of the source), $V_T = 29\ \text{mV}$, $\delta V_T = 2\ \text{mV}$.
3. You cannot drop the -1 here. The shortcut $I \approx I_s e^{V/V_T}$ only works when $V \gg V_T$. At this bias
$$\frac{V}{V_T} = \frac{20}{29} = 0.68966, \qquad e^{V/V_T} = 1.99303, \qquad e^{V/V_T} - 1 = 0.99303$$
The exponential is only about twice the 1, so the -1 roughly halves the current and has to be kept.
4. Partial derivatives:
$$\frac{\partial I}{\partial V} = \frac{I_s}{V_T}e^{V/V_T}, \qquad \frac{\partial I}{\partial V_T} = -\frac{I_s V}{V_T^2}e^{V/V_T}$$
Divide each by $I = I_s\left(e^{V/V_T} - 1\right)$. Both relative sensitivities then carry the same factor
$$k = \frac{e^{V/V_T}}{e^{V/V_T} - 1} = \frac{1.99303}{0.99303} = 2.00702$$
5. Relative uncertainty in I due to V:
$$\frac{1}{I}\frac{\partial I}{\partial V}\delta V = k\,\frac{\delta V}{V_T} = 2.00702 \times \frac{1}{29} = 0.069208$$
6. Relative uncertainty in I due to V_T:
$$\frac{1}{I}\left|\frac{\partial I}{\partial V_T}\right|\delta V_T = k\,\frac{V\,\delta V_T}{V_T^2} = 2.00702 \times \frac{20 \times 2}{29^2} = 2.00702 \times 0.047562 = 0.095459$$
7. V (set by the source) and V_T (a property of the diode) are measured independently, so the two contributions add as a root-sum-square:
$$\frac{\delta I}{I} = \sqrt{\left(k\,\frac{\delta V}{V_T}\right)^2 + \left(k\,\frac{V\,\delta V_T}{V_T^2}\right)^2}$$
8. $$\frac{\delta I}{I} = \sqrt{(0.069208)^2 + (0.095459)^2} = \sqrt{0.0047897 + 0.0091124} = \sqrt{0.0139021} = 0.11791$$
9. The percentage uncertainty in the measured current is $\approx 11.79\ \%$.

KEY IDEA

Propagate the uncertainty through the full diode law $I = I_s\left(e^{V/V_T} - 1\right)$. At $V = 20\ \text{mV}$ the bias is smaller than $V_T = 29\ \text{mV}$, so the -1 term matters. It multiplies both relative sensitivities by $e^{V/V_T}/(e^{V/V_T} - 1) \approx 2.007$. The independent contributions from δV and δV_T then combine as a root-sum-square.

WATCH OUT FOR

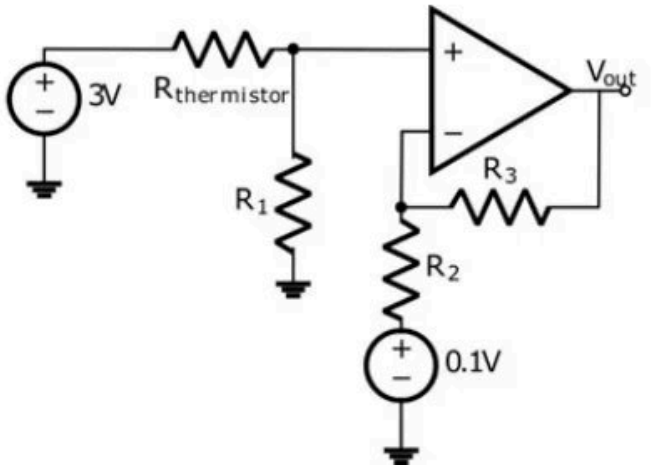
The usual mistake is to use $I \approx I_s e^{V/V_T}$, which is only valid for $V \gg V_T$. At $20\ \text{mV}$ it gives $5.87\ \%$ (root-sum-square) or $8.20\ \%$ (linear sum), both far from the key. Two other slips: adding the two contributions linearly with the full equation gives $16.47\ \%$, and taking half the source resolution ($0.5\ \text{mV}$) as δV gives $10.15\ \%$.

Q.53 NAT · 2 MARKS

Analog Electronics › Op-amps

Q.No. 53

The temperature of the coolant oil bath for a transformer is monitored using the circuit shown. It contains a thermistor with a temperature-dependent resistance, $R_{thermistor} = 2\,(1 + \alpha\,T)\ \text{k}\Omega$, where T is the temperature in °C. The temperature coefficient, α, is $-(4 \pm 0.25)$ %/°C. Circuit parameters: $R_1 = 1\ \text{k}\Omega$, $R_2 = 1.3\ \text{k}\Omega$, $R_3 = 2.6\ \text{k}\Omega$. The error in the output signal (in V, rounded off to 2 decimal places) at 150°C is _____.



Official answer: **Marks to all**

SOLUTION

1. **Assumptions.** The op-amp is ideal: no input current, a virtual short so $V_- = V_+$, and it stays out of saturation (no supply rails are given). The nominal coefficient is $\alpha = -4\,\%/^\circ\text{C} = -0.04\ /^\circ\text{C}$ and its tolerance is $\Delta\alpha = 0.25\,\%/^\circ\text{C} = 0.0025\ /^\circ\text{C}$. All resistances are in $\text{k}\Omega$ and all voltages in V.
2. **Non-inverting input (voltage divider).** No current flows into the input, so R_{th} and R_1 form a plain divider across the 3 V source:
$$V_+ = 3 \cdot \frac{R_1}{R_1 + R_{th}} = \frac{3}{1 + R_{th}}$$
3. **KCL at the inverting node.**
$$\frac{V_- - 0.1}{R_2} + \frac{V_- - V_{out}}{R_3} = 0 \;\Rightarrow\; V_{out} = \left(1 + \frac{R_3}{R_2}\right) V_+ - \frac{R_3}{R_2}(0.1) = 3V_+ - 0.2$$
because $V_- = V_+$ and $R_3/R_2 = 2.6/1.3 = 2$.
4. **Output as a function of α.** With $R_{th} = 2(1 + \alpha T)$ we get $1 + R_{th} = 3 + 2\alpha T$, so

$$V_{out} = \frac{9}{3 + 2\alpha T} - 0.2$$

5. **The printed data do not describe a physical circuit.** With $\alpha = -0.04\ /^{\circ}\mathrm{C}$, $R_{th} = 2(1 - 0.04T)$ is zero at $T = 25^{\circ}\mathrm{C}$ and negative at every higher temperature. At $T = 150^{\circ}\mathrm{C}$, $\alpha T = -6$ and $R_{th} = 2(1 - 6) = -10\ \mathrm{k\Omega}$. A thermistor cannot have a negative resistance, and a resistive divider fed from $+3\ \mathrm{V}$ cannot give $V_{+} = 3/(1 - 10) = -0.333\ \mathrm{V}$. This is the most likely reason marks were awarded to all candidates. The rest of the solution takes the printed formula and numbers at face value, which is all a candidate in the exam hall could do. The error-propagation method is standard, but the number it gives does not describe any real circuit.
6. **Nominal output at $T = 150^{\circ}\mathrm{C}$.**
$$V_{out} = \frac{9}{3 + 2(-6)} - 0.2 = \frac{9}{-9} - 0.2 = -1.2\ \mathrm{V}$$
7. **Sensitivity to α.** Differentiate the expression from step 4:
$$\frac{\partial V_{out}}{\partial \alpha} = -\frac{9\,(2T)}{(3 + 2\alpha T)^2} = -\frac{9 \times 300}{(-9)^2} = -33.33\ \mathrm{V\ per\ unit\ of}\ \alpha$$
8. **Limiting (first-order) error.**
$$\Delta V_{out} = \left|\frac{\partial V_{out}}{\partial \alpha}\right| \Delta\alpha = 33.33 \times 0.0025 = 0.0833\ \mathrm{V}$$
9. **Check at the end points.** At $\alpha = -0.0425$: $3 + 2\alpha T = -9.75$, so $V_{out} = 9/(-9.75) - 0.2 = -1.123\ \mathrm{V}$. At $\alpha = -0.0375$: $3 + 2\alpha T = -8.25$, so $V_{out} = 9/(-8.25) - 0.2 = -1.291\ \mathrm{V}$. Half the spread is $(1.291 - 1.123)/2 = 0.084\ \mathrm{V}$, which agrees with the first-order result. The spread is not symmetric about the nominal value, because V_{out} is not linear in α: the deviations are $+0.077\ \mathrm{V}$ and $-0.091\ \mathrm{V}$. The conventional limiting error is therefore $\Delta V_{out} \approx \pm 0.08\ \mathrm{V}$. Quoting the larger one-sided deviation instead would give $0.09\ \mathrm{V}$.

KEY IDEA

Write the output as a closed-form function of the uncertain parameter. Here that is V_out = 9/(3 + 2αT) − 0.2, from a divider followed by a non-inverting stage with an offset. Differentiate with respect to α and multiply by Δα to get the limiting error. Always check the operating point first: in this question the printed α makes the thermistor resistance negative above 25 °C.

WATCH OUT FOR

Using the tolerance in percent, Δα = 0.25 instead of 0.0025 per °C, which inflates the error a hundredfold. Quoting the full spread (about 0.17 V) instead of the ± error. Not noticing that the printed α gives a negative thermistor resistance at 150 °C, which is why the question could not be keyed.

Q.54 NAT · 2 MARKS

Digital Electronics › Processors & memory

An 8085 microprocessor accesses two memory locations (2001H) and (2002H), that contain 8-bit numbers 98H and B1H, respectively. The following program is executed:

```
LXI H,2001H
MVI A, 21H
INX H
ADD M
INX H
MOV M, A
HLT
```

At the end of this program, the memory location 2003H contains the number in decimal (base 10) form _____.

Official answer: **210**

SOLUTION

1. $\mathrm{LXI\ H,2001H}$ loads the register pair with the address: $HL = 2001\mathrm{H}$, which points to the location holding $98\mathrm{H}$.
2. $\mathrm{MVI\ A,\ 21H}$ loads the immediate byte into the accumulator: $A = 21\mathrm{H}$.
3. $\mathrm{INX\ H}$ increments the 16-bit pair: $HL = 2002\mathrm{H}$. M now refers to the location holding $\mathrm{B1H}$, not $98\mathrm{H}$.
4. $\mathrm{ADD\ M}$ computes $A \leftarrow A + M$:
$$21\mathrm{H} + \mathrm{B1H} = \mathrm{D2H}$$
In binary, $(0010\,0001)_2 + (1011\,0001)_2 = (1101\,0010)_2$. There is no carry out of bit 7, so the sum fits in 8 bits.
5. $\mathrm{INX\ H}$ increments the pointer again: $HL = 2003\mathrm{H}$.
6. $\mathrm{MOV\ M,\ A}$ stores the accumulator at the address in HL, so location $2003\mathrm{H}$ now holds $\mathrm{D2H}$. HLT stops the program.
7. Convert to decimal: $(\mathrm{D2})_{16} = 13 \times 16 + 2 = 210$. Check: $21\mathrm{H} = 33$, $\mathrm{B1H} = 177$, and $33 + 177 = 210$.

KEY IDEA

In the 8085, M always means the memory byte at the address in HL. Follow HL through each $\mathrm{INX\ H}$: the first one runs before $\mathrm{ADD\ M}$, so the byte added is the one at $2002\mathrm{H}$ ($\mathrm{B1H}$), and $98\mathrm{H}$ at $2001\mathrm{H}$ is never read. The second $\mathrm{INX\ H}$ moves HL to $2003\mathrm{H}$, and the sum is stored there.

WATCH OUT FOR

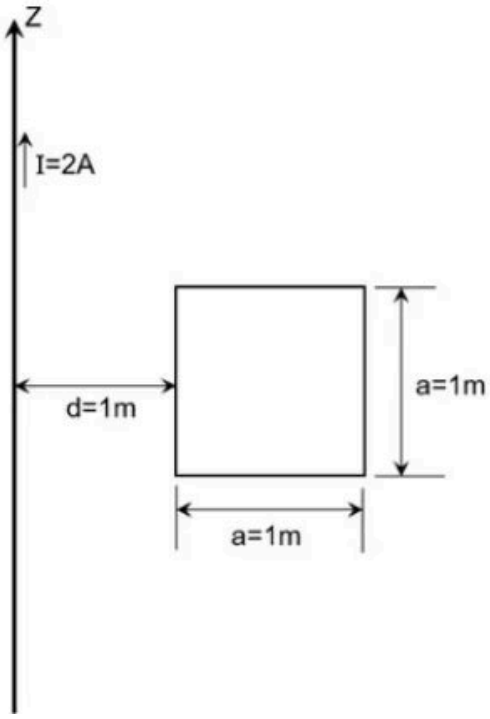
Adding $98\mathrm{H}$ as well, or adding it instead of $\mathrm{B1H}$, because the question names both locations. $98\mathrm{H} + \mathrm{B1H} + 21\mathrm{H}$ overflows 8 bits and gives the wrong answer. The other common slip is writing the answer in hex (D2), or converting $\mathrm{D2H}$ as $12 \times 16 + 2 = 194$ by mistaking the value of the digit D, which is 13.

Q.55 NAT · 2 MARKS

Electromagnetics › Magnetostatics

Q.No. 55

A conducting square loop of side length 1 m is placed at a distance of 1 m from a long straight wire carrying a current $I = 2$ A as shown below. The mutual inductance, in nH (rounded off to 2 decimal places), between the conducting loop and the long wire is _____.



Official answer: **138.10 to 139.20**

SOLUTION

1. **Field of the wire (Ampere's circuital law):** at perpendicular distance ρ from an infinitely long wire carrying current I,

$$\mathbf{B} = \frac{\mu_0 I}{2\pi\rho}\,\hat{a}_\phi, \qquad \mu_0 = 4\pi \times 10^{-7}\ \mathrm{H/m}$$

   Over the loop, which lies in the same plane as the wire, $\hat{a}_\phi$ is normal to that plane, so $\mathbf{B}$ is perpendicular to the loop's surface everywhere.

2. **Flux through the loop:** split the loop into strips parallel to the wire, each of height a and width $d\rho$ at distance ρ. B depends only on ρ, so it is constant over each strip:

$$\Phi_{21} = \int_d^{d+a} \frac{\mu_0 I}{2\pi\rho}\, a\, d\rho = \frac{\mu_0 I a}{2\pi} \ln\left(\frac{d+a}{d}\right)$$

3. **Mutual inductance $M = \Phi_{21}/I$:** the current cancels, so the given $I = 2$ A does not affect the answer:

$$M = \frac{\mu_0 a}{2\pi} \ln\left(\frac{d+a}{d}\right)$$

4. **Substitute $a = 1$ m, $d = 1$ m:** $\dfrac{\mu_0}{2\pi} = 2 \times 10^{-7}$ H/m, so

$$M = 2 \times 10^{-7} \times 1 \times \ln 2 = 2 \times 10^{-7} \times 0.693147 = 1.3863 \times 10^{-7}\ \mathrm{H}$$

5. **Convert to nH:** $M = 138.63$ nH (rounded to 2 decimal places).

KEY IDEA

The wire's field $B = \mu_0 I/(2\pi\rho)$ changes only with distance from the wire. Integrate it over the loop in strips parallel to the wire to get $\Phi = \dfrac{\mu_0 I a}{2\pi}\ln\dfrac{d+a}{d}$. Then $M = \Phi/I$ depends only on geometry, so the 2 A current is a distractor.

WATCH OUT FOR

Taking B at one distance (the near side, the far side or the centre, 1.5 m) and multiplying by the area a^2 instead of integrating across $1\ \text{m} \le \rho \le 2\ \text{m}$. At the centre this gives $2\times10^{-7}/1.5 = 133.33\ \text{nH}$, outside the key range. The other common slips are multiplying by $I = 2$ A, which gives flux rather than inductance, and using $\log_{10}$ where the natural log is needed.

FROM IAE ACADEMY

Preparing for GATE EE?

IAE Academy teaches GATE EE in Ballygunge, Kolkata, and online. Counselling is free: ask us about batches, fees or your preparation.

FREE COUNSELLING

Chat on WhatsApp

Call +91 84204 36524

Open every day, 10:30 am to 7:30 pm.

Scan to chat on WhatsApp

THE GATE EE COURSE

Classroom batches in Ballygunge and online batches. Fees and details are on the course page.

Scan: course page

See the course

FREE ON IAEACADEMY.IN

More GATE EE papers, question by question with the official answers, and free notes by subject.

Scan: more papers

More solved papers

Found this useful? Send it to a friend preparing for GATE.

Send to a friend on WhatsApp

IAE Academy free study material. The questions are those of the GATE EE 2020 question paper, organised by IIT Delhi; the answers are from its official answer key. Prepared and published by IAE Academy, Kolkata. Free to read and free to share, unaltered and with attribution. Not affiliated with, endorsed by or produced by IIT Delhi or the GATE committee.

IAE ACADEMY INSTITUTE FOR ENGINEERS

GATE coaching in Kolkata since 2012 · EC, EE, IN, CS and IT, DA, BT and CE · in the classroom and online

227A Rash Behari Ave, Ballygunge, Kolkata 700019

+91 84204 36524 · iaeacademy.in · Updated 27 October 2026

Get directions