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GATE EE · 2018

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GATE EE 2018 Solved Paper

INSIDE

- All 65 questions, in the order of the paper
- The official answer to every question
- Step-by-step worked solutions
- The answer key on one page

Question paper and official answer key: IIT Guwahati

Online, with the questions linked to their topic notes: **iaeacademy.in/study/papers/ee-2018**

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About this paper

65 questions, 100 marks. Question types: 40 MCQ · 25 NAT.

Official answer key: IIT Guwahati

This paper numbers General Aptitude GA Q.1 to GA Q.10 and the subject questions Q.1 to Q.55: 65 questions in all.

We placed each question under a subject and topic ourselves; the exam paper does not label them.

This paper online, with the questions linked to their topic notes:
iaeacademy.in/study/papers/ee-2018

Answer key

Q	TYPE	MARKS	ANSWER
GA Q.1	MCQ	1	**A**
GA Q.2	MCQ	1	**D**
GA Q.3	MCQ	1	**D**
GA Q.4	MCQ	1	**C**
GA Q.5	MCQ	1	**A**
GA Q.6	MCQ	2	**C**
GA Q.7	MCQ	2	**C**
GA Q.8	MCQ	2	**Marks to all**
GA Q.9	MCQ	2	**C**
GA Q.10	MCQ	2	**C**
Q.1	MCQ	1	**A**
Q.2	MCQ	1	**B**
Q.3	MCQ	1	**C**
Q.4	MCQ	1	**B**
Q.5	MCQ	1	**D**
Q.6	MCQ	1	**B**
Q.7	MCQ	1	**B**
Q.8	MCQ	1	**C**
Q.9	MCQ	1	**C**
Q.10	MCQ	1	**D**
Q.11	MCQ	1	**D**
Q.12	MCQ	1	**A**
Q.13	MCQ	1	**A**
Q.14	MCQ	1	**D**
Q.15	MCQ	1	**B**
Q.16	MCQ	1	**A**
Q.17	NAT	1	**5.5**
Q.18	NAT	1	**0.49 to 0.51**
Q.19	NAT	1	**0.45 to 0.55**
Q.20	NAT	1	**7.0**
Q.21	NAT	1	**73.0 to 74.0**
Q.22	NAT	1	**3500.0**
Q.23	NAT	1	**16.90 to 17.70**
Q.24	NAT	1	**600.0**
Q.25	NAT	1	**0.65 to 0.69**
Q.26	MCQ	2	**D**
Q.27	MCQ	2	**A**
Q.28	MCQ	2	**A**
Q.29	MCQ	2	**C**
Q.30	MCQ	2	**C**
Q.31	MCQ	2	**B**
Q.32	MCQ	2	**A**
Q.33	MCQ	2	**C**
Q.34	MCQ	2	**A**
Q.35	MCQ	2	**B**
Q.36	MCQ	2	**B**
Q.37	MCQ	2	**B**
Q.38	MCQ	2	**A**
Q.39	MCQ	2	**D**
Q.40	NAT	2	**9.5 to 10.5**
Q.41	NAT	2	**89.0 to 91.5**
Q.42	NAT	2	**0.0**
Q.43	NAT	2	**11.5 to 12.5**
Q.44	NAT	2	**0.9 to 1.1**
Q.45	NAT	2	**2.50 to 2.55**
Q.46	NAT	2	**8.0**
Q.47	NAT	2	**5.70 to 5.85**
Q.48	NAT	2	**10.0**
Q.49	NAT	2	**90.0**
Q.50	NAT	2	**11.50 to 13.50**
Q.51	NAT	2	**97.20 to 97.55**
Q.52	NAT	2	**823 to 827**
Q.53	NAT	2	**12.0**
Q.54	NAT	2	**0.104 to 0.112**
Q.55	NAT	2	**50.0**

General Aptitude GA Q.1 to Q.10

GA Q.1 MCQ · 1 MARK

General Aptitude › Grammar & vocabulary

"Since you have gone off the _____, the _____ sand is likely to damage the car."

The words that best fill the blanks in the above sentence are

(A) course, coarse

(B) course, course

(C) coarse, course

(D) coarse, coarse

Official answer: **A**

SOLUTION

1. The options use two homophones, words that sound the same but mean different things. **Course** is a noun meaning a path, route or track. **Coarse** is an adjective meaning rough, or made of large grains.
2. First blank: "gone off the _____" needs a noun after the article "the", because the phrase is the object of "gone off". To leave the path or track you were meant to follow is to go off the **course**. "Coarse" is an adjective and cannot stand alone as that object, so (C) and (D) are ruled out.
3. Second blank: "the _____ sand" needs an adjective describing the sand. Rough, large-grained sand is **coarse** sand, and that is the kind likely to damage a car. "Course sand" makes no sense, so (B) is ruled out.
4. Check the whole sentence: "Since you have gone off the course, the coarse sand is likely to damage the car." It is grammatical and it makes sense, so the answer is (A).

KEY IDEA

Homophones: tell them apart by what each blank needs grammatically. After "the" with nothing following, the blank needs a noun (course = path or track). Before a noun such as "sand", it needs an adjective (coarse = rough, large-grained).

WATCH OUT FOR

Picking by sound or by spelling habit, for example choosing (B) or (D) on the idea that the same word fills both blanks. Also mixing the two up and writing "coarse" for a route. Always ask whether the gap needs a noun or an adjective.

GA Q.2 MCQ · 1 MARK

General Aptitude › Grammar & vocabulary

"A common misconception among writers is that sentence structure mirrors thought; the more _____ the structure, the more complicated the ideas."

The word that best fills the blank in the above sentence is

(A) detailed

(B) simple

(C) clear

(D) convoluted

Official answer: **D**

SOLUTION

1. **Read the sentence as two parts.** The clause after the semicolon spells out the "misconception" named in the first part: the belief that sentence structure *mirrors* thought.
2. **What "mirrors" means here.** A mirror gives back the same image. So the belief is that structure and thought match: whatever quality the structure has, the ideas are taken to have the same quality.

3. **The "the more X, the more Y" pattern.** This pattern links two qualities that increase together. Y is "complicated" and describes the ideas. For the structure to mirror the ideas, X must be the matching quality of a *structure*: complicated, twisted, hard to follow.
4. **Remove the opposites.** (B) *simple* and (C) *clear* mean the reverse of complicated. "The more simple the structure, the more complicated the ideas" describes an inverse relationship, not a mirror, so it contradicts the first clause.
5. **Remove the near miss.** (A) *detailed* means having many parts or specifics. It does not mean tangled or hard to follow, and a detailed sentence can still be perfectly clear. It is not the structural counterpart of "complicated".
6. **Confirm the answer.** (D) *convoluted* means intricately twisted and hard to follow, which is exactly the structural counterpart of "complicated". The sentence now reads: the more convoluted the structure, the more complicated the ideas. That is the mirror belief the writer calls a misconception.
7. **Answer: (D) convoluted.**

KEY IDEA

The semicolon clause explains the "misconception" that structure *mirrors* thought. In "the more _____ the structure, the more complicated the ideas", the blank must therefore be the structural equivalent of "complicated". That word is *convoluted* (twisted, hard to follow).

WATCH OUT FOR

Picking (B) *simple* or (C) *clear* because good writing should be simple. The question asks what the misconception *says*, not what good writing advice is, and a mirror needs matching qualities, not opposite ones. Another trap is (A) *detailed*: a detailed sentence has many parts, but it is not necessarily complicated or hard to follow.

GA Q.3 MCQ · 1 MARK

General Aptitude › Numerical

The three roots of the equation $f(x) = 0$ are $x = \{-2, 0, 3\}$. What are the three values of x for which $f(x - 3) = 0$?

(A) $-5, -3, 0$

(B) $-2, 0, 3$

(C) $0, 6, 8$

(D) $1, 3, 6$

Official answer: **D**

SOLUTION

1. $f(u) = 0$ exactly when $u \in \{-2, 0, 3\}$, because these are the three roots of f.
2. Put $u = x - 3$. Then $f(x - 3) = 0$ exactly when $x - 3 \in \{-2, 0, 3\}$.
3. Solve each case: $x - 3 = -2 \Rightarrow x = 1$; $x - 3 = 0 \Rightarrow x = 3$; $x - 3 = 3 \Rightarrow x = 6$.
4. Another way to see it: the graph of $f(x - 3)$ is the graph of $f(x)$ moved 3 units to the right, so every root goes up by 3: $\{-2 + 3,\ 0 + 3,\ 3 + 3\} = \{1, 3, 6\}$.
5. Check: $f(1 - 3) = f(-2) = 0$, $f(3 - 3) = f(0) = 0$, $f(6 - 3) = f(3) = 0$. The three values are $1, 3, 6$, which is option (D).

KEY IDEA

If r is a root of f, then $f(x-3)=0$ when $x-3=r$, that is, when $x=r+3$. Replacing x by $x-3$ moves the graph 3 units to the right, so every root goes up by 3.

WATCH OUT FOR

Seeing the minus sign in $x-3$ and subtracting 3 from each root. That gives $-5,-3,0$, which is option (A). Another slip is to think the shift leaves the roots alone and pick option (B), $-2,0,3$. In $f(x-h)$ the graph moves right by h, so the roots go up by h. A quick check settles it: substitute the candidate x into $x-3$ and see whether you get an original root.

GA Q.4 MCQ · 1 MARK

General Aptitude › Numerical

For what values of k given below is $\dfrac{(k+2)^2}{k-3}$ an integer?

(A) 4, 8, 18

(B) 4, 10, 16

(C) 4, 8, 28

(D) 8, 26, 28

Official answer: **C**

SOLUTION

1. Write the numerator in terms of the denominator: $k+2=(k-3)+5$.
2. Expand: $(k+2)^2=((k-3)+5)^2=(k-3)^2+10(k-3)+25$.
3. Divide term by term:
$$\frac{(k+2)^2}{k-3}=(k-3)+10+\frac{25}{k-3}$$
4. Every option lists integer values of k, so $(k-3)+10$ is an integer. The whole expression is therefore an integer exactly when $\dfrac{25}{k-3}$ is an integer, that is, when $(k-3)$ divides 25.
5. The divisors of 25 are $\pm1,\pm5,\pm25$. So $k-3\in\{1,-1,5,-5,25,-25\}$, which gives $k\in\{4,2,8,-2,28,-22\}$.
6. Check the options. The values 18, 10, 16 and 26 give $k-3=15,7,13,23$, and none of these divides 25, so options (A), (B) and (D) each contain a value that fails. Only option (C), $4,8,28$, has all three values in the set.
7. Verify directly: $k=4$ gives $\dfrac{36}{1}=36$; $k=8$ gives $\dfrac{100}{5}=20$; $k=28$ gives $\dfrac{900}{25}=36$. All three are integers, so the answer is (C).

KEY IDEA

Write the numerator in powers of the denominator (put $k+2=(k-3)+5$). This leaves a polynomial part plus a constant remainder over $(k-3)$, so the fraction is an integer exactly when $(k-3)$ divides that remainder, here 25.

WATCH OUT FOR

Checking only one or two values in an option. Option (A) passes for 4 and 8, but $k=18$ gives $\dfrac{400}{15}$, which is not an integer. Every value in the option has to work. A second slip is to expand $(k+2)^2$ and divide by k instead of by $(k-3)$, which gives a wrong remainder.

GA Q.5 MCQ · 1 MARK

General Aptitude › Numerical

Functions $F(a,b)$ and $G(a,b)$ are defined as follows:
$F(a,b)=(a-b)^2$ and $G(a,b)=|a-b|$, where $|x|$ represents the absolute value of x.
What would be the value of $G(F(1,3),G(1,3))$?

(A) 2

(B) 4

(C) 6

(D) 36

Official answer: **A**

SOLUTION

1. Work from the inside out: evaluate both inner function values first, then feed them into the outer G.
2. Inner F: $F(1,3) = (1-3)^2 = (-2)^2 = 4$ (squaring a negative number gives a positive result).
3. Inner G: $G(1,3) = |1-3| = |-2| = 2$ (the absolute value drops the sign).
4. Outer G, with $a = F(1,3) = 4$ and $b = G(1,3) = 2$:
$$G(4,2) = |4-2| = 2$$
5. So $G(F(1,3), G(1,3)) = 2$, which is option (A).

KEY IDEA

In a composite function, evaluate the innermost calls first. Each result then becomes an argument of the outer function, in the same position: the first argument is a and the second is b.

WATCH OUT FOR

Stopping at the inner value $F(1,3) = 4$ and picking (B), or squaring at the outer level as well, e.g. computing $F(4,2)$ or $(4-2)^2$, or mixing up which function goes outside. Another slip is forgetting the absolute value, which only matters when the order of the arguments is reversed; here $4-2$ is already positive.

GA Q.6 MCQ · 2 MARKS

General Aptitude › Numerical

An e-mail password must contain three characters. The password has to contain one numeral from 0 to 9, one upper case and one lower case character from the English alphabet. How many distinct passwords are possible?

(A) 6,760

(B) 13,520

(C) 40,560

(D) 1,05,456

Official answer: **C**

SOLUTION

1. Count the choices for each type of character. The numerals 0 to 9 give 10 choices. The English alphabet has 26 letters, so there are 26 upper case choices and 26 lower case choices.
2. By the multiplication principle, the set of three characters (one of each type) can be chosen in $10 \times 26 \times 26 = 6760$ ways.
3. A password is an ordered string, so the same three characters can be arranged in different orders. The three characters are of different types, so every arrangement gives a different password. Three distinct characters can be arranged in $3! = 6$ ways.
4. Total number of passwords:
$$6760 \times 3! = 6760 \times 6 = 40{,}560$$
5. This is option (C).

KEY IDEA

First choose which character of each type to use ($10 \times 26 \times 26$). Then count the positions: the numeral, the upper case letter and the lower case letter can sit in any order, which gives $3!$ arrangements. Every arrangement is a different password because the three characters are of different types.

WATCH OUT FOR

Stopping at $10 \times 26 \times 26 = 6760$ (option A), which silently fixes the order as numeral, then upper case, then lower case. Some students multiply by 2 instead of $3! = 6$ and get 13,520 (option B). Others count every 3-character string over all 62 symbols, which gives 62^3 and ignores the rule that the password needs one character of each type.

GA Q.7 MCQ · 2 MARKS

General Aptitude › Reasoning

In a certain code, AMCF is written as EQGJ and NKUF is written as ROYJ. How will DHLP be written in that code?

(A) RSTN

(B) TLPH

(C) HLPT

(D) XSVR

Official answer: **C**

SOLUTION

1. Write each letter of AMCF and EQGJ as its position in the alphabet: A(1)→E(5), M(13)→Q(17), C(3)→G(7), F(6)→J(10). Every letter moves forward by $+4$.
2. Check the rule on the second example: N(14)→R(18), K(11)→O(15), U(21)→Y(25), F(6)→J(10). Again every letter moves $+4$. So the code shifts each letter 4 places forward and keeps the letters in the same order.
3. Apply the same shift to DHLP: D(4)→H(8), H(8)→L(12), L(12)→P(16), P(16)→T(20).
4. So DHLP is written as HLPT, which is option (C).

KEY IDEA

Turn each letter into its alphabet position (A = 1, ..., Z = 26) and compare the pairs one by one. Both examples show the same shift, $+4$, for every letter with the order unchanged. Once the rule is fixed, the first letter alone decides the answer: D + 4 = H, and only option (C) starts with H.

WATCH OUT FOR

Picking (B) TLPH. It has the right shifted letters (H, L, P, T) but in reverse order. The code keeps the letters in order: the first letter of AMCF gives the first letter of EQGJ, and so on. A second trap is checking only one example. You need both examples to confirm the shift really is $+4$ for every position.

GA Q.8 MCQ · 2 MARKS

General Aptitude › Statistics & probability

A class of twelve children has two more boys than girls. A group of three children are randomly picked from this class to accompany the teacher on a field trip. What is the probability that the group accompanying the teacher contains more girls than boys?

(A) 0

(B) $\frac{325}{864}$

(C) $\frac{525}{864}$

(D) $\frac{5}{12}$

Official answer: **Marks to all**

GA Q.9 MCQ · 2 MARKS

General Aptitude › Numerical

Q.9 A designer uses marbles of four different colours for his designs. The cost of each marble is the same, irrespective of the colour. The table below shows the percentage of marbles of each colour used in the current design. The cost of each marble increased by 25%. Therefore, the designer decided to reduce equal numbers of marbles of each colour to keep the total cost unchanged. What is the percentage of blue marbles in the new design?

Blue	Black	Red	Yellow
40%	25%	20%	15%

(A) 35.75 (B) 40.25 (C) 43.75 (D) 46.25

Official answer: **C**

SOLUTION

1. Take 100 marbles in the current design, each costing c. Any total works because the answer is a ratio; 100 just makes the percentages whole numbers. The table gives Blue 40, Black 25, Red 20, Yellow 15 marbles, so the total cost is $100c$.
2. The new price per marble is $1.25c$. To keep the total cost the same, the new number of marbles n must satisfy $n \times 1.25c = 100c$, which gives $n = \dfrac{100}{1.25} = 80$.
3. So $100 - 80 = 20$ marbles are removed. They are removed in equal numbers from the four colours, so $\dfrac{20}{4} = 5$ marbles of each colour go.
4. Blue marbles left: $40 - 5 = 35$.
5. The percentage is taken over the new total, not the old one:
$$\frac{35}{80} \times 100 = 43.75\%$$
6. Check: the other colours become $25 - 5 = 20$, $20 - 5 = 15$ and $15 - 5 = 10$, and $35 + 20 + 15 + 10 = 80$, as required. The answer is (C) 43.75.

KEY IDEA

A fixed budget with a 25% higher price per marble buys $\frac{1}{1.25} = 0.8$ times as many marbles, so 20% of the marbles go. Because they are removed in equal numbers from each of the four colours, each colour loses 5% of the original total. That leaves blue at $40 - 5 = 35$ out of 80.

WATCH OUT FOR

Removing the same percentage from every colour. That leaves blue at 40%, which is not an option. The question says equal numbers, not equal proportions. Two other slips: dividing 35 by the old total of 100 to get 35% (option A, 35.75, is close to this), and treating a 25% price rise as a 25% cut in the number of marbles. The count falls to $100/1.25 = 80$, not 75.

GA Q.10 MCQ · 2 MARKS

General Aptitude › Reasoning

P, Q, R and S crossed a lake in a boat that can hold a maximum of two persons, with only one set of oars. The following additional facts are available.

(i) The boat held two persons on each of the three forward trips across the lake and one person on each of the two return trips.

(ii) P is unable to row when someone else is in the boat.

(iii) Q is unable to row with anyone else except R.

(iv) Each person rowed for at least one trip.

(v) Only one person can row during a trip.

Who rowed twice?

(A) P

(B) Q

(C) R

(D) S

Official answer: **C**

SOLUTION

1. **Count the rowing turns.** There are 3 forward trips with 2 people each and 2 return trips with 1 person each, so 5 trips in all. Exactly one person rows each trip (v), and each of the 4 people rows at least once (iv). So exactly one person rows twice.
2. **What P can row.** By (ii), P rows only when alone in the boat. Forward trips always carry two people, so P can row only a return trip. Everyone starts on the near side and ends on the far side, so each person crosses forward exactly once more than they cross back.
3. **P rows exactly one return.** Suppose P rowed both returns. Then P crosses forward 3 times, so P is on every forward trip. Q cannot row a return because both are taken, so Q must row a forward trip, and by (iii) R must be the other person on it. But every forward trip already holds P and one other person, so there is no room for both Q and R. Contradiction. So P rows exactly one return and crosses forward twice. The other return is made, and rowed, by someone else.
4. **Q never makes a return trip.** Suppose Q made the other return. Then Q also crosses forward twice, so the six forward seats are P, P, Q, Q, R, S. A trip $\{P, Q\}$ has nobody who can row it: P cannot row with a passenger (ii), and Q rows only with R (iii). So P's two trips must be $\{P, R\}$ and $\{P, S\}$, which leaves $\{Q, Q\}$ for the third trip. That is impossible. So Q never goes back and crosses exactly once, forward. Q must row at least once (iv), so Q rows that one trip, and by (iii) it is the trip $\{Q, R\}$ with Q rowing.
5. **Who makes the second return?** It is R or S, and that person crosses forward twice. The other of R and S never goes back, so they cross forward only once. Suppose S made the second return. Then R crosses forward only once, and that crossing is the trip $\{Q, R\}$, which Q rows. R makes no return either, because P and S have them. So R rows no trip at all, which breaks (iv). Therefore R makes and rows the second return, and R crosses forward twice.
6. **P's two crossings.** The six forward seats are P, P, Q, R, R, S. The trip $\{Q, R\}$ uses one Q seat and one R seat, which leaves P, P, R, S, so P's two crossings are $\{P, R\}$ and $\{P, S\}$. P cannot row either one (ii), so R rows $\{P, R\}$ and S rows $\{P, S\}$.
7. **Tally and check.** P rows 1 trip (a return), Q rows 1 ($\{Q, R\}$), S rows 1 ($\{P, S\}$) and R rows 2 ($\{P, R\}$ and a return). That makes $1 + 1 + 1 + 2 = 5$ trips. One order that works: Q rows Q and R across; R rows back; S rows S and P across; P rows back; R rows R and P across. Every condition (i) to (v) holds. So **R** rowed twice, which is option (C).

KEY IDEA

There are 5 trips and 4 rowers, and each person rows at least once, so exactly one person rows twice. Each person crosses forward exactly once more than they come back. P can row only when alone, which means only on a return. So P rows one return and crosses forward twice. Q can then only row a forward trip with R. Counting the six forward seats (who fills them) leaves R as the only person who can take the second return, and R also has to row P across once.

WATCH OUT FOR

Two errors are common. The first is to assume that P, who can row only alone, rows both returns. That puts P on every forward trip and leaves no trip where Q can row with R. The second is to build one schedule where S rows twice without checking condition (iv): if S makes the second return, R's only crossing is the trip Q rows, so R never rows. A full case check also has to rule out R rowing both of P's crossings. That would need R on three forward trips, which the seat count does not allow.

Electrical Engineering Q.1 to Q.55

Q.1 MCQ · 1 MARK

Electrical Machines › Transformers

A single-phase 100 kVA, 1000 V / 100 V, 50 Hz transformer has a voltage drop of 5% across its series impedance at full load. Of this, 3% is due to resistance. The percentage regulation of the transformer at full load with 0.8 lagging power factor is

(A) 4.8

(B) 6.8

(C) 8.8

(D) 10.8

Official answer: **A**

SOLUTION

1. Step 1. Split the impedance drop into resistive and reactive parts: The total per-unit impedance voltage drop at full load is $e_z = 5\%$, and the resistive part is $e_r = 3\%$. Since e_r and e_x combine as quadrature phasor components to give e_z,
$$e_z^2 = e_r^2 + e_x^2 \implies e_x = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4\%$$
2. Step 2. Resolve the load power factor: For 0.8 lagging power factor, $\cos\phi = 0.8$ and $\sin\phi = \sqrt{1 - 0.8^2} = 0.6$.
3. Step 3. Apply the approximate regulation formula for a lagging power factor load:
$$\%\text{Reg} = e_r \cos\phi + e_x \sin\phi$$
(the reactive-drop term is added for a lagging load; it would be subtracted only for a leading load).
4. Step 4. Substitute the values:
$$\%\text{Reg} = 3(0.8) + 4(0.6) = 2.4 + 2.4 = 4.8\%$$
5. This matches option (A).

KEY IDEA

Total per-unit impedance drop e_z splits into a resistive part e_r and reactive part e_x via $e_z^2 = e_r^2 + e_x^2$ (right-triangle relation, since resistance drop and reactance drop are in phase quadrature relative to the load current phasor). For lagging power factor, the approximate percentage regulation is $\%\text{Reg} \approx e_r \cos\phi + e_x \sin\phi$ (a second-order correction term is negligible here and does not change the answer).

WATCH OUT FOR

Using the plus/minus sign wrong for the reactive term (lagging power factor requires the reactive-drop term to be added, not subtracted; subtraction applies to leading pf), or mis-splitting the 5% total impedance drop into resistive/reactive components using an arithmetic sum instead of $\sqrt{e_z^2 - e_r^2}$.

Q.2 MCQ · 1 MARK

Electrical Machines › Synchronous machines

In a salient pole synchronous motor, the developed reluctance torque attains the maximum value when the load angle in electrical degrees is

(A) 0

(B) 45

(C) 60

(D) 90

Official answer: **B**

SOLUTION

1. Power developed by a salient-pole synchronous motor has two components: an excitation (magnet) component and a reluctance component:
$$P(\delta) = \frac{EV}{X_d}\sin\delta + \frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta$$
2. The second term, the reluctance power (and hence reluctance torque), exists only because of rotor saliency ($X_d \neq X_q$) and is proportional to $\sin 2\delta$, independent of the excitation emf E.
3. The magnitude of $\sin 2\delta$ is maximum (equal to 1) when $2\delta = 90^\circ$, i.e. when $\delta = 45^\circ$.
4. Therefore the developed reluctance torque attains its maximum value at a load angle of 45° electrical, independent of the excitation torque component (which instead peaks at $\delta = 90^\circ$).
5. This corresponds to option (B).

KEY IDEA

In a salient-pole synchronous machine, $X_d \neq X_q$, so besides the excitation (fundamental) power/torque term proportional to $\sin\delta$, there is a reluctance power/torque term proportional to $\sin 2\delta$ arising purely from the rotor saliency ($X_d > X_q$). This reluctance term is maximum when $\sin 2\delta = 1$, i.e. at $\delta = 45^\circ$ electrical, independent of excitation.

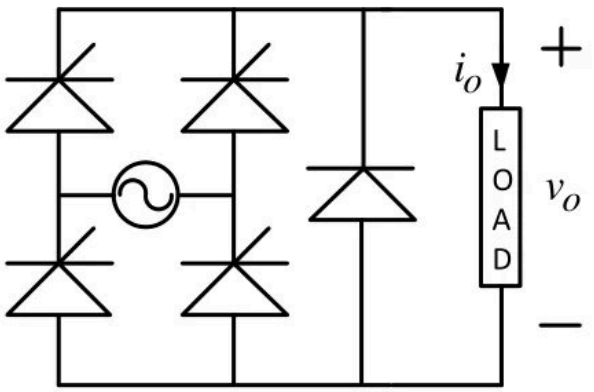
WATCH OUT FOR

Confusing the reluctance-torque maximum (at $\delta = 45^\circ$) with the excitation-torque maximum (at $\delta = 90^\circ$), or assuming the total developed torque and the reluctance-torque component peak at the same angle.

Q.3 MCQ · 1 MARK

Power Electronics › Rectifiers

Q.3 A single phase fully controlled rectifier is supplying a load with an anti-parallel diode as shown in the figure. All switches and diodes are ideal. Which one of the following is true for instantaneous load voltage and current?



(A) $v_o \geq 0$ & $i_o < 0$
(B) $v_o < 0$ & $i_o < 0$
(C) $v_o \geq 0$ & $i_o \geq 0$
(D) $v_o < 0$ & $i_o \geq 0$

Official answer: **C**

SOLUTION

1. The circuit is a single-phase fully controlled bridge of four thyristors, with a diode D_f across the DC output. D_f has its cathode on the top (positive) rail and its anode on the bottom rail, so it is a freewheeling diode. The firing angle α may take any value. The duty ratio of D_f depends on α and the load, and the argument below does not need it.

2. Current direction: thyristors and diodes conduct only from anode to cathode. The upper devices have their cathodes on the top rail and the lower devices have their anodes on the bottom rail. So the bridge can only push current into the top rail and out of the bottom rail, and D_f only carries current from the bottom rail to the top rail, in parallel with the load. The load current i_o is drawn downward through the load, so it is the sum of the bridge current and the D_f current, both non-negative. Hence $i_o \geq 0$ at every instant. Option (A) and option (B) are therefore ruled out.
3. Voltage limit: v_o is the voltage from the top rail to the bottom rail. If v_o tried to become negative, the top rail would be below the bottom rail. D_f would then be forward biased, because its anode is on the bottom rail and its cathode is on the top rail. An ideal diode in conduction holds $v_o = 0$. So v_o can never be negative.
4. While a diagonal pair of thyristors conducts, the output is the rectified source voltage, $v_o = v_s = V_m \sin \omega t$ for the pair that conducts in the positive half cycle, and $v_o = -v_s = |V_m \sin \omega t|$ for the other pair in the negative half cycle. Both are non-negative. When the thyristors stop conducting, the load current freewheels through D_f and $v_o = 0$.
5. So at every instant $v_o \geq 0$ and $i_o \geq 0$. This is option (C). The bridge is effectively a half-controlled rectifier, because the freewheeling diode removes the negative-voltage (inverting) interval.

KEY IDEA

A bridge of thyristors can only carry current in one direction, so $i_o \geq 0$. An ideal freewheeling diode across the output clamps v_o at 0 whenever it would go negative, so $v_o \geq 0$. Together these give $v_o \geq 0$ and $i_o \geq 0$.

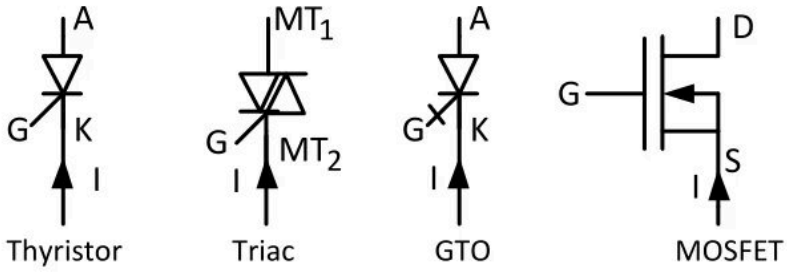
WATCH OUT FOR

Treating this as an ordinary fully controlled bridge, where v_o goes negative after π for $\alpha > 0$ with an inductive load (inverter-like operation). The anti-parallel diode across the load removes that interval. Another slip is to read the diode as reversing the load current. It cannot, because every device in the circuit conducts in one direction only.

Q.4 MCQ · 1 MARK

Power Electronics › Switches

Q.4 Four power semiconductor devices are shown in the figure along with their relevant terminals. The device(s) that can carry dc current continuously in the direction shown when gated appropriately is (are)



(A) Triac only
(B) Triac and MOSFET
(C) Triac and GTO
(D) Thyristor and Triac

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Official answer: **B**

SOLUTION

1. The marked current I enters each device at its lower terminal and flows upward. That is from K to A in the thyristor and the GTO, from MT_2 to MT_1 in the triac, and from S to D in the MOSFET.

2. Thyristor: it is a unidirectional device. It conducts only from A to K, and in the K to A direction it blocks. It cannot carry the marked current, whatever the gate signal.
3. GTO: it is also asymmetric. It conducts only from A to K, and gate turn-off action does not change that. It cannot carry current from K to A.
4. Triac: it is a bidirectional switch, equivalent to two thyristors in anti-parallel. With a suitable gate pulse it conducts in either direction, so it can carry the current from MT_2 to MT_1.
5. MOSFET: the gated channel conducts from D to S. The device also has an intrinsic body diode with anode at S and cathode at D. This diode conducts from S to D whatever the gate signal, so the MOSFET can carry the marked current continuously. In the reverse direction the channel, once turned on, also conducts and shares the current.
6. The devices that can carry the current in the direction shown are therefore the triac and the MOSFET, which is option (B).

KEY IDEA
Compare the direction of the marked current with each device's conduction direction. A thyristor and a GTO conduct only from anode to cathode. A triac conducts both ways. A MOSFET has an intrinsic body diode that conducts from source to drain.

WATCH OUT FOR
Treating the MOSFET as a one-way device from D to S and forgetting its body diode. Also assuming the GTO can carry current both ways because it can be turned off by the gate.

Q.5 MCQ · 1 MARK

Electrical and Electronic Measurements › V, I, power & energy

Two wattmeter method is used for measurement of power in a balanced three-phase load supplied from a balanced three-phase system. If one of the wattmeters reads half of the other (both positive), then the power factor of the load is

(A) 0.532
(B) 0.632
(C) 0.707
(D) 0.866

Official answer: **D**

SOLUTION

1. Call the larger reading $W_1 = 2W$ and the smaller one $W_2 = W$. The question says both readings are positive.
2. For a balanced load measured by the two-wattmeter method, $\tan\phi = \dfrac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2}$, where W_1 is the larger reading.
3. Substitute the readings:
$$\tan\phi = \frac{\sqrt{3}\,(2W - W)}{2W + W} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$
4. So $\phi = \tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right) = 30^\circ$.
5. Power factor $= \cos 30^\circ = \dfrac{\sqrt{3}}{2} \approx 0.866$. This is option (D).
6. Check: the readings are $W_1 = V_L I_L \cos(30^\circ - \phi)$ and $W_2 = V_L I_L \cos(30^\circ + \phi)$. With $\phi = 30^\circ$ these become $V_L I_L \cos 0^\circ = V_L I_L$ and $V_L I_L \cos 60^\circ = 0.5\, V_L I_L$. The ratio is 2 : 1 with both readings positive, as the question states. Whether the load is lagging or leading only changes which meter gives the larger reading, so the power factor is 0.866 either way.

KEY IDEA

In the two-wattmeter method on a balanced load, the power factor depends only on the ratio of the two readings: $\tan\phi = \dfrac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2}$. When one reading is half the other and both are positive, $\tan\phi = \dfrac{1}{\sqrt{3}}$, so $\phi = 30^\circ$ and the power factor is 0.866. Points worth remembering: equal readings give a power factor of 1, a ratio of 2 : 1 gives 0.866, one reading at zero gives 0.5, and one negative reading means the power factor is below 0.5.

WATCH OUT FOR

Leaving out the $\sqrt{3}$ gives $\tan\phi = \dfrac{1}{3}$ and a power factor of about 0.949, which is not among the options. Another slip is treating the smaller reading as negative even though the question says both are positive. That gives $\tan\phi = \dfrac{\sqrt{3}\,(1.5)}{0.5} = 3\sqrt{3}$ and a power factor of about 0.189. Some students also put the sum on top and the difference below, which gives $\tan\phi = 3\sqrt{3}$ as well.

Q.6 MCQ · 1 MARK

Power Systems › Stability

Consider a lossy transmission line with V_1 and V_2 as the sending and receiving end voltages, respectively. Z and X are the series impedance and reactance of the line, respectively. The steady-state stability limit for the transmission line will be

(A) greater than $\left|\dfrac{V_1V_2}{X}\right|$

(B) less than $\left|\dfrac{V_1V_2}{X}\right|$

(C) equal to $\left|\dfrac{V_1V_2}{X}\right|$

(D) equal to $\left|\dfrac{V_1V_2}{Z}\right|$

Official answer: **B**

SOLUTION

1. Let the line have series impedance $Z = R + jX = |Z|\angle\theta$ with $\theta = \tan^{-1}(X/R)$. Because the line is lossy, $R > 0$, so $\theta < 90^\circ$ and $\cos\theta > 0$. Take the shunt admittance as negligible, so the ABCD constants are $A = D = 1$, $B = Z$.
2. Power received at the receiving end, with δ the angle by which V_1 leads V_2:
$$P_R = \frac{|V_1||V_2|}{|Z|}\cos(\theta - \delta) - \frac{|V_2|^2}{|Z|}\cos\theta$$
3. P_R is largest when $\cos(\theta - \delta) = 1$, that is $\delta = \theta$. The steady-state stability limit is this maximum:
$$P_{max} = \frac{|V_1||V_2|}{|Z|} - \frac{|V_2|^2}{|Z|}\cos\theta$$
4. Compare with the lossless value $\left|\dfrac{V_1V_2}{X}\right|$. Since $|Z| = \sqrt{R^2 + X^2} > X$, the first term satisfies $\dfrac{|V_1V_2|}{|Z|} < \dfrac{|V_1V_2|}{X}$.
5. The subtracted term $\dfrac{|V_2|^2}{|Z|}\cos\theta$ is positive because $\cos\theta > 0$, so it lowers the limit further. Hence $P_{max} < \left|\dfrac{V_1V_2}{X}\right|$.
6. Check: as $R \to 0$, $|Z| \to X$, $\theta \to 90^\circ$ and $\cos\theta \to 0$, so $P_{max} \to \left|\dfrac{V_1V_2}{X}\right|$, the lossless value. Any resistance lowers the limit, so the answer is (B).

KEY IDEA

For a lossy line the received power is $P_R = \frac{V_1 V_2}{|Z|}\cos(\theta - \delta) - \frac{V_2^2}{|Z|}\cos\theta$. Its maximum, at $\delta = \theta$, is smaller than the lossless limit $V_1 V_2 / X$ because $|Z| > X$ and the second term is subtracted.

WATCH OUT FOR

Using the lossless formula $P_{max} = V_1 V_2 / X$ (the stability limit at $\delta = 90^\circ$) even though the line has resistance. In a lossy line the maximum occurs at $\delta = \theta$, not 90°, and its value is lower.

Q.7 MCQ · 1 MARK

Networks (Electric Circuits) › Laws & node/mesh

The graph of a network has 8 nodes and 5 independent loops. The number of branches of the graph is

(A) 11

(B) 12

(C) 13

(D) 14

Official answer: **B**

SOLUTION

1. Take the network graph as connected, which is the standard assumption for a single network. For a connected graph with n nodes, every tree has $n - 1$ branches (twigs): $n - 1 = 8 - 1 = 7$.
2. Every branch not in the tree is a link. Adding a link to the tree closes exactly one fundamental loop, and these loops are independent, so the number of independent loops equals the number of links:
$$L = b - (n - 1)$$
3. Rearranging for the number of branches:
$$b = L + n - 1$$
4. Substituting $L = 5$ and $n = 8$:
$$b = 5 + 8 - 1 = 12$$
5. Check: 7 twigs $+$ 5 links $=$ 12 branches. The answer is option (B).

KEY IDEA

In a connected graph, the number of independent loops equals the number of links: $L = b - n + 1$. A tree uses $n - 1$ branches and each remaining branch closes one fundamental loop, so $b = L + n - 1$.

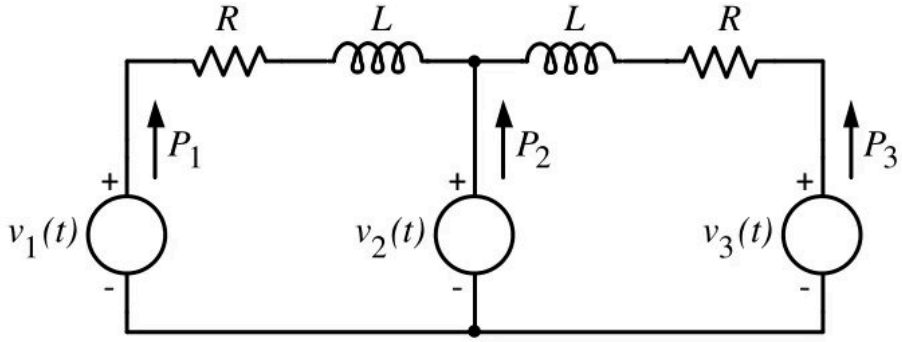
WATCH OUT FOR

Using $L = b - n$, which leaves out the $+1$ and gives 13 (option C), or confusing independent loops with meshes or node equations ($n - 1 = 7$). Another mistake is adding $L + n = 13$ without subtracting 1 for the tree.

Q.8 MCQ · 1 MARK

Networks (Electric Circuits) › AC, power & resonance

Q.8 In the figure, the voltages are $v_1(t) = 100\cos(\omega t)$, $v_2(t) = 100\cos(\omega t + \pi/18)$ and $v_3(t) = 100\cos(\omega t + \pi/36)$. The circuit is in sinusoidal steady state, and $R << \omega L$. P_1, P_2 and P_3 are the average power outputs. Which one of the following statements is true?



(A) $P_1 = P_2 = P_3 = 0$
(B) $P_1 < 0, P_2 > 0, P_3 > 0$
(C) $P_1 < 0, P_2 > 0, P_3 < 0$
(D) $P_1 > 0, P_2 < 0, P_3 > 0$

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Official answer: **C**

SOLUTION

1. Phasors (rms): $\pi/18 = 10^\circ$ and $\pi/36 = 5^\circ$, so $V_1 = 70.71\angle 0^\circ$ V, $V_2 = 70.71\angle 10^\circ$ V, $V_3 = 70.71\angle 5^\circ$ V. Write $V = 100/\sqrt{2} = 70.71$ V, so $V^2 = 5000$ V^2. All three magnitudes are equal; only the phase angles differ.
2. The bottom terminals of all three sources are joined, so each outer source is connected straight to the top of v_2 through one series branch $Z = R + jX$, where $X = \omega L$. The order of R and L inside a branch makes no difference. Because $R \ll \omega L$, each branch is almost a pure reactance.
3. Power output of v_1: the current leaving its $+$ terminal is $I_1 = (V_1 - V_2)/Z$, so $P_1 = \mathrm{Re}(V_1 I_1^*) = \mathrm{Re}\left[\dfrac{V^2 - V_1 V_2^*}{Z^*}\right]$. Put $V_1 V_2^* = V^2\angle(-\delta)$, where $\delta = 10^\circ$ is the angle by which V_2 leads V_1, and $\dfrac{1}{Z^*} = \dfrac{R + jX}{|Z|^2}$:
$$P_1 = \frac{V^2}{|Z|^2}\,\mathrm{Re}\left[(1 - \cos\delta + j\sin\delta)(R + jX)\right] = \frac{V^2\left[R(1 - \cos\delta) - X\sin\delta\right]}{|Z|^2}$$
4. Sign of P_1: $P_1 < 0$ exactly when $R(1 - \cos\delta) < X\sin\delta$, that is when $\dfrac{R}{X} < \dfrac{\sin\delta}{1 - \cos\delta} = \cot\dfrac{\delta}{2} = \cot 5^\circ \approx 11.4$. Since $R \ll \omega L$, this certainly holds. Dropping R (so $|Z|^2 \approx X^2$) gives $P_1 \approx -\dfrac{V^2\sin 10^\circ}{\omega L} = -\dfrac{868}{\omega L}$ W < 0. So v_1 absorbs power.
5. Power output of v_3: it is connected only to v_2, through the same Z. With $I_3 = (V_3 - V_2)/Z$ leaving its $+$ terminal, and V_2 leading V_3 by $\delta = 10^\circ - 5^\circ = 5^\circ$, the same algebra gives $P_3 = \dfrac{V^2\left[R(1 - \cos 5^\circ) - X\sin 5^\circ\right]}{|Z|^2}$. This is negative because $R/X < \cot 2.5^\circ \approx 22.9$. Approximately, $P_3 \approx -\dfrac{V^2\sin 5^\circ}{\omega L} = -\dfrac{436}{\omega L}$ W < 0. So v_3 also absorbs power.
6. Power output of v_2: the current leaving its $+$ terminal is $\dfrac{V_2 - V_1}{Z} + \dfrac{V_2 - V_3}{Z}$. Each branch now contributes $+X\sin\delta$ instead of $-X\sin\delta$:
$$P_2 = \frac{V^2\left[R(1 - \cos 10^\circ) + X\sin 10^\circ\right]}{|Z|^2} + \frac{V^2\left[R(1 - \cos 5^\circ) + X\sin 5^\circ\right]}{|Z|^2} > 0$$

since every term is positive. Approximately, $P_2 \approx \dfrac{1304}{\omega L}$ W. Check: $P_1 + P_2 + P_3 = \dfrac{2RV^2\left[(1-\cos 10^\circ)+(1-\cos 5^\circ)\right]}{|Z|^2}$. That is exactly the loss $|I_1|^2R + |I_3|^2R$ in the two resistors, so energy balances.

7. So $P_1 < 0$, $P_2 > 0$ and $P_3 < 0$, which is option (C). This matches the rule of thumb for a nearly lossless inductive link: real power flows from the leading voltage to the lagging one. v_2 leads both of its neighbours, so it supplies both.

KEY IDEA

When sources of equal magnitude are joined by a mostly inductive branch ($R \ll \omega L$), real power flows from the source whose voltage leads to the one that lags, $P \approx \dfrac{V^2 \sin\delta}{X}$. Compare each source only with the source it is actually wired to. Here v_1 and v_3 are each wired to v_2, and v_2 leads both of them.

WATCH OUT FOR

Picking (A) because all three magnitudes are equal, when it is the phase difference that drives real power through a reactance. Another mistake is picking (B): comparing v_3 with v_1 (it leads v_1 by 5°) and calling it a supplier. But v_3 is connected only to v_2, which leads it by 5°, so v_3 absorbs power.

Q.9 MCQ · 1 MARK

Control Systems › Time response

Match the transfer functions of the second-order systems with the nature of the systems given below.

Transfer functions

P: $\dfrac{15}{s^2+5s+15}$

Q: $\dfrac{25}{s^2+10s+25}$

R: $\dfrac{35}{s^2+18s+35}$

Nature of system

I: Overdamped

II: Critically damped

III: Underdamped

(A) P-I, Q-II, R-III

(B) P-II, Q-I, R-III

(C) P-III, Q-II, R-I

(D) P-III, Q-I, R-II

Official answer: **C**

SOLUTION

1. Compare each denominator with the standard form $s^2 + 2\zeta\omega_n s + \omega_n^2$. This gives $\omega_n = \sqrt{\text{constant term}}$ and $\zeta = \dfrac{\text{coefficient of } s}{2\omega_n}$.
2. Classification rule: $\zeta < 1$ is underdamped, $\zeta = 1$ is critically damped and $\zeta > 1$ is overdamped. For a denominator $s^2 + bs + c$ the same test can be run on the discriminant $b^2 - 4c$: negative, zero or positive respectively.
3. P: $\omega_n = \sqrt{15} \approx 3.873$, so $\zeta = \dfrac{5}{2\sqrt{15}} \approx 0.645 < 1$. Check: the discriminant is $25 - 60 = -35 < 0$, so the poles are complex, $s = -2.5 \pm j\dfrac{\sqrt{35}}{2} \approx -2.5 \pm j2.958$. **Underdamped (III).**

4. Q: $s^2 + 10s + 25 = (s+5)^2$, so $\omega_n = 5$ and $\zeta = \dfrac{10}{2 \times 5} = 1$. There is a repeated real pole at $s = -5$. **Critically damped (II).**
5. R: $\omega_n = \sqrt{35} \approx 5.916$, so $\zeta = \dfrac{18}{2\sqrt{35}} \approx 1.52 > 1$. Check: the discriminant is $324 - 140 = 184 > 0$, giving two distinct real poles $s = -9 \pm \sqrt{46} \approx -2.22$ and -15.78. **Overdamped (I).**
6. So the matching is P-III, Q-II, R-I, which is option (C).

KEY IDEA

The damping ratio of a second-order system comes from its denominator alone: for $s^2 + bs + c$, $\zeta = \dfrac{b}{2\sqrt{c}}$. Equivalently, compare b^2 with $4c$. If $b^2 < 4c$ the poles are complex (underdamped), if $b^2 = 4c$ they are repeated (critically damped), and if $b^2 > 4c$ they are real and distinct (overdamped). The numerator plays no part.

WATCH OUT FOR

Taking ζ to be the coefficient of s divided by 2, forgetting the division by ω_n. That gives $\zeta = 2.5$, 5 and 9, which would wrongly call all three systems overdamped. A second mistake is to rank the systems by the size of the s coefficient alone. R has the largest s coefficient, but you only know it is overdamped once you check $18^2 = 324 > 4 \times 35 = 140$.

Q.10 MCQ · 1 MARK

Electromagnetics › Electrostatics

A positive charge of 1 nC is placed at (0, 0, 0.2) where all dimensions are in metres. Consider the x-y plane to be a conducting ground plane. Take $\epsilon_0 = 8.85 \times 10^{-12}$ F/m. The z component of the E field at (0, 0, 0.1) is closest to

(A) 899.18 V/m

(B) −899.18 V/m

(C) 999.09 V/m

(D) −999.09 V/m

Official answer: **D**

SOLUTION

1. **Method of images:** replace the grounded plane $z = 0$ with an image charge $Q' = -1$ nC at $(0, 0, -0.2)$. The pair keeps $V = 0$ everywhere on the plane, so the field in the region $z > 0$ is unchanged. The field point $(0, 0, 0.1)$ lies in that region.
2. Coulomb constant, with $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$:
$$\frac{1}{4\pi\epsilon_0} = \frac{1}{4\pi(8.85 \times 10^{-12})} \approx 8.992 \times 10^{9}\ \mathrm{m/F}$$
3. **Real charge** $+1$ nC at $z = 0.2$: the field point is 0.1 m *below* it. The field of a positive charge points away from the charge, so here it points in $-\hat{a}_z$:
$$E_1 = \frac{8.992 \times 10^{9} \times 10^{-9}}{(0.1)^2} = 899.18\ \mathrm{V/m}, \quad \mathbf{E}_1 = -899.18\,\hat{a}_z\ \mathrm{V/m}$$
4. **Image charge** -1 nC at $z = -0.2$: the field point is 0.3 m *above* it. The field of a negative charge points toward the charge, so this one also points in $-\hat{a}_z$:
$$E_2 = \frac{8.992 \times 10^{9} \times 10^{-9}}{(0.3)^2} = 99.91\ \mathrm{V/m}, \quad \mathbf{E}_2 = -99.91\,\hat{a}_z\ \mathrm{V/m}$$
5. **Superposition:**
$$E_z = -(899.18 + 99.91) \approx -999.09\ \mathrm{V/m}$$
which is option (D).

KEY IDEA

A point charge above a grounded conducting plane gives the same field (above the plane) as the charge plus an equal and opposite image charge placed at its mirror point. At a point between the real charge and the plane, both the real charge (pushing away) and the negative image (pulling toward) give a field along −z, so the two magnitudes add.

WATCH OUT FOR

Leaving out the image charge gives −899.18 V/m (option B). Getting the direction wrong gives a positive value (option A or C). Another mistake is to subtract the image contribution instead of adding it. Directly below a positive charge, and above a negative image, both fields point along −z.

Q.11 MCQ · 1 MARK

Engineering Mathematics › Calculus

Let f be a real-valued function of a real variable defined as $f(x) = x^2$ for $x \geq 0$, and $f(x) = -x^2$ for $x < 0$. Which one of the following statements is true?

(A) $f(x)$ is discontinuous at $x = 0$.

(B) $f(x)$ is continuous but not differentiable at $x = 0$.

(C) $f(x)$ is differentiable but its first derivative is not continuous at $x = 0$.

(D) $f(x)$ is differentiable but its first derivative is not differentiable at $x = 0$.

Official answer: **D**

SOLUTION

1. The two branches combine into one formula, $f(x) = x\,|x|$. For $x \geq 0$, $x \cdot x = x^2$, and for $x < 0$, $x \cdot (-x) = -x^2$.
2. Continuity at 0: $\lim_{x\to 0^-}(-x^2) = 0 = \lim_{x\to 0^+} x^2 = f(0)$, so f is continuous at 0. This rules out (A).
3. Differentiability at 0, from the definition of the derivative:
$$f'(0) = \lim_{h\to 0} \frac{f(h) - f(0)}{h} = \lim_{h\to 0} \frac{h|h|}{h} = \lim_{h\to 0} |h| = 0.$$
The limit exists, so f is differentiable at 0. This rules out (B).
4. The first derivative everywhere: $f'(x) = 2x$ for $x > 0$, $f'(x) = -2x$ for $x < 0$, and $f'(0) = 0$. Taken together, $f'(x) = 2|x|$.
5. Continuity of f' at 0: $\lim_{x\to 0} 2|x| = 0 = f'(0)$, so f' is continuous at 0. This rules out (C).
6. Differentiability of f' at 0, from the one-sided derivatives:
$$\lim_{h\to 0^+} \frac{2|h| - 0}{h} = \lim_{h\to 0^+} \frac{2h}{h} = 2, \qquad \lim_{h\to 0^-} \frac{2|h| - 0}{h} = \lim_{h\to 0^-} \frac{-2h}{h} = -2.$$
The two sides differ, so f' is not differentiable at 0. The graph of $f'(x) = 2|x|$ has a corner there.
7. So f is differentiable and f' is continuous at $x = 0$, but f' is not differentiable there. The answer is (D).

KEY IDEA

$f(x) = x|x|$. Its derivative is $f'(x) = 2|x|$, which is continuous at 0 but has a corner there. So f has a continuous first derivative, but no second derivative exists at 0: the one-sided second derivatives are $+2$ and -2. When a function is defined in pieces, test each level of smoothness at the joint separately with one-sided limits.

WATCH OUT FOR

Mixing up $x|x|$ with $|x|$ and choosing (B). Another mistake is to see that the branch derivatives $2x$ and $-2x$ come from different formulas and conclude that f' jumps, which gives (C). In fact both branches give 0 at $x = 0$, so f' is continuous there. What fails is the next level: the slopes of f' are $+2$ and -2.

Q.12 MCQ · 1 MARK

Engineering Mathematics › Vector calculus

The value of the directional derivative of the function $\Phi(x, y, z) = xy^2 + yz^2 + zx^2$ at the point $(2, -1, 1)$ in the direction of the vector $\mathbf{p} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ is

(A) 1

(B) 0.95

(C) 0.93

(D) 0.9

Official answer: **A**

SOLUTION

1. The directional derivative of Φ along $\mathbf{p}$ is $D_{\mathbf{p}}\Phi = \nabla\Phi \cdot \hat{\mathbf{p}}$, where $\hat{\mathbf{p}} = \dfrac{\mathbf{p}}{|\mathbf{p}|}$ is the unit vector along $\mathbf{p}$.
2. Partial derivatives of $\Phi = xy^2 + yz^2 + zx^2$: $\dfrac{\partial\Phi}{\partial x} = y^2 + 2zx$, $\dfrac{\partial\Phi}{\partial y} = 2xy + z^2$, $\dfrac{\partial\Phi}{\partial z} = 2yz + x^2$.
3. At $(2, -1, 1)$: $\dfrac{\partial\Phi}{\partial x} = (-1)^2 + 2(1)(2) = 1 + 4 = 5$, $\dfrac{\partial\Phi}{\partial y} = 2(2)(-1) + 1^2 = -4 + 1 = -3$, $\dfrac{\partial\Phi}{\partial z} = 2(-1)(1) + 2^2 = -2 + 4 = 2$, so $\nabla\Phi = 5\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$.
4. Unit vector: $|\mathbf{p}| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = 3$, so $\hat{\mathbf{p}} = \dfrac{1}{3}(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$.
5. Dot product:

$$D_{\mathbf{p}}\Phi = \frac{(5)(1) + (-3)(2) + (2)(2)}{3} = \frac{5 - 6 + 4}{3} = \frac{3}{3} = 1$$

6. The directional derivative is 1, which is option (A).

KEY IDEA

Directional derivative = gradient at the point dotted with the UNIT vector in the given direction: $D_{\mathbf{p}}\Phi = \nabla\Phi \cdot \mathbf{p}/|\mathbf{p}|$. Here $\nabla\Phi(2, -1, 1) = (5, -3, 2)$ and $|\mathbf{p}| = 3$, giving $3/3 = 1$.

WATCH OUT FOR

Dotting the gradient with $\mathbf{p}$ itself instead of the unit vector gives 3, which is not an option. The other trap is differentiating the cyclic terms carelessly: zx^2 contributes $2zx$ to $\partial\Phi/\partial x$ and x^2 to $\partial\Phi/\partial z$. Also watch the sign of $y = -1$ in $2xy$ and $2yz$.

Q.13 MCQ · 1 MARK

Engineering Mathematics › Complex analysis

The value of the integral $\oint_C \dfrac{z+1}{z^2-4}\,dz$ in counter clockwise direction around a circle C of radius 1 with center at the point $z = -2$ is

(A) $\dfrac{\pi i}{2}$

(B) $2\pi i$

(C) $-\dfrac{\pi i}{2}$

(D) $-2\pi i$

Official answer: **A**

SOLUTION

1. The contour is the circle $|z + 2| = 1$, traversed counter-clockwise (the positive direction).
2. Factor the denominator: $z^2 - 4 = (z - 2)(z + 2)$. The numerator $z + 1$ vanishes at neither $z = 2$ nor $z = -2$, so the integrand has simple poles at $z = 2$ and $z = -2$ and is analytic everywhere else.
3. Locate the poles: $z = -2$ is the centre of C, so it lies inside. For $z = 2$, $|2 + 2| = 4 > 1$, so it lies outside C and contributes nothing.
4. Residue at the simple pole $z = -2$:

$$\text{Res}_{z=-2} = \lim_{z\to-2}(z+2)\frac{z+1}{(z-2)(z+2)} = \frac{-2+1}{-2-2} = \frac{-1}{-4} = \frac{1}{4}$$

5. Residue theorem, $\oint_C f(z)\,dz = 2\pi i \sum \text{Res}$ (poles inside C):

$$\oint_C \frac{z+1}{z^2-4}\,dz = 2\pi i \cdot \frac{1}{4} = \frac{\pi i}{2}$$

6. Check with Cauchy's integral formula: write the integrand as $\dfrac{g(z)}{z+2}$ with $g(z) = \dfrac{z+1}{z-2}$, which is analytic on and inside C. Then $\oint_C = 2\pi i\, g(-2) = 2\pi i \cdot \dfrac{1}{4} = \dfrac{\pi i}{2}$, the same result.
7. This matches option (A).

KEY IDEA

Only poles inside the contour count. $|z+2| = 1$ encloses $z = -2$ but not $z = 2$, so the integral is $2\pi i$ times the residue at $z = -2$, which is $\frac{z+1}{z-2}$ evaluated at $z = -2$, that is $\frac{1}{4}$.

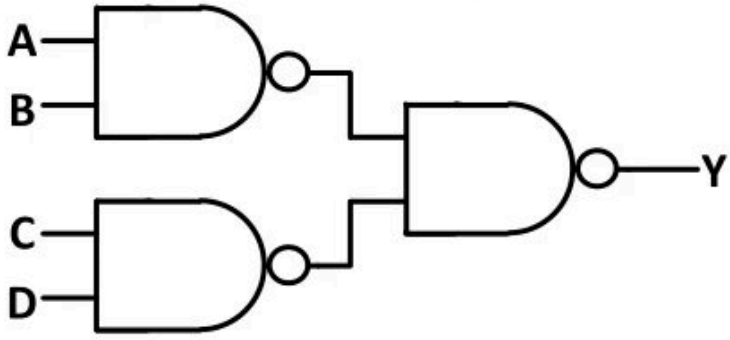
WATCH OUT FOR

Three slips are common here. The first is counting the pole at $z = 2$ as well: it lies at distance 4 from the centre, outside the unit circle. The second is losing a sign in $\frac{-1}{-4}$, which gives $-\frac{\pi i}{2}$ (option C). The third is forgetting the residue altogether and answering $2\pi i$ (option B).

Q.14 MCQ · 1 MARK

Digital Electronics › Boolean logic

Q.14 In the logic circuit shown in the figure, Y is given by



(A) Y = ABCD
(B) Y = (A + B)(C + D)
(C) Y = A + B + C + D
(D) Y = AB + CD

Official answer: **D**

SOLUTION

1. All three gates are two-input NAND gates: each is an AND body with a bubble on its output. Complement is written with an overline, -.
2. First-level outputs: the upper gate gives $P = \overline{AB}$ and the lower gate gives $Q = \overline{CD}$.
3. The output gate is a NAND of P and Q:
$$Y = \overline{P \cdot Q} = \overline{\overline{AB} \cdot \overline{CD}}$$
4. By De Morgan's theorem, $\overline{X \cdot Z} = \overline{X} + \overline{Z}$, with $X = \overline{AB}$ and $Z = \overline{CD}$:
$$Y = \overline{\overline{AB}} + \overline{\overline{CD}}$$
5. Double complementation, $\overline{\overline{X}} = X$, gives
$$Y = AB + CD$$
This is the standard NAND-NAND form of a sum of products.
6. Check with $A = B = 1$, $C = D = 0$: $P = 0$ and $Q = 1$, so $Y = \overline{0 \cdot 1} = 1$, and $AB + CD = 1$ agrees. Option (A) gives $ABCD = 0$ and option (B) gives $(1+1)(0+0) = 0$, so both are ruled out.
7. Check with $A = 1$, $B = C = D = 0$: $P = 1$ and $Q = 1$, so $Y = \overline{1 \cdot 1} = 0$, and $AB + CD = 0$ agrees. Option (C) gives $A + B + C + D = 1$, so it is ruled out. Option (D) is correct.

KEY IDEA

A two-level NAND-NAND circuit carries out an AND-OR (sum-of-products) function. By De Morgan's theorem, the output gate's NAND turns into an OR of the complemented inputs, and those complements cancel the bubbles of the first-level gates. So each first-level NAND gives one product term, and the output is the OR of those terms: $AB + CD$.

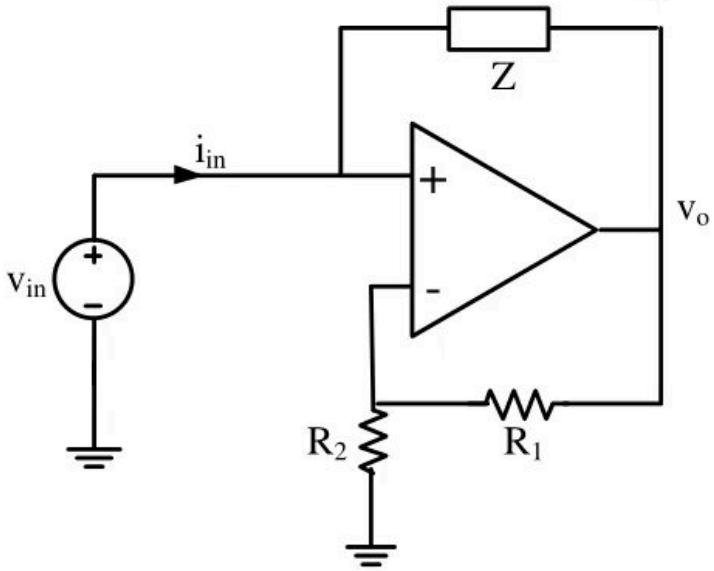
WATCH OUT FOR

Treating each bubble as a separate inversion of the whole circuit and ending up with $\overline{AB + CD}$, or applying De Morgan to the inner NANDs as well, which gives $(\overline{A} + \overline{B})$-type terms and leads to option (B) or (C). Only the output NAND needs De Morgan. Its complements cancel the first-level bubbles, so the inner products AB and CD come through unchanged.

Q.15 MCQ · 1 MARK

Analog Electronics › Op-amps

Q.15 The op-amp shown in the figure is ideal. The input impedance $\frac{v_{in}}{i_{in}}$ is given by



(A) $Z\frac{R_1}{R_2}$ (B) $-Z\frac{R_2}{R_1}$ (C) Z (D) $-Z\frac{R_1}{R_1+R_2}$

Official answer: **B**

SOLUTION

1. **Assumptions.** The op-amp is ideal (infinite gain, zero input current, as the question states) and works in its linear region. So there is a virtual short, $v_+ = v_-$, and no current flows into either input terminal.
2. **Inverting input.** No current enters the $-$ terminal, so R_1 and R_2 form a plain voltage divider from V_o to ground:
$$v_- = V_o \frac{R_2}{R_1 + R_2}$$
3. **Non-inverting input.** The source is connected straight to the $+$ terminal, so $v_+ = v_{in}$. The virtual short gives
$$v_{in} = V_o \frac{R_2}{R_1 + R_2} \quad \Rightarrow \quad V_o = v_{in}\left(1 + \frac{R_1}{R_2}\right)$$
4. **KCL at the non-inverting input node.** The ideal op-amp draws no input current, so all of i_{in} flows through Z from that node to the output:
$$i_{in} = \frac{v_{in} - V_o}{Z}$$
5. **Substitute for V_o:**

$$i_{in} = \frac{v_{in} - v_{in}\left(1 + \dfrac{R_1}{R_2}\right)}{Z} = -\frac{v_{in}\,R_1}{R_2\,Z}$$

6. **Input impedance:**

$$\frac{v_{in}}{i_{in}} = -Z\,\frac{R_2}{R_1}$$

This is option **(B)**. The circuit is a negative impedance converter. Because $V_o > v_{in}$, current flows *out of* the input node, back toward the source.

KEY IDEA

The R_1–R_2 network makes a non-inverting amplifier, so $V_o = v_{in}(1 + R_1/R_2)$ is larger than v_{in}. The only path for the input current is through Z to the output. Since the output sits above the input, current is pushed back into the source, so the input impedance comes out negative: $-ZR_2/R_1$. This is a negative impedance converter.

WATCH OUT FOR

Answering Z (option C) by treating Z as if it simply loads the source, which ignores the fact that the far end of Z is the amplified output V_o, not ground. Another slip is writing the current through Z as $(V_o - v_{in})/Z$ while still calling it i_{in}. That drops the minus sign, the key feature of the result.

Q.16 MCQ · 1 MARK

Signals and Systems › LTI & convolution

A continuous-time input signal $x(t)$ is an eigenfunction of an LTI system, if the output is

(A) $k\,x(t)$, where k is an eigenvalue

(B) $k\,e^{j\omega t}x(t)$, where k is an eigenvalue and $e^{j\omega t}$ is a complex exponential signal

(C) $x(t)\,e^{j\omega t}$, where $e^{j\omega t}$ is a complex exponential signal

(D) $k\,H(\omega)$, where k is an eigenvalue and $H(\omega)$ is a frequency response of the system

Official answer: **A**

SOLUTION

1. **What an eigenfunction is.** For a system operator $T\{\cdot\}$, an input $x(t)$ is an eigenfunction if the output is the same signal multiplied by a constant:

$$y(t) = T\{x(t)\} = k\,x(t)$$

The constant k may be complex but must not depend on t. It is called the eigenvalue.

2. **Check with the standard LTI eigenfunction.** Take $x(t) = e^{j\omega_0 t}$. The convolution integral gives

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\,e^{j\omega_0(t-\tau)}\,d\tau = e^{j\omega_0 t}\int_{-\infty}^{\infty} h(\tau)\,e^{-j\omega_0\tau}\,d\tau = H(\omega_0)\,x(t)$$

The output is $k\,x(t)$ with $k = H(\omega_0)$, a constant. This is the form in (A).

3. **Option (B):** $k\,e^{j\omega t}x(t)$ multiplies the input by $e^{j\omega t}$, which changes with time (for $\omega \neq 0$). The output is therefore not a constant times $x(t)$, so (B) fails the definition.

4. **Option (C):** $x(t)\,e^{j\omega t}$ is the input modulated, i.e. shifted in frequency. The multiplying factor again depends on t, so (C) fails too.

5. **Option (D):** $k\,H(\omega)$ does not contain $x(t)$ at all. It is not a scaled copy of the input, so it cannot be the output for an eigenfunction input.

6. (A) is the only option where the output equals the input times a constant. Answer: **(A)**.

KEY IDEA

An eigenfunction passes through the system unchanged in shape: $T\{x(t)\} = k\,x(t)$, where k is a constant (possibly complex) that does not depend on t. For an LTI system, $e^{j\omega_0 t}$ is an eigenfunction and its eigenvalue is $H(\omega_0)$.

WATCH OUT FOR

Picking (D) because the frequency response is involved. For the input $e^{j\omega_0 t}$, $H(\omega_0)$ is itself the eigenvalue, and the output is $H(\omega_0)\,x(t)$, not $k\,H(\omega)$. The output always has to contain the input signal. Picking (B) or (C) is the other mistake: both multiply the input by a factor that changes with time, which is modulation, not scaling.

Q.17 NAT · 1 MARK

Engineering Mathematics › Linear algebra

Consider a non-singular 2×2 square matrix $\mathbf{A}$. If $trace(\mathbf{A}) = 4$ and $trace(\mathbf{A}^2) = 5$, the determinant of the matrix $\mathbf{A}$ is _____ (up to 1 decimal place).

Official answer: **5.5**

SOLUTION

1. Let the eigenvalues of $\mathbf{A}$ be λ_1, λ_2, counted over the complex numbers. The trace is the sum of the eigenvalues: $\lambda_1 + \lambda_2 = \mathrm{tr}(\mathbf{A}) = 4$.
2. The eigenvalues of $\mathbf{A}^2$ are λ_1^2, λ_2^2, so $\lambda_1^2 + \lambda_2^2 = \mathrm{tr}(\mathbf{A}^2) = 5$.
3. The determinant is the product of the eigenvalues: $\det(\mathbf{A}) = \lambda_1\lambda_2$.
4. Use the identity $(\lambda_1 + \lambda_2)^2 = \lambda_1^2 + \lambda_2^2 + 2\lambda_1\lambda_2$:
$$16 = 5 + 2\lambda_1\lambda_2 \;\Rightarrow\; \lambda_1\lambda_2 = \frac{11}{2} = 5.5$$
5. The same result without eigenvalues: by the Cayley–Hamilton theorem every 2×2 matrix satisfies $\mathbf{A}^2 - \mathrm{tr}(\mathbf{A})\,\mathbf{A} + \det(\mathbf{A})\,\mathbf{I} = \mathbf{0}$. Taking the trace of both sides (with $\mathrm{tr}(\mathbf{I}) = 2$) gives
$$\det(\mathbf{A}) = \frac{[\mathrm{tr}(\mathbf{A})]^2 - \mathrm{tr}(\mathbf{A}^2)}{2} = \frac{16 - 5}{2} = 5.5$$
6. Check: the eigenvalues solve $\lambda^2 - 4\lambda + 5.5 = 0$, so they are the complex pair $2 \pm j\sqrt{1.5}$. That is allowed, because a real matrix can have complex eigenvalues, and the sum and product relations still hold. For example, $\mathbf{A} = \begin{bmatrix} 2 & -1.5 \\ 1 & 2 \end{bmatrix}$ has trace 4, $\mathbf{A}^2 = \begin{bmatrix} 2.5 & -6 \\ 4 & 2.5 \end{bmatrix}$ has trace 5, and $\det(\mathbf{A}) = 4 + 1.5 = 5.5$. The determinant is non-zero, as a non-singular matrix requires, so $\det(\mathbf{A}) = 5.5$.

KEY IDEA

For a 2×2 matrix, trace = $\lambda_1 + \lambda_2$ and determinant = $\lambda_1\lambda_2$, while $\mathrm{tr}(\mathbf{A}^2) = \lambda_1^2 + \lambda_2^2$. The identity $(\lambda_1 + \lambda_2)^2 = \lambda_1^2 + \lambda_2^2 + 2\lambda_1\lambda_2$ therefore gives $\det(\mathbf{A}) = \frac{1}{2}\left[(\mathrm{tr}\,\mathbf{A})^2 - \mathrm{tr}(\mathbf{A}^2)\right]$. You get the same formula by taking the trace of the Cayley–Hamilton equation $\mathbf{A}^2 - (\mathrm{tr}\,\mathbf{A})\mathbf{A} + (\det\mathbf{A})\mathbf{I} = \mathbf{0}$.

WATCH OUT FOR

Assuming $\mathrm{tr}(\mathbf{A}^2) = [\mathrm{tr}(\mathbf{A})]^2$ (this would give 16, not 5), or dropping the factor 2 and answering $16 - 5 = 11$. Another trap is to try to find real eigenvalues, get a negative discriminant ($16 - 22 = -6$) and decide the data are inconsistent. The eigenvalues are simply the complex pair $2 \pm j\sqrt{1.5}$, and the sum and product relations still hold for them.

Q.18 NAT · 1 MARK

Engineering Mathematics › Calculus

Let f be a real-valued function of a real variable defined as $f(x) = x - [x]$, where $[x]$ denotes the largest

integer less than or equal to x. The value of $\int_{0.25}^{1.25} f(x)\,dx$ is _____ (up to 2 decimal places).

Official answer: **0.49 to 0.51**

SOLUTION

1. $f(x) = x - [x]$ is the fractional part of x. The greatest integer function $[x]$ stays constant between consecutive integers and jumps at each integer. The only integer inside $[0.25,\ 1.25]$ is $x = 1$.
2. Split the range at $x = 1$. On $0.25 \le x < 1$, $[x] = 0$, so $f(x) = x$. On $1 \le x \le 1.25$, $[x] = 1$, so $f(x) = x - 1$. The value of f at the single point $x = 1$ does not affect the integral.
3. First piece:
$$\int_{0.25}^{1} x\,dx = \left[\frac{x^2}{2}\right]_{0.25}^{1} = \frac{1 - 0.0625}{2} = 0.46875$$
4. Second piece:
$$\int_{1}^{1.25} (x-1)\,dx = \left[\frac{(x-1)^2}{2}\right]_{1}^{1.25} = \frac{0.0625 - 0}{2} = 0.03125$$
5. Add the pieces:
$$\int_{0.25}^{1.25} f(x)\,dx = 0.46875 + 0.03125 = 0.5$$
6. Check with periodicity: $[x+1] = [x] + 1$, so $f(x+1) = f(x)$ and f has period 1. A periodic function has the same integral over every interval that is one period long, and $[0.25,\ 1.25]$ is exactly one period long. So the integral equals $\int_0^1 x\,dx = \frac{1}{2} = 0.50$.

KEY IDEA

$x - [x]$ is the fractional part of x: a sawtooth with period 1 that rises from 0 towards 1 on every unit interval. Split the integral where $[x]$ jumps (here at $x = 1$), or note that $[0.25,\ 1.25]$ is exactly one period long, so the integral equals $\int_0^1 x\,dx = \frac{1}{2}$. Geometrically, it is the area of a triangle with base 1 and height 1.

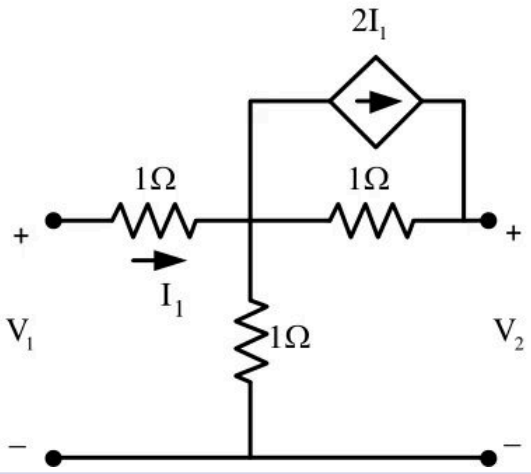
WATCH OUT FOR

Treating $[x]$ as 0 across the whole range and integrating x from 0.25 to 1.25, which gives 0.75. Another slip is reading $[x]$ as rounding to the nearest integer rather than the floor. On $[1,\ 1.25]$ the integrand is $x - 1$, not x.

Q.19 NAT · 1 MARK

Networks (Electric Circuits) › Two-port

Q.19 In the two-port network shown, the h_{11} parameter $\left(\text{where, } h_{11}=\frac{V_1}{I_1}, \text{ when } V_2=0\right)$ in ohms is _____ (up to 2 decimal places).



Official answer: **0.45 to 0.55**

SOLUTION

1. **Apply the test condition.** h_{11} is measured with $V_2 = 0$, so short port 2: node B (the port-2 + terminal) is now at the potential of the bottom rail. Let V_A be the voltage of the central node A with respect to the bottom rail.
2. **KCL at node A** (current in = current out): I_1 enters through the input 1 Ω resistor. Current leaves through the 1 Ω shunt ($V_A/1$), through the 1 Ω resistor to the shorted node B ($(V_A - 0)/1$), and through the dependent source, which carries $2I_1$ from A towards B:
$$I_1 = \frac{V_A}{1} + \frac{V_A - 0}{1} + 2I_1$$
3. **Solve for the node voltage:**
$$2V_A = I_1 - 2I_1 = -I_1 \quad \Rightarrow \quad V_A = -\frac{I_1}{2}$$
The node voltage is negative because the dependent source takes more current out of node A ($2I_1$) than I_1 brings in, so the two resistors must return the difference to node A.
4. **KVL around the input loop** (port-1 + terminal, through the 1 Ω resistor, to node A, then to the bottom rail):
$$V_1 = 1 \cdot I_1 + V_A = I_1 - \frac{I_1}{2} = 0.5\, I_1$$
5. **The h-parameter:**
$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = 0.50\ \Omega$$

KEY IDEA

h_{11} is the input impedance with the output shorted. Set $V_2 = 0$ so that node B is grounded, write one KCL equation at the central node with the controlled source current $2I_1$ included, and then $V_1 = 1 \cdot I_1 + V_A$. You cannot simplify the network with series/parallel reduction because the controlled source depends on I_1. The dependent source lowers the input impedance from the passive value $1 + (1 \parallel 1) = 1.5\ \Omega$ to $0.5\ \Omega$.

WATCH OUT FOR

Ignoring the dependent source, or treating it as disappearing when port 2 is shorted, and reporting the passive value $1 + (1 \parallel 1) = 1.5\ \Omega$. With $V_2 = 0$ the source is not removed. It still takes $2I_1$ out of node A (into the short), so it must stay in the KCL equation. A second slip is getting its direction wrong (taking $2I_1$ as entering node A). That gives $V_A = 1.5I_1$ and $h_{11} = 2.5\ \Omega$.

Q.20 NAT · 1 MARK

Power Systems › Faults

The series impedance matrix of a short three-phase transmission line in phase coordinates is [[Z_s, Z_m, Z_m],[Z_m, Z_s, Z_m],[Z_m, Z_m, Z_s]]. If the positive sequence impedance is (1 + j10) Ω, and the zero sequence is (4 + j31) Ω, then the imaginary part of Z_m (in Ω) is _____ (up to 2 decimal places).

Official answer: **7.0**

SOLUTION

1. The matrix has Z_s on the diagonal and Z_m in every off-diagonal position, so the line is fully symmetric (balanced mutual coupling). Its sequence impedances are found by the symmetrical-component transformation $Z_{012} = A^{-1} Z_{abc} A$, which gives a diagonal matrix.
2. Positive and negative sequence: $Z_1 = Z_2 = Z_s - Z_m = (1 + j10)\ \Omega$.
3. Zero sequence: $Z_0 = Z_s + 2Z_m = (4 + j31)\ \Omega$.
4. Subtract the first from the second so that Z_s cancels: $Z_0 - Z_1 = 3Z_m$.
5. $$Z_m = \frac{(4 + j31) - (1 + j10)}{3} = \frac{3 + j21}{3} = (1 + j7)\ \Omega$$
6. The imaginary part of Z_m is $7.00\ \Omega$.

KEY IDEA

For a symmetric line matrix with Z_s on the diagonal and Z_m off it, $Z_1 = Z_2 = Z_s - Z_m$ and $Z_0 = Z_s + 2Z_m$. Hence $Z_m = (Z_0 - Z_1)/3$, and Z_s cancels.

WATCH OUT FOR

Writing $Z_0 = Z_s + Z_m$, or $Z_1 = Z_s + Z_m$, instead of $Z_s + 2Z_m$ and $Z_s - Z_m$. Another slip is dividing the difference by 2 rather than 3, or taking the real part instead of the imaginary part.

Q.21 NAT · 1 MARK

Power Systems › Faults

The positive, negative and zero sequence impedances of a 125 MVA, three-phase, 15.5 kV, star-grounded, 50 Hz generator are j0.1 pu, j0.05 pu and j0.01 pu respectively on the machine rating base. The machine is unloaded and working at the rated terminal voltage. If the grounding impedance of the generator is j0.01 pu, then the magnitude of fault current for a b-phase to ground fault (in kA) is _____ (up to 2 decimal places).

Official answer: **73.0 to 74.0**

SOLUTION

1. Per-unit bases: $S_b = 125\ \text{MVA}$, $V_b = 15.5\ \text{kV}$, so $I_b = \dfrac{125 \times 10^3}{\sqrt{3} \times 15.5}\ \text{A} \approx 4656.1\ \text{A}$. The machine is unloaded at rated voltage, so the prefault voltage is $E = 1\angle 0^\circ$ pu.
2. A fault from one phase (here phase b) to ground is a single line-to-ground (LG) fault. For an LG fault the three sequence networks are connected in series, and the fault current is $I_f = 3I_0$. By symmetry the result is the same for any phase, so only the magnitude matters.

3. The grounding impedance carries three times the zero-sequence current, so it enters the zero-sequence network as $3Z_n = 3 \times j0.01 = j0.03$ pu. The total zero-sequence impedance is $Z_0^{\text{total}} = j0.01 + j0.03 = j0.04$ pu.
4. The sequence currents are equal: $I_1 = I_2 = I_0 = \dfrac{E}{Z_1 + Z_2 + Z_0^{\text{total}}} = \dfrac{1}{j(0.1 + 0.05 + 0.04)} = \dfrac{1}{j0.19}$ pu.
5. Fault current: $|I_f| = 3|I_0| = \dfrac{3}{0.19} = 15.789$ pu.
6. Convert to amperes: $|I_f| = 15.789 \times 4656.1\ \text{A} \approx 73.52\ \text{kA}$.

KEY IDEA

For a line-to-ground fault, connect the positive, negative and zero sequence networks in series, with $3Z_n$ added in the zero-sequence network. Then $I_f = 3E/(Z_1 + Z_2 + Z_0 + 3Z_n)$, and the per-unit value is multiplied by the base current.

WATCH OUT FOR

Adding the grounding impedance Z_n to the zero-sequence network without the factor of 3. That gives $Z_0^{\text{total}} = j0.02$, a total of $j0.17$ pu and about 82 kA. Another slip is forgetting that $I_f = 3I_0$, or using the wrong base current.

Q.22 NAT · 1 MARK

Power Systems › Per unit & load flow

A 1000 × 1000 bus admittance matrix for an electric power system has 8000 non-zero elements. The minimum number of branches (transmission lines and transformers) in this system are ____ (up to 2 decimal places).

Official answer: **3500.0**

SOLUTION

1. The bus admittance matrix Y_{bus} is 1000×1000 with 8000 non-zero elements. Split these into diagonal and off-diagonal entries.
2. Each diagonal entry Y_{ii} is the sum of the admittances of everything connected to bus i. In a connected system every bus has at least one branch, so all 1000 diagonal entries are non-zero.
3. The off-diagonal non-zero entries number $8000 - 1000 = 7000$. If any diagonal entry were zero, more of the 8000 would be off-diagonal and the branch count would rise, so assuming all diagonals non-zero gives the minimum.
4. A branch between buses i and k gives the two off-diagonal entries $Y_{ik} = Y_{ki} = -y_{ik}$. The matrix is symmetric when there are no phase-shifting transformers.
5. So $n_{\text{branches}} \geq \dfrac{\text{off-diagonal non-zeros}}{2}$. Equality holds when no two branches join the same pair of buses. Parallel branches between one pair share a single pair of entries, so they would need more branches for the same 7000 entries.
6. $$n_{\text{branches,min}} = \frac{7000}{2} = 3500$$

KEY IDEA

Each diagonal entry of Y-bus is non-zero, and every branch fills exactly two symmetric off-diagonal entries, so the branch count is (non-zeros minus buses) divided by 2 when no branches are parallel.

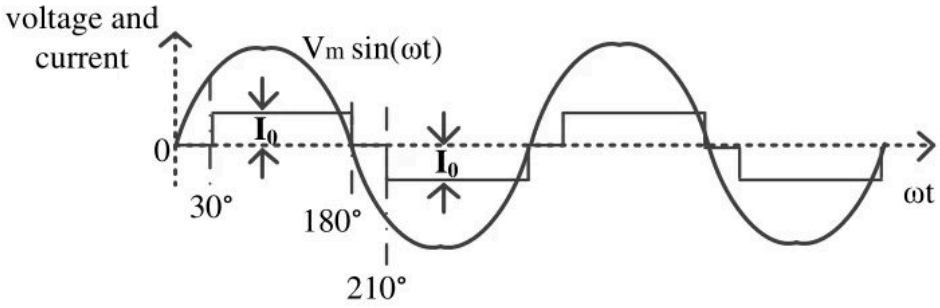
WATCH OUT FOR

Dividing all 8000 non-zero elements by 2 and getting 4000, which forgets that the 1000 diagonal entries do not come from separate branches.

Q.23 NAT · 1 MARK

Power Electronics › Harmonics & PF

Q.23 The waveform of the current drawn by a semi-converter from a sinusoidal AC voltage source is shown in the figure. If $I_0 = 20$ A, the rms value of fundamental component of the current is _____A (up to 2 decimal places).



EE 5/15

Official answer: **16.90 to 17.70**

SOLUTION

1. Read the waveform: the current is $+I_0$ for $30^\circ < \omega t < 180^\circ$, zero from 180° to 210°, $-I_0$ for $210^\circ < \omega t < 360^\circ$, and zero from 0° to 30°. The firing angle is $\alpha = 30^\circ$, so conduction lasts $180^\circ - \alpha = 150^\circ$ in each half-cycle, a duty ratio of $D = \dfrac{150^\circ}{180^\circ} = \dfrac{5}{6}$ of each half-cycle (each pulse is $\dfrac{150}{360} = \dfrac{5}{12}$ of the full period). The average value over a full cycle is zero.
2. The wave has half-wave symmetry, $i(\theta + \pi) = -i(\theta)$, so only odd harmonics exist and the fundamental coefficients are

$$a_1 = \frac{2}{\pi}\int_{\alpha}^{\pi} I_0 \cos\theta \, d\theta, \qquad b_1 = \frac{2}{\pi}\int_{\alpha}^{\pi} I_0 \sin\theta \, d\theta.$$

3. $$a_1 = \frac{2I_0}{\pi}\Big[\sin\theta\Big]_{30^\circ}^{180^\circ} = \frac{2I_0}{\pi}\left(0 - \frac{1}{2}\right) = -\frac{I_0}{\pi} = -6.366 \text{ A}$$

4. $$b_1 = \frac{2I_0}{\pi}\Big[-\cos\theta\Big]_{30^\circ}^{180^\circ} = \frac{2I_0}{\pi}(1 + \cos 30^\circ) = \frac{40}{\pi}(1.8660) = 23.76 \text{ A}$$

5. Peak of the fundamental:

$$I_{1m} = \sqrt{a_1^2 + b_1^2} = \sqrt{6.366^2 + 23.76^2} \approx 24.60 \text{ A}.$$

Check with the closed form: $a_1^2 + b_1^2 = \left(\dfrac{2I_0}{\pi}\right)^2\left[(1+\cos\alpha)^2 + \sin^2\alpha\right] = \left(\dfrac{2I_0}{\pi}\right)^2 \cdot 4\cos^2\dfrac{\alpha}{2}$, so $I_{1m} = \dfrac{4I_0}{\pi}\cos\dfrac{\alpha}{2} = 25.465\cos 15^\circ = 24.60$ A.

6. RMS value of the fundamental:

$$I_1 = \frac{I_{1m}}{\sqrt{2}} = \frac{24.60}{1.4142} \approx 17.39 \text{ A}.$$

KEY IDEA

For a semi-converter input current (quasi-square wave with firing angle α), the fundamental has peak $\dfrac{4I_0}{\pi}\cos\dfrac{\alpha}{2}$, so its rms value is $\dfrac{2\sqrt{2}\,I_0}{\pi}\cos\dfrac{\alpha}{2}$.

WATCH OUT FOR

Using the full-conduction square-wave result $\dfrac{2\sqrt{2}}{\pi}I_0 = 18.03$ A and ignoring the $\cos(\alpha/2)$ factor, or taking $\cos\alpha$ instead of $\cos(\alpha/2)$.

Q.24 NAT · 1 MARK

Electrical Machines › DC machines

A separately excited dc motor has an armature resistance $R_a = 0.05\ \Omega$. The field excitation is kept constant. At an armature voltage of 100 V, the motor produces a torque of 500 Nm at zero speed. Neglecting all

mechanical losses, the no-load speed of the motor (in radians/s) for an armature voltage of 150 V is _____ (up to 2 decimal places).

Official answer: **600.0**

SOLUTION

1. At zero speed the back-emf is zero ($E = K\phi\omega = 0$), so the entire applied voltage drops across the armature resistance:
$$I_a = \frac{V_a}{R_a} = \frac{100}{0.05} = 2000 \text{ A}$$
2. Using the stall torque to find the torque constant (which also relates back-emf to speed, since the field excitation is unchanged):
$$T = K\phi I_a \quad \Rightarrow \quad K\phi = \frac{500}{2000} = 0.25 \text{ Nm/A}$$
3. At no load, with all mechanical losses neglected, the developed torque must be zero. Since $K\phi \neq 0$, this forces the no-load armature current to be zero:
$$T = K\phi I_a = 0 \quad \Rightarrow \quad I_a = 0$$
4. With $I_a = 0$ there is no drop across R_a, so the back-emf equals the applied voltage:
$$E = V_a - I_a R_a = 150 - 0 = 150 \text{ V}$$
5. The no-load speed follows from $E = K\phi\omega$:
$$\omega = \frac{E}{K\phi} = \frac{150}{0.25} = 600 \text{ rad/s}$$

KEY IDEA
At zero speed the back-emf is zero, so the entire applied voltage drops across armature resistance, fixing the stall current and hence the torque constant $K\phi$ (from $T = K\phi I_a$). At no load (no mechanical losses, so no-load torque = 0), the armature current must be zero, so the full new armature voltage appears as back-emf; speed then follows from $E = K\phi\omega$.

WATCH OUT FOR
Assuming the armature current found from the stall condition (2000 A) is reused at the new voltage, or forgetting that at no-load with no mechanical losses the torque (and hence I_a) must be exactly zero, which is what makes $E = V_a$ at no load.

Q.25 NAT · 1 MARK

Control Systems › Time response

Consider a unity feedback system with forward transfer function given by
$$G(s) = \frac{1}{(s+1)(s+2)}$$
The steady-state error in the output of the system for a unit-step input is _____ (up to 2 decimal places).

Official answer: **0.65 to 0.69**

SOLUTION

1. Check stability first, because the final value theorem only applies to a stable closed loop. The characteristic equation is $1 + G(s) = 0$, which gives $(s+1)(s+2) + 1 = s^2 + 3s + 3 = 0$. For a second-order polynomial, all coefficients positive is necessary and sufficient for both roots to lie in the left half-plane. So the closed loop is stable.
2. Find the system type. $G(s)$ has no pole at $s = 0$, so the system is type 0. A type-0 system tracks a step input with a finite, non-zero steady-state error.
3. Position error constant:
$$K_p = \lim_{s \to 0} G(s) = \frac{1}{(0+1)(0+2)} = \frac{1}{2} = 0.5$$

4. For unity feedback the error is $E(s) = \dfrac{R(s)}{1+G(s)}$, with $R(s) = \dfrac{1}{s}$ for a unit step. Apply the final value theorem:

$$e_{ss} = \lim_{s\to 0} s \cdot \frac{1}{s} \cdot \frac{1}{1+G(s)} = \frac{1}{1+K_p} = \frac{1}{1+0.5} = \frac{2}{3}$$

5. $e_{ss} = 0.6667 \approx 0.67$. Cross-check: the closed-loop DC gain is $\dfrac{G(0)}{1+G(0)} = \dfrac{0.5}{1.5} = \dfrac{1}{3}$, so the output settles at $\dfrac{1}{3}$ and the error is $1 - \dfrac{1}{3} = \dfrac{2}{3}$.

KEY IDEA

In a stable unity-feedback type-0 system, the unit-step steady-state error is $e_{ss} = \dfrac{1}{1+K_p}$, where $K_p = G(0)$ is the DC gain of the open-loop transfer function. You can check it quickly: the closed-loop DC gain is $\dfrac{G(0)}{1+G(0)}$, and the error is 1 minus that.

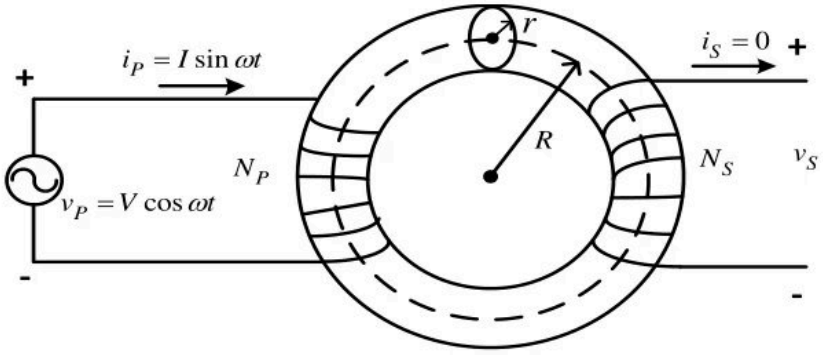
WATCH OUT FOR

Writing $e_{ss} = \dfrac{1}{K_p} = 2$ (the formula for ramp error with K_v, or a slip in the step formula). Another error is to give the steady-state output, $\dfrac{1}{3} \approx 0.33$, as the error. A third is to skip the stability check. The final value theorem only holds when the closed loop is stable, and here that means confirming $s^2 + 3s + 3$ has both roots in the left half-plane.

Q.26 MCQ · 2 MARKS

Electromagnetics › Magnetostatics

Q.26 A transformer with toroidal core of permeability μ is shown in the figure. Assuming uniform flux density across the circular core cross-section of radius $r \ll R$, and neglecting any leakage flux, the best estimate for the mean radius R is



(A) $\dfrac{\mu V r^2 N_P^2 \omega}{I}$ (B) $\dfrac{\mu I r^2 N_P N_S \omega}{V}$ (C) $\dfrac{\mu V r^2 N_P^2 \omega}{2I}$ (D) $\dfrac{\mu I r^2 N_P^2 \omega}{2V}$

Official answer: **D**

SOLUTION

1. Since $i_S = 0$, the secondary carries no current, so the core is magnetised by the primary ampere-turns $N_P i_P$ alone.
2. Apply Ampere's circuital law along the mean circular path of radius R: $H \cdot 2\pi R = N_P i_P \Rightarrow B = \mu H = \dfrac{\mu N_P i_P}{2\pi R}$. Because $r \ll R$, this B is taken as uniform over the cross-section.
3. The core flux through the circular cross-section of area πr^2 is

$$\Phi = B\,\pi r^2 = \frac{\mu N_P r^2}{2R} I \sin \omega t$$

4. Apply Faraday's law to the primary, neglecting leakage and winding resistance:

$$v_P = N_P \frac{d\Phi}{dt} = \frac{\mu N_P^2 r^2 \omega I}{2R} \cos \omega t$$

This has the same $\cos\omega t$ form as $v_P = V\cos\omega t$. That fits a purely inductive magnetising current, which lags the voltage by 90°. Equivalently, the primary self-inductance is $L = \dfrac{\mu N_P^2 r^2}{2R}$ and $v_P = L\dfrac{di_P}{dt}$.

5. Equate the amplitudes: $V = \dfrac{\mu N_P^2 r^2 \omega I}{2R}$.
6. Solve for the mean radius:

$$R = \frac{\mu I r^2 N_P^2 \omega}{2V}$$

This is option (D).

KEY IDEA

With the secondary open, the primary behaves as a toroidal inductor with $L = \dfrac{\mu N_P^2(\pi r^2)}{2\pi R} = \dfrac{\mu N_P^2 r^2}{2R}$. Its voltage and current amplitudes satisfy $V = \omega L I$. Solving this for R gives the answer.

WATCH OUT FOR

There are three easy slips. (1) Using the mutual inductance, which gives $N_P N_S$ as in option (B), when it is the PRIMARY voltage and current that are given. The self-inductance goes as N_P^2. (2) Inverting the relation to $R \propto V/I$ as in options (A) and (C). A larger radius means a longer magnetic path and so a SMALLER inductance, so R must grow with I/V. (3) Dropping the factor 2 that comes from $\pi r^2/(2\pi R)$.

Q.27 MCQ · 2 MARKS

Electrical and Electronic Measurements › V, I, power & energy

A 0-1 Ampere moving iron ammeter has an internal resistance of 50 mΩ and inductance of 0.1 mH. A shunt coil is connected to extend its range to 0-10 Ampere for all operating frequencies. The time constant in milliseconds and resistance in mΩ of the shunt coil respectively are

(A) 2, 5.55

(B) 2, 1

(C) 2.18, 0.55

(D) 11.1, 2

Official answer: **A**

SOLUTION

1. Time constant of the meter coil: $\tau_m = \dfrac{L_m}{R_m} = \dfrac{0.1\times10^{-3}\ \text{H}}{50\times10^{-3}\ \Omega} = 2\times10^{-3}\ \text{s} = 2\ \text{ms}$.
2. The meter and the shunt are in parallel, so the current divides inversely as the branch impedances:

$$\frac{I_m}{I_{sh}} = \frac{R_{sh}+j\omega L_{sh}}{R_m+j\omega L_m} = \frac{R_{sh}}{R_m}\cdot\frac{1+j\omega\tau_{sh}}{1+j\omega\tau_m}$$

   For this ratio to be the same at every frequency, the factor $\dfrac{1+j\omega\tau_{sh}}{1+j\omega\tau_m}$ must equal 1 for all ω, so the two time constants must be equal: $\dfrac{L_{sh}}{R_{sh}} = \dfrac{L_m}{R_m}$.
3. So the shunt coil's time constant is $\tau_{sh} = \tau_m = 2\ \text{ms}$.
4. With equal time constants, $\dfrac{I_m}{I_{sh}} = \dfrac{R_{sh}}{R_m}$ at every frequency, so the ordinary DC shunt formula applies. The multiplying power is $m = \dfrac{I}{I_m} = \dfrac{10}{1} = 10$, and $I = I_m + I_{sh} = I_m\left(1+\dfrac{R_m}{R_{sh}}\right)$ gives $R_{sh} = \dfrac{R_m}{m-1}$.
5. $R_{sh} = \dfrac{50\ \text{m}\Omega}{10-1} = \dfrac{50}{9}\ \text{m}\Omega \approx 5.56\ \text{m}\Omega$ (option (A) prints this as 5.55, cutting off rather than rounding the last digit). The shunt inductance follows as $L_{sh} = \tau_{sh}R_{sh} = 2\times10^{-3}\times\dfrac{50}{9}\times10^{-3} \approx 0.0111\ \text{mH}$.
6. The pair (2 ms, 5.55 mΩ) is option (A).

KEY IDEA

For a shunt to divide the current the same way at every frequency, the shunt branch must have the same L/R time constant as the meter coil. The impedance ratio is then a pure resistance ratio, so the DC formula $R_{sh} = R_m/(m-1)$ still holds.

WATCH OUT FOR

One common slip is to use $R_{sh} = R_m/m = 5\ \text{m}\Omega$ instead of $R_m/(m-1)$. The meter carries 1 A of the 10 A, so the shunt carries only 9 A. Another is to think frequency independence means the shunt must be purely resistive. A non-inductive shunt makes the reading depend on frequency, because the meter's reactance ωL_m grows with ω. The shunt needs the same time constant as the meter, not zero inductance.

Q.28 MCQ · 2 MARKS

Power Systems › Faults

The positive, negative and zero sequence impedances of a three phase generator are Z_1, Z_2 and Z_0 respectively. For a line-to-line fault with fault impedance Z_f, the fault current is I_{f1} = k I_f, where I_f is the fault current with zero fault impedance. The relation between Z_f and k is

(A) Z_f = (Z_1+Z_2)(1-k)/k

(B) Z_f = (Z_1+Z_2)(1+k)/k

(C) Z_f = (Z_1+Z_2)k/(1-k)

(D) Z_f = (Z_1+Z_2)k/(1+k)

Official answer: **A**

SOLUTION

1. For a line-to-line fault between phases b and c through a fault impedance Z_f, the zero-sequence current is zero, so Z_0 plays no part. The positive- and negative-sequence networks are connected in series with Z_f:
$$I_{a1} = -I_{a2} = \frac{E}{Z_1 + Z_2 + Z_f}$$
2. The faulted-phase current is $I_b = -I_c = -j\sqrt{3}\, I_{a1}$. This is a fixed multiple of I_{a1} whatever the value of Z_f. So the ratio of the fault current with Z_f to the fault current with $Z_f = 0$ equals the ratio of the corresponding positive-sequence currents, and $I_{f1} = kI_f$ can be read as a ratio of I_{a1} values.
3. With $Z_f = 0$:
$$I_{a1} = \frac{E}{Z_1 + Z_2}$$
4. Take the ratio:
$$k = \frac{E/(Z_1 + Z_2 + Z_f)}{E/(Z_1 + Z_2)} = \frac{Z_1 + Z_2}{Z_1 + Z_2 + Z_f}$$
5. Cross-multiply: $k(Z_1 + Z_2) + kZ_f = Z_1 + Z_2$, so $kZ_f = (Z_1 + Z_2)(1 - k)$.
6. Hence
$$Z_f = \frac{(Z_1 + Z_2)(1 - k)}{k}$$
which is option (A). Check: for $k = 1$ this gives $Z_f = 0$, and as $k \to 0$, $Z_f \to \infty$, both as expected.

KEY IDEA

In a line-to-line fault the sequence networks Z_1, Z_2 and Z_f are in series and Z_0 is not involved. So $k = \dfrac{Z_1 + Z_2}{Z_1 + Z_2 + Z_f}$, and solving for Z_f gives the answer.

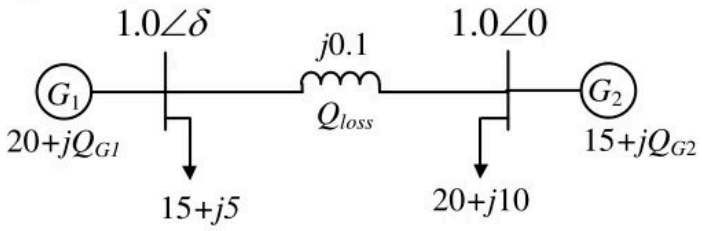
WATCH OUT FOR

Putting Z_0 into the fault loop, or using the line-to-ground connection (all three sequence networks in series, with $3Z_f$). A line-to-line fault has no zero-sequence path. Another slip is a rearrangement error that gives $(1 + k)$ or $k/(1 - k)$ in place of $(1 - k)/k$.

Q.29 MCQ · 2 MARKS

Power Systems › Per unit & load flow

Q.29 Consider the two bus power system network with given loads as shown in the figure. All the values shown in the figure are in per unit. The reactive power supplied by generator G_1 and G_2 are Q_{G1} and Q_{G2} respectively. The per unit values of Q_{G1}, Q_{G2}, and line reactive power loss (Q_{loss}) respectively are



(A) 5. 00, 12.68, 2.68
(B) 6.34, 10.00, 1.34
(C) 6.34, 11.34, 2.68
(D) 5.00, 11.34, 1.34

Official answer: **C**

SOLUTION

1. Take per-unit values throughout. The line is a pure reactance $X = 0.1$ pu with $V_1 = 1\angle\delta$ and $V_2 = 1\angle 0^\circ$. Bus 1 has generation $20 + jQ_{G1}$ and load $15 + j5$. Bus 2 has generation $15 + jQ_{G2}$ and load $20 + j10$.
2. Real power sent into the line from bus 1 is $P_{12} = 20 - 15 = 5$ pu. At bus 2 the net real injection is $15 - 20 = -5$ pu, so the line delivers 5 pu there. The two ends agree because the line has no resistance and so no real-power loss.
3. Real power over a reactance: $P_{12} = \dfrac{V_1 V_2}{X}\sin\delta = \dfrac{1\cdot 1}{0.1}\sin\delta = 5$. Hence $\sin\delta = 0.5$, $\delta = 30^\circ$ and $\cos\delta = 0.866$.
4. Reactive power leaving bus 1 into the line: $Q_{12} = \dfrac{V_1^2 - V_1 V_2\cos\delta}{X} = \dfrac{1 - 0.866}{0.1} = 1.34$ pu.
5. Reactive power leaving bus 2 into the line: $Q_{21} = \dfrac{V_2^2 - V_1 V_2\cos\delta}{X} = \dfrac{1 - 0.866}{0.1} = 1.34$ pu. Both ends send reactive power into the line because the two voltage magnitudes are equal.
6. Line reactive loss: $Q_{loss} = Q_{12} + Q_{21} = 1.34 + 1.34 = 2.68$ pu. Check with $Q_{loss} = XI^2$: $|I| = \dfrac{|V_1 - V_2|}{X} = \dfrac{2\sin 15^\circ}{0.1} = 5.176$ pu, so $Q_{loss} = 0.1 \times 26.79 = 2.68$ pu.
7. Reactive balance at bus 1: $Q_{G1} = Q_{load,1} + Q_{12} = 5 + 1.34 = 6.34$ pu.
8. Reactive balance at bus 2: $Q_{G2} = Q_{load,2} + Q_{21} = 10 + 1.34 = 11.34$ pu.
9. Therefore $(Q_{G1}, Q_{G2}, Q_{loss}) = (6.34,\ 11.34,\ 2.68)$ pu, which is option (C).

KEY IDEA

A lossless reactive line carries real power $P = \dfrac{V_1 V_2}{X}\sin\delta$, which fixes δ. Each end then sends $\dfrac{V^2 - V_1 V_2\cos\delta}{X}$ of reactive power into the line, and the generators supply their bus loads plus that amount.

WATCH OUT FOR

Giving the whole line loss to one bus, or treating the line as reactive-lossless so that $Q_{loss} = 0$. Another slip is taking $Q_{loss} = 1.34$ (one end's figure) instead of the sum of both ends, which gives option (B) or (D).

Q.30 MCQ · 2 MARKS

Engineering Mathematics › Numerical methods

The per-unit power output of a salient-pole generator which is connected to an infinite bus, is given by the expression, $P = 1.4\sin\delta + 0.15\sin 2\delta$, where δ is the load angle. Newton-Raphson method is used to

calculate the value of δ for $P = 0.8$ pu. If the initial guess is 30°, then its value (in degree) at the end of the first iteration is

(A) 15°

(B) 28.48°

(C) 28.74°

(D) 31.20°

Official answer: **C**

SOLUTION

1. Write the equation as a root-finding problem: $f(\delta) = 1.4\sin\delta + 0.15\sin 2\delta - 0.8 = 0$.
2. Differentiate, keeping the chain-rule factor of 2 on the second term: $f'(\delta) = 1.4\cos\delta + 0.3\cos 2\delta$.
3. Evaluate at $\delta_0 = 30^\circ$: $f(\delta_0) = 1.4(0.5) + 0.15(0.8660) - 0.8 = 0.7 + 0.1299 - 0.8 = 0.0299$.
4. $f'(\delta_0) = 1.4(0.8660) + 0.3(0.5) = 1.2124 + 0.15 = 1.3624$.
5. Newton-Raphson update: $\delta_1 = \delta_0 - \dfrac{f(\delta_0)}{f'(\delta_0)}$. The correction is $\Delta\delta = -\dfrac{0.0299}{1.3624} = -0.02195$ rad. It is in radians because f' is the derivative with respect to δ measured in radians.
6. Convert to degrees: $\Delta\delta = -0.02195 \times \dfrac{180}{\pi} = -1.258^\circ$.
7. $\delta_1 = 30^\circ - 1.258^\circ = 28.74^\circ$, which is option (C).

KEY IDEA

Newton-Raphson on $f(\delta) = P(\delta) - 0.8$ gives $\delta_1 = \delta_0 - f(\delta_0)/f'(\delta_0)$. Differentiating the sine terms gives f' per radian, so the correction comes out in radians and has to be converted to degrees before you subtract it from 30°.

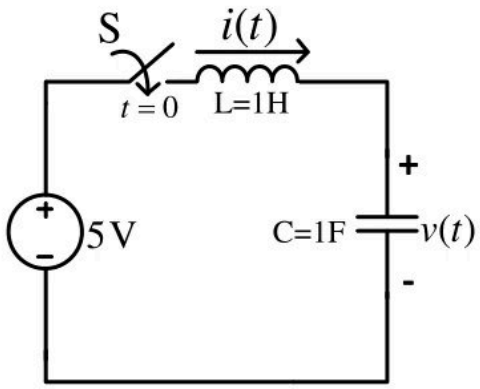
WATCH OUT FOR

There are two traps. (1) Subtracting the radian correction 0.02195 straight from 30 gives 29.98°. (2) Dropping the chain-rule factor, i.e. writing $\frac{d}{d\delta}(0.15\sin 2\delta) = 0.15\cos 2\delta$ instead of $0.3\cos 2\delta$, gives about 28.67°. Neither result is among the options.

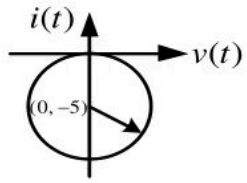
Q.31 MCQ · 2 MARKS

Networks (Electric Circuits) › Transients

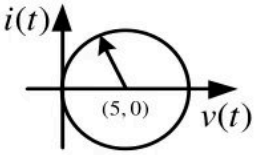
Q.31 A DC voltage source is connected to a series L-C circuit by turning on the switch S at time $t=0$ as shown in the figure. Assume $i(0)=0$, $v(0)=0$. Which one of the following circular loci represents the plot of $i(t)$ versus $v(t)$?



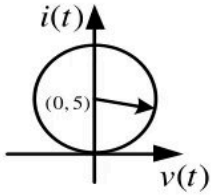
(A)



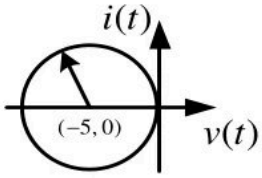
(B)



(C)



(D)



EE 8/15

Official answer: **B**

SOLUTION

1. **Set up the loop equations for $t > 0$.** With S closed, the current $i(t)$ flows clockwise: out of the source's + terminal, through L, and into the + terminal of C. That gives the passive sign convention at the capacitor, so $i = C\dfrac{dv}{dt}$. KVL around the loop gives $L\dfrac{di}{dt} + v = 5$. With $L = 1\ \text{H}$ and $C = 1\ \text{F}$, substitute $i = \dfrac{dv}{dt}$:

$$\frac{d^2v}{dt^2} + v = 5, \qquad \omega_0 = \frac{1}{\sqrt{LC}} = 1\ \text{rad/s}$$

2. **General solution** (particular plus natural): $v(t) = 5 + A\cos t + B\sin t$.
3. **Apply the initial conditions.** $v(0) = 0$ gives $5 + A = 0$, so $A = -5$. $i(0) = C\,v'(0) = B = 0$. Hence

$$v(t) = 5(1 - \cos t)\ \text{V}$$

4. **Current:** $i(t) = C\dfrac{dv}{dt} = 5\sin t\ \text{A}$. Check: at $t = 0^+$, $\dfrac{di}{dt} = \dfrac{5 - v(0)}{L} = 5\ \text{A/s}$, which agrees with $\dfrac{d}{dt}(5\sin t)\big|_0 = 5$.
5. **Eliminate t.** From the two results, $\cos t = \dfrac{5 - v}{5}$ and $\sin t = \dfrac{i}{5}$. Then $\sin^2 t + \cos^2 t = 1$ gives

$$\left(\frac{v-5}{5}\right)^2 + \left(\frac{i}{5}\right)^2 = 1 \ \Rightarrow\ (v-5)^2 + i^2 = 25$$

6. **Cross-check with energy (the circuit is lossless).** Multiply $L\dfrac{di}{dt} = 5 - v$ by $i = C\dfrac{dv}{dt}$ to get $L\,i\,\dfrac{di}{dt} = C(5 - v)\dfrac{dv}{dt}$. Integrate from $t = 0$, where $i = 0$ and $v = 0$:

$$\frac{1}{2}Li^2 = C\left(5v - \frac{1}{2}v^2\right) \;\Rightarrow\; i^2 = 10v - v^2 \;\Rightarrow\; (v-5)^2 + i^2 = 25$$

This is the same circle.

7. **Read off the locus.** It is a circle of radius 5 centred at $(v, i) = (5, 0)$. It passes through the origin at $t = 0$ and touches the $i(t)$ axis there. Over each cycle, v swings from 0 to 10 V and i from -5 to $+5$ A. This is option (B).

KEY IDEA

A DC step applied to a lossless series LC circuit makes the capacitor voltage oscillate about the source voltage. Here $v(t) = 5 - 5\cos t$, so the voltage has amplitude 5 V around a centre of 5 V. The current $i(t) = 5\sin t$ is 90 degrees out of phase with that oscillation, with amplitude $V_s/\sqrt{L/C}$. Plotting two quadrature sinusoids against each other gives an ellipse centred at $(V_s, 0)$. It is a circle here only because $\sqrt{L/C} = 1\ \Omega$, which makes both amplitudes equal to 5.

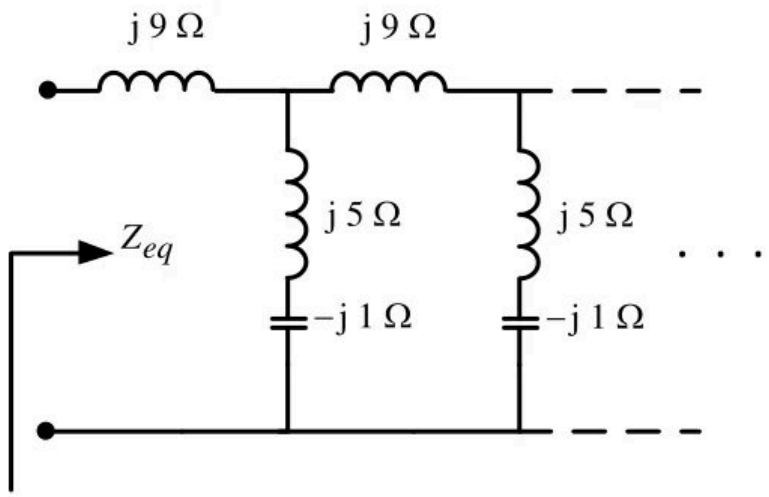
WATCH OUT FOR

One mistake is assuming the capacitor charges only up to 5 V, as it would in an RC circuit. With no resistance, it overshoots to 10 V, so the circle is centred at $v = 5$, not at the origin. Another is mixing up the axes (the vertical axis is i) and choosing the circle centred at $(0, 5)$. A third is a polarity slip that gives the centre $(-5, 0)$. $v(t) = 5(1 - \cos t)$ is never negative, so every point of the locus has $v \geq 0$.

Q.32 MCQ · 2 MARKS

Networks (Electric Circuits) › AC, power & resonance

Q.32 The equivalent impedance Z_{eq} for the infinite ladder circuit shown in the figure is



(A) j12 Ω (B) –j12 Ω (C) j13 Ω (D) 13 Ω

Official answer: **A**

SOLUTION

1. **Reduce one shunt branch.** The inductor and the capacitor are in series, so $Z_2 = j5 + (-j1) = j4\ \Omega$. The series element in each section is $Z_1 = j9\ \Omega$.
2. **Use the self-similarity of an infinite ladder.** Removing the first section leaves the same infinite ladder, so the impedance seen to the right of the first series arm is Z_{eq} again. It sits in parallel with the first shunt branch. Hence

$$Z_{eq} = Z_1 + Z_2 \parallel Z_{eq} = j9 + \frac{j4\, Z_{eq}}{j4 + Z_{eq}}$$

3. **Form the quadratic.** Multiply both sides by $(j4 + Z_{eq})$:

$$Z_{eq}(j4 + Z_{eq}) = j9(j4 + Z_{eq}) + j4\, Z_{eq}$$

Using $j9 \cdot j4 = -36$:

$$Z_{eq}^2 + j4\,Z_{eq} = -36 + j9\,Z_{eq} + j4\,Z_{eq}$$
$$Z_{eq}^2 - j9\,Z_{eq} + 36 = 0$$

4. **Solve it.**
$$Z_{eq} = \frac{j9 \pm \sqrt{(j9)^2 - 4(36)}}{2} = \frac{j9 \pm \sqrt{-225}}{2} = \frac{j9 \pm j15}{2}$$
This gives $Z_{eq} = j12\ \Omega$ or $Z_{eq} = -j3\ \Omega$.
5. **Pick the physical root.** At this frequency every element of a section is inductive: $j9$ in series and a net $j4$ in shunt. Two positive reactances combine to a positive reactance whether they are in series ($jX_1 + jX_2$) or in parallel ($j\frac{X_1X_2}{X_1+X_2}$). So a ladder cut off after any number of sections is inductive, and so is its limit. That rules out $-j3\ \Omega$.
6. **Check by iteration.** Put $Z_{eq} = jX$, so $X = 9 + \frac{4X}{4+X}$. Start from the one-section value $X = 9 + 4 = 13$ and repeat: $13 \to 12.06 \to 12.004 \to \cdots \to 12$. This confirms $Z_{eq} = j12\ \Omega$. The iteration settles on 12 and not on -3 because $f(X) = 9 + \frac{4X}{4+X}$ has slope $f'(X) = \frac{16}{(4+X)^2}$. That slope is $\frac{1}{16}$ at $X = 12$, so each step shrinks the error and the iteration converges there. At $X = -3$ the slope is 16, so the error grows and -3 is never reached. **Answer: (A)** $j12\ \Omega$.

KEY IDEA
An infinite ladder looks the same after you remove its first section. So $Z_{eq} = Z_1 + (Z_2 \parallel Z_{eq})$, which is a quadratic in Z_{eq}. Of its two roots, keep the one a physical ladder can actually have. Here every element is inductive, so Z_{eq} must be a positive reactance.

WATCH OUT FOR
One mistake is to stop after one section and answer $j9 + j4 = j13\ \Omega$ (option C); that is only the first step of the iteration. Another is to take the wrong quadratic root ($-j3\ \Omega$), or to slip a sign in $j \cdot j = -1$ and land on $-j12\ \Omega$ (option B). A third is to forget that the shunt branch is a series L–C pair and must be combined first, giving $j5 - j1 = j4\ \Omega$.

Q.33 MCQ · 2 MARKS

Control Systems › State space

Consider a system governed by the following equations

$$\frac{dx_1(t)}{dt} = x_2(t) - x_1(t)$$

$$\frac{dx_2(t)}{dt} = x_1(t) - x_2(t)$$

The initial conditions are such that $x_1(0) < x_2(0) < \infty$. Let $x_{1f} = \lim_{t\to\infty} x_1(t)$ and $x_{2f} = \lim_{t\to\infty} x_2(t)$. Which one of the following is true?

(A) $x_{1f} < x_{2f} < \infty$
(B) $x_{2f} < x_{1f} < \infty$
(C) $x_{1f} = x_{2f} < \infty$
(D) $x_{1f} = x_{2f} = \infty$

Official answer: **C**

SOLUTION

1. Write the equations in state-space form: $\dot{x} = Ax$ with $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $A = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$.
2. Eigenvalues: $\det(\lambda I - A) = (\lambda+1)^2 - 1 = \lambda(\lambda+2) = 0$, so $\lambda = 0, -2$. Neither eigenvalue has a positive real part, and $\lambda = 0$ is a simple (non-repeated) root, so it contributes only a constant term. No state can grow without bound, which rules out (D).

3. Conserved quantity: adding the two equations gives $\frac{d}{dt}(x_1 + x_2) = (x_2 - x_1) + (x_1 - x_2) = 0$, so $x_1(t) + x_2(t) = x_1(0) + x_2(0)$ for all t. This is the $\lambda = 0$ mode.
4. Difference mode: let $e = x_2 - x_1$. Then $\dot{e} = \dot{x}_2 - \dot{x}_1 = (x_1 - x_2) - (x_2 - x_1) = -2e$, so $e(t) = e(0)\, e^{-2t}$. This is the $\lambda = -2$ mode.
5. Since $e(0) = x_2(0) - x_1(0) > 0$, the gap stays positive for every finite t but decays to zero, so $x_{2f} - x_{1f} = 0$. That rules out (A) and (B).
6. Solving the sum and the difference together gives the final values:
$$x_{1f} = x_{2f} = \frac{x_1(0) + x_2(0)}{2}$$
This is finite because both initial values are finite.
7. So $x_{1f} = x_{2f} < \infty$, which is option (C).

KEY IDEA

Break the system into its modes. The sum $x_1 + x_2$ is conserved (the $\lambda = 0$ mode) and the difference $x_2 - x_1$ decays as e^{-2t} (the $\lambda = -2$ mode). The states therefore meet at the average of their initial values. The system is marginally stable, not asymptotically stable: it stays bounded but does not return to zero.

WATCH OUT FOR

Reading $\lambda = 0$ as instability and choosing (D). A simple eigenvalue at the origin gives a constant mode, not a growing one; only a repeated eigenvalue at zero (with a Jordan block) would give a term that grows like t. The other mistake is assuming the ordering $x_1 < x_2$ survives to the limit and choosing (A). The gap stays positive for every finite t but goes to zero as $t \to \infty$.

Q.34 MCQ · 2 MARKS

Control Systems › Routh–Hurwitz

Q.34 The number of roots of the polynomial, $s^7 + s^6 + 7s^5 + 14s^4 + 31s^3 + 73s^2 + 25s + 200$, in the open left half of the complex plane is

(A) 3

(B) 4

(C) 5

(D) 6

Official answer: **A**

SOLUTION

1. **Routh array, first two rows** (from the coefficients): $s^7 :\ 1,\ 7,\ 31,\ 25$ and $s^6 :\ 1,\ 14,\ 73,\ 200$.
2. **Row s^5:** $\frac{1 \cdot 7 - 1 \cdot 14}{1} = -7,\ \frac{1 \cdot 31 - 1 \cdot 73}{1} = -42,\ \frac{1 \cdot 25 - 1 \cdot 200}{1} = -175$. Dividing a whole row by a positive number does not change any sign, so divide by 7: $-1,\ -6,\ -25$.
3. **Row s^4** (from the s^6 and s^5 rows): $\frac{(-1)(14) - (1)(-6)}{-1} = 8,\ \frac{(-1)(73) - (1)(-25)}{-1} = 48,\ \frac{(-1)(200) - (1)(0)}{-1} = 200$. Divide by 8: $1,\ 6,\ 25$.
4. **Row s^3:** $\frac{(1)(-6) - (-1)(6)}{1} = 0$ and $\frac{(1)(-25) - (-1)(25)}{1} = 0$. The whole row is zero, so form the **auxiliary polynomial** from the s^4 row: $A(s) = s^4 + 6s^2 + 25$.
5. **Replace the zero row with dA/ds:** $\frac{dA}{ds} = 4s^3 + 12s$, which gives $s^3 :\ 1,\ 3$ after dividing by 4. Then $s^2 :\ \frac{1 \cdot 6 - 1 \cdot 3}{1} = 3,\ 25$; $s^1 :\ \frac{3 \cdot 3 - 1 \cdot 25}{3} = -\frac{16}{3}$; $s^0 :\ 25$.
6. **First column** (rows s^7 to s^0, after the positive scalings): $1,\ 1,\ -1,\ 1,\ 1,\ 3,\ -\frac{16}{3},\ 25$. It changes sign four times: $1 \to -1$, $-1 \to 1$, $3 \to -\frac{16}{3}$ and $-\frac{16}{3} \to 25$. So there are **4 roots in the right half-plane**.

7. **Check for roots on the $j\omega$ axis:** a zero row means the auxiliary polynomial's roots are also roots of $P(s)$. From $A(s)=0$: $s^2=\dfrac{-6\pm\sqrt{36-100}}{2}=-3\pm j4$. Since $(1+j2)^2=-3+j4$ and $(1-j2)^2=-3-j4$, the roots are $s=\pm(1+j2),\ \pm(1-j2)$, i.e. $1\pm j2$ and $-1\pm j2$. None lies on the imaginary axis. Two are in the RHP and two in the LHP, which matches the two sign changes from the s^4 row downward $(1,\ 1,\ 3,\ -\frac{16}{3},\ 25)$.
8. **Cross-check by division:** $P(s)=(s^4+6s^2+25)(s^3+s^2+s+8)$; expanding gives $s^7+s^6+7s^5+14s^4+31s^3+73s^2+25s+200$. For the cubic factor, the Routh column is $1,\ 1,\ \dfrac{1\cdot1-1\cdot8}{1}=-7,\ 8$: two sign changes, so the cubic has 2 RHP roots and 1 LHP root.
9. **Count:** RHP roots $=2+2=4$, $j\omega$-axis roots $=0$, so LHP roots $=7-4-0=3$ (1 from the cubic and 2 from $A(s)$, namely $-1\pm j2$). The answer is **(A) 3**.

KEY IDEA

Routh–Hurwitz with an all-zero row: the zero row at s^3 means an even auxiliary polynomial $A(s)=s^4+6s^2+25$ divides $P(s)$. Replace the zero row with dA/ds and continue. The sign changes in the first column count the RHP roots (4). Then check whether any roots of $A(s)$ lie on the $j\omega$ axis (none here; they are $\pm1\pm j2$). LHP count = total − RHP − $j\omega$ = 7 − 4 − 0 = 3.

WATCH OUT FOR

Seeing a zero row and assuming it always means roots on the $j\omega$ axis. That makes a student subtract four imaginary-axis roots, or count $A(s)$'s roots as marginal, and so pick a wrong option. An even polynomial's roots are symmetric about the origin: they can be real pairs $\pm a$, imaginary pairs $\pm j\omega$, or complex quadruples $\pm\sigma\pm j\omega$. Here $s^2=-3\pm j4$ gives the quadruple $\pm1\pm j2$, so none are on the axis. A second trap: people read "4 sign changes" as 4 LHP roots (option B). The sign changes count right-half-plane roots.

Q.35 MCQ · 2 MARKS

Engineering Mathematics › Complex analysis

If C is a circle $|z|=4$ and $f(z)=\dfrac{z^2}{(z^2-3z+2)^2}$, then $\oint_C f(z)\,dz$ is

(A) 1

(B) 0

(C) -1

(D) -2

Official answer: **B**

SOLUTION

1. Factor the denominator: $z^2-3z+2=(z-1)(z-2)$, so $f(z)=\dfrac{z^2}{(z-1)^2(z-2)^2}$. The numerator is not zero at $z=1$ or $z=2$, so f has poles of order 2 at $z=1$ and $z=2$. Both lie inside $|z|=4$.
2. For a pole of order 2 at $z=a$, the residue is $\operatorname{Res}_{z=a}f=\dfrac{d}{dz}\left[(z-a)^2f(z)\right]\Big|_{z=a}$.
3. At $z=1$: $(z-1)^2f(z)=\dfrac{z^2}{(z-2)^2}$, and $\dfrac{d}{dz}\left[\dfrac{z^2}{(z-2)^2}\right]=\dfrac{2z(z-2)-2z^2}{(z-2)^3}=\dfrac{-4z}{(z-2)^3}$. At $z=1$ this is $\dfrac{-4}{(-1)^3}=4$.
4. At $z=2$: $(z-2)^2f(z)=\dfrac{z^2}{(z-1)^2}$, and $\dfrac{d}{dz}\left[\dfrac{z^2}{(z-1)^2}\right]=\dfrac{2z(z-1)-2z^2}{(z-1)^3}=\dfrac{-2z}{(z-1)^3}$. At $z=2$ this is $\dfrac{-4}{1^3}=-4$.
5. By Cauchy's residue theorem,

$$\oint_C f(z)\,dz = 2\pi i\,\big(4 + (-4)\big) = 0.$$

6. Quick check: for large $|z|$, $f(z) = \dfrac{1}{z^2}\left(1 - \dfrac{3}{z} + \dfrac{2}{z^2}\right)^{-2} = \dfrac{1}{z^2} + \dfrac{6}{z^3} + \ldots$ There is no $\dfrac{1}{z}$ term, so the residue at infinity is 0. The residues at all finite poles plus the residue at infinity add up to zero, so the finite residues must add up to 0. This agrees with $4 + (-4) = 0$.
7. The answer is 0, which is option (B).

KEY IDEA

Both double poles, $z = 1$ and $z = 2$, lie inside $|z| = 4$. Their residues are $+4$ and -4, which cancel. There is a shortcut: when a contour encloses every singularity of a rational function whose denominator degree is at least 2 more than its numerator degree (here 4 against 2), the integral is 0, because the function has no $1/z$ term at infinity.

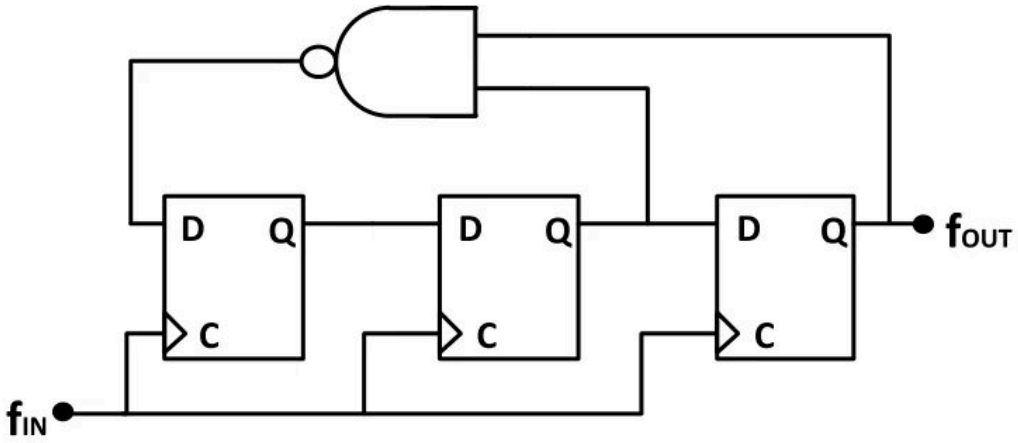
WATCH OUT FOR

Using the simple-pole formula $\lim(z-a)f(z)$ at what are actually double poles, since the whole denominator is squared. A second mistake is a sign error in the quotient rule, for example writing $(z-2)^3$ where it should be $(z-2)^2$, which gives residues that do not cancel and leads to ± 1 or -2.

Q.36 MCQ · 2 MARKS

Digital Electronics › Sequential

Q.36 Which one of the following statements is true about the digital circuit shown in the figure



(A) It can be used for dividing the input frequency by 3.
(B) It can be used for dividing the input frequency by 5.
(C) It can be used for dividing the input frequency by 7.
(D) It cannot be reliably used as a frequency divider due to disjoint internal cycles.

Official answer: **B**

SOLUTION

1. Read the circuit as a 3-bit shift register with NAND feedback. A prime ($'$) means complement. The inputs are $D_1 = (Q_2Q_3)'$, $D_2 = Q_1$ and $D_3 = Q_2$. A D flip-flop's next state equals its D input, so $Q_1^+ = (Q_2Q_3)'$, $Q_2^+ = Q_1$ and $Q_3^+ = Q_2$.
2. Start from $Q_1Q_2Q_3 = 100$. Here $Q_2Q_3 = 0$, so $D_1 = 1$. The next state is 110.
3. At 110, $Q_2Q_3 = 0$, so the next state is 111. At 111, $Q_2Q_3 = 1$, so $D_1 = 0$ and the next state is 011.
4. At 011, $Q_2Q_3 = 1$, so the next state is 001. At 001, $Q_2Q_3 = 0$, so the next state is 100, which is where we started. The main cycle is

$$100 \to 110 \to 111 \to 011 \to 001 \to 100$$

and it has 5 states.

5. Option (D) is about lock-out, so check the three unused states. $000 \rightarrow 100$, which joins the cycle. $010 \rightarrow 101$, because $Q_3 = 0$ makes $D_1 = 1$. $101 \rightarrow 110$, which joins the cycle. None of them forms a separate loop, so there are no disjoint cycles and the counter starts itself from any power-up state.
6. The output is $f_{OUT} = Q_3$. Around the cycle it reads $0, 0, 1, 1, 1$ and repeats every 5 clock pulses. So $f_{OUT} = f_{IN}/5$, with a duty cycle of $3/5 = 60\%$.
7. The circuit is a self-starting mod-5 counter, so it divides the input frequency by 5. The answer is (B).

KEY IDEA

A shift register with feedback is a counter. Its division ratio is the length of the state cycle, which you find by stepping through the next-state equations $Q_1^+ = (Q_2Q_3)'$, $Q_2^+ = Q_1$, $Q_3^+ = Q_2$. Here the cycle has 5 states. To rule out lock-out, check where every unused state goes. All three unused states here lead into the main cycle, so the counter starts itself and divides by 5.

WATCH OUT FOR

Seeing three flip-flops and assuming divide-by-3 (a ring counter) or divide-by-7/8. Another trap is picking (D) without tracing the unused states. 000, 010 and 101 all feed into the 5-state cycle, so there is no disjoint loop. A third slip is reading the NAND inputs as Q_1 and Q_3 instead of Q_2 and Q_3.

Q.37 MCQ · 2 MARKS

Digital Electronics › Boolean logic

Digital input signals A, B, C with A as the MSB and C as the LSB are used to realize the Boolean function $F = m_0 + m_2 + m_3 + m_5 + m_7$, where m_i denotes the i^{th} minterm. In addition, F has a don't care for m_1. The simplified expression for F is given by

(A) $\overline{A}\,\overline{C} + \overline{B}C + AC$

(B) $\overline{A} + C$

(C) $\overline{C} + A$

(D) $\overline{A}C + BC + A\overline{C}$

Official answer: **B**

SOLUTION

1. Here a bar ($\overline{X}$) means complement. With A as the MSB and C as the LSB, the minterms in ABC order are $m_0 = 000$, $m_2 = 010$, $m_3 = 011$, $m_5 = 101$ and $m_7 = 111$, where $F = 1$. The minterm $m_1 = 001$ is a don't care (X). That leaves $F = 0$ only for $m_4 = 100$ and $m_6 = 110$.
2. Draw the 3-variable K-map with rows $A = 0, 1$ and columns $BC = 00, 01, 11, 10$ in Gray order. Row $A = 0$: $m_0 = 1$, $m_1 = X$, $m_3 = 1$, $m_2 = 1$. Row $A = 1$: $m_4 = 0$, $m_5 = 1$, $m_7 = 1$, $m_6 = 0$.
3. Take the don't care m_1 as 1. The whole row $A = 0$ (m_0, m_1, m_3, m_2) is then a quad. Only A is constant across it (it is 0), so the quad gives $\overline{A}$.
4. The two middle columns $BC = 01$ and $BC = 11$ (m_1, m_3, m_5, m_7) form a second quad. $C = 1$ in every one of these cells while A and B both change, so this quad gives C.
5. The two quads together cover every 1 (m_0, m_2, m_3, m_5, m_7), and neither includes a 0 (m_4 or m_6). Each quad is the largest group possible, and one quad alone cannot cover all the 1s. So the minimal SOP is
$$F = \overline{A} + C$$
Check: $\overline{A} + C = 0$ only when $A = 1$ and $C = 0$, which is exactly m_4 and m_6.
6. Checking the other options: (A) $\overline{A}\,\overline{C} + \overline{B}C + AC$ covers m_0, m_2 (from $\overline{A}\,\overline{C}$), m_1, m_5 (from $\overline{B}C$) and m_5, m_7 (from AC), but it misses $m_3 = 011$. (C) $\overline{C} + A$ equals 1 at m_4 and m_6, where $F = 0$. (D) $\overline{A}C + BC + A\overline{C}$ contains $A\overline{C}$, which is 1 at m_4 and m_6, and it also misses m_0, m_2 and m_5. Only (B) matches.

KEY IDEA

Set the don't care m_1 to 1 where it makes a group bigger. Here it completes two quads: the row $A = 0$ gives $\overline{A}$, and the columns with $C = 1$ give C. Before accepting the answer, check that F is 0 exactly at m_4 and m_6.

WATCH OUT FOR

Leaving the don't care m_1 as 0. The 1s then break into smaller groups, such as $\overline{A}\,\overline{C} + \overline{A}B + C$ or something close to option (A), and it looks as if no option simplifies far enough. Another slip is to use B's weight for A and get the minterm numbers wrong, which puts the 0s in the wrong cells and makes (C) look right.

Q.38 MCQ · 2 MARKS

Signals and Systems › Signals basics

Consider the two continuous-time signals defined below:

$$x_1(t) = \begin{cases} |t|, & -1 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}, \qquad x_2(t) = \begin{cases} 1 - |t|, & -1 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$$

These signals are sampled with a sampling period of $T = 0.25$ seconds to obtain discrete-time signals $x_1[n]$ and $x_2[n]$, respectively. Which one of the following statements is true?

(A) The energy of $x_1[n]$ is greater than the energy of $x_2[n]$.

(B) The energy of $x_2[n]$ is greater than the energy of $x_1[n]$.

(C) $x_1[n]$ and $x_2[n]$ have equal energies.

(D) Neither $x_1[n]$ nor $x_2[n]$ is a finite-energy signal.

Official answer: **A**

SOLUTION

1. Sampling: $x[n] = x(nT) = x(0.25n)$. Both signals are nonzero only for $-1 \le t \le 1$, so only $-4 \le n \le 4$ can give nonzero samples. Both are finite-length sequences with finite values, so both have finite energy, which rules out (D).
2. $x_1[n] = |0.25n|$ for $|n| \le 4$. At $n = 0, \pm1, \pm2, \pm3, \pm4$ the values are $0,\ 0.25,\ 0.5,\ 0.75,\ 1$. The endpoints $t = \pm1$ lie inside the closed interval $-1 \le t \le 1$, so $x_1[\pm4] = 1$.
3. $x_2[n] = 1 - |0.25n|$ for $|n| \le 4$. At $n = 0, \pm1, \pm2, \pm3, \pm4$ the values are $1,\ 0.75,\ 0.5,\ 0.25,\ 0$.
4. Discrete-time energy is $E = \sum_n |x[n]|^2$. For x_1:
$$E_1 = 0 + 2\left(0.25^2 + 0.5^2 + 0.75^2 + 1^2\right) = 2(0.0625 + 0.25 + 0.5625 + 1) = 2(1.875) = 3.75$$
5. For x_2:
$$E_2 = 1^2 + 2\left(0.75^2 + 0.5^2 + 0.25^2 + 0^2\right) = 1 + 2(0.5625 + 0.25 + 0.0625) = 1 + 2(0.875) = 2.75$$
6. $E_1 = 3.75 > E_2 = 2.75$. The two sequences take the same set of values, but the largest value 1 occurs twice in $x_1[n]$ (at $n = \pm4$) and only once in $x_2[n]$ (at $n = 0$), while the value 0 occurs once in $x_1[n]$ and twice in $x_2[n]$. So the energy of $x_1[n]$ is greater, which is option (A).

KEY IDEA

Sample at $t = nT$, list the finite set of samples (including the endpoints $t = \pm1$, which are inside the closed interval), and add their squares: $E = \sum_n |x[n]|^2$. Here $x_1[n] + x_2[n] = 1$ for $|n| \le 4$. Both sequences take the values $0, 0.25, 0.5, 0.75, 1$, but $x_1[n]$ reaches the peak value 1 at two samples ($n = \pm4$) and $x_2[n]$ at only one ($n = 0$).

WATCH OUT FOR

Choosing (C) by reasoning from the continuous-time signals. Their energies are equal, $\int |x_1(t)|^2\,dt = \int |x_2(t)|^2\,dt = \frac{2}{3}$, but sampling does not keep that equality: the samples at the interval edges $t = \pm1$ are where x_1 is largest and x_2 is zero, so $x_1[n]$ gets two peak samples and $x_2[n]$ only one. Dropping the endpoint samples $n = \pm4$ (treating the interval as open) gives the same kind of miscount, with $E_1 = 1.75$ against $E_2 = 2.75$, which points to (B).

Q.39 MCQ · 2 MARKS

Signals and Systems › Signals basics

The signal energy of the continuous-time signal

$$x(t) = [(t-1)u(t-1)] - [(t-2)u(t-2)] - [(t-3)u(t-3)] + [(t-4)u(t-4)]$$

is

(A) $11/3$

(B) $7/3$

(C) $1/3$

(D) $5/3$

Official answer: **D**

SOLUTION

1. Write each term as a shifted unit ramp $r(t) = t\,u(t)$:
$$x(t) = r(t-1) - r(t-2) - r(t-3) + r(t-4)$$
2. Build the waveform piece by piece. Each ramp switches on at its breakpoint, so the slope changes by $+1, -1, -1, +1$ at $t = 1, 2, 3, 4$.
3. $t < 1$: $x(t) = 0$. For $1 \le t < 2$: $x(t) = t - 1$, rising from 0 to 1 (slope $+1$).
4. $2 \le t < 3$: $x(t) = (t-1) - (t-2) = 1$ (slope 0). For $3 \le t < 4$: $x(t) = 1 - (t-3) = 4 - t$, falling from 1 to 0 (slope -1).
5. $t \ge 4$: $x(t) = (4-t) + (t-4) = 0$ (slope 0). So $x(t)$ is a trapezoid: 0 at $t = 1$, up to 1 at $t = 2$, flat until $t = 3$, back to 0 at $t = 4$.
6. Energy definition: $E = \int_{-\infty}^{\infty} |x(t)|^2\,dt$. Split the integral over the three non-zero pieces:
$$E = \int_1^2 (t-1)^2\,dt + \int_2^3 1^2\,dt + \int_3^4 (4-t)^2\,dt$$
7. Substituting $\tau = t - 1$ in the first integral and $\tau = 4 - t$ in the third, each ramp edge gives $\int_0^1 \tau^2\,d\tau = \dfrac{1}{3}$, and the flat top gives 1:
$$E = \frac{1}{3} + 1 + \frac{1}{3} = \frac{5}{3}$$
8. This matches option (D).

KEY IDEA

A sum of shifted ramps $(t-a)\,u(t-a)$ is a piecewise-linear signal: at each breakpoint the slope changes by that ramp's coefficient. Sketch the waveform first, then integrate $x^2(t)$ one piece at a time. A linear edge that rises or falls by A over a width T contributes $A^2T/3$, and a flat segment of height A and width T contributes A^2T.

WATCH OUT FOR

Students often find the energy of each ramp term separately and add the results. Each ramp on its own has infinite energy, and energy does not add across overlapping terms because x^2 has cross terms. Another slip is to take the energy as the area under $x(t)$, which is 2, not $5/3$. A third is to mix up the signs at $t = 3$ and $t = 4$, which gives a waveform that never returns to zero.

Q.40 NAT · 2 MARKS

Signals and Systems › Fourier transform

The Fourier transform of a continuous-time signal $x(t)$ is given by

$$X(\omega) = \frac{1}{(10 + j\omega)^2}, \quad -\infty < \omega < \infty,$$

where $j = \sqrt{-1}$ and ω denotes frequency. Then the value of $|\ln x(t)|$ at $t = 1$ is _____ (up to 1 decimal place). ($\ln$ denotes the logarithm to base e)

Official answer: **9.5 to 10.5**

SOLUTION

1. Standard pair: $e^{-at}u(t) \longleftrightarrow \dfrac{1}{a + j\omega}$ for $a > 0$. Here $a = 10$.
2. Frequency-differentiation property: $t\,x(t) \longleftrightarrow j\dfrac{dX(\omega)}{d\omega}$. For $X_1(\omega) = \dfrac{1}{10 + j\omega}$, $\dfrac{dX_1}{d\omega} = \dfrac{-j}{(10 + j\omega)^2}$, so $j\dfrac{dX_1}{d\omega} = j \cdot \dfrac{-j}{(10 + j\omega)^2} = \dfrac{1}{(10 + j\omega)^2}$.
3. Hence
$$x(t) = t\,e^{-10t}\,u(t).$$
The same result follows from convolving $e^{-10t}u(t)$ with itself, since a product (here a square) in frequency is a convolution in time: $\int_0^t e^{-10\tau}e^{-10(t-\tau)}\,d\tau = t\,e^{-10t}$ for $t > 0$.
4. At $t = 1$: $x(1) = 1 \cdot e^{-10} = e^{-10}$, which is positive, so its logarithm is defined.
5. $\ln x(1) = \ln e^{-10} = -10$, so $|\ln x(1)| = 10.0$.

KEY IDEA

A squared first-order factor $\dfrac{1}{(a + j\omega)^2}$ in frequency is $t\,e^{-at}u(t)$ in time. You can get it from frequency differentiation ($t\,x(t) \leftrightarrow j\,dX/d\omega$) or from convolving $e^{-at}u(t)$ with itself. After that, the logarithm simply undoes the exponential.

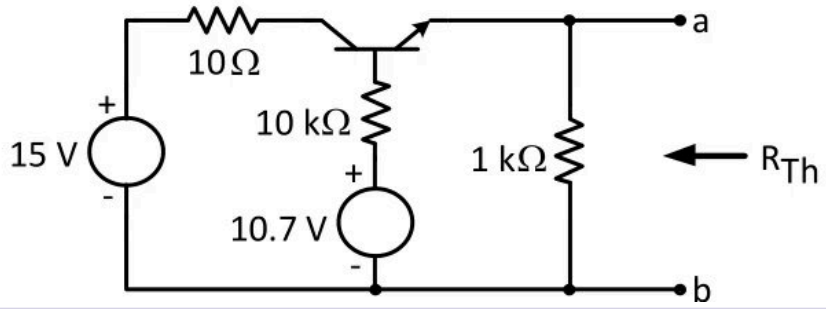
WATCH OUT FOR

Writing $x(t) = e^{-10t}u(t)$ or $e^{-20t}u(t)$ and forgetting the factor t that the squared denominator brings. Luckily it is 1 at $t = 1$, so the answer comes out as 10 anyway, but the e^{-20t} slip gives 20, which is wrong. Another trap is using $1/(a - j\omega)$, the anticausal pair, and getting a signal that is zero at $t = 1$, whose logarithm does not exist.

Q.41 NAT · 2 MARKS

Analog Electronics › Biasing

Q.41 In the circuit shown in the figure, the bipolar junction transistor (BJT) has a current gain $\beta = 100$. The base-emitter voltage drop is a constant, $V_{BE} = 0.7\,V$. The value of the Thevenin equivalent resistance R_{Th} (in Ω) as shown in the figure is _____ (up to 2 decimal places).



Official answer: **89.0 to 91.5**

SOLUTION

1. **Identify the terminals.** The emitter is node a and ground is node b. The base is fed from 10.7 V through $R_B = 10$ kΩ. The collector goes to 15 V through $R_C = 10\ \Omega$. The resistor across a–b is $R_E = 1$ kΩ.
2. **Check the active region (KVL around the base-emitter loop).**
$$10.7 = I_B(10\ \text{k}\Omega) + 0.7 + (\beta + 1)I_B(1\ \text{k}\Omega) \;\Rightarrow\; I_B = \frac{10\ \text{V}}{111\ \text{k}\Omega} = 90.09\ \mu\text{A}$$

So $I_C = 9.009\ \text{mA}$, $I_E = 9.099\ \text{mA}$ and $V_E = I_E R_E = 9.099\ \text{V}$. Then $V_C = 15 - 10(9.009\ \text{mA}) = 14.91\ \text{V}$, so $V_{CE} \approx 5.81\ \text{V} > 0$. The transistor is in the active region, where (with constant V_{BE} and constant β) the circuit is linear, so a Thevenin equivalent exists.

3. **Small-signal model with a constant V_{BE}.** A constant V_{BE} means the base-emitter junction has no incremental resistance ($r_\pi = 0$): the base and emitter move together. The collector acts as a controlled current source βi_b (r_o neglected), so the $10\ \Omega$ collector resistor and the $15\ \text{V}$ source have no effect on the emitter. Both DC sources are set to zero.
4. **Resistance looking into the emitter (resistance reflection).** Apply a test voltage v_x at the emitter. Since $v_{be} = 0$, the base is also at v_x, so a current v_x/R_B flows out of the base into R_B; the base current into the transistor is $i_b = -v_x/R_B$. The current flowing into the emitter from the test source is $-i_e = -(\beta+1)i_b = (\beta+1)v_x/R_B$. So
$$R_{\text{emitter}} = \frac{R_B + r_\pi}{\beta + 1} = \frac{10\,000}{101} = 99.01\ \Omega$$
5. **Combine with the 1 kΩ resistor across a–b.**
$$R_{Th} = 99.01\ \Omega \parallel 1000\ \Omega = \frac{(10^4/101)(10^3)}{10^4/101 + 10^3} = \frac{10^7}{111\,000} = 90.09\ \Omega$$
6. **Cross-check with V_{oc}/I_{sc}.** Open circuit: $V_{Th} = V_E = 9.099\ \text{V}$ (step 2). Short a–b: the emitter is grounded, so $I_B = (10.7 - 0.7)/10\ \text{k}\Omega = 1\ \text{mA}$, $I_C = 100\ \text{mA}$ and $I_{sc} = I_E = 101\ \text{mA}$ (the shorted $1\ \text{k}\Omega$ carries nothing). Then $V_C = 15 - 10(0.1) = 14\ \text{V}$, so the transistor is still active.
$$R_{Th} = \frac{9.099\ \text{V}}{0.101\ \text{A}} = 90.09\ \Omega$$

KEY IDEA

Because V_{BE} is constant, the base-emitter junction has zero incremental resistance ($r_\pi = 0$). Looking into the emitter you therefore see only the base resistance divided by $(\beta+1)$, which is $10\ \text{k}\Omega/101 = 99.01\ \Omega$. The collector branch is a current source and drops out. R_{Th} is this value in parallel with the $1\ \text{k}\Omega$ across a–b. Dividing the open-circuit voltage by the short-circuit current gives the same answer, because the circuit is linear while the BJT stays active.

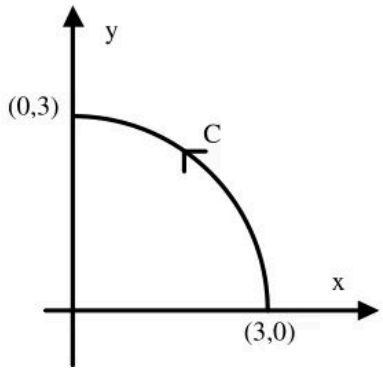
WATCH OUT FOR

Common mistakes: (1) Taking $R_{Th} = 1\ \text{k}\Omega$ and ignoring the transistor. (2) Multiplying R_B by $(\beta+1)$ instead of dividing: that is the rule for looking into the base, not the emitter. (3) Including the $10\ \Omega$ collector resistor, which sits behind an ideal current source (r_o neglected). (4) Adding a nonzero $r_\pi = V_T/I_B$, when the question says V_{BE} is constant.

Q.42 NAT · 2 MARKS

Engineering Mathematics › Vector calculus

Q.42 As shown in the figure, C is the arc from the point (3,0) to the point (0,3) on the circle $x^2 + y^2 = 9$. The value of the integral $\int_C (y^2 + 2yx)dx + (2xy + x^2)dy$ is _____ (up to 2 decimal places).



Official answer: **0.0**

SOLUTION

1. Write the integral as $\int_C P\,dx + Q\,dy$ with $P = y^2 + 2xy$ and $Q = 2xy + x^2$.
2. Check whether the field is conservative. The integrand is exact when $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$. Here $\frac{\partial P}{\partial y} = 2y + 2x$ and $\frac{\partial Q}{\partial x} = 2y + 2x$. They are equal at every point of the plane, and the plane is simply connected, so the integral does not depend on the path.
3. Find a potential ϕ with $\phi_x = P$ and $\phi_y = Q$. Integrating P with respect to x gives $\phi = xy^2 + x^2y + g(y)$. Then $\phi_y = 2xy + x^2 + g'(y)$, and setting this equal to Q gives $g'(y) = 0$. So
$$\phi(x, y) = xy^2 + x^2y = xy(x + y)$$
4. Apply the fundamental theorem for line integrals, $\int_C \nabla\phi \cdot d\mathbf{r} = \phi(\text{end}) - \phi(\text{start})$. The path starts at $(3, 0)$ and ends at $(0, 3)$:
$$\int_C P\,dx + Q\,dy = \phi(0, 3) - \phi(3, 0) = 0 - 0 = 0$$
5. Check: $\phi = xy(x + y)$ is zero wherever $x = 0$ or $y = 0$, so both endpoints give 0, and the direction of travel along the quarter circle makes no difference. The integral is 0.00.

KEY IDEA

If $\partial P/\partial y = \partial Q/\partial x$ on a simply connected region, then $P\,dx + Q\,dy$ is the exact differential of a potential ϕ. The line integral is then $\phi(\text{end}) - \phi(\text{start})$, whatever the path. Here $\phi = xy(x + y)$ is zero on both coordinate axes, and both endpoints lie on an axis.

WATCH OUT FOR

Parametrising the arc as $x = 3\cos t$, $y = 3\sin t$ and grinding through the trigonometric integral, which wastes time and invites sign or limit errors. Another mistake is applying Green's theorem to the open arc as if it were a closed curve. The exactness test takes seconds and shows the answer is simply $\phi(0, 3) - \phi(3, 0) = 0$.

Q.43 NAT · 2 MARKS

Engineering Mathematics › Calculus

Let $f(x) = 3x^3 - 7x^2 + 5x + 6$. The maximum value of $f(x)$ over the interval $[0, 2]$ is _____ (up to 1

decimal place).

Official answer: **11.5 to 12.5**

SOLUTION

1. A continuous function on a closed interval reaches its absolute maximum either at a critical point inside the interval or at an endpoint (closed-interval method). $f(x)$ is a polynomial, so it is continuous on $[0, 2]$, and we check both kinds of point.
2. Critical points: $f'(x) = 9x^2 - 14x + 5 = (9x - 5)(x - 1) = 0 \Rightarrow x = \dfrac{5}{9},\ x = 1$. Both lie in $[0, 2]$.
3. Second-derivative test: $f''(x) = 18x - 14$. $f''\left(\frac{5}{9}\right) = 10 - 14 = -4 < 0$, so $x = \frac{5}{9}$ is a local maximum. $f''(1) = 4 > 0$, so $x = 1$ is a local minimum.
4. Value at the local maximum:
$$f\left(\tfrac{5}{9}\right) = 3 \cdot \frac{125}{729} - 7 \cdot \frac{25}{81} + 5 \cdot \frac{5}{9} + 6 = \frac{125}{243} - \frac{525}{243} + \frac{675}{243} + \frac{1458}{243} = \frac{1733}{243} \approx 7.13$$
5. Value at the local minimum: $f(1) = 3 - 7 + 5 + 6 = 7$.
6. Endpoints: $f(0) = 6$ and $f(2) = 3(8) - 7(4) + 5(2) + 6 = 24 - 28 + 10 + 6 = 12$.
7. Compare all candidates: $\max\{6,\ 7.13,\ 7,\ 12\} = 12$, reached at the endpoint $x = 2$. Maximum value $= 12.0$.

KEY IDEA

For the absolute maximum on a closed interval, compare f at every critical point inside the interval AND at both endpoints. A local maximum found with $f'' < 0$ is only a candidate. Here the cubic has a positive leading coefficient, so it is rising again after its local minimum at $x = 1$ and ends higher at $x = 2$ than at its local peak.

WATCH OUT FOR

Setting $f'(x) = 0$, finding the local maximum at $x = 5/9$ with $f'' < 0$, and answering $f(5/9) \approx 7.1$ without checking the endpoints. The absolute maximum on $[0, 2]$ is at the endpoint $x = 2$, where $f(2) = 12$.

Q.44 NAT · 2 MARKS

Engineering Mathematics › Linear algebra

Let $A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ and $B = A^3 - A^2 - 4A + 5I$, where I is the 3×3 identity matrix. The determinant of B is _____ (up to 1 decimal place).

Official answer: **0.9 to 1.1**

SOLUTION

1. Find the characteristic polynomial $\det(A - \lambda I)$. Row 3 of $A - \lambda I$ is $\begin{bmatrix} 0 & 0 & -2 - \lambda \end{bmatrix}$, so it has only one non-zero entry. Expand along row 3:
$$\det(A - \lambda I) = (-2 - \lambda)\begin{vmatrix} 1 - \lambda & 0 \\ -1 & 2 - \lambda \end{vmatrix} = (-2 - \lambda)(1 - \lambda)(2 - \lambda) = -(\lambda - 1)(\lambda - 2)(\lambda + 2)$$
This gives the eigenvalues $\lambda = 1,\ 2,\ -2$.
2. Multiply out: $(\lambda - 1)(\lambda - 2)(\lambda + 2) = (\lambda^2 - 3\lambda + 2)(\lambda + 2) = \lambda^3 - \lambda^2 - 4\lambda + 4$. The characteristic equation is therefore $\lambda^3 - \lambda^2 - 4\lambda + 4 = 0$.
3. By the Cayley-Hamilton theorem, every square matrix satisfies its own characteristic equation: $A^3 - A^2 - 4A + 4I = 0$.
4. Split $5I$ as $4I + I$: $B = A^3 - A^2 - 4A + 5I = (A^3 - A^2 - 4A + 4I) + I = 0 + I = I$.
5. So $\det B = \det I = 1$.

6. Check with the eigenvalues of a matrix polynomial. Let $p(\lambda) = \lambda^3 - \lambda^2 - 4\lambda + 5$. The eigenvalues of $B = p(A)$ are $p(1) = 1 - 1 - 4 + 5 = 1$, $p(2) = 8 - 4 - 8 + 5 = 1$ and $p(-2) = -8 - 4 + 8 + 5 = 1$. The determinant is the product of the eigenvalues, so $\det B = 1 \cdot 1 \cdot 1 = 1.0$.

KEY IDEA

The polynomial in B differs from A's characteristic polynomial $\lambda^3 - \lambda^2 - 4\lambda + 4$ only in the constant term. By Cayley-Hamilton that characteristic polynomial evaluated at A is the zero matrix, so $B = 0 + I = I$ without computing any powers of A. The same thing shows up in the eigenvalues: $p(\lambda_i) = 1$ for every eigenvalue λ_i of A.

WATCH OUT FOR

Students often take $\det B$ as $\det(A^3) - \det(A^2) - 4\det A + 5$. The determinant is not linear, so that is wrong. Other slips: getting a sign wrong when expanding $\det(A - \lambda I)$ (for example, writing the eigenvalue -2 as $+2$), or computing A^2 and A^3 by hand and making an arithmetic error. Also, $\det(4A)$ for a 3×3 matrix is $4^3 \det A$, not $4 \det A$.

Q.45 NAT · 2 MARKS

Electromagnetics › Electrostatics

The capacitance of an air-filled parallel-plate capacitor is 60 pF. When a dielectric slab whose thickness is half the distance between the plates, is placed on one of the plates covering it entirely, the capacitance becomes 86 pF. Neglecting the fringing effects, the relative permittivity of the dielectric is _____ (up to 2 decimal places).

Official answer: **2.50 to 2.55**

SOLUTION

1. Let the plate area be A and the plate separation d. The air-filled capacitor has $C_0 = \dfrac{\epsilon_0 A}{d} = 60\ \text{pF}$, where $\epsilon_0 = 8.854 \times 10^{-12}\ \text{F/m}$. Its value cancels out, so we never need it.
2. The slab covers the whole plate and fills half the gap, so the gap now holds two stacked layers: a dielectric layer of thickness $d/2$ and an air layer of thickness $d/2$. Each layer is parallel to the plates. The normal flux density $\mathbf{D}$ is continuous across the dielectric-air boundary, so the same $\mathbf{D}$ (the same charge per unit area) passes through both layers. The two layer capacitors are therefore in **series**.
3. Dielectric layer: $C_1 = \dfrac{\epsilon_0 \epsilon_r A}{d/2} = 2\epsilon_r C_0$. Air layer: $C_2 = \dfrac{\epsilon_0 A}{d/2} = 2C_0$.
4. Series combination:
$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{(2\epsilon_r C_0)(2C_0)}{2\epsilon_r C_0 + 2C_0} = \frac{2\epsilon_r C_0}{\epsilon_r + 1}$$
5. Put in $C = 86\ \text{pF}$ and $C_0 = 60\ \text{pF}$:
$$\frac{120\,\epsilon_r}{\epsilon_r + 1} = 86 \ \Rightarrow\ 120\,\epsilon_r = 86\,\epsilon_r + 86 \ \Rightarrow\ 34\,\epsilon_r = 86$$
6. $\epsilon_r = \dfrac{86}{34} = 2.5294\ldots \approx 2.53$
7. Check: $\epsilon_r = 2.53$ gives $C = \dfrac{120 \times 2.53}{3.53} = \dfrac{303.6}{3.53} \approx 86.0\ \text{pF}$, which matches the given value.

KEY IDEA

A slab parallel to the plates that covers the whole plate area splits the gap into layers stacked along the field. The same $\mathbf{D}$ passes through every layer, so the layer capacitors add in series. Each half-thickness layer has twice the capacitance of the full gap filled with the same medium, which gives $C = \dfrac{2\epsilon_r C_0}{\epsilon_r + 1}$.

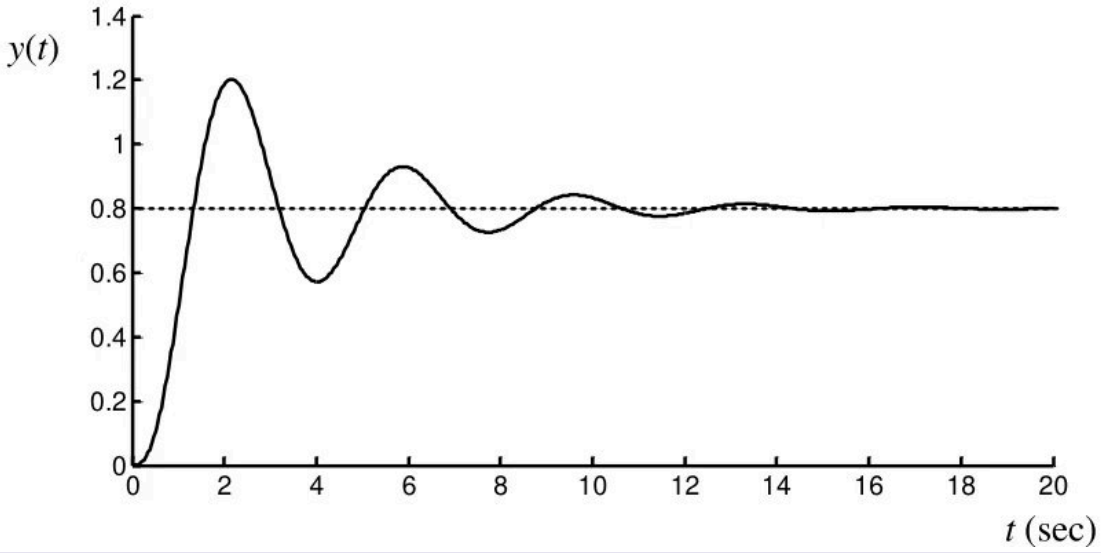
WATCH OUT FOR

Treating the two layers as capacitors in parallel, as if the slab covered half the plate area over the full gap. That gives $C = \dfrac{C_0(1 + \epsilon_r)}{2}$ and $\epsilon_r \approx 1.87$. A second mistake is forgetting that each layer is only $d/2$ thick, so its capacitance is $2C_0$ or $2\epsilon_r C_0$, not C_0 or $\epsilon_r C_0$.

Q.46 NAT · 2 MARKS

Control Systems › Time response

Q.46 The unit step response $y(t)$ of a unity feedback system with open loop transfer function $G(s)H(s)=\dfrac{K}{(s+1)^2(s+2)}$ is shown in the figure. The value of K is _____ (up to 2 decimal places).



Official answer: **8.0**

SOLUTION

1. Read the steady-state value off the graph. The dashed line the response settles onto is at $y(\infty)=0.8$, not 1, so the closed loop has a steady-state error to a unit step.
2. The open loop $G(s)H(s)=\dfrac{K}{(s+1)^2(s+2)}$ has no pole at $s=0$, so the system is type 0. Its position error constant is
$$K_p=\lim_{s\to 0}G(s)H(s)=\frac{K}{(1)^2(2)}=\frac{K}{2}$$
3. With unity feedback and a unit step input, $y(\infty)=\dfrac{K_p}{1+K_p}$. The final value theorem gives the same thing directly: the closed loop is $T(s)=\dfrac{K}{(s+1)^2(s+2)+K}$, so
$$y(\infty)=T(0)=\frac{K}{(1)^2(2)+K}=\frac{K}{K+2}$$
4. Set this equal to the value read from the graph:
$$\frac{K}{K+2}=0.8\ \Rightarrow\ K=0.8K+1.6\ \Rightarrow\ 0.2K=1.6\ \Rightarrow\ K=8$$
5. Check stability, because the final value theorem only holds for a stable closed loop. Expanding $(s+1)^2(s+2)=s^3+4s^2+5s+2$, the characteristic equation is $s^3+4s^2+5s+(2+K)=0$. For a cubic $s^3+a_2s^2+a_1s+a_0$ the Routh condition is $a_2a_1>a_0$ with all coefficients positive, so $4\cdot 5>2+K$, which gives $-2<K<18$. $K=8$ lies inside this range, so the loop is stable and the final value theorem applies.
6. Check against the shape of the curve. With $K=8$ the characteristic equation $s^3+4s^2+5s+10=0$ factors approximately as $(s+3.39)(s^2+0.61s+2.95)$. The complex pair is $s\approx -0.30\pm j1.69$, so $\omega_d\approx 1.69\ \text{rad/s}$ and the oscillation period is $2\pi/\omega_d\approx 3.7$ s. That matches the spacing of the peaks at about 2.2, 5.9 and 9.6 s. The slow decay, set by the real part 0.30, fits the ringing lasting until about 16 s.

7. Answer: $K = 8.00$.

KEY IDEA

Only one number on the graph matters: the steady-state value. For a stable unity-feedback loop, $y(\infty) = T(0) = \dfrac{G(0)H(0)}{1 + G(0)H(0)}$ for a unit step. Here $G(0)H(0) = K/2$, so $0.8 = \dfrac{K/2}{1 + K/2}$ fixes K. The overshoot and ringing are there to distract you, or at most to confirm the answer afterwards.

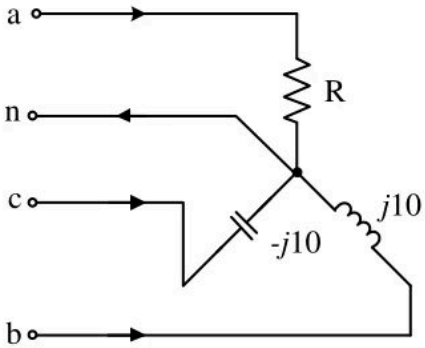
WATCH OUT FOR

1) Reading the 1.2 peak and trying to extract K from percent overshoot with second-order formulas. The system is third order, and the overshoot has to be measured from 0.8, not from 1. 2) Taking $G(0)H(0) = K$ instead of $K/2$, forgetting the factor $(0 + 2) = 2$, which gives $K = 4$. 3) Assuming the response settles at 1, the usual unit-step reflex. A type-0 loop always leaves a steady-state error. 4) Skipping the stability check. The final value theorem is only valid if K is in the stable range, here $K < 18$.

Q.47 NAT · 2 MARKS

Networks (Electric Circuits) › Three-phase (EE)

Q.47 A three-phase load is connected to a three-phase balanced supply as shown in the figure. If $V_{an} = 100\angle 0°$ V, $V_{bn} = 100\angle -120°$ V and $V_{cn} = 100\angle -240°$ V (angles are considered positive in the anti-clockwise direction), the value of R for zero current in the neutral wire is _____ Ω (up to 2 decimal places).



EE 12/15

Official answer: **5.70 to 5.85**

SOLUTION

1. Read the figure. The star point is tied to the supply neutral, so each branch has its own phase voltage across it. The branch impedances are $Z_a = R$, $Z_b = j10\ \Omega$ (inductor) and $Z_c = -j10\ \Omega$ (capacitor).
2. KCL at the star point gives $I_n = I_a + I_b + I_c = \dfrac{V_{an}}{R} + \dfrac{V_{bn}}{j10} + \dfrac{V_{cn}}{-j10}$. For zero neutral current this sum must be 0.
3. Phase b current: $I_b = \dfrac{100\angle -120°}{10\angle 90°} = 10\angle -210° = 10\angle 150°$ A.
4. Phase c current: $I_c = \dfrac{100\angle -240°}{10\angle -90°} = 10\angle -150° = 10\angle 210°$ A.
5. Add them. The imaginary parts cancel because $\sin 150° + \sin 210° = 0$:
$$I_b + I_c = 10(\cos 150° + \cos 210°) = 10\left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right) = -10\sqrt{3} = -17.32\ \text{A}$$
6. So I_a must be $+10\sqrt{3}\angle 0° = 17.32\angle 0°$ A. That current is in phase with $V_{an} = 100\angle 0°$ V, so a pure (positive) resistor can carry it: $\dfrac{100}{R} = 10\sqrt{3}$.
7. $$R = \frac{100}{10\sqrt{3}} = \frac{10}{\sqrt{3}} = 5.7735\ \Omega \approx 5.77\ \Omega$$

KEY IDEA

When the star point is tied to the neutral, each branch current is its own phase voltage divided by its own impedance, and the neutral carries their phasor sum. The j10 and -j10 branches each draw 10 A. Their quadrature parts cancel and leave a real current of $-10\sqrt{3}$ A. Phase a must therefore draw $+10\sqrt{3}\angle 0^\circ$ A, which is in phase with V_{an}, so a resistor can supply it: $R = 100/(10\sqrt{3})$.

WATCH OUT FOR

The usual slip is swapping the reactances between phases, i.e. putting the capacitor on b and the inductor on c. Then $I_b + I_c = +17.32$ A, which needs $R = -5.77\ \Omega$. A negative value like this shows the phase assignment is wrong, but candidates often just enter its magnitude without noticing. Other slips are forgetting that dividing by $-j10$ adds $+90^\circ$ to the angle, and using line voltages ($100\sqrt{3}$ V) when a neutral-connected star has phase voltages across its branches.

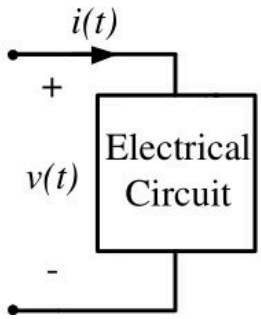
Q.48 NAT · 2 MARKS

Networks (Electric Circuits) › AC, power & resonance

Q.48 The voltage across the circuit in the figure, and the current through it, are given by the following expressions:

$$v(t) = 5 - 10\cos(\omega t + 60^\circ)\ \text{V}$$
$$i(t) = 5 + X\cos(\omega t)\ \text{A}$$

where $\omega = 100\,\pi$ radian/s. If the average power delivered to the circuit is zero, then the value of X (in Ampere) is _____ (up to 2 decimal places).



Official answer: **10.0**

SOLUTION

1. **Passive sign convention.** In the figure $i(t)$ enters the $+$ terminal of $v(t)$, so the average power delivered to the circuit is the time average of $v(t)\,i(t)$: $P = \overline{v(t)\,i(t)}$.
2. **Power adds across different frequencies.** Multiplying out $v(t)\,i(t)$ gives a DC×DC term, two DC×AC cross terms and one AC×AC term. Each cross term is a constant times a sinusoid at ω, whose average over a period is zero. So $P = P_{\text{DC}} + P_\omega$.
3. **DC part:** $P_{\text{DC}} = V_0 I_0 = 5 \times 5 = 25$ W.
4. **Write the AC voltage as a cosine with a positive amplitude:** $-10\cos(\omega t + 60^\circ) = 10\cos(\omega t + 60^\circ - 180^\circ) = 10\cos(\omega t - 120^\circ)$ V. So $\theta_v = -120^\circ$ and $\theta_i = 0^\circ$.
5. **AC part, using peak amplitudes** ($V_m = 10$ V, $I_m = X$ A):
$$P_\omega = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = \frac{10\,X}{2}\cos(-120^\circ) = 5X\left(-\frac{1}{2}\right) = -2.5X\ \text{W}$$
(Same result with rms values: $\frac{10}{\sqrt{2}} \cdot \frac{X}{\sqrt{2}}\cos 120^\circ = -2.5X$ W.)
6. **Set the average power to zero:** $25 - 2.5X = 0 \Rightarrow X = \dfrac{25}{2.5} = 10.00$ A.
7. **Check:** with $X = 10$, the circuit absorbs 25 W at DC and $P_\omega = -2.5 \times 10 = -25$ W at $\omega = 100\pi$ rad/s (50 Hz), meaning it sends 25 W back. The net average power is 0.

KEY IDEA

When voltage and current contain components at different frequencies (here DC and ω), the average power is the sum of the powers at each frequency, because products of components at different frequencies average to zero. At DC, $P = V_0 I_0$. At ω, $P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$, and only after both signals are written as cosines with positive amplitudes.

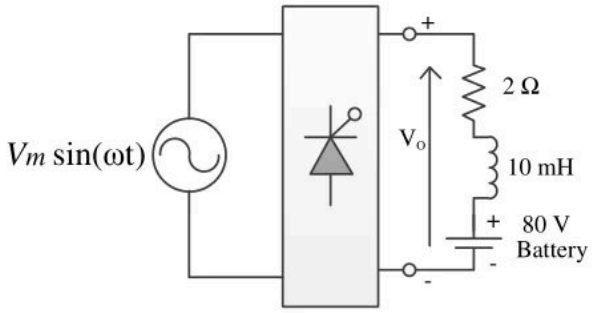
WATCH OUT FOR

Taking the voltage angle as $+60^\circ$ and ignoring the minus sign in $-10\cos(\omega t + 60^\circ)$. That gives $P_\omega = +2.5X$ and $X = -10$, a sign error. The minus sign is a 180° phase shift, so the angle difference is 120°, not 60°. A second mistake is to leave out the DC power $5 \times 5 = 25$ W, or to add cross terms such as $5 \cdot X\cos\omega t$, which average to zero.

Q.49 NAT · 2 MARKS

Power Electronics › Rectifiers

Q.49 A phase controlled single phase rectifier, supplied by an AC source, feeds power to an R-L-E load as shown in the figure. The rectifier output voltage has an average value given by $V_o = \frac{V_m}{2\pi}(3 + \cos\alpha)$, where $V_m = 80\pi$ volts and α is the firing angle. If the power delivered to the lossless battery is 1600 W, α in degree is_____ (up to 2 decimal places).



Official answer: **90.0**

SOLUTION

1. The battery is lossless, so all the power it receives is $P = E\,I_o$ with $E = 80$ V. Hence $I_o = \frac{1600}{80} = 20$ A (the average load current).
2. In steady state the average voltage across the 10 mH inductor is zero, so only the resistor and the battery drop average voltage: $V_o = E + I_o R = 80 + 20 \times 2 = 120$ V.
3. The given average output voltage is $V_o = \frac{V_m}{2\pi}(3 + \cos\alpha)$ with $V_m = 80\pi$ V, so $V_o = \frac{80\pi}{2\pi}(3 + \cos\alpha) = 40\,(3 + \cos\alpha)$ V.
4. Equate the two expressions for V_o: $40(3 + \cos\alpha) = 120 \Rightarrow 3 + \cos\alpha = 3 \Rightarrow \cos\alpha = 0$.
5. The firing angle is therefore $\alpha = \cos^{-1}(0) = 90^\circ$, that is 90.00°.

KEY IDEA

The power into a lossless battery is E times the average current. Because the inductor has zero average voltage in steady state, the average output voltage is the battery voltage plus the resistor drop, and this is equated to the given expression for the average output voltage.

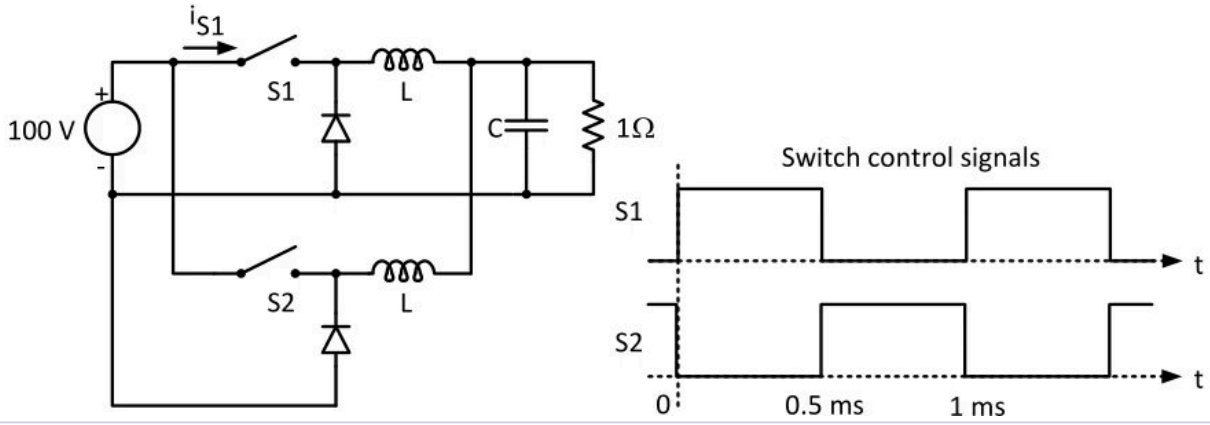
WATCH OUT FOR

Including the inductor in the average-voltage balance, or taking the power as $I_{rms}^2 R$ or $V_o I_o$. The 1600 W is the battery's power only, so it equals EI_o. The resistor loss is extra and is not part of it.

Q.50 NAT · 2 MARKS

Power Electronics › DC-DC converters

Q.50 The figure shows two buck converters connected in parallel. The common input dc voltage for the converters has a value of 100 V. The converters have inductors of identical value. The load resistance is 1 Ω. The capacitor voltage has negligible ripple. Both converters operate in the continuous conduction mode. The switching frequency is 1 kHz, and the switch control signals are as shown. The circuit operates in the steady state. Assuming that the converters share the load equally, the average value of i_{S1}, the current of switch S1 (in Ampere), is _____ (up to 2 decimal places).



Official answer: **11.50 to 13.50**

SOLUTION

1. Switch S1 is on from 0 to 0.5 ms in a period of $T = \dfrac{1}{1\ \text{kHz}} = 1\ \text{ms}$, so the duty ratio is $D = \dfrac{0.5}{1} = 0.5$. S2 is on in the other half of the period, so it has the same $D = 0.5$.
2. Both converters run in continuous conduction from the same 100 V input with the same D. With negligible capacitor ripple, $V_o = D\,V_s = 0.5 \times 100 = 50$ V.
3. Load current: $I_o = \dfrac{V_o}{R} = \dfrac{50}{1} = 50$ A.
4. The converters share the load equally, so each average inductor current is $I_L = \dfrac{50}{2} = 25$ A.
5. In a buck converter the switch carries the inductor current only while it is on, a fraction D of each period. The diode carries the inductor current for the rest of the period. The ripple does not change the average, because the average of the switch current over a period is D times the average inductor current.
6. Average switch current: $I_{S1,\text{avg}} = D\,I_L = 0.5 \times 25 = 12.5$ A.
7. Check by power balance: each converter delivers $50 \times 25 = 1250$ W, so the current drawn from the 100 V source is $\dfrac{1250}{100} = 12.5$ A. This current flows only through S1, so it is the average of i_{S1}.

KEY IDEA

Each buck converter has $D = 0.5$, so $V_o = 50$ V and the load takes 50 A. Each converter supplies 25 A through its inductor, and the switch conducts that current for the fraction D of the period, giving DI_L.

WATCH OUT FOR

Giving 25 A, the average inductor current, as the average switch current. The switch conducts only during the on-time, so its average is DI_L. Another slip is taking the full 50 A load current as the share of one converter.

Q.51 NAT · 2 MARKS

Electrical Machines › Transformers

A 3-phase 900 kVA, 3 kV / √3 kV (Δ/Y), 50 Hz transformer has primary (high voltage side) resistance per phase of 0.3 Ω and secondary (low voltage side) resistance per phase of 0.02 Ω. Iron loss of the transformer is 10 kW. The full load % efficiency of the transformer operated at unity power factor is _____ (up to 2 decimal places).

Official answer: **97.20 to 97.55**

SOLUTION

1. Identify winding phase voltages: primary is Δ-connected at line voltage 3 kV, so per-phase voltage = 3 kV = 3000 V; secondary is Y-connected at line voltage √3 kV, so per-phase voltage = √3 kV / √3 = 1 kV = 1000 V.
2. Full-load primary phase current: $I_1 = \dfrac{900000}{3 \times 3000} = 100\ \text{A}$.
3. Full-load secondary phase current: $I_2 = \dfrac{900000}{3 \times 1000} = 300\ \text{A}$.
4. Consistency check: phase turns ratio $= 3000/1000 = 3$, and $I_2/I_1 = 300/100 = 3$ (matches, confirming the current directions and side assignment are correct).
5. Total full-load copper loss (sum over all three phases, both windings):
$$P_{cu} = 3\left(I_1^2 R_1 + I_2^2 R_2\right) = 3\left(100^2 \times 0.3 + 300^2 \times 0.02\right)\ \text{W}$$
6. $P_{cu} = 3(3000 + 1800) = 14400\ \text{W} = 14.4\ \text{kW}$.
7. Total full-load losses $= P_{cu} + P_{iron} = 14.4 + 10 = 24.4\ \text{kW}$.
8. Full-load output at unity power factor: $P_{out} = 900 \times 1 = 900\ \text{kW}$.
9. $$\eta = \frac{P_{out}}{P_{out} + \text{total losses}} = \frac{900}{900 + 24.4} = \frac{900}{924.4} = 0.97360$$
10. % efficiency $= 97.36\%$.

KEY IDEA

For a Δ/Y three-phase transformer, phase voltage equals line voltage on the Δ side but line voltage/√3 on the Y side; full-load phase currents on each side then follow from rated kVA = 3 × V_phase × I_phase for that side. Total copper loss is the sum of I²R loss in all three phases of both windings, and efficiency at any power factor is (output)/(output + copper loss + iron loss), with output = rated kVA × cosφ at full load.

WATCH OUT FOR

Using the line voltage as the phase voltage on the Y (secondary) side (forgetting to divide by √3), or computing copper loss for only one phase instead of multiplying by 3, both of which give a wrong current/loss and shift the efficiency outside the key range.

Q.52 NAT · 2 MARKS

Electrical Machines › DC machines

A 200 V DC series motor, when operating from rated voltage while driving a certain load, draws 10 A current and runs at 1000 r.p.m. The total series resistance is 1 Ω. The magnetic circuit is assumed to be linear. At the same supply voltage, the load torque is increased by 44%. The speed of the motor in r.p.m. (rounded to the nearest integer) is _____.

Official answer: **823 to 827**

SOLUTION

1. Given: rated voltage $V = 200\ \text{V}$, armature current $I_{a1} = 10\ \text{A}$, speed $N_1 = 1000\ \text{r.p.m.}$, total series resistance $R = 1\ \Omega$. The magnetic circuit is linear, so flux $\phi \propto I_a$.
2. Since $T \propto \phi I_a \propto I_a^2$ for a series motor with linear magnetics, a 44% increase in load torque requires $I_{a2}^2 = 1.44\, I_{a1}^2$, giving $I_{a2} = 1.2\, I_{a1} = 1.2 \times 10 = 12\ \text{A}$.

3. Back-EMF at state 1: $E_{b1} = V - I_{a1}R = 200 - 10 \times 1 = 190$ V.
4. Back-EMF at state 2: $E_{b2} = V - I_{a2}R = 200 - 12 \times 1 = 188$ V.
5. Since $\phi \propto I_a$, the back-EMF $E_b \propto \phi N \propto I_a N$, so $\dfrac{E_{b1}}{E_{b2}} = \dfrac{N_1 I_{a1}}{N_2 I_{a2}}$.
6. Substituting values: $\dfrac{190}{188} = \dfrac{1000 \times 10}{N_2 \times 12} \Rightarrow N_2 = \dfrac{188 \times 10000}{190 \times 12} = \dfrac{1880000}{2280} \approx 824.56$ r.p.m.
7. Rounded to the nearest integer, $N_2 \approx 825$ r.p.m.

KEY IDEA

In a linearly-saturated (unsaturated, linear) DC series motor, flux is proportional to armature current, so torque is proportional to the square of armature current. Equating motor torque to the new load torque gives the new armature current directly; back-EMF at each operating point then follows from V − IaR, and since Eb ∝ φN ∝ IaN, the ratio of back-EMFs gives the new speed.

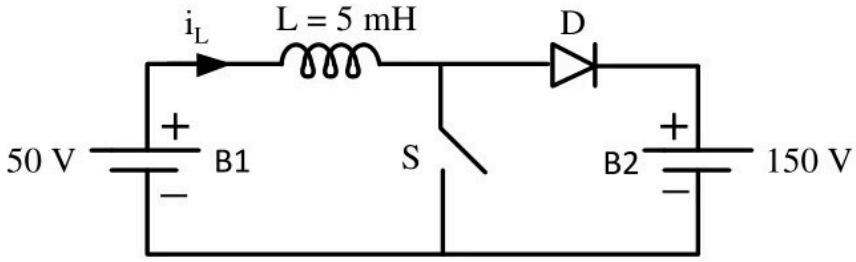
WATCH OUT FOR

Assuming speed is inversely proportional to current alone (as in a shunt motor with constant flux) instead of properly forming the ratio Eb1/Eb2 = (N1 Ia1)/(N2 Ia2), which correctly accounts for flux also changing with armature current in a series motor.

Q.53 NAT · 2 MARKS

Power Electronics › DC-DC converters

Q.53 A dc to dc converter shown in the figure is charging a battery bank, B2 whose voltage is constant at 150 V. B1 is another battery bank whose voltage is constant at 50 V. The value of the inductor, L is 5 mH and the ideal switch, S is operated with a switching frequency of 5 kHz with a duty ratio of 0.4. Once the circuit has attained steady state and assuming the diode D to be ideal, the power transferred from B1 to B2 (in Watt) is _____ (up to 2 decimal places).



EE 14/15

Official answer: **12.0**

SOLUTION

1. The circuit is a boost converter: source $V_s = 50$ V (B1) feeds $L = 5$ mH, the switch S shunts the node after L, and the diode D leads to the 150 V battery B2. Duty ratio $D = 0.4$, $T = \dfrac{1}{5\text{ kHz}} = 200\ \mu$s, so $DT = 80\ \mu$s.
2. Which mode? In continuous conduction, volt-second balance would need $V_o = \dfrac{V_s}{1-D} = \dfrac{50}{0.6} \approx 83.3$ V. The battery holds the output at 150 V, which is higher. The inductor therefore demagnetises faster than in CCM, and its current reaches zero before the period ends. The diode blocks reverse current, so the converter runs in discontinuous conduction (DCM). Steps 3 to 5 confirm this.
3. S on for $DT = 80\ \mu$s: $v_L = V_s = 50$ V, so the current rises linearly from zero to
$$I_{pk} = \frac{V_s\, DT}{L} = \frac{50 \times 80 \times 10^{-6}}{5 \times 10^{-3}} = 0.8 \text{ A}.$$
4. S off, D conducting: $v_L = V_s - V_o = -100$ V, so the current falls to zero in

$$t_f = \frac{L\,I_{pk}}{V_o - V_s} = \frac{5 \times 10^{-3} \times 0.8}{100} = 40\ \mu\text{s}.$$

5. Since $DT + t_f = 120\ \mu\text{s} < T = 200\ \mu\text{s}$, the current stays zero for the last $80\ \mu\text{s}$ of the period. This confirms DCM and is consistent with the assumption in step 2.
6. The source current is the inductor current. Its average is the area of the triangle (base $DT + t_f$, height I_{pk}) divided by T:
$$I_L = \frac{1}{2} I_{pk}\,\frac{DT + t_f}{T} = \frac{1}{2}(0.8)\frac{120}{200} = 0.24\ \text{A}.$$
7. Power taken from B1: $P = V_s I_L = 50 \times 0.24 = 12\ \text{W}$.
8. Cross-check at B2: the diode carries current only during t_f, so $I_D = \frac{1}{2}(0.8)\frac{40}{200} = 0.08\ \text{A}$ and $V_o I_D = 150 \times 0.08 = 12\ \text{W}$. This equals the power drawn from B1, as it must for ideal components.

KEY IDEA

When the output is a fixed battery higher than $V_s/(1-D)$, the boost converter cannot stay in CCM. Find the peak current from $V_s DT/L$, find the fall time from $LI_{pk}/(V_o - V_s)$, and average the triangular current over the full period.

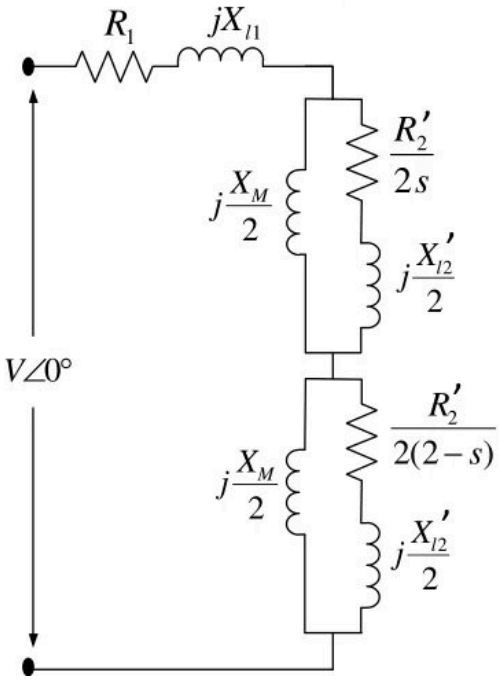
WATCH OUT FOR

Using the CCM relation, which gives a power of $V_s^2 D/(\ldots)$-style or $I_L = P/V_s$ with I_{pk} treated as a flat current. Another slip is averaging the triangle over $DT + t_f$ instead of the full period T, which gives 0.4 A and 20 W instead of 12 W.

Q.54 NAT · 2 MARKS

Electrical Machines › Induction machines

Q.54 The equivalent circuit of a single phase induction motor is shown in the figure, where the parameters are $R_1 = R_2' = X_{l1} = X_{l2}' = 12\,\Omega$, $X_M = 240\,\Omega$ and s is the slip. At no-load, the motor speed can be approximated to be the synchronous speed. The no-load lagging power factor of the motor is_____ (up to 3 decimal places).



Official answer: **0.104 to 0.112**

SOLUTION

1. At no-load, the motor runs essentially at synchronous speed, so the slip s → 0.

2. Forward-field branch: as s → 0, the rotor term $\frac{R_2'}{2s} \to \infty$ (open circuit), so this branch reduces to just the magnetizing reactance $j\frac{X_M}{2} = j\frac{240}{2} = j120\ \Omega$.
3. Backward-field branch: as s → 0, $2 - s \to 2$, so the rotor resistance term becomes $\frac{R_2'}{2(2-s)} = \frac{12}{4} = 3\ \Omega$, in series with $j\frac{X_{l2}'}{2} = j6\ \Omega$, giving $3 + j6\ \Omega$; this is in parallel with $j\frac{X_M}{2} = j120\ \Omega$.
4. Compute the backward-branch impedance:

$$Z_b = \frac{(j120)(3+j6)}{3+j6+j120} = \frac{-720+j360}{3+j126} = 2.720 + j5.779\ \Omega$$

5. Total input impedance seen at no-load:

$$Z = R_1 + jX_{l1} + j\frac{X_M}{2} + Z_b = 12 + j12 + j120 + (2.720 + j5.779) = 14.720 + j137.779\ \Omega$$

6. Magnitude:

$$|Z| = \sqrt{14.720^2 + 137.779^2} = 138.564\ \Omega$$

7. No-load power factor (lagging, since the net reactance is inductive):

$$\cos\phi = \frac{R}{|Z|} = \frac{14.720}{138.564} = 0.106$$

KEY IDEA

At no-load the slip s → 0, so in the forward-field branch of the double-revolving-field equivalent circuit the rotor term $R_2'/(2s)$ tends to infinity (open circuit), leaving only the magnetizing reactance $jX_M/2$ in the forward branch; in the backward-field branch, $2 - s \to 2$, so it stays a finite parallel combination that must be computed and added to the series stator and forward-branch impedances to get the total no-load impedance and hence the power factor.

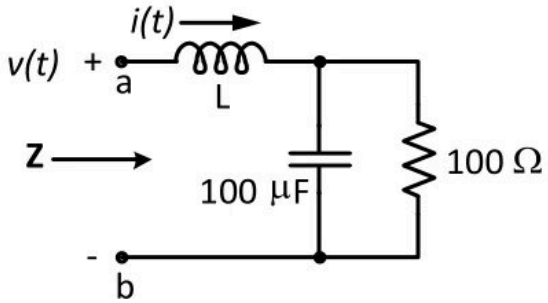
WATCH OUT FOR

Forgetting that the backward-field branch does NOT vanish or simplify away at no load (only the forward branch's rotor resistance term goes to infinity because of s → 0 in the denominator; the backward branch's denominator is $2 - s \to 2$, a finite number), so students either drop the backward branch entirely (giving a much higher, wrong power factor) or incorrectly send $2 - s \to 0$ as well.

Q.55 NAT · 2 MARKS

Networks (Electric Circuits) › AC, power & resonance

Q.55 The voltage $v(t)$ across the terminals a and b as shown in the figure, is a sinusoidal voltage having a frequency $\omega = 100$ radian/s. When the inductor current $i(t)$ is in phase with the voltage $v(t)$, the magnitude of the impedance Z (in Ω) seen between the terminals a and b is _____ (up to 2 decimal places).



Official answer: **50.0**

SOLUTION

1. **Capacitor reactance at** $\omega = 100\ \text{rad/s}$**:** $X_C = \frac{1}{\omega C} = \frac{1}{100 \times 100 \times 10^{-6}} = 100\ \Omega$, so the capacitor's impedance is $-j100\ \Omega$.
2. **Parallel combination of R and C:**

$$Z_p = \frac{100\,(-j100)}{100 - j100} = \frac{-j100}{1 - j} = \frac{-j100\,(1 + j)}{(1 - j)(1 + j)} = \frac{100 - j100}{2} = 50 - j50\ \Omega$$

3. **Total impedance seen from a-b (L in series with Z_p):** $Z = j\omega L + 50 - j50 = 50 + j(100L - 50)\ \Omega$.
4. **In-phase condition:** the inductor sits in series with the terminals, so the inductor current $i(t)$ is the terminal current. i in phase with v therefore means $Z = V/I$ is purely resistive: $\mathrm{Im}(Z) = 0 \Rightarrow 100L = 50 \Rightarrow L = 0.5\ \mathrm{H}$.
5. **Magnitude at that condition:** $Z = 50 + j0\ \Omega$, so $|Z| = 50.00\ \Omega$.

KEY IDEA

Because the inductor carries the terminal current, "i in phase with v" means the whole input impedance is purely resistive. First reduce the parallel R-C branch to its series form, $50 - j50\ \Omega$. The inductor's $+j\omega L$ has to cancel the $-j50$, so the resistive part left over, $50\ \Omega$, is $|Z|$. You don't need to find L to get the answer.

WATCH OUT FOR

A common mistake is to resonate L with the capacitor alone, using $\omega^2 LC = 1$, which gives $L = 1\ \mathrm{H}$. That ignores the parallel resistor. With $L = 1\ \mathrm{H}$ you get $Z = 50 + j50\ \Omega$ and $|Z| = 70.71\ \Omega$, and there the current is not in phase with the voltage. Another mistake is to quote $|Z_p| = |50 - j50| = 70.71\ \Omega$ as the answer, which leaves out the inductor.

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