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GATE EC · 2026

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GATE EC 2026 Solved Paper

INSIDE

- All 65 questions, in the order of the paper
- The official answer to every question
- Step-by-step worked solutions
- The answer key on one page

Question paper and official answer key: IIT Guwahati

Online, with the questions linked to their topic notes: **iaeacademy.in/study/papers/ec-2026**

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About this paper

65 questions, 100 marks. Question types: 42 MCQ · 10 MSQ · 13 NAT.

Official answer key: IIT Guwahati · https://gate2026.iitg.ac.in/doc/download/2026/Keys/EC_Keys.pdf

We placed each question under a subject and topic ourselves; the exam paper does not label them.

This paper online, with the questions linked to their topic notes:
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Answer key

Q	TYPE	MARKS	ANSWER
Q.1	MCQ	1	**B**
Q.2	MCQ	1	**A**
Q.3	MCQ	1	**D**
Q.4	MCQ	1	**B**
Q.5	MCQ	1	**B**
Q.6	MCQ	2	**A**
Q.7	MCQ	2	**C**
Q.8	MCQ	2	**C**
Q.9	MCQ	2	**C**
Q.10	MCQ	2	**C**
Q.11	MCQ	1	**A**
Q.12	MCQ	1	**D**
Q.13	MCQ	1	**D**
Q.14	MCQ	1	**C**
Q.15	MCQ	1	**D**
Q.16	MCQ	1	**A**
Q.17	MCQ	1	**C**
Q.18	MCQ	1	**D**
Q.19	MCQ	1	**B**
Q.20	MCQ	1	**A**
Q.21	MCQ	1	**B**
Q.22	MCQ	1	**C**
Q.23	MCQ	1	**A**
Q.24	MCQ	1	**A**
Q.25	MSQ	1	**A**
Q.26	MSQ	1	**B, D**
Q.27	MSQ	1	**B**
Q.28	MSQ	1	**A, B**
Q.29	MSQ	1	**C, D**
Q.30	NAT	1	**0.95 to 1.05**
Q.31	NAT	1	**71.50 to 72.50**
Q.32	NAT	1	**0.95 to 1.05**
Q.33	NAT	1	**20.50 to 21.50**
Q.34	NAT	1	**20.00 to 21.00**
Q.35	NAT	1	**4.90 to 5.10**
Q.36	MCQ	2	**A**
Q.37	MCQ	2	**A**
Q.38	MCQ	2	**A**
Q.39	MCQ	2	**B**
Q.40	MCQ	2	**B**
Q.41	MCQ	2	**C**
Q.42	MCQ	2	**C**
Q.43	MCQ	2	**C**
Q.44	MCQ	2	**B**
Q.45	MCQ	2	**B**
Q.46	MCQ	2	**B**
Q.47	MCQ	2	**B**
Q.48	MCQ	2	**C**
Q.49	MCQ	2	**A**
Q.50	MCQ	2	**A**
Q.51	MCQ	2	**D**
Q.52	MCQ	2	**B**
Q.53	MCQ	2	**C**
Q.54	MSQ	2	**A, B, D**
Q.55	MSQ	2	**C**
Q.56	MSQ	2	**A, D**
Q.57	MSQ	2	**A, B, C**
Q.58	MSQ	2	**A, C, D**
Q.59	NAT	2	**1386**
Q.60	NAT	2	**2.50 to 2.60**
Q.61	NAT	2	**0.60 to 0.70**
Q.62	NAT	2	**2.90 to 3.10**
Q.63	NAT	2	**0.10 to 0.20**
Q.64	NAT	2	**-36.50 to -34.50, or 34.50 to 36.50**
Q.65	NAT	2	**3.70 to 4.30**

General Aptitude Q.1 to Q.10

Q.1 MCQ · 1 MARK

General Aptitude › Grammar & vocabulary

Among the following options, the antonym of the word ‘nocturnal’ is _____.

(A) normal

(B) diurnal

(C) abnormal

(D) exceptional

Official answer: **B**

SOLUTION

1. **Meaning of the word.** *Nocturnal* (from Latin *nox, noctis* = night) means active or happening at night, as in "owls are nocturnal birds".
2. **What an antonym must be.** An antonym is the opposite on the *same* scale of meaning. Here the scale is time of day, so the answer has to mean "active or happening during the day".
3. **Check (A) normal.** It means usual or ordinary. It has nothing to do with night or day, so it is not the opposite of nocturnal.
4. **Check (C) abnormal and (D) exceptional.** Both mean unusual or out of the ordinary. They describe how common something is, not the time of day, so neither can be the opposite.
5. **Check (B) diurnal.** From Latin *dies* = day: active or happening during the daytime, as in "most butterflies are diurnal". This is the exact opposite of nocturnal on the day–night scale.
6. **Conclusion.** The antonym of ‘nocturnal’ is ‘diurnal’, so the answer is option (B).

KEY IDEA

A true antonym is the opposite on the same scale of meaning. Nocturnal is about the time of day (night), so its antonym must also be about time of day: diurnal (by day). The Latin roots help here: nox = night, dies = day.

WATCH OUT FOR

Seeing the shared ending "-nal" and choosing 'normal', or treating nocturnal as meaning "unusual" and then picking its opposite. Options (A), (C) and (D) are about how ordinary something is, not about night and day, so none of them can be the antonym.

Q.2 MCQ · 1 MARK

General Aptitude › Reasoning

The statements (S1), (S2), and (S3) pertain to the scores obtained by students in an exam. The maximum possible marks in the exam is 150.

(S1) The highest score is 100.

(S2) The fourth highest score is 76.

(S3) There are at least four students whose scores are within 25 of each other.

Which one of the following options is necessarily correct?

(A) (S1) and (S2) together imply (S3)

(B) (S1) and (S3) together imply (S2)

(C) (S2) and (S3) together imply (S1)

(D) (S1) implies (S3)

Official answer: **A**

SOLUTION

1. Test each option by asking whether the two given statements force the third. A single set of scores that satisfies the given statements but not the third is enough to rule an implication out. "Within 25 of each other" means every pair of the four scores differs by at most 25.
2. (A) Suppose (S1) and (S2) hold. Since a fourth highest score exists, there are at least four students. The top four scores are no higher than the highest score (100) and no lower than the fourth highest (76), so all four lie in $[76,\ 100]$.
3. Any two of these four scores therefore differ by at most $100 - 76 = 24 \leq 25$. So at least four students have scores within 25 of each other, which is (S3). (S1) and (S2) always force (S3), so (A) is correct.
4. (B) Counterexample: four students scoring 100, 95, 92 and 90. The highest score is 100, so (S1) holds, and all four lie within $100 - 90 = 10$ of each other, so (S3) holds. But the fourth highest score is 90, not 76, so (S2) fails. (B) is not necessarily correct.
5. (C) Counterexample: four students scoring 90, 80, 78 and 76. The fourth highest score is 76, so (S2) holds, and all four lie within $90 - 76 = 14$ of each other, so (S3) holds. But the highest score is 90, not 100, so (S1) fails. (C) is not necessarily correct.
6. (D) Counterexample: four students scoring 100, 70, 40 and 10. The highest score is 100, so (S1) holds. With only these four students, the four scores span $100 - 10 = 90 > 25$, so no four students are within 25 of each other and (S3) fails. (D) is not necessarily correct.
7. Only (A) holds for every possible set of scores. The maximum of 150 marks plays no part beyond keeping every score used here valid.

KEY IDEA

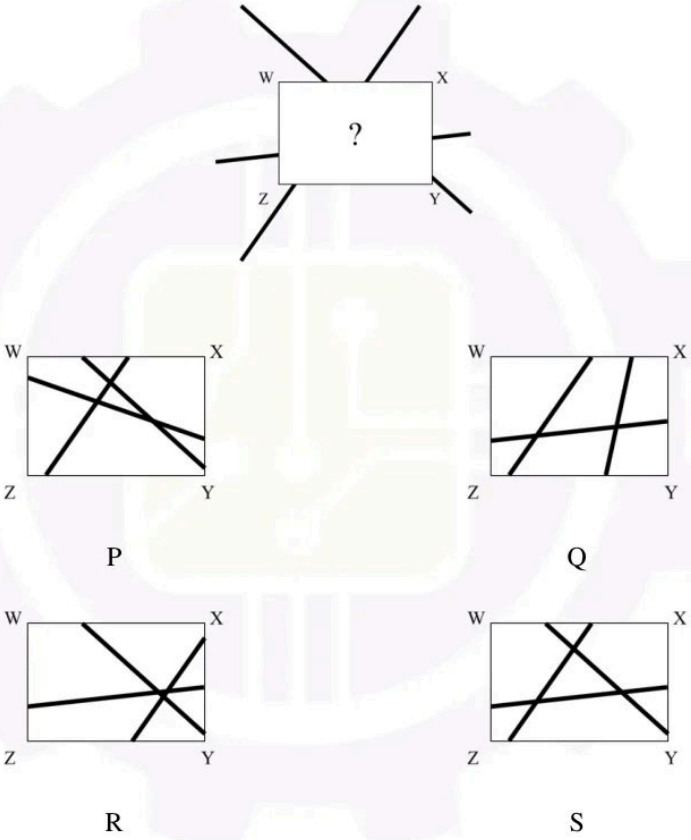
Knowing the highest and the fourth highest score boxes the top four students into an interval whose width is their difference. Here that width is $100 - 76 = 24 \leq 25$, so (S3) is forced. Each of the other implications falls to a single counterexample.

WATCH OUT FOR

Treating an implication as true because it holds in one convenient example (for example, choosing (C) or (D) because some score list fits), instead of asking whether it holds in every case. The other slip is reading (S3) as a claim about the highest score. (S3) fixes no particular value, so on its own it can never tell you the highest or the fourth highest score.

Q.3 MCQ · 1 MARK

General Aptitude › Spatial

Q.3	The figure below has exactly three intersecting line segments with a rectangular portion WXYZ missing. Which one of the following options P, Q, R, and S is the missing portion WXYZ? 
(A)	P
(B)	Q
(C)	R
(D)	S

Official answer: **D**

SOLUTION

1. **Two stubs make one segment.** A straight line stays straight through the gap. So each stub outside the gap must join a stub on another edge that has the **same slope** and lies on the same straight line.
2. **Pair the six stubs.** Line 1: the top-edge stub at about 31% that falls to the right continues to the stub on the right edge next to Y. Line 2: the top-edge stub at about 57% that rises to the right continues to the bottom-edge stub near Z (about 10%). Line 3: the left-edge stub (about 70% down) continues to the right-edge stub (about 54% down), so this near-horizontal line **rises** slightly to the right.

3. **What the gap must show.** Lines 1 and 2 enter the top edge close together and lean towards each other, so they cross just below the top edge. Line 3 stays between about 54% and 70% of the way down across the whole gap, so it cannot pass through that crossing point. It crosses line 2 in the left part of the gap and line 1 in the right part. That gives **three separate crossing points, forming a small triangle**.
4. **P fails.** Lines 1 and 2 are right, but its third line falls to the right, from high on the left edge (about 19% down) to low on the right edge (about 69% down). The outside stubs need the opposite: low on the left, higher on the right.
5. **Q fails.** Line 3 is right, but both steep lines rise to the right and never cross. No line reaches the right edge next to Y, and no line enters the top edge falling to the right, so line 1 is missing.
6. **R fails.** Lines 1 and 3 are right, but its rising line runs from the middle of the bottom edge (about 60%) to the right edge near X. The outside stubs need a line from the top edge at about 57% to the bottom edge near Z. R's three lines also pass through a single point, not a triangle.
7. **S matches all six edge points.** Top at about 31% to near Y (falling), top at about 57% to bottom at about 10% (rising), and left edge about 71% down to right edge about 54% down (rising slightly). The three crossings form a triangle, as required. Answer: **S**, option (D).

KEY IDEA

Each line has to pass straight through the gap. Join every stub to the stub on another edge with the same slope, which gives three lines: top 31% to the right edge near Y, top 57% to the bottom edge near Z, and left 70% to right 54%. Then check each option at the edges. P, Q and R each get exactly one of the three lines wrong. Only S has all three.

WATCH OUT FOR

Choosing P because its two top-edge lines and their crossing look right, without noticing that its near-horizontal line slopes the wrong way (high on the left, low on the right). Another trap is choosing R, whose single triple crossing looks neat, without checking that its rising line leaves through the right edge near X instead of the top edge.

Q.4 MCQ · 1 MARK

General Aptitude › Numerical

Real numbers y, p, and n (all greater than 1) satisfy

$$\left(\log_{p^{1/n}} y\right)\left(\log_{y^{1/n}} p\right) = 16,$$

where the logarithms are taken to the bases $p^{1/n}$ and $y^{1/n}$.

The value of n is _____

(A) 2

(B) 4

(C) 8

(D) 16

Official answer: **B**

SOLUTION

1. Power-of-base rule: $\log_{a^k} x = \dfrac{\ln x}{k \ln a} = \dfrac{1}{k}\log_a x$. With $k = \dfrac{1}{n}$ this gives $\log_{a^{1/n}} x = n \log_a x$.
2. So $\log_{p^{1/n}} y = n \log_p y$ and $\log_{y^{1/n}} p = n \log_y p$.
3. Multiply the two:
$$\left(\log_{p^{1/n}} y\right)\left(\log_{y^{1/n}} p\right) = n^2 \left(\log_p y\right)\left(\log_y p\right)$$
4. Reciprocal rule: $\log_p y \cdot \log_y p = \dfrac{\ln y}{\ln p} \cdot \dfrac{\ln p}{\ln y} = 1$. This is valid because $p, y > 1$, so $\ln p \neq 0$ and $\ln y \neq 0$: both bases are valid and neither log is zero.
5. The equation becomes $n^2 = 16$, so $n = \pm 4$.
6. The question says $n > 1$, so $n = 4$. This is option (B).

KEY IDEA

Taking the nth root of a log's base multiplies the log by n: $\log_{a^{1/n}} x = n \log_a x$. Each factor therefore gains a factor of n, and $\log_p y \cdot \log_y p = 1$. The whole product is n^2, so $n^2 = 16$.

WATCH OUT FOR

Treating $\log_{p^{1/n}} y$ as $\frac{1}{n} \log_p y$ (a root in the base multiplies the log by n, it does not divide it). That gives $\frac{1}{n^2} = 16$, which no option satisfies. The other common slip is stopping at $n = 16$ and forgetting that each of the two factors contributes an n, so the product holds n^2, not n.

Q.5 MCQ · 1 MARK

General Aptitude › Reasoning

The following observation is made about the scores obtained by 100 students in an exam:

'For each student, there exists another student in the class such that their scores are at most ten marks away.'

If the above statement is **false**, which one of the following statements is necessarily true?

(A) For each student, the scores of all the other students are more than 10 marks away.

(B) There exists at least one student in the class for whom the scores of all the other students are more than 10 marks away.

(C) There is exactly one student in the class for whom the scores of some students are more than 10 marks away.

(D) For each student, the score of exactly one other student is more than 10 marks away.

Official answer: **B**

Q.6 MCQ · 2 MARKS

General Aptitude › Grammar & vocabulary

Each one of the following clues contains a keyword that is partially filled.

Clue 1: Synonym of recognize (8 letters): _ D _ NT_ FY

Clue 2: A story long enough to fill a book (5 letters): _ _ _ EL

Clue 3: Two of something (6 letters): _ _ _ PLE

Clue 4: A fraction of something, split equally into two parts (4 letters): _ _ _ F

The first letter of each of the keywords can be rearranged to form a four-letter word.

Which one of the options below is a possible choice for the four-letter word?

(A) CHIN

(B) COIN

(C) ITCH

(D) NOSE

Official answer: **A**

SOLUTION

1. Clue 1: an 8-letter synonym of recognize that fits _ D _ NT_ FY (slots: _ D _ N T _ F Y) is **IDENTIFY** (I-D-E-N-T-I-F-Y). Every given letter sits in its printed slot. First letter: **I**.
2. Clue 2: a story long enough to fill a book, 5 letters, fitting _ _ _ EL, is **NOVEL**. First letter: **N**.
3. Clue 3: two of something, 6 letters, fitting _ _ _ PLE, is **COUPLE**. (TRIPLE fits the letter pattern but means three, so it does not match the clue.) First letter: **C**.
4. Clue 4: one of two equal parts of something, 4 letters, fitting _ _ _ F, is **HALF**. First letter: **H**.

5. The four first letters are I, N, C, H. Rearranged, they spell CHIN (and also INCH, which is not an option).
6. Check each option against the letter set {C, H, I, N}. CHIN uses exactly C, H, I and N, so it works. COIN needs an O, ITCH needs a T, and NOSE needs O, S and E. None of those letters is in the set, so B, C and D are ruled out.
7. The four-letter word is CHIN, which is option (A).

KEY IDEA

Solve each clue from both its meaning and its letter pattern (length plus the fixed letters), take the first letter of each keyword, then keep only the option that uses exactly those four letters, each once.

WATCH OUT FOR

Stopping after two or three clues and guessing. Every wrong option except NOSE shares three letters with the answer (COIN has C, I, N; ITCH has I, C, H), so you need all four keywords to decide. Another slip is taking TRIPLE for Clue 3 because it fits _ _ _ PLE. The clue says two, so the word is COUPLE.

Q.7 MCQ · 2 MARKS

General Aptitude › Reasoning

Q.7

Three children P, Q, R and two grown-ups X, Y play a badminton doubles tournament. X and Y are parents to two of the children playing. The child of X is not the same as the child of Y. Exactly one of the children does not have a parent playing in the tournament. The following rules are followed:

(i) A parent and his/her child cannot be on the same team.
(ii) A match can feature at most one parent and his/her child, that is, a maximum of one parent-child pair can play in a match.

The following matches were played:

	TEAM 1	**TEAM 2**
MATCH 1	P and X	Q and R
MATCH 2	P and R	X and Y
MATCH 3	R and X	Q and Y

Which one of the following options is correct?

(A)	P does not have any parent playing
(B)	Q does not have any parent playing
(C)	R does not have any parent playing
(D)	X does not have a child playing

Official answer: **C**

SOLUTION

1. Set-up: X and Y each have one child among P, Q, R, and the two children are different. So exactly one child has no parent playing, and option (D) is false from the start: the question says X is a parent of one of the children playing.
2. Rule (i): a parent and child are never on the same team. Match 1 puts P with X, so X is not P's parent. Match 3 puts R with X, so X is not R's parent. That leaves only one possibility: **X is Q's parent**.

3. Rule (i) again: in Match 3, Q is on Y's team, so Y is not Q's parent. (This also follows from X and Y having different children.) Y's child must therefore be P or R.
4. Rule (ii): a match can have at most one parent-child pair. In Match 3, X (Team 1) plays against Q (Team 2), so that match already has its one pair. If Y were R's parent, Y (Team 2) playing against R (Team 1) would be a second pair in the same match. So Y is not R's parent, and **Y is P's parent**.
5. Check every match with X as Q's parent and Y as P's parent. Match 1 (P and X vs Q and R): Y is not playing, so the only pair is X and Q, on opposite teams. Match 2 (P and R vs X and Y): Q is not playing, so the only pair is Y and P, on opposite teams. Match 3 (R and X vs Q and Y): P is not playing, so the only pair is X and Q, on opposite teams. No parent is ever on the same team as their own child, and no match has more than one pair. Both rules hold.
6. So X is Q's parent and Y is P's parent, which leaves R as the one child with no parent playing. Option (C) is correct.

KEY IDEA

Use rule (i) first, because it removes candidates directly: anyone who has partnered a grown-up cannot be that grown-up's child. Partnering P and R leaves X only one possible child. Then use rule (ii) on a match that already has one parent-child pair: it cannot hold a second one, and that decides Y's child.

WATCH OUT FOR

Using only rule (i). It fixes X as Q's parent, but Y could still seem to be the parent of P or R, which leaves a guess between (A) and (C). Only rule (ii) applied to Match 3, where X already faces Q, rules out Y being R's parent. Another slip is reading 'X and Y are parents to two of the children' as both grown-ups sharing the same children and so choosing (D). The question says the child of X is not the child of Y.

Q.8 MCQ · 2 MARKS

General Aptitude › Numerical

Let P_k represent the perimeter of a square with sides of length k. The value of the expression $P_1 + P_2 + P_3 + \cdots + P_{10}$ is _____

(A) 55

(B) 110

(C) 220

(D) 440

Official answer: **C**

SOLUTION

1. A square has 4 equal sides, so the perimeter of a square of side k is $P_k = 4k$.
2. So the sum is $P_1 + P_2 + \cdots + P_{10} = 4(1 + 2 + \cdots + 10)$. Take the common factor 4 out first.
3. The sum of the first n natural numbers is $\frac{n(n+1)}{2}$. With $n = 10$: $1 + 2 + \cdots + 10 = \frac{10 \times 11}{2} = 55$.
4. $$P_1 + P_2 + \cdots + P_{10} = 4 \times 55 = 220$$
5. Check by adding the terms directly: $4 + 8 + 12 + 16 + 20 + 24 + 28 + 32 + 36 + 40 = 220$. So the answer is 220, option (C).

KEY IDEA

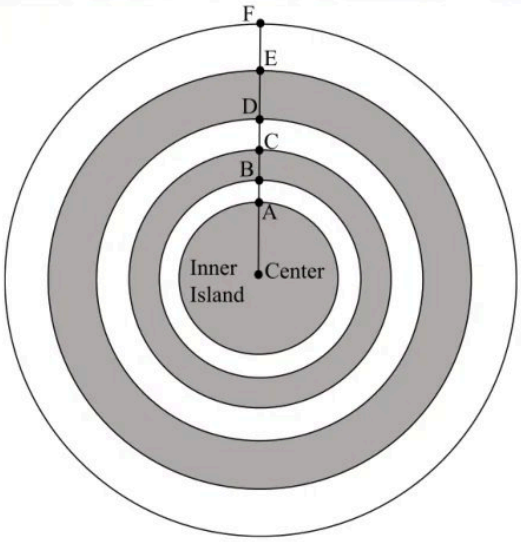
Write each term in terms of k ($P_k = 4k$), take the common factor 4 out, and use $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$.

WATCH OUT FOR

Summing the side lengths and stopping there gives $1 + 2 + \cdots + 10 = 55$ (option A). Using $2k$ per square, i.e. only two sides or a semi-perimeter, gives 110 (option B). Using the square's area-like factor or doubling the perimeter by mistake gives 440 (option D).

Q.9 MCQ · 2 MARKS

General Aptitude › Geometry

Q.9	The city of Atlantis was crafted by the God of the seas, Poseidon. It was made of alternating concentric circular rings of land (shaded) and water (not shaded) as represented in the figure (not to scale). The radius of Inner Island was 2.5 stades (a unit of length used in ancient Greece). The water surrounding Inner Island was one stade wide (length AB). This was surrounded by two pairs of alternating rings of land and water. The first pair of land and water was two stades wide each (lengths BC and CD), and the outer pair is three stades wide each (lengths DE and EF). The ratio of the surface area of the land to that of the water in the city of Atlantis is _____ (round off to two decimal places). 
(A)	0.45
(B)	0.60
(C)	0.75
(D)	0.90

Official answer: **C**

SOLUTION

1. Build the radii outward from the centre by adding the widths (all in stades): Inner Island $r_A = 2.5$; $r_B = 2.5 + 1 = 3.5$; $r_C = 3.5 + 2 = 5.5$; $r_D = 5.5 + 2 = 7.5$; $r_E = 7.5 + 3 = 10.5$; $r_F = 10.5 + 3 = 13.5$.
2. The area of a ring between radii $r_1 < r_2$ is $\pi(r_2^2 - r_1^2)$, and the Inner Island is a full disc of area πr_A^2. Every area carries the factor π, which cancels in the ratio, so work with the r^2 values only.

3. Squares of the radii: $2.5^2 = 6.25$, $3.5^2 = 12.25$, $5.5^2 = 30.25$, $7.5^2 = 56.25$, $10.5^2 = 110.25$, $13.5^2 = 182.25$.
4. Land (shaded: the Inner Island disc, ring BC and ring DE):
$$6.25 + (30.25 - 12.25) + (110.25 - 56.25) = 6.25 + 18 + 54 = 78.25$$
5. Water (unshaded: rings AB, CD and EF):
$$(12.25 - 6.25) + (56.25 - 30.25) + (182.25 - 110.25) = 6 + 26 + 72 = 104$$
6. Check: land + water must equal the whole city, a disc of radius 13.5: $78.25 + 104 = 182.25 = 13.5^2$. It does. (Each ring can also be checked as $(r_2 - r_1)(r_2 + r_1)$, e.g. ring DE: $3 \times 18 = 54$.)
7. Ratio of land to water:
$$\frac{78.25}{104} = 0.7524\ldots \approx 0.75$$
which is option (C).

KEY IDEA

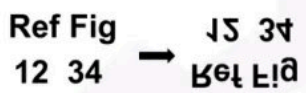
A ring of inner radius r_1 and outer radius r_2 has area $\pi(r_2^2 - r_1^2)$. Work out each boundary radius by adding the widths from the centre outward. The factor π cancels in a ratio of areas, so only the differences of squared radii matter. A quick check is that all the pieces together must add up to the area of the full disc.

WATCH OUT FOR

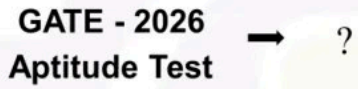
Comparing widths instead of areas: land widths $2.5 + 2 + 3 = 7.5$ against water widths $1 + 2 + 3 = 6$ gives 1.25, but outer rings of the same width cover much more area. Another slip is leaving the Inner Island out of the land total, which gives $72/104 \approx 0.69$. Others stop the city at E and drop the outer water ring EF, or divide land by the whole city ($78.25/182.25 \approx 0.43$) instead of by the water.

Q.10 MCQ · 2 MARKS

Q.10 If



Then





(A)	P
(B)	Q
(C)	R
(D)	S

Official answer: **C**

SOLUTION

1. **Read the reference pair.** On the left, "Ref Fig" is on top and "12 34" is below. On the right, "12 34" is on top and "Ref Fig" is below, and every glyph is upside down: f looks like t, g hangs upward, the 1 has its flag at the bottom, the 4 has its triangle at the bottom, and the 3 looks unchanged.
2. **Check the left-to-right order.** It has not changed. "Ref" still comes before "Fig", "12" still comes before "34", and the letters inside each word are in their original order. So the rule is not a 180° rotation, which

would reverse the reading order and swap the lines. It is not a left-right mirror either, which would reverse the order and leave the lines where they were.

3. **Name the transformation.** It is a water image: a reflection in a horizontal mirror line. Such a reflection swaps top and bottom but keeps every point's left-right position. So the two lines change places, each character turns upside down, and the reading order stays the same.
4. **Apply it to the question.** Take "GATE - 2026" (top) and "Aptitude Test" (bottom). In the image, the top line must be "Aptitude Test" with "Aptitude" first, and the bottom line must be "GATE - 2026" with "GATE" first. Every glyph is flipped vertically.
5. **Rule out the wrong options.** P has "Test Aptitude" on top, so the word order is reversed. Q has "2026-GATE" at the bottom, so the word order is reversed. S reverses the word order on both lines.
6. **Conclude.** Only R has "Aptitude Test" on top and "GATE - 2026" at the bottom, both in their original left-to-right order with vertically flipped glyphs. So R is the required figure, which is option (C).

KEY IDEA

A water image (reflection in a horizontal line) swaps top and bottom and turns each character upside down. It never changes left-to-right order. A left-right mirror reverses the order and keeps the lines where they are. A 180° rotation does both: it reverses the order and swaps the lines.

WATCH OUT FOR

Seeing upside-down text and assuming a 180° rotation, then choosing an option with the word order reversed (S, or P or Q, which each reverse one line). Check the reference: "Ref" still comes before "Fig" and "12" before "34", so the reading order is kept and only the vertical direction changes.

Electronics and Communication Engineering Q.11 to Q.65

Q.11 MCQ · 1 MARK

Engineering Mathematics › Differential equations

Consider the differential equation $\dot{\vec{w}} = A\vec{w}$, with $\vec{w}(t=0) = \begin{bmatrix}1\\1\end{bmatrix}$.

If $\vec{w}(t) = e^{t}\vec{u}_x + e^{-2t}\vec{u}_y$ be the solution to the equation where $\vec{u}_x$ and $\vec{u}_y$ are unit vectors along the positive x and y axes respectively, then which of the following options is the correct matrix representing A?

(A) $\begin{bmatrix}1 & 0\\ 0 & -2\end{bmatrix}$

(B) $\begin{bmatrix}-1 & 0\\ 0 & 2\end{bmatrix}$

(C) $\begin{bmatrix}0 & -2\\ 1 & 0\end{bmatrix}$

(D) $\begin{bmatrix}0 & 2\\ -1 & 0\end{bmatrix}$

Official answer: **A**

SOLUTION

1. Write the solution in components: $\vec{w}(t) = \begin{bmatrix}e^{t}\\ e^{-2t}\end{bmatrix}$. At $t = 0$ this is $\begin{bmatrix}1\\1\end{bmatrix}$, so it satisfies the initial condition.
2. Differentiate each component: $\dot{\vec{w}}(t) = \begin{bmatrix}e^{t}\\ -2e^{-2t}\end{bmatrix}$.
3. Split the solution into its modes: $\vec{w}(t) = e^{t}\begin{bmatrix}1\\0\end{bmatrix} + e^{-2t}\begin{bmatrix}0\\1\end{bmatrix}$. Substituting into $\dot{\vec{w}} = A\vec{w}$ gives $e^{t}\,A\begin{bmatrix}1\\0\end{bmatrix} + e^{-2t}\,A\begin{bmatrix}0\\1\end{bmatrix} = e^{t}\begin{bmatrix}1\\0\end{bmatrix} - 2e^{-2t}\begin{bmatrix}0\\1\end{bmatrix}$ for every t.
4. The functions e^{t} and e^{-2t} are linearly independent, so their coefficients must match separately: $A\begin{bmatrix}1\\0\end{bmatrix} = 1\cdot\begin{bmatrix}1\\0\end{bmatrix}$ and $A\begin{bmatrix}0\\1\end{bmatrix} = -2\begin{bmatrix}0\\1\end{bmatrix}$. So $\vec{u}_x$ and $\vec{u}_y$ are eigenvectors of A with eigenvalues 1 and -2. (A mode $e^{\lambda t}\vec{v}$ solves the system only when $A\vec{v} = \lambda\vec{v}$.)
5. The columns of A are $A\vec{u}_x$ and $A\vec{u}_y$, so
$$A = \begin{bmatrix}1 & 0\\ 0 & -2\end{bmatrix}.$$
6. Check: $\begin{bmatrix}1 & 0\\ 0 & -2\end{bmatrix}\begin{bmatrix}e^{t}\\ e^{-2t}\end{bmatrix} = \begin{bmatrix}e^{t}\\ -2e^{-2t}\end{bmatrix} = \dot{\vec{w}}$. The other options fail. (B) gives $\begin{bmatrix}-e^{t}\\ 2e^{-2t}\end{bmatrix}$, which has the wrong signs. (C) gives $\begin{bmatrix}-2e^{-2t}\\ e^{t}\end{bmatrix}$ and (D) gives $\begin{bmatrix}2e^{-2t}\\ -e^{t}\end{bmatrix}$, and both swap the two components. Also, (C) and (D) both have trace 0 and determinant 2, so their eigenvalues are $\pm j\sqrt{2}$. That would give oscillatory solutions, not real exponentials. So the answer is (A).

KEY IDEA

For $\dot{\vec{w}} = A\vec{w}$, each term $e^{\lambda t}\vec{v}$ of the solution is a mode, and a mode only works when $A\vec{v} = \lambda\vec{v}$. Here the modes lie along $\vec{u}_x$ (with $\lambda = 1$) and $\vec{u}_y$ (with $\lambda = -2$). The eigenvectors are the standard basis vectors, so A is simply $\mathrm{diag}(1, -2)$: its eigenvalues sit on the diagonal in the same order as the axes.

WATCH OUT FOR

One mistake is picking (B) by reading the exponent e^{-2t} as a pole at $+2$, which flips the signs. The eigenvalue is the coefficient of t in the exponent itself, so it is -2 and not $+2$. Another is choosing (C) or (D) because they contain the right numbers 1 and -2 in off-diagonal places. But those matrices mix x and y, and their eigenvalues $\pm j\sqrt{2}$ would give oscillation, not e^{t} and e^{-2t}.

Q.12 MCQ · 1 MARK

Engineering Mathematics › Vector calculus

A surface is given by $z^2 = 2x^2 - y^2$ and $\vec{n}$ and $-\vec{n}$ are unit normal vectors to the surface at the point $\vec{P} = \hat{\imath} + \sqrt{2}\,\hat{k}$.

Which of the following vectors can be $\vec{n}$, where $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ are the unit vectors along x, y and z axes, respectively?

(A) $\hat{\imath} - \sqrt{2}\,\hat{k}$

(B) $\frac{2}{3}\hat{\imath} - \frac{1}{3}\hat{k}$

(C) $\sqrt{2}\,\hat{\imath} - \sqrt{3}\,\hat{k}$

(D) $\dfrac{\sqrt{2}\,\hat{\imath} - \hat{k}}{\sqrt{3}}$

Official answer: **D**

SOLUTION

1. Write the surface as a level surface: $\phi(x, y, z) = 2x^2 - y^2 - z^2 = 0$. The point is $P = (1, 0, \sqrt{2})$. It lies on the surface because $2(1)^2 - 0^2 - (\sqrt{2})^2 = 2 - 0 - 2 = 0$.
2. The gradient of ϕ is normal to the level surface $\phi = c$: $\nabla\phi = \dfrac{\partial\phi}{\partial x}\hat{\imath} + \dfrac{\partial\phi}{\partial y}\hat{\jmath} + \dfrac{\partial\phi}{\partial z}\hat{k} = 4x\,\hat{\imath} - 2y\,\hat{\jmath} - 2z\,\hat{k}$.
3. At $P = (1, 0, \sqrt{2})$: $\nabla\phi = 4\,\hat{\imath} + 0\,\hat{\jmath} - 2\sqrt{2}\,\hat{k}$.
4. Its magnitude is $|\nabla\phi| = \sqrt{4^2 + 0^2 + (2\sqrt{2})^2} = \sqrt{16 + 8} = \sqrt{24} = 2\sqrt{6}$.
5. Divide by the magnitude to get the unit normal:
$$\vec{n} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{4\,\hat{\imath} - 2\sqrt{2}\,\hat{k}}{2\sqrt{6}} = \frac{2}{\sqrt{6}}\hat{\imath} - \frac{\sqrt{2}}{\sqrt{6}}\hat{k} = \frac{\sqrt{2}}{\sqrt{3}}\hat{\imath} - \frac{1}{\sqrt{3}}\hat{k} = \frac{\sqrt{2}\,\hat{\imath} - \hat{k}}{\sqrt{3}}$$
6. Check: $|\vec{n}|^2 = \dfrac{(\sqrt{2})^2 + 1^2}{3} = \dfrac{3}{3} = 1$, so this is a unit vector. It is option (D), and $-\vec{n}$ is the other unit normal.
7. Rule out the others. (A) has magnitude $\sqrt{1 + 2} = \sqrt{3}$ and (C) has magnitude $\sqrt{2 + 3} = \sqrt{5}$, so neither is a unit vector. (B) has magnitude $\sqrt{\frac{4}{9} + \frac{1}{9}} = \frac{\sqrt{5}}{3} \neq 1$, and it points the wrong way too: its $\hat{\imath} : \hat{k}$ ratio is $2 : -1$, but the normal needs $\sqrt{2} : -1$. The answer is (D).

KEY IDEA

Write the surface as a level surface $\phi(x, y, z) = 0$. The gradient $\nabla\phi$ at a point on it is normal to the surface there, and $\pm\nabla\phi/|\nabla\phi|$ are the two unit normals. An option has to pass two tests: its components must be in the same ratio as $\nabla\phi$, and its magnitude must be 1.

WATCH OUT FOR

Stopping at $\nabla\phi = 4\,\hat{\imath} - 2\sqrt{2}\,\hat{k}$ and choosing an option by direction alone, without dividing by $|\nabla\phi| = 2\sqrt{6}$. The question asks for a unit normal, and (A) and (C) are not unit vectors. Other slips are reading the point as $(1, \sqrt{2}, 0)$ instead of $(1, 0, \sqrt{2})$, and getting the sign of the z term wrong when moving z^2 across. Either one puts a spurious $\hat{\jmath}$ component into the normal or changes its direction.

Q.13 MCQ · 1 MARK

Signals and Systems › Laplace

The Laplace Transform of the signal $x(t) = u(t - 2) * (t\,u(t))$ is given by which of the following expressions?

["$*$" represents convolution operator]

(A) $\dfrac{e^{-2s}}{s^2(s - 2)}$

(B) $\dfrac{e^{-2(s-2)}}{s^3}$

(C) $\dfrac{s\,e^{-2s}}{(s-2)^2}$

(D) $\dfrac{e^{-2s}}{s^3}$

Official answer: **D**

SOLUTION

1. **Convolution property.** Convolution in time becomes multiplication in s:
$$X(s) = \mathcal{L}\{u(t-2)\} \cdot \mathcal{L}\{t\,u(t)\}$$
2. **Time-shifting property.** $\mathcal{L}\{u(t)\} = \dfrac{1}{s}$ for $\mathrm{Re}(s) > 0$. Delaying a signal by 2 multiplies its transform by e^{-2s}, so
$$\mathcal{L}\{u(t-2)\} = \frac{e^{-2s}}{s}, \quad \mathrm{Re}(s) > 0$$
3. **Standard pair (ramp).**
$$\mathcal{L}\{t\,u(t)\} = \frac{1}{s^2}, \quad \mathrm{Re}(s) > 0$$
4. **Multiply.** Both transforms exist for $\mathrm{Re}(s) > 0$, so the product is valid there:
$$X(s) = \frac{e^{-2s}}{s} \cdot \frac{1}{s^2} = \frac{e^{-2s}}{s^3}, \quad \mathrm{Re}(s) > 0$$
5. **Check in time.** Convolving with $u(t)$ integrates a signal from 0 to t, so $u(t) * t\,u(t) = \dfrac{t^2}{2}u(t)$. With the step delayed by 2, the result is delayed by 2: $x(t) = \dfrac{(t-2)^2}{2}u(t-2)$. Since $\mathcal{L}\{t^n u(t)\} = \dfrac{n!}{s^{n+1}}$, we get $\mathcal{L}\left\{\dfrac{t^2}{2}u(t)\right\} = \dfrac{1}{s^3}$. The delay multiplies this by e^{-2s}, which gives $\dfrac{e^{-2s}}{s^3}$, option (D).

KEY IDEA

Convolution in time becomes a product in s, and a time delay of t_0 becomes the factor e^{-st_0}. Here $\frac{e^{-2s}}{s} \cdot \frac{1}{s^2} = \frac{e^{-2s}}{s^3}$. In the time domain, convolving with a delayed step integrates the ramp once and delays the result: $x(t) = \frac{(t-2)^2}{2}u(t-2)$.

WATCH OUT FOR

Mixing up the two shifting properties. A delay in time, $x(t-2)$, multiplies the transform by e^{-2s}. A shift in s, such as $(s-2)$ or $e^{-2(s-2)}$, comes from multiplying by e^{2t}, and this signal has no such factor. That confusion produces options (A), (B) and (C). A second slip is to multiply the two signals in time instead of their transforms.

Q.14 MCQ · 1 MARK

Electronic Devices › P-N junction

Consider carrier transport in a Zener diode in the breakdown region.

Which is the dominant transport mechanism for current flow in this case?

(A) Drift

(B) Diffusion

(C) Tunneling

(D) Ballistic transport

Official answer: **C**

SOLUTION

1. Zener diodes are fabricated with heavily doped p and n regions (N_A and N_D both large). The depletion width W scales as $1/\sqrt{N}$ (dominated by the more lightly doped side), so with both sides heavily

doped, W is extremely small.

2. Under reverse bias V_R, this very thin depletion width produces an extremely high electric field, $E = V_R/W$, across the junction even for a modest reverse voltage.
3. This strong field tilts the energy bands so steeply that the valence band edge on the p-side comes into energetic alignment with the conduction band edge on the n-side, separated only by a very thin barrier.
4. Electrons filling the valence band on the p-side can then pass directly, without gaining kinetic energy, into empty conduction-band states on the n-side through this thin barrier. This direct quantum-mechanical tunneling through the barrier is the Zener effect, distinct from carrier acceleration followed by impact ionization (avalanche breakdown).
5. Hence the dominant carrier transport mechanism in the breakdown region of a (heavily doped) Zener diode is tunneling: option (C).

KEY IDEA

A Zener diode is deliberately made with heavily doped p and n regions, so its depletion width is extremely thin; under reverse bias this thin barrier develops a very high electric field, which allows electrons to tunnel directly from the valence band on the p-side to the conduction band on the n-side. This is the Zener (tunneling) breakdown mechanism, distinct from avalanche breakdown (drift + impact ionization) that dominates in lightly doped, lightly biased junctions.

WATCH OUT FOR

Confusing the Zener (tunneling) breakdown mechanism with avalanche breakdown (impact ionization due to carrier drift), or assuming all diode breakdown is "avalanche" regardless of doping level.

Q.15 MCQ · 1 MARK

Signals and Systems › Sampling

Two analog signals $x_1(t)$ and $x_2(t)$ (t in second), are sampled at a rate $F_s = 40$ Hz, where $x_1(t) = \cos(20\pi t),\ t \geq 0$, and $x_2(t) = \cos(100\pi t),\ t \geq 0$.

The first ten samples (starting from $t = 0$) are considered for the analysis.

Which of the following statements is TRUE?

(A) All of the first three samples of $x_1(t)$ are greater than the corresponding samples of $x_2(t)$.

(B) All of the last three samples of $x_1(t)$ are greater than the corresponding samples of $x_2(t)$.

(C) All of the samples of $x_2(t)$ are greater than the corresponding samples of $x_1(t)$.

(D) All of the fourth to seventh samples of $x_1(t)$ are equal to the corresponding samples of $x_2(t)$.

Official answer: **D**

SOLUTION

1. Sampling at $F_s = 40$ Hz gives sampling instants $t = nT_s = \dfrac{n}{40}$ s. The first ten samples, starting from $t = 0$, are $n = 0, 1, \ldots, 9$.
2. Samples of x_1: $x_1[n] = \cos\left(20\pi \cdot \dfrac{n}{40}\right) = \cos\left(\dfrac{\pi n}{2}\right)$.
3. Samples of x_2: $x_2[n] = \cos\left(100\pi \cdot \dfrac{n}{40}\right) = \cos\left(\dfrac{5\pi n}{2}\right)$.
4. Because n is an integer, $2\pi n$ is a whole number of cycles, and cosine has period 2π:

$$\cos\left(\frac{5\pi n}{2}\right) = \cos\left(2\pi n + \frac{\pi n}{2}\right) = \cos\left(\frac{\pi n}{2}\right).$$

So $x_2[n] = x_1[n]$ for every n.

5. Aliasing view of the same fact: x_2 is a $50\ \text{Hz}$ tone, above $F_s/2 = 20\ \text{Hz}$, so after sampling it appears at $|50 - 40| = 10\ \text{Hz}$, which is exactly the frequency of x_1. The two sampled sequences cannot be told apart.
6. Both sequences, for $n = 0, \ldots, 9$, are $1, 0, -1, 0, 1, 0, -1, 0, 1, 0$.
7. Every pair of corresponding samples is equal, so no sample of one signal is greater than the other: (A), (B) and (C) are all false. The fourth to seventh samples ($n = 3$ to 6) are $0, 1, 0, -1$ for both signals, so they are equal and (D) is TRUE.

KEY IDEA

A sinusoid at f_0 sampled at F_s gives the same samples as one at $f_0 - kF_s$ for any integer k, because the discrete-time frequency $2\pi f_0/F_s$ is only defined modulo 2π. Here $50\ \text{Hz} - 40\ \text{Hz} = 10\ \text{Hz}$, so x_2 aliases exactly onto x_1 and every sample matches.

WATCH OUT FOR

Comparing the continuous-time waveforms instead of the samples. $\cos(100\pi t)$ oscillates five times faster than $\cos(20\pi t)$, so at most instants the two signals differ, and that tempts students towards (A), (B) or (C). At the sampling instants $t = n/40$, though, the extra $80\ \text{Hz}$ ($2\pi n$ in phase) adds a whole number of cycles, and the samples are identical. A second slip is counting the fourth sample as $n = 4$; since sampling starts at $n = 0$, the fourth to seventh samples are $n = 3$ to 6. Here that makes no difference because every sample is equal.

Q.16 MCQ · 1 MARK

Signals and Systems › z-transform

The response of a discrete time system $y[n]$ obeys the following relation:

$$y[n] = \frac{5}{6}y[n-1] - \frac{1}{6}y[n-2] + x[n].$$

The input to the system is $x[n] = \delta[n] - \frac{1}{3}\delta[n-1]$.

Which of the following options is TRUE for $y[n]$?

(A) Stable and causal response

(B) Stable and non-causal response

(C) Unstable and causal response

(D) Unstable and non-causal response

Official answer: **A**

SOLUTION

1. Take the z-transform of the difference equation, assuming initial rest (the system is run forward from $y[-1] = y[-2] = 0$):
$$Y(z)\left(1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}\right) = X(z)$$
2. Factor the denominator. The roots of $z^2 - \frac{5}{6}z + \frac{1}{6} = 0$ are $z = \frac{1}{2}$ and $z = \frac{1}{3}$, since $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ and $\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$. So
$$H(z) = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{3}z^{-1}\right)}$$
3. Transform the input using $\delta[n-k] \leftrightarrow z^{-k}$: $X(z) = 1 - \frac{1}{3}z^{-1}$.
4. Multiply. The input's zero at $z = \frac{1}{3}$ cancels the pole at $z = \frac{1}{3}$:
$$Y(z) = H(z)X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}$$
5. A recursive difference equation run forward from rest gives a right-sided output, so the ROC is $|z| > \frac{1}{2}$. Inverting, $y[n] = \left(\frac{1}{2}\right)^n u[n]$.

6. Check by direct recursion: $y[0] = x[0] = 1$, $y[1] = \frac{5}{6} \cdot 1 - \frac{1}{3} = \frac{1}{2}$, $y[2] = \frac{5}{6} \cdot \frac{1}{2} - \frac{1}{6} \cdot 1 + 0 = \frac{1}{4}$. This matches $\left(\frac{1}{2}\right)^n$.
7. $y[n] = 0$ for $n < 0$, so the response is causal. Also $\sum_n |y[n]| = \sum_{n \geq 0} \left(\frac{1}{2}\right)^n = 2 < \infty$, and the ROC $|z| > \frac{1}{2}$ contains the unit circle $|z| = 1$, so the response is stable (absolutely summable). The answer is (A).

KEY IDEA

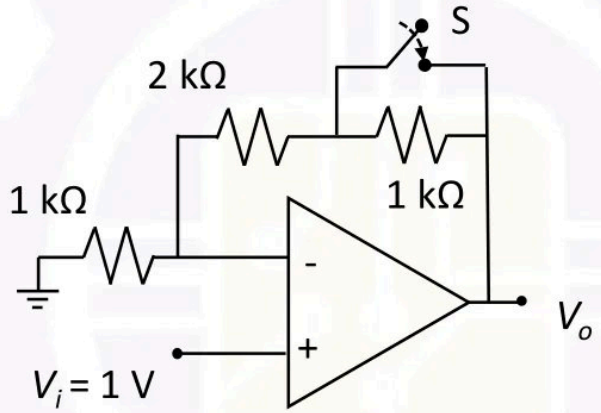
Find $Y(z) = H(z)X(z)$. The input's zero at $z = \frac{1}{3}$ cancels one system pole, which leaves one pole at $z = \frac{1}{2}$. A recursion run forward from rest gives a right-sided output, so the ROC is $|z| > \frac{1}{2}$. That ROC contains the unit circle, so $y[n] = \left(\frac{1}{2}\right)^n u[n]$ is both causal and absolutely summable.

WATCH OUT FOR

Taking the left-sided ROC $|z| < \frac{1}{2}$ gives $y[n] = -\left(\frac{1}{2}\right)^n u[-n-1]$, which grows like $2^{|n|}$ as $n \to -\infty$. That points to option (D), "unstable and non-causal". But a difference equation computed forward in time from rest always gives a right-sided response. A second slip is to judge the response by the system's poles alone and miss the pole-zero cancellation. Here both poles ($\frac{1}{2}$ and $\frac{1}{3}$) lie inside the unit circle anyway, so the conclusion is the same, but the cancellation is what makes $y[n]$ a single geometric sequence.

Q.17 MCQ · 1 MARK

Analog Electronics › Op-amps

Q.17	The ideal OP-AMP circuit shown in the Figure produces output voltage $V_o = x$ when the Switch, S, is open. Which of the options represents the output voltage when S is closed? 
(A)	x
(B)	$\frac{2}{3}x$
(C)	$\frac{3}{4}x$
(D)	$\frac{1}{2}x$

Official answer: **C**

SOLUTION

1. Assume an ideal op-amp: the virtual short gives $V_- = V_+ = V_i = 1$ V, and no current flows into either input. The circuit is a non-inverting amplifier with $R_1 = 1\ \text{k}\Omega$ from the inverting node to ground and a feedback resistance R_f from the inverting node to the output.

2. KCL at the inverting node: the current through R_1 is $\frac{V_-}{R_1}$ and all of it flows through R_f, so $\frac{V_o - V_-}{R_f} = \frac{V_-}{R_1}$. This gives the non-inverting gain
$$\frac{V_o}{V_i} = 1 + \frac{R_f}{R_1}.$$
3. S open: the feedback path is the $2\ \text{k}\Omega$ resistor in series with the $1\ \text{k}\Omega$ resistor, so $R_f = 3\ \text{k}\Omega$ and
$$x = V_o = \left(1 + \frac{3\ \text{k}\Omega}{1\ \text{k}\Omega}\right)(1\ \text{V}) = 4\ \text{V}.$$
4. S closed: S is wired from the junction of the two feedback resistors straight to the output, which puts it across the output-side $1\ \text{k}\Omega$ resistor. Closing it shorts that resistor, leaving $R_f = 2\ \text{k}\Omega$.
5. Output with S closed:
$$V_o' = \left(1 + \frac{2\ \text{k}\Omega}{1\ \text{k}\Omega}\right)(1\ \text{V}) = 3\ \text{V}.$$
6. Ratio: $\frac{V_o'}{x} = \frac{3\ \text{V}}{4\ \text{V}} = \frac{3}{4}$, so $V_o' = \frac{3}{4}x$. This is option (C).

KEY IDEA

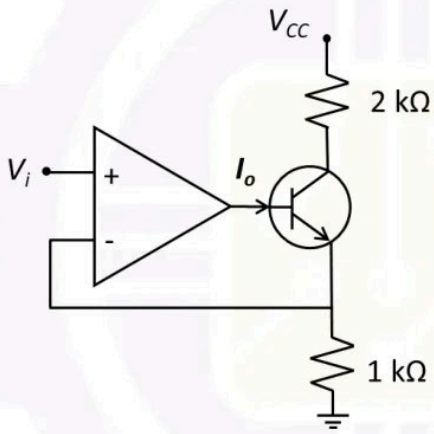
For an ideal non-inverting amplifier, $V_o = \left(1 + \frac{R_f}{R_1}\right) V_i$. Closing S shorts the $1\ \text{k}\Omega$ feedback resistor, so R_f drops from $3\ \text{k}\Omega$ to $2\ \text{k}\Omega$ and the gain drops from 4 to 3. The ratio of the two outputs is the ratio of the two gains, $3/4$.

WATCH OUT FOR

Taking the ratio of the feedback resistances ($R_f'/R_f = 2/3$, option B) instead of the ratio of the gains. A non-inverting gain is $1 + R_f/R_1$, not R_f/R_1, so the '1 +' term has to be kept. Another slip is using the inverting-amplifier formula $-R_f/R_1$, which also gives $2/3$ for the ratio.

Q.18 MCQ · 1 MARK

Analog Electronics › Biasing

Q.18	Consider the circuit shown in the Figure with $V_i = 3$ V and $V_{CC} = 12$ V. Assume $V_{BE} = 0.7$ V and $\beta_{dc} = 99$ for the BJT. Which of the following options is the correct value of the current I_o? 
(A)	60 μA
(B)	6 μA
(C)	3 μA
(D)	30 μA

Official answer: **D**

SOLUTION

1. Assume an ideal op-amp: it draws no input current, and its gain is large enough to hold both inputs at the same voltage. The emitter is wired back to the inverting input. If the emitter voltage rises, the op-amp output (the base) falls and pulls the emitter back down, so this is negative feedback and the virtual short holds: $V_E = V_i = 3$ V.

2. The inverting input draws no current, so all of the emitter current flows through the $1\ \text{k}\Omega$ resistor. By Ohm's law, $I_E = \dfrac{V_E}{1\ \text{k}\Omega} = \dfrac{3\ \text{V}}{1\ \text{k}\Omega} = 3\ \text{mA}$.
3. I_o is the current from the op-amp output into the base, so $I_o = I_B$. In the active region $I_E = (\beta_{dc} + 1)I_B$, which gives
$$I_o = I_B = \frac{I_E}{\beta_{dc} + 1} = \frac{3\ \text{mA}}{99 + 1} = 30\ \mu\text{A}$$
4. Check that the BJT really is in the active region. $I_C = \beta_{dc} I_B = 99 \times 30\ \mu\text{A} = 2.97\ \text{mA}$. KVL on the collector side gives $V_C = 12\ \text{V} - (2.97\ \text{mA})(2\ \text{k}\Omega) = 12 - 5.94 = 6.06\ \text{V}$.
5. The op-amp output sits at $V_B = V_E + V_{BE} = 3 + 0.7 = 3.7\ \text{V}$ (the ideal op-amp is assumed able to reach this level). Then $V_{CB} = 6.06 - 3.7 = 2.36\ \text{V} > 0$, so the collector-base junction is reverse biased, and $V_{CE} = 6.06 - 3 = 3.06\ \text{V}$ is well above $V_{CE(sat)}$. The transistor is in the active region, so $I_E = (\beta_{dc} + 1)I_B$ was valid.
6. So $I_o = 30\ \mu\text{A}$, which is option (D).

KEY IDEA

This is an op-amp driving a BJT, the standard voltage-to-current converter. Negative feedback through the emitter forces $V_E = V_i$. The op-amp raises its output to $V_i + V_{BE}$, which cancels V_{BE}. So the emitter current is set only by $V_i / R_E = 3\ \text{mA}$, whatever β or V_{BE} are. The op-amp only has to supply the base current, $I_E/(\beta_{dc} + 1)$. The $2\ \text{k}\Omega$ collector resistor and V_{CC} play no part in I_o; they matter only for checking that the transistor stays active.

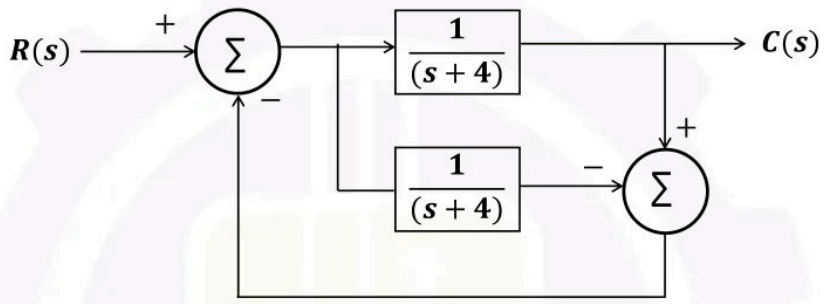
WATCH OUT FOR

The first mistake is to treat the circuit as a plain emitter follower and write $V_E = V_i - V_{BE} = 2.3\ \text{V}$. The feedback loop puts the base at $3.7\ \text{V}$, so the emitter sits at exactly $V_i = 3\ \text{V}$. The second is to take I_o as the emitter or collector current (about $3\ \text{mA}$) rather than the base current the op-amp supplies. The third is to divide by β instead of $\beta + 1$: that gives $30.3\ \mu\text{A}$, which is close but not exact. With $\beta_{dc} = 99$, $\beta + 1 = 100$ gives exactly $30\ \mu\text{A}$.

Q.19 MCQ · 1 MARK

Q.19 A control system is shown in the Figure.

Which option represents the correct transfer function of the system?



(A) $\frac{1}{(s+4)^2}$

(B) $\frac{1}{(s+4)}$

(C) $\frac{2}{(s+4)}$

(D) $\frac{1}{(s^2+8s+17)}$

Official answer: **B**

SOLUTION

1. Call the signal leaving the first summing junction $E(s)$. The top block and the bottom block both take $E(s)$ as input, so the top block gives $C(s) = \frac{E(s)}{s+4}$ and the bottom block also gives $\frac{E(s)}{s+4}$.
2. Summing junction 2 takes $+C(s)$ and subtracts the bottom-block output, so the feedback signal is

$$F(s) = C(s) - \frac{E(s)}{s+4} = \frac{E(s)}{s+4} - \frac{E(s)}{s+4} = 0$$

3. Summing junction 1 gives $E(s) = R(s) - F(s) = R(s)$. The feedback path carries no signal because the two identical branches cancel exactly.
4. Check with the general reduction. Take G_1 as the top block and G_2 as the bottom block. Since $E = C/G_1$, the feedback is $F = C - G_2E = C\left(1 - \dfrac{G_2}{G_1}\right)$, so $H_{\text{eq}} = 1 - \dfrac{G_2}{G_1}$ and
$$\frac{C}{R} = \frac{G_1}{1 + G_1H_{\text{eq}}} = \frac{G_1}{1 + G_1 - G_2}$$
With $G_1 = G_2 = \dfrac{1}{s+4}$ the denominator is 1.
5. Therefore
$$\frac{C(s)}{R(s)} = \frac{1}{s+4}$$
which is option (B).

KEY IDEA

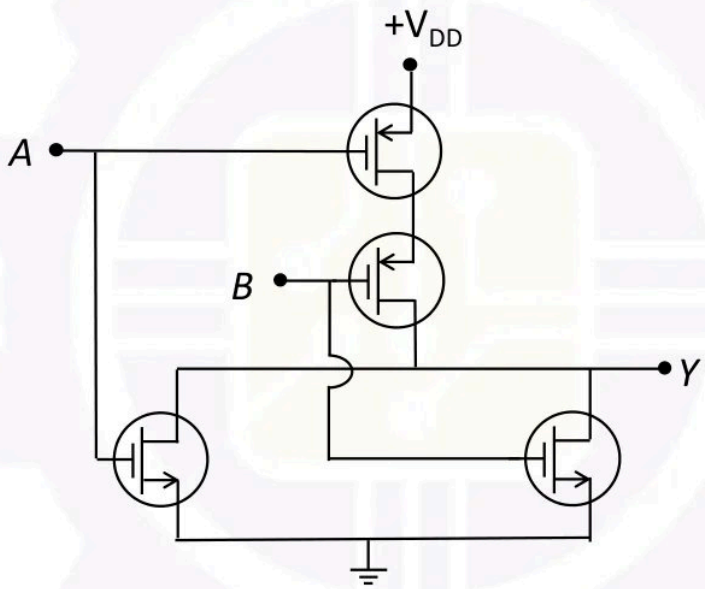
Work out what signal actually reaches each summing junction before you reach for a reduction formula. Summing junction 2 subtracts two copies of the same signal, $E(s)/(s+4)$, so the feedback is exactly zero. The loop is effectively open and C/R is just the forward block $\dfrac{1}{s+4}$.

WATCH OUT FOR

Treating the two $\dfrac{1}{s+4}$ blocks as a parallel pair whose outputs add, which gives $\dfrac{2}{s+4}$ (option C). Only the top block feeds $C(s)$. The other common error is using the standard $\dfrac{G}{1+GH}$ with $H = \dfrac{1}{s+4}$ and ignoring the $+C$ input at summing junction 2. That misses the cancellation, and the resulting denominator is not in the options.

Q.20 MCQ · 1 MARK

Digital Electronics › Logic families

Q.20	In the circuit shown in the Figure, A and B are logic inputs and Y is the logic output. Which of the following logic operations is realized by the circuit? 
(A)	NOR
(B)	OR
(C)	XOR
(D)	AND

Official answer: **A**

SOLUTION

1. Identify the devices from the source arrows. The two upper transistors have the arrow on the source pointing into the channel, and the top one's source is at $+V_{DD}$. They are enhancement PMOS devices and form the pull-up network. The two lower transistors have the arrow pointing out of the channel, with their sources on the ground rail. They are enhancement NMOS devices and form the pull-down network.

2. Read the topology. The PMOS gated by A and the PMOS gated by B are in **series** between V_{DD} and Y: the drain of the upper one feeds the source of the lower one. The NMOS gated by A and the NMOS gated by B are in **parallel** between Y and ground, with both drains on the Y line and both sources on the ground rail. The wire from B hops over the Y line, so it does not connect to Y. It only drives the gate of the right NMOS.
3. Switching rules: an NMOS conducts when its gate is at logic 1, and a PMOS conducts when its gate is at logic 0.
4. Pull-down network: the parallel NMOS pair connects Y to ground when $A = 1$ **or** $B = 1$. So $Y = 0$ exactly when $A + B = 1$.
5. Pull-up check: the series PMOS pair connects Y to V_{DD} only when $A = 0$ **and** $B = 0$, which is $\overline{A}\,\overline{B}$. By De Morgan, $\overline{A}\,\overline{B} = \overline{A + B}$, the exact complement of the pull-down condition. So for every input exactly one network conducts, as a static CMOS gate requires.
6. Truth table: $AB = 00 \Rightarrow Y = 1$. For $AB = 01, 10, 11$, $Y = 0$.
7. So $Y = \overline{A + B}$ (overline means complement). This is the 2-input NOR gate, option (A).

KEY IDEA

In a static CMOS gate, the NMOS pull-down network directly implements the complement of the output. Parallel NMOS devices give OR, and series NMOS devices give AND. The PMOS pull-up network is the dual: series where the NMOS are parallel, and parallel where they are in series. Parallel NMOS with series PMOS gives $Y = \overline{A + B}$, which is NOR.

WATCH OUT FOR

Reading the output from the pull-down function without inverting it. The parallel NMOS pair computes $A + B$, but it pulls Y LOW, so the answer is NOR, not OR. A second slip is mixing up the dual pair: series PMOS with parallel NMOS gives NOR, while parallel PMOS with series NMOS gives NAND. It also helps to notice the hop on the B wire, which shows B does not connect to the output line.

Q.21 MCQ · 1 MARK

Electromagnetics › Antennas

Consider the Friis' transmission equation $P_R = (P_T G_T G_R \lambda^2)/(4\pi D)^2$, where P_R and P_T are the received and the transmitted powers, respectively. G_T and G_R are the gain of transmitting and receiving antennas, respectively, D is the distance between the transmitting and receiving antennas, and λ is the wavelength in free space.

Given: $G_T = G_R = 1.0$, $\lambda = 0.30$ m and $P_T = +10$ dBm.

Choose the distance (D), in km, from the following options at which the received power, $P_R = -90$ dBm?

(A) $\frac{15}{4\pi}$

(B) $\frac{15}{2\pi}$

(C) $\frac{75}{2\pi}$

(D) $\frac{3}{4\pi}$

Official answer: **B**

SOLUTION

1. With $G_T = G_R = 1$, the Friis equation becomes
$$\frac{P_R}{P_T} = \left(\frac{\lambda}{4\pi D}\right)^2$$
2. Both powers are given in dBm, so their difference is the power ratio in dB: $P_R - P_T = -90 - (+10) = -100$ dB. That gives $\frac{P_R}{P_T} = 10^{-100/10} = 10^{-10}$. (Check: $P_T = 10$ mW and $P_R = 10^{-9}$ mW $= 10^{-12}$ W,

and their ratio is 10^{-10}.)

3. Take the square root: $\frac{\lambda}{4\pi D} = \sqrt{10^{-10}} = 10^{-5}$. In dB terms, the free-space path loss is $20\log_{10}\left(\frac{4\pi D}{\lambda}\right) = 100$ dB, so $\frac{4\pi D}{\lambda} = 10^{5}$.
4. Solve for the distance:

$$D = \frac{\lambda}{4\pi \times 10^{-5}} = \frac{0.30 \times 10^{5}}{4\pi}\ \text{m} = \frac{30000}{4\pi}\ \text{m}$$

5. Convert to km: $D = \frac{30}{4\pi}\ \text{km} = \frac{15}{2\pi}\ \text{km} \approx 2.39\ \text{km}$. This is option (B).

KEY IDEA

Subtracting one dBm value from another gives a dB ratio. Here $P_R - P_T = -100$ dB is the free-space path loss with unity-gain antennas, and $\left(\frac{\lambda}{4\pi D}\right)^2 = 10^{-10}$. Because Friis loss depends on the square of distance, 100 dB of power loss means $\frac{4\pi D}{\lambda} = 10^{5}$, not 10^{10}.

WATCH OUT FOR

The usual mistakes are about the square. One is forgetting the square root, which leaves $\frac{4\pi D}{\lambda} = 10^{10}$ and a distance far larger than any option. Another is converting the 100 dB power ratio with $10^{-100/20}$ and then taking a square root as well, which applies the halving twice. A third is leaving D in metres ($\frac{30000}{4\pi}$ m) and not turning it into km before comparing with the options. It also helps to remember that dBm minus dBm is a plain dB ratio, so there is no need to convert to watts.

Q.22 MCQ · 1 MARK

Communications › Digital communication

Consider a discrete memoryless source with an alphabet of four source symbols. s(t) is a multi-level (-1, 0, +1, +2) signal representing a long sequence of random symbols from the above source which is generating 10^4 symbols per second. Which of the following options is the correct value of equivalent Nyquist bandwidth of s(t)?

(A) 10 kHz

(B) 64 kHz

(C) 5 kHz

(D) 20 kHz

Official answer: **C**

SOLUTION

1. The source has four symbols, mapped one-to-one to the four amplitude levels $-1, 0, +1, +2$, so $s(t)$ is a 4-level baseband PAM signal.
2. The symbol rate is $R_s = 10^{4}$ symbols/s, so the symbol period is $T_s = 10^{-4}$ s.
3. The number of levels only sets the bits per symbol, $\log_2 4 = 2$, giving a bit rate of 2×10^{4} bit/s. It does not enter the Nyquist bandwidth, which depends only on the symbol rate.
4. For ISI-free baseband signalling at R_s symbols per second, the minimum (equivalent Nyquist) bandwidth is $B_N = \frac{R_s}{2}$ (one-sided, in Hz).
5. $$B_N = \frac{10^{4}}{2}\ \text{Hz} = 5\ \text{kHz}$$
6. This is option (C).

KEY IDEA
Nyquist bandwidth of baseband signalling depends only on the symbol rate: $B_N = R_s/2$. The number of amplitude levels changes the bit rate, not the bandwidth.

WATCH OUT FOR
Using the bit rate 2×10^4 bit/s instead of the symbol rate and getting 10 kHz, or taking $B = R_s$ (also 10 kHz) or $2R_s$ (20 kHz).

Q.23 MCQ · 1 MARK

Networks (Electric Circuits) › Transients

The relation between the input current (I) and the output voltage (V) of a circuit is governed by the equation: $C\dfrac{dV}{dt} = I(t) - m(t)$. The circuit is excited by $I(t) = q\delta(t)$, where q is a real valued constant. V at $t = 0^-$ is V_0.

Which of the following is an equivalent representation of the above case?

(A) $C\dfrac{dV}{dt} = -m(t)$, with $V(t = 0^-) = V_0 + q/C$

(B) $C\dfrac{dV}{dt} = -m(t)$, with $V(t = 0^-) = V_0 + q/C + m(t = 0^-)$

(C) $C\dfrac{dV}{dt} = -m(t)$, with $V(t = 0^-) = V_0 - q/C + m(t = 0^-)$

(D) $C\dfrac{dV}{dt} = -m(t)$, with $V(t = 0^-) = q/C$

Official answer: **A**

SOLUTION

1. Read the equation as KCL at a capacitor node: $C\,\dfrac{dV}{dt}$ is the current into a capacitor C, $I(t)$ is the source current and $m(t)$ is the current drawn by the rest of the circuit. The impulse $q\,\delta(t)$ delivers a charge q (in C) in zero time.
2. Integrate both sides across the impulse, from $t = 0^-$ to $t = 0^+$:
$$C\left[V(0^+) - V(0^-)\right] = \int_{0^-}^{0^+} q\,\delta(t)\,dt - \int_{0^-}^{0^+} m(t)\,dt$$
3. By the sifting property, $\int_{0^-}^{0^+} q\,\delta(t)\,dt = q$. The rest of the circuit draws a finite current (the standard assumption, implicit in the question: $m(t)$ contains no impulse), and a finite function integrated over an interval of zero width gives 0.
4. So the voltage jumps by the injected charge divided by C:
$$V(0^+) = V_0 + \frac{q}{C}$$
where q/C is in V.
5. For $t > 0$ we have $\delta(t) = 0$, so the circuit obeys $C\,\dfrac{dV}{dt} = -m(t)$, starting from $V_0 + q/C$.
6. Equivalent picture: remove the impulse source and give the capacitor the initial voltage $V_0 + q/C$. With no impulse in this equivalent system, V is continuous at $t = 0$, so its value at $t = 0^-$ equals its value at $t = 0^+$: $V(0^-) = V_0 + q/C$. This is option (A).
7. Check in the Laplace domain (initial conditions taken at 0^-). The original equation transforms to
$$C\left[sV(s) - V_0\right] = q - M(s)$$
Moving q to the left:
$$C\left[sV(s) - \left(V_0 + \frac{q}{C}\right)\right] = -M(s)$$
which is exactly the transform of $C\,\dfrac{dV}{dt} = -m(t)$ with $V(0^-) = V_0 + q/C$. Both systems give the same $V(s)$, so they are equivalent.

8. The other options fail: (B) and (C) add $m(0^-)$, a current in A, to a voltage, which is dimensionally inconsistent; (C) also has the wrong sign on q/C (a positive charge raises the capacitor voltage). (D) drops V_0 and is right only when $V_0 = 0$.

KEY IDEA

A current impulse $q\,\delta(t)$ into a capacitor dumps a charge q on it instantly, so the voltage jumps by q/C. The impulse can be removed and replaced by the initial condition $V_0 + q/C$. In the Laplace domain this is just moving q into the initial-condition term: $C[sV(s) - V_0] = q - M(s)$ is the same as $C[sV(s) - (V_0 + q/C)] = -M(s)$.

WATCH OUT FOR

Being put off by the label $t = 0^-$ in option (A) and expecting $V_0 + q/C$ to belong only at 0^+. In the equivalent system there is no impulse, so V is continuous and its 0^- and 0^+ values are the same. Other traps: picking (B) or (C), which add a current $m(0^-)$ to a voltage (a quick unit check rules them out), or picking (D), which forgets the voltage V_0 the capacitor already had.

Q.24 MCQ · 1 MARK

Electromagnetics › Plane waves

The electric field of a monochromatic plane wave travelling in a lossless isotropic and homogenous medium is given by

$$\vec{E}(z,t) = E_0\left[\hat{x}\cos(\omega t - kz) + \hat{y}\sin(\omega t - kz)\right]$$

in a right-handed orthogonal co-ordinate system.

Which of the following is the correct polarization of the electromagnetic wave?

(A) Right-handed circularly polarized

(B) Left-handed circularly polarized

(C) Linearly polarized

(D) Linearly polarized with -45° angle to $\hat{x}$

Official answer: **A**

SOLUTION

1. **Direction of travel.** Write $\hat{a}_x \equiv \hat{x}$ and $\hat{a}_y \equiv \hat{y}$. Both components carry the phase $(\omega t - kz)$, so the wave travels along $+\hat{a}_z$.
2. **Amplitudes and phase.** Write $\sin(\omega t - kz) = \cos(\omega t - kz - 90^\circ)$. The two components have equal amplitude E_0 and differ in phase by 90°. The magnitude is constant: $|\mathbf{E}|^2 = E_0^2\left[\cos^2(\omega t - kz) + \sin^2(\omega t - kz)\right] = E_0^2$, so the tip of $\mathbf{E}$ traces a circle. The wave is circularly polarized, not linearly polarized, which rules out (C) and (D).
3. **Sense of rotation.** Fix the plane $z = 0$, so $\mathbf{E} = E_0\left[\hat{a}_x\cos\omega t + \hat{a}_y\sin\omega t\right]$. At $\omega t = 0$, $\mathbf{E} = E_0\hat{a}_x$. At $\omega t = 90^\circ$, $\mathbf{E} = E_0\hat{a}_y$. As time goes on, the field turns from $+\hat{a}_x$ towards $+\hat{a}_y$.
4. **Right-hand rule (IEEE convention).** In a right-handed system, curl the fingers of the right hand from $\hat{a}_x$ to $\hat{a}_y$ and the thumb points along $\hat{a}_x \times \hat{a}_y = +\hat{a}_z$, which is the direction of travel. The rotation and the propagation obey the right-hand rule, so the wave is right-hand circularly polarized.
5. **Phasor check.** Since $\sin(\omega t - kz) = \cos(\omega t - kz - 90^\circ)$, the phasor is $\mathbf{E}_s = E_0(\hat{a}_x - j\hat{a}_y)e^{-jkz}$. The y-component lags the x-component by 90° in a wave travelling along $+z$. That is the standard form of a right-hand circularly polarized wave, so the answer is (A).

KEY IDEA

Equal amplitudes with a 90° phase difference give circular polarization. To find the sense, fix one plane (say $z = 0$) and follow $\mathbf{E}$ as time increases. If the right-hand fingers curl the way $\mathbf{E}$ turns and the thumb points along the direction of travel, the wave is right-handed (IEEE convention). Equivalently, for travel along $+z$ the phasor $(\hat{a}_x - j\hat{a}_y)e^{-jkz}$ is RHCP and $(\hat{a}_x + j\hat{a}_y)e^{-jkz}$ is LHCP.

WATCH OUT FOR

Two errors lead to (B). (1) Tracking the rotation in space at a fixed instant (letting z vary) rather than in time at a fixed plane: along z the vector turns the opposite way, so the sense comes out reversed. (2) Using the classical optics convention, which names the sense as seen by an observer facing the oncoming wave. Under that convention the same wave is called left-handed. GATE uses the IEEE (engineering) convention, where the sense is judged with the thumb along the direction of travel. The stem's mention of a right-handed co-ordinate system points to that convention.

Q.25 MSQ · 1 MARK

Electronic Devices › P-N junction

Consider a p-n junction diode when it is forward biased with 2 V. Which of the following is/are the correct magnitude(s) of the energy difference between quasi Fermi-levels, E_{fn} in the n-side and E_{fp} in the p-side?

(A) 2 eV

(B) 1 eV

(C) 2 V

(D) 1 V

Official answer: **A**

SOLUTION

1. Under thermal equilibrium (V = 0), electron and hole populations across the p-n junction are both described by a single, common Fermi level E_F: $E_{fn} = E_{fp}$.
2. Under forward bias, minority-carrier injection at the junction pushes both carrier populations far from their equilibrium values, so a single Fermi level can no longer describe both. Two separate quasi-Fermi levels are introduced: E_{fn} for electrons and E_{fp} for holes.
3. Deep in the quasi-neutral bulk regions on each side (away from the depletion region), majority carriers remain close to local equilibrium, so E_{fn} and E_{fp} are each essentially flat (constant with position) through most of each neutral region.
4. The applied forward bias V does work qV per carrier as it is transported across the junction; this work manifests exactly as the separation between the two flat quasi-Fermi levels in the neutral regions: $E_{fn} - E_{fp} = qV$.
5. Substituting the given bias $V = 2\ \text{V}$: $E_{fn} - E_{fp} = q(2\ \text{V}) = 2\ \text{eV}$.
6. Since the question asks for an ENERGY difference, the answer must be expressed in eV, not V. This immediately rules out (C) 2 V and (D) 1 V on dimensional grounds, independent of their numeric value.
7. Therefore the magnitude of the quasi-Fermi level splitting is exactly $2\ \text{eV}$, matching option (A); option (B) 1 eV would require an applied bias of 1 V, which is not the value given in the problem.

KEY IDEA

Under forward bias V, the depletion region is no longer in equilibrium, so a single Fermi level no longer describes it; separate quasi-Fermi levels E_{fn} (electrons) and E_{fp} (holes) are needed. In the quasi-neutral bulk on each side, each carrier type stays close to local equilibrium, so E_{fn} and E_{fp} are each nearly flat there. The applied bias does work qV on carriers crossing the junction, and this work appears entirely as the split between the two (flat, bulk-side) quasi-Fermi levels: $E_{fn} - E_{fp} = qV$. With $V = 2\ \text{V}$, the magnitude is $2\ \text{eV}$.

WATCH OUT FOR

Reporting the answer as "2 V" (option C) or "1 V" (option D) by treating the quasi-Fermi level separation as a voltage rather than an energy, a dimensional slip, since $E_{fn} - E_{fp}$ is an energy and must be quoted in eV (numerically equal to the bias in volts only because of the eV/V unit correspondence, $E = qV$). A second common mistake is halving the bias (giving 1 eV, option B) by mistakenly splitting the applied voltage symmetrically between the two quasi-Fermi levels relative to the equilibrium Fermi level, instead of recognizing that the full qV appears as their mutual separation.

Q.26 MSQ · 1 MARK

Engineering Mathematics › Linear algebra

Consider the matrix $M = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 0 \\ -1 & a & b \end{bmatrix}$.

Which of the following options is/ are TRUE if $\det(M) \neq 0$?

(A) $a = -\dfrac{1}{2}$ and $b = -\dfrac{1}{2}$

(B) $a = \dfrac{1}{2}$ and $b = \dfrac{1}{2}$

(C) $a = -3$ and $b = 0$

(D) $a = \dfrac{1}{2}$ and $b = -3$

Official answer: **B, D**

SOLUTION

1. Expand $\det(M)$ along the third column, because it contains a zero. The cofactor signs down that column are $+, -, +$:
$$\det(M) = 1 \cdot \begin{vmatrix} 1 & 3 \\ -1 & a \end{vmatrix} - 0 + b \cdot \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix}.$$
2. Evaluate the minors: $\begin{vmatrix} 1 & 3 \\ -1 & a \end{vmatrix} = a - (3)(-1) = a + 3$ and $\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 6 - 1 = 5$.
3. Therefore
$$\det(M) = a + 5b + 3.$$
An option is TRUE exactly when its values make $a + 5b + 3 \neq 0$.
4. (A) $a = -\frac{1}{2},\ b = -\frac{1}{2}$: $-\frac{1}{2} - \frac{5}{2} + 3 = 0$. M is singular, so (A) is not TRUE.
5. (B) $a = \frac{1}{2},\ b = \frac{1}{2}$: $\frac{1}{2} + \frac{5}{2} + 3 = 6 \neq 0$. TRUE.
6. (C) $a = -3,\ b = 0$: $-3 + 0 + 3 = 0$. M is singular, so (C) is not TRUE.
7. (D) $a = \frac{1}{2},\ b = -3$: $\frac{1}{2} - 15 + 3 = -\frac{23}{2} \neq 0$. TRUE.
8. Check by expanding along the first row: $2(3b - 0) - 1(b - 0) + 1(a + 3) = a + 5b + 3$. Both expansions agree, so the options with $\det(M) \neq 0$ are (B) and (D).

KEY IDEA

Expand along the row or column with the most zeros to get $\det(M)$ as a linear expression in a and b, here $a + 5b + 3$. Then substitute each option. Every option that makes the expression zero gives a singular matrix and must be rejected.

WATCH OUT FOR

Getting a cofactor sign wrong. In the 2×2 minor $\begin{vmatrix} 1 & 3 \\ -1 & a \end{vmatrix}$, the -1 makes it $a + 3$, not $a - 3$. The sign pattern on the middle element also matters. Either slip changes which options give zero. For example, $a - 3 + 5b$ would wrongly reject (B) instead of (C). Since this is an MSQ, stopping after finding one option that works also loses the mark.

Q.27 MSQ · 1 MARK

Digital Electronics › Sequential

A binary ripple counter is designed to count $(0)_{10}$ to $(64)_{10}$.

Which of the following is/are the number of flip-flops required to design the counter?

(A) 6

(B) 7

(C) 4

(D) 5

Official answer: **B**

SOLUTION

1. Count the states. A counter that runs from $(0)_{10}$ to $(64)_{10}$ must pass through $64 - 0 + 1 = 65$ distinct states.
2. Apply the flip-flop count rule. n flip-flops can hold at most 2^n distinct states, so the counter needs the smallest n with $2^n \geq 65$.
3. Check $n = 6$: $2^6 = 64 < 65$. Six flip-flops count only up to $(63)_{10} = (111111)_2$, so they cannot represent the state 64.
4. Check $n = 7$: $2^7 = 128 \geq 65$. The largest count needed, $(64)_{10} = (1000000)_2$, is a 7-bit number, so a seventh flip-flop is needed.
5. Check the options. 4, 5 and 6 flip-flops all give fewer than 65 states. Of the numbers offered, only 7 is enough. The 7-bit ripple counter is cut down to a MOD-65 counter by clearing all flip-flops as soon as it reaches $(65)_{10} = (1000001)_2$, the first state where Q_6 and Q_0 are both 1.
6. Answer: (B) only.

KEY IDEA

A counter with N distinct states needs the smallest n with $2^n \geq N$. Counting from 0 to M gives $N = M + 1$ states, so the number of flip-flops equals the number of bits in the largest count M: here $(64)_{10} = (1000000)_2$ has 7 bits.

WATCH OUT FOR

Taking $N = 64$ instead of 65 and picking 6 because $2^6 = 64$. Six flip-flops count 0 to 63, which is 64 states, and cannot hold the state 64 itself. When counting starts at 0, the final count needs one more bit than you might first expect.

Q.28 MSQ · 1 MARK

Electromagnetics › Plane waves

Which option(s) represents/ represent the dielectric loss tangent of a substrate?

(A) Ratio of the real to imaginary parts of the total displacement current

(B) $(\omega\epsilon'' + \sigma)/(\omega\epsilon')$

(C) Ratio of the electric susceptibility to permittivity

(D) Ratio of the polarization vector $\vec{P}$ to the displacement vector $\vec{D}$

Official answer: **A, B**

SOLUTION

1. Write the complex permittivity of a lossy dielectric, with $e^{j\omega t}$ time dependence, as $\epsilon = \epsilon' - j\epsilon''$. Here ϵ'' accounts for polarisation (damping) loss and σ for conduction loss.
2. Write the Ampère–Maxwell law in phasor form and group the terms in phase with $\mathbf{E}$ and those in quadrature with it:
$$\nabla \times \mathbf{H} = \sigma\mathbf{E} + j\omega(\epsilon' - j\epsilon'')\mathbf{E} = (\omega\epsilon'' + \sigma)\mathbf{E} + j\omega\epsilon'\mathbf{E}$$
At high frequencies the conduction term $\sigma\mathbf{E}$ cannot be told apart from the damping term $\omega\epsilon''\mathbf{E}$, so the whole right-hand side is treated as the total displacement current.
3. The loss tangent is the in-phase (lossy) part of this current divided by the quadrature (reactive) part:
$$\tan\delta = \frac{\omega\epsilon'' + \sigma}{\omega\epsilon'}$$
This is exactly option (B), so (B) is correct.
4. Option (A): take $\mathbf{E}$ as the phase reference. The real part of the total displacement current is then $(\omega\epsilon'' + \sigma)\mathbf{E}$ and the imaginary part is $\omega\epsilon'\mathbf{E}$. Their ratio is $\dfrac{\omega\epsilon'' + \sigma}{\omega\epsilon'} = \tan\delta$. If only the pure displacement term $j\omega\epsilon\mathbf{E} = \omega\epsilon''\mathbf{E} + j\omega\epsilon'\mathbf{E}$ is used, the ratio is ϵ''/ϵ', which is the dielectric loss tangent of a non-conducting substrate. Either way, (A) describes the loss tangent, so (A) is correct.
5. Option (C): χ_e/ϵ has units of $1/(\mathrm{F/m})$, so it is not dimensionless. It also contains no loss term, so it cannot be $\tan\delta$. Incorrect.
6. Option (D): $\mathbf{P} = \epsilon_0\chi_e\mathbf{E}$ and $\mathbf{D} = \epsilon\mathbf{E}$, so $\dfrac{P}{D} = \dfrac{\epsilon_0\chi_e}{\epsilon} = \dfrac{\epsilon_r - 1}{\epsilon_r}$, with $\epsilon_0 = 8.854 \times 10^{-12}\ \mathrm{F/m}$. This measures how strongly the material polarises, not how much energy it loses per cycle. Incorrect.
7. The correct options are (A) and (B).

KEY IDEA

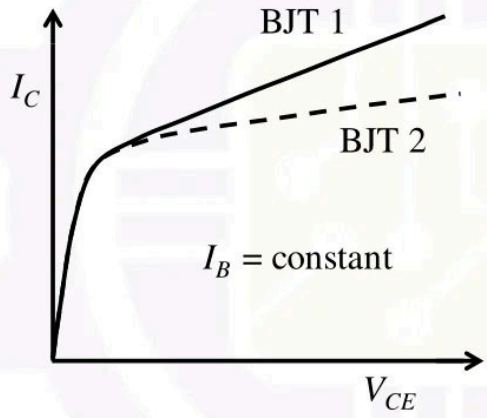
Put $\epsilon = \epsilon' - j\epsilon''$ into $\nabla \times \mathbf{H} = \sigma\mathbf{E} + j\omega\epsilon\mathbf{E}$ and split the current into a part in phase with $\mathbf{E}$, $(\omega\epsilon'' + \sigma)\mathbf{E}$, and a part in quadrature, $j\omega\epsilon'\mathbf{E}$. The loss tangent is their ratio, $\tan\delta = (\omega\epsilon'' + \sigma)/(\omega\epsilon')$. That single formula is both option (B) and the wording of option (A).

WATCH OUT FOR

Rejecting (A) because the word 'displacement' seems to leave out conduction, or because it names 'real to imaginary' rather than 'imaginary to real'. With $\mathbf{E}$ as the phase reference, the lossy part of $\sigma\mathbf{E} + j\omega\epsilon\mathbf{E}$ is the real part and the stored-energy part is the imaginary part, so real/imaginary is exactly $\tan\delta$. Another trap is picking (D) because P/D looks like a dimensionless material ratio. It is $(\epsilon_r - 1)/\epsilon_r$, which measures polarisation, not loss.

Q.29 MSQ · 1 MARK

Electronic Devices › BJT

Q.29	Figure shows the output characteristics of two different Bipolar Junction Transistors (BJT), BJT 1 with magnitude of Early voltage $\lvert V_{A1}\rvert$, and BJT 2 with magnitude of Early voltage $\lvert V_{A2}\rvert$. Which of the following options is/are correct regarding the Early voltages? 
(A)	$\lvert V_{A1}\rvert > \lvert V_{A2}\rvert$
(B)	$\lvert V_{A1}\rvert$ is infinitely large
(C)	$\lvert V_{A1}\rvert < \lvert V_{A2}\rvert$
(D)	$\lvert V_{A2}\rvert$ is finite

Official answer: **C, D**

SOLUTION

1. The Early effect gives $I_C = I_{C0}\left(1 + \frac{V_{CE}}{V_A}\right)$ at fixed I_B, so each output curve's active-region segment is a straight line of slope $\frac{I_{C0}}{V_A}$.
2. Extrapolated backward, this line intersects the V_{CE}-axis at $V_{CE} = -V_A$. Since both curves branch from the same knee point (same I_{C0}) in the figure, a steeper active-region slope corresponds to a smaller

$|V_A|$, and a flatter slope corresponds to a larger $|V_A|$.

3. From the figure, BJT 1's curve (solid) rises noticeably faster with V_{CE} than BJT 2's curve (dashed), which is nearly flat.
4. Steeper slope for BJT 1 $\Rightarrow$ smaller $|V_{A1}|$; flatter slope for BJT 2 $\Rightarrow$ larger $|V_{A2}|$, so $|V_{A1}| < |V_{A2}|$: option (C) is correct and (A) is wrong.
5. An ideal, perfectly horizontal output curve would correspond to $V_A \to \infty$; but BJT 1's curve visibly has a finite positive slope, so $|V_{A1}|$ cannot be infinite: option (B) is wrong.
6. BJT 2's curve, though much flatter than BJT 1's, still has a small nonzero positive slope (it is not perfectly horizontal), so $|V_{A2}|$ is large but finite: option (D) is correct.
7. Hence the correct options are (C) and (D).

KEY IDEA

In the Early-effect model, $I_C = I_{C0}\left(1 + \frac{V_{CE}}{V_A}\right)$ at fixed I_B, so each output curve's active-region straight-line segment has slope $\frac{I_{C0}}{V_A}$; extrapolated backward, that line meets the V_{CE}-axis at $V_{CE} = -V_A$. Since both curves in the figure branch from the same knee point (same I_{C0}), a flatter active-region slope means a larger $|V_A|$, and a steeper slope means a smaller $|V_A|$.

WATCH OUT FOR

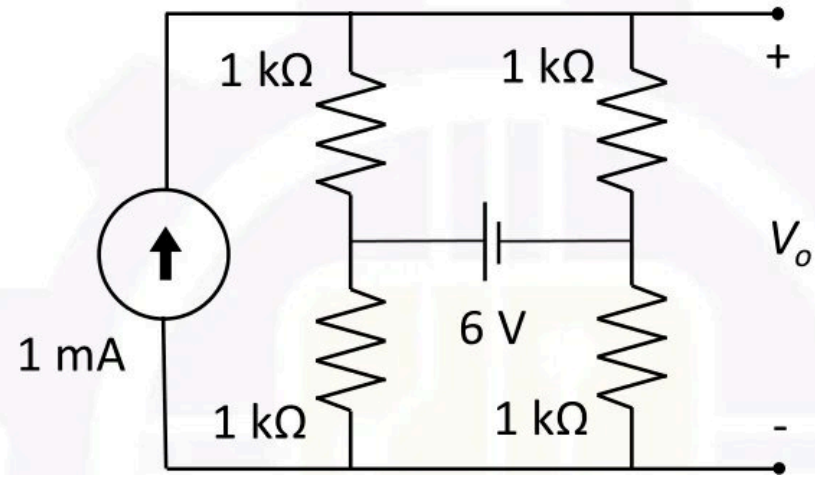
Assuming a nearly-flat curve on a printed figure must mean $V_A \to \infty$, and so marking (B) or dismissing (D). The figure clearly shows BJT 2's line still has a small positive slope (it is not perfectly horizontal), so $|V_{A2}|$ is large but finite, not infinite; only a truly horizontal line would signal $V_A \to \infty$.

Q.30 NAT · 1 MARK

Networks (Electric Circuits) › Laws & node/mesh

Q.30 The output voltage V_o (in Volt) for the network given in the Figure is __.

(rounded off to two decimal places)



Official answer: **0.95 to 1.05**

SOLUTION

1. Label the nodes. Take the bottom rail G as the reference and call the top-rail voltage V_o. The middle nodes are A (left) and B (right). The battery's long plate faces A, so it fixes $V_A - V_B = 6\ \text{V}$.
2. KCL at the top node T, working in mA and kΩ. The source pushes 1 mA into T, and that current leaves through the two upper resistors:

$$1 = \frac{V_o - V_A}{1} + \frac{V_o - V_B}{1} \Rightarrow 2V_o - (V_A + V_B) = 1$$

3. Draw a supernode around A, B and the battery. The current entering it through the two upper resistors must equal the current leaving it through the two lower resistors:
$$(V_o - V_A) + (V_o - V_B) = V_A + V_B$$
The left side is 1 mA by the KCL at T, so $V_A + V_B = 1\ \text{V}$.
4. Substitute back into the KCL at T: $2V_o - 1 = 1 \ \Rightarrow\ V_o = 1\ \text{V}$.
5. Check by superposition. With the battery acting alone (current source opened), T is joined to A and B by equal 1 kΩ resistors, and so is G. Each rail therefore sits at $\dfrac{V_A + V_B}{2}$, so the battery contributes nothing to V_o. With the current source acting alone (battery shorted), A and B merge into one node. The resistance seen by the source is then $(1 \parallel 1) + (1 \parallel 1) = 0.5 + 0.5 = 1\ \text{k}\Omega$, so $V_o = 1\ \text{mA} \times 1\ \text{k}\Omega = 1\ \text{V}$.
6. Consistency check. From $V_A - V_B = 6$ and $V_A + V_B = 1$: $V_A = 3.5\ \text{V}$ and $V_B = -2.5\ \text{V}$. The upper resistors carry $(1 - 3.5) + (1 + 2.5) = -2.5 + 3.5 = 1\ \text{mA}$ out of T. The lower resistors carry $3.5 + (-2.5) = 1\ \text{mA}$ into G. Both match the 1 mA source, so $V_o = 1.00\ \text{V}$.

KEY IDEA

The four equal resistors form a balanced bridge, with the battery across the middle arm. By symmetry the battery shifts V_A and V_B equally in opposite directions, so it adds nothing to the rail-to-rail voltage. With the battery shorted, A and B become one node and the source sees $(1 \parallel 1) + (1 \parallel 1) = 1\ \text{k}\Omega$. That gives $V_o = 1\ \text{mA} \times 1\ \text{k}\Omega = 1\ \text{V}$. The supernode equations confirm it directly: $V_A + V_B = 1$ and $2V_o - (V_A + V_B) = 1$.

WATCH OUT FOR

Assuming the 6 V battery must appear in V_o and adding or subtracting it (for example, answering 4 V or 7 V). In a balanced bridge the battery only splits the middle nodes to $V_A = 3.5\ \text{V}$ and $V_B = -2.5\ \text{V}$, and its effect cancels between the two rails. Another slip is to write a KCL at A or at B alone and then try to include the battery current, which is unknown. Enclose the battery in a supernode instead.

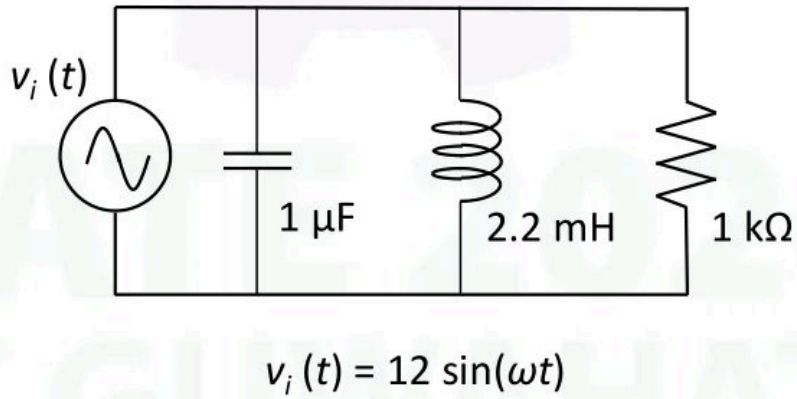
Q.31 NAT · 1 MARK

Networks (Electric Circuits) › AC, power & resonance

Q.31 Consider the circuit shown in the Figure, where the input $v_i(t)$ is in Volt.

The average power (in mW) dissipated in the load resistance of 1 kΩ at the resonant frequency is __.

(rounded off to two decimal places)



Official answer: **71.50 to 72.50**

SOLUTION

1. **Topology check.** The ideal source, the $1\ \mu\text{F}$ capacitor, the $2.2\ \text{mH}$ inductor and the $1\ \text{k}\Omega$ resistor all connect between the same two nodes, so all four are in parallel. The voltage across the resistor is therefore the source voltage, $v_R(t) = v_i(t) = 12\sin(\omega t)\ \text{V}$, at every frequency.
2. **Resonant frequency, for reference.** $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{2.2 \times 10^{-3} \times 10^{-6}}} \approx 2.132 \times 10^4\ \text{rad/s}$, about $3.39\ \text{kHz}$. At ω_0 the currents I_L and I_C are equal and opposite, so the parallel LC pair draws no net current. That changes the current the source supplies. It does not change the voltage across R, because an ideal voltage source fixes that voltage.
3. **Peak and rms.** In $12\sin(\omega t)$ the amplitude is a peak value: $V_m = 12\ \text{V}$, so $V_{\text{rms}} = \dfrac{12}{\sqrt{2}}\ \text{V}$.
4. **Average power in the resistor.**

$$P = \frac{V_{\text{rms}}^2}{R} = \frac{V_m^2}{2R} = \frac{12^2}{2 \times 1000} = \frac{144}{2000}\ \text{W}$$

5. $P = 0.072\ \text{W} = 72.00\ \text{mW}$.

KEY IDEA

An element connected straight across an ideal voltage source always has the source voltage across it. Here R is in parallel with the source, so resonance changes only the current the source supplies, not the power in R: $P = V_m^2/(2R)$ at any frequency.

WATCH OUT FOR

Using the peak value as if it were rms, $12^2/1000 = 144\ \text{mW}$, which is twice the correct answer. A second trap is to assume resonance must change the answer and start computing currents through an equivalent impedance. The parallel LC only changes the source current, while the voltage across R stays fixed by the ideal source.

Q.32 NAT · 1 MARK

Communications › Digital communication

A wireless digital transmission scheme is using 16-QAM over an additive white Gaussian noise channel and a maximum-likelihood receiver. Consider the information bit rate from source to be 4×10^6 bits per second. The minimum transmission bandwidth (in MHz) of the modulated signal necessary for optimum recovery of information at the receiver is ____. (rounded off to two decimal places)

Official answer: **0.95 to 1.05**

SOLUTION

1. 16-QAM has $M = 16$ signal points, so each symbol carries $k = \log_2 16 = 4$ bits.
2. Symbol rate: $R_s = \dfrac{R_b}{k} = \dfrac{4 \times 10^6}{4} = 1 \times 10^6\ \text{symbols/s}$.
3. The smallest bandwidth that allows distortion-free (ISI-free) recovery is the Nyquist limit, reached with zero roll-off. Each of the in-phase and quadrature baseband pulse streams then needs $R_s/2$ (one-sided, in Hz).
4. The modulated signal is a band-pass signal. The two quadrature carriers share the same band, and the band-pass spectrum extends $R_s/2$ on each side of the carrier, so $B_{\min} = 2 \cdot \dfrac{R_s}{2} = R_s$.
5. $B_{\min} = 1 \times 10^6\ \text{Hz} = 1.00\ \text{MHz}$.

KEY IDEA

For M-ary QAM the symbol rate is $R_s = R_b/\log_2 M$. The minimum (Nyquist, zero roll-off) band-pass bandwidth equals the symbol rate, $B_{\min} = R_s$.

WATCH OUT FOR

Using $B = R_b$ (ignoring that each 16-QAM symbol carries 4 bits), or using the baseband figure $R_s/2$, which gives 0.5 MHz. $R_s/2$ is the baseband bandwidth of each of the I and Q streams, not the band-pass bandwidth of the modulated signal.

Q.33 NAT · 1 MARK

Electromagnetics › Waveguides & fibres

The cutoff frequency (in GHz) for the dominant TE_{10} mode of an air-filled rectangular waveguide of inner dimension 0.28 inch $\times$ 0.14 inch is ____. (rounded off to two decimal places).

Official answer: **20.50 to 21.50**

SOLUTION

1. For a rectangular waveguide with $a > b$, the dominant mode is TE_{10}. Its cut-off depends only on the broad dimension: $f_{c,10} = \dfrac{c}{2a}$, where $c = \dfrac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8\ \text{m/s}$ for air ($\mu_0 = 4\pi \times 10^{-7}\ \text{H/m}$, $\epsilon_0 = 8.854 \times 10^{-12}\ \text{F/m}$).
2. Take the broad wall $a = 0.28\ \text{inch}$. The narrow wall $b = 0.14\ \text{inch}$ plays no part in TE_{10}.
3. Convert to SI units with $1\ \text{inch} = 2.54\ \text{cm}$: $a = 0.28 \times 2.54\ \text{cm} = 0.7112\ \text{cm} = 7.112 \times 10^{-3}\ \text{m}$.
4. Cut-off wavelength: $\lambda_{c,10} = 2a = 1.4224 \times 10^{-2}\ \text{m}$.

5. $$f_{c,10} = \frac{c}{2a} = \frac{3 \times 10^8}{1.4224 \times 10^{-2}}\ \text{Hz} \approx 2.1091 \times 10^{10}\ \text{Hz} = 21.09\ \text{GHz}$$

6. Check: the next modes are TE_{20} and TE_{01}, both cut off at $c/a = c/2b \approx 42.18$ GHz (here $b = a/2$). TE_{10} has the lowest cut-off, so it is indeed the dominant mode. Answer: 21.09 GHz.

KEY IDEA

For TE_{mn} in an air-filled rectangular guide, $f_{c,mn} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$. With $m = 1$, $n = 0$ this becomes $f_{c,10} = c/2a$, which depends only on the broad dimension a. The whole problem comes down to converting a from inches to metres correctly.

WATCH OUT FOR

Using the narrow dimension $b = 0.14$ inch, or writing $f_c = c/a$ without the factor 2. Both give about 42.18 GHz, which is the $\text{TE}_{01}/\text{TE}_{20}$ cut-off, not TE_{10}. Another slip is forgetting to convert inches to metres, which is off by a factor of about 39. (Taking $c = 2.998 \times 10^8$ m/s gives 21.08 GHz, which is still inside the key range.)

Q.34 NAT · 1 MARK

Electromagnetics › Transmission lines

For a lossless passive two-port network, $|S_{11}|$ and $|S_{21}|$ intersect at -3 dB.

For a lossy passive two-port network, $|S_{11}|$ and $|S_{21}|$ intersect at -4 dB.

The percentage of power dissipated in the lossy network at the intersection frequency is ___. (rounded off to two decimal places)

Official answer: **20.00 to 21.00**

SOLUTION

1. Send unit power into port 1 with port 2 terminated in a matched load. Then $|S_{11}|^2$ is the fraction of power reflected and $|S_{21}|^2$ is the fraction transmitted. Whatever is neither reflected nor transmitted is dissipated inside the network:
$$P_{\text{diss}} = 1 - |S_{11}|^2 - |S_{21}|^2$$
2. Check this against the lossless case. A lossless network has a unitary S-matrix, so $|S_{11}|^2 + |S_{21}|^2 = 1$. Where the two curves cross, $|S_{11}| = |S_{21}|$, so each squared magnitude is $\frac{1}{2}$, and $10\log_{10}(0.5) \approx -3$ dB. That agrees with the first statement in the question.
3. In the lossy network the curves cross at -4 dB, so $|S_{11}|_{\text{dB}} = |S_{21}|_{\text{dB}} = -4$ dB. Since $20\log_{10}|S| = 10\log_{10}|S|^2 = -4$,
$$|S_{11}|^2 = |S_{21}|^2 = 10^{-4/10} = 10^{-0.4} = 0.39811$$
4. Reflected plus transmitted power: $|S_{11}|^2 + |S_{21}|^2 = 2 \times 0.39811 = 0.79621$.
5. Dissipated power:
$$P_{\text{diss}} = 1 - 0.79621 = 0.20379 \ \Rightarrow\ 20.38\%$$

KEY IDEA

S-parameter magnitudes squared are power fractions. In a passive two-port driven at port 1 with port 2 matched, reflected power is $|S_{11}|^2$ and transmitted power is $|S_{21}|^2$. Any shortfall from 1 is power lost inside the network. At a crossing point both curves have the same dB value, so both power fractions are $10^{\text{dB}/10}$.

WATCH OUT FOR

A common slip is converting -4 dB with $10^{-4/20} = 0.631$. That is the magnitude $|S|$, not the power fraction. Adding the two magnitudes, or subtracting magnitudes from 1, gives a meaningless result. Another slip is to count only one of the two terms and write $1 - 0.398 = 60.2\%$. The reflected and transmitted powers must both be subtracted.

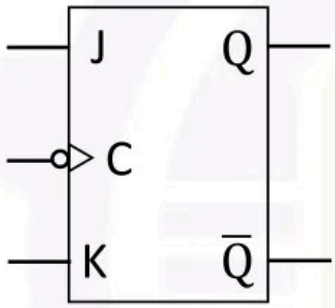
Q.35 NAT · 1 MARK

Digital Electronics › Sequential

Q.35 The negative edge triggered JK flip-flop in the Figure has J and K inputs tied to Logic High and a square wave of 10 cycles/second is applied to its clock (C) input.

The frequency of the output Q (in cycles/second) is __.

(rounded off to two decimal places)



Official answer: **4.90 to 5.10**

SOLUTION

1. Characteristic equation of the JK flip-flop: $Q_{n+1} = J\,\overline{Q_n} + \overline{K}\,Q_n$, where the overline means complement.
2. With $J = K = 1$: $Q_{n+1} = 1 \cdot \overline{Q_n} + 0 \cdot Q_n = \overline{Q_n}$. The flip-flop is in toggle mode.
3. The flip-flop is negative-edge triggered, so Q toggles once on every falling edge of the clock. A square wave has one falling edge per cycle, so there are 10 falling edges every second.
4. One full cycle of Q needs two toggles (low to high, then high back to low), so one cycle of Q takes 2 clock cycles.
5. $$f_Q = \frac{f_{clk}}{2} = \frac{10}{2} = 5 \text{ cycles/second}$$
6. Rounded to two decimal places: $f_Q = 5.00 \text{ cycles/second}$. Which edge triggers the flip-flop changes when Q's edges happen, but not how often they happen, so the frequency is unchanged.

KEY IDEA

With $J = K = 1$, a JK flip-flop toggles on every active clock edge ($Q_{n+1} = \overline{Q_n}$). A full output cycle needs two toggles, so the flip-flop works as a divide-by-2 counter: $f_Q = f_{clk}/2$. The triggering edge (rising or falling) only moves where the output transitions fall in time. It does not change their rate.

WATCH OUT FOR

One mistake is to count each toggle as a full output cycle and answer 10, which is the clock frequency. Another is to think negative-edge triggering changes the frequency, for example by counting both clock edges and answering 10 or 20. An edge-triggered flip-flop responds to only one edge per clock cycle, and it takes two toggles to complete one cycle of Q.

Q.36 MCQ · 2 MARKS

Engineering Mathematics › Calculus

Consider the two series, S_A and S_B, where

$$S_A = \sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

$$S_B = 1 + \frac{1}{2} + \frac{1}{8} + \frac{1}{16} + \frac{1}{64} + \frac{1}{128} + \frac{1}{512} + \cdots$$

Which of the following statements is correct for the two given series?

(A) Both S_A and S_B converge.

(B) Neither S_A nor S_B converges.

(C) S_A converges but S_B does not converge.

(D) S_B converges but S_A does not converge.

Official answer: **A**

SOLUTION

1. S_A**, ratio test.** With $a_n = \dfrac{n^2}{2^n}$:
$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^2}{2^{n+1}} \cdot \frac{2^n}{n^2} = \frac{1}{2}\left(1 + \frac{1}{n}\right)^2 \to \frac{1}{2} < 1,$$
so S_A converges.
2. Check with the standard sum $\sum_{n=1}^{\infty} n^2 x^n = \dfrac{x(1+x)}{(1-x)^3}$ for $|x| < 1$. At $x = \frac{1}{2}$:
$$S_A = \frac{\frac{1}{2} \cdot \frac{3}{2}}{\left(\frac{1}{2}\right)^3} = \frac{3}{4} \cdot 8 = 6,$$
which is finite.
3. S_B**, spot the pattern.** The terms are $2^0, 2^{-1}, 2^{-3}, 2^{-4}, 2^{-6}, 2^{-7}, 2^{-9}, \ldots$: powers of 2 in pairs, with every third power ($2^{-2}, 2^{-5}, 2^{-8}, \ldots$) left out.
4. **Comparison test.** Every term of S_B is positive, and the terms are distinct terms of the geometric series $\sum_{k=0}^{\infty} 2^{-k} = 2$. So the partial sums of S_B increase and never exceed 2. A bounded increasing sequence converges, so S_B converges.
5. Exact value (a check, not needed for the answer): since all terms are positive, grouping consecutive pairs does not change the sum. The k-th pair ($k = 0, 1, 2, \ldots$) is $2^{-3k}\left(1 + \frac{1}{2}\right) = \frac{3}{2} \cdot 8^{-k}$, so
$$S_B = \frac{3}{2}\left(1 + \frac{1}{8} + \frac{1}{64} + \cdots\right) = \frac{3}{2} \cdot \frac{1}{1 - \frac{1}{8}} = \frac{3}{2} \cdot \frac{8}{7} = \frac{12}{7}.$$
6. Both series converge, so option (A) is correct.

KEY IDEA

An exponential in the denominator beats any power of n in the numerator: for $\sum n^k / r^n$ with $r > 1$ the ratio test gives the limit $1/r < 1$, so the series converges. A series of positive terms, each a distinct term of a convergent geometric series, has bounded increasing partial sums and therefore converges, however irregular the pattern of left-out terms.

WATCH OUT FOR

Two traps. First, seeing n^2 in the numerator of S_A and deciding the terms grow, when 2^n grows much faster and the terms tend to 0. Second, deciding that S_B is not geometric because of the gaps ($\frac{1}{4}, \frac{1}{32}, \ldots$ are missing) and so it cannot be judged. Leaving out positive terms from a convergent geometric series can only make the sum smaller, so it still converges.

Q.37 MCQ · 2 MARKS

Signals and Systems › Fourier series

The continuous time signal $x(t)$ is real, periodic with period T and satisfies the Dirichlet conditions.

The Fourier series representation of $x(t) = \sum_{-\infty}^{\infty} a_n e^{j\left(\frac{2\pi nt}{T}\right)}$ and $x(t)$ satisfies the following: $x\left(t - \frac{T}{2}\right) = -x(t)$.

For any integer m, which of the following options is correct?

(A) $a_{2m} = 0$

(B) $a_{2m} = 1$

(C) $a_{2m} = a_{2m+1}$

(D) $a_{2m} = -1$

Official answer: **A**

SOLUTION

1. **Time-shift property of the Fourier series.** Let $\omega_0 = \dfrac{2\pi}{T}$. If $x(t) \leftrightarrow a_n$, then $x(t - t_0) \leftrightarrow a_n e^{-jn\omega_0 t_0}$. This follows directly from the series:
$$x(t - t_0) = \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0(t-t_0)} = \sum_{n=-\infty}^{\infty} \left(a_n e^{-jn\omega_0 t_0}\right) e^{jn\omega_0 t}$$
2. Put $t_0 = \dfrac{T}{2}$:
$$e^{-jn\omega_0 T/2} = e^{-jn\pi} = (-1)^n$$
so $x\left(t - \frac{T}{2}\right) \leftrightarrow (-1)^n a_n$.
3. **Match coefficients.** The Fourier coefficients of a signal are unique. The given condition $x\left(t - \frac{T}{2}\right) = -x(t)$ therefore means that, for every n,
$$(-1)^n a_n = -a_n \;\Rightarrow\; \left[1 + (-1)^n\right] a_n = 0$$
4. Odd n: $1 + (-1)^n = 0$, so the equation holds whatever a_n is. Odd harmonics can be present.
5. Even $n = 2m$: $1 + (-1)^{2m} = 2 \neq 0$, so $a_{2m} = 0$ for every integer m. This includes $m = 0$, so $a_0 = 0$ and the signal has zero average.
6. **Check the options.** (B) and (D) contradict $a_{2m} = 0$. If (C) held for every m, it would force $a_{2m+1} = a_{2m} = 0$ as well. Every coefficient would then be zero and $x(t) \equiv 0$. A nonzero signal can satisfy the condition, for example $x(t) = \cos\left(\frac{2\pi t}{T}\right)$: here $a_1 = \frac{1}{2} \neq 0 = a_0$, so (C) fails. **Answer: (A)**, $a_{2m} = 0$.

KEY IDEA

Shifting by half a period multiplies the n-th coefficient by $e^{-jn\pi} = (-1)^n$. With half-wave symmetry, $x\left(t - \frac{T}{2}\right) = -x(t)$, you need $(-1)^n a_n = -a_n$, so only odd harmonics survive. Every even-indexed coefficient is zero, and that includes a_0.

WATCH OUT FOR

Students often mix up half-wave symmetry, $x\left(t - \frac{T}{2}\right) = -x(t)$, with odd symmetry, $x(-t) = -x(t)$. Odd symmetry only makes the coefficients purely imaginary. It does not remove the even harmonics. Another slip is to use a shift of a full period T instead of $T/2$, which gives $e^{-j2\pi n} = 1$ and seems to tell you nothing. Some students also forget that $m = 0$ is included, so the DC value a_0 is zero as well.

Q.38 MCQ · 2 MARKS

Engineering Mathematics › Probability & statistics

Let X, N, Y and Z be random variables. The variables X and N are independent of each other. X is uniformly distributed between -1 and 1; N follows Normal distribution with zero mean and unity variance.

Y and Z are defined as, $Y = X + N$ and $Z = X^2 + N$.

Which of the following pairs represents the values of correlation between X and Y and that between X and Z?

(A) 1/3 and 0

(B) 1/3 and 1/9

(C) 1/3 and 1/3

(D) 1 and 0

Official answer: **A**

SOLUTION

1. Here "correlation" means the correlation $R_{XY} = E[XY]$, as used in signals and communication. It is not the normalised correlation coefficient. (Step 6 checks this reading.)
2. Moments of $X \sim U(-1, 1)$: the density is $\frac{1}{2}$ on $[-1, 1]$. The density is symmetric about zero, so every odd moment is zero: $E[X] = 0$ and $E[X^3] = 0$. Also
$$E[X^2] = \int_{-1}^{1} \frac{x^2}{2}\, dx = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$$
3. Moments of $N \sim \mathcal{N}(0, 1)$: $E[N] = 0$. Because X and N are independent, $E[XN] = E[X]\,E[N] = 0$.
4. Correlation of X and Y, by linearity of expectation:
$$E[XY] = E[X(X + N)] = E[X^2] + E[XN] = \frac{1}{3} + 0 = \frac{1}{3}$$
5. Correlation of X and Z:
$$E[XZ] = E[X(X^2 + N)] = E[X^3] + E[XN] = 0 + 0 = 0$$
6. Checking the reading: since $E[X] = 0$, the covariances equal these values, $\mathrm{Cov}(X, Y) = \frac{1}{3}$ and $\mathrm{Cov}(X, Z) = 0$. The normalised coefficient would be $\rho_{XY} = \dfrac{1/3}{\sqrt{1/3}\,\sqrt{1/3 + 1}} = \dfrac{1}{2}$, and $\frac{1}{2}$ is not in any option. So the question means $E[XY]$. Under either reading X and Z are uncorrelated, even though Z depends on X through X^2.
7. The pair is $\left(\frac{1}{3},\, 0\right)$, which is option (A).

KEY IDEA

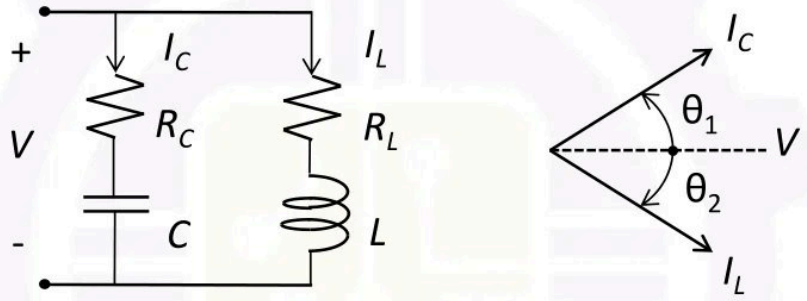
Expand each correlation with linearity of expectation. Independence makes the cross term zero, $E[XN] = E[X]E[N] = 0$. A uniform distribution symmetric about zero has every odd moment zero, so $E[XY] = E[X^2] = \frac{1}{3}$ and $E[XZ] = E[X^3] = 0$.

WATCH OUT FOR

Thinking that because Z is built from X, the two must be correlated. Z depends on X only through X^2, and $E[X \cdot X^2] = E[X^3] = 0$ for a symmetric X. So they are uncorrelated but not independent. A second trap is computing the normalised coefficient $\rho_{XY} = \frac{1}{2}$ and finding it in no option. In this question "correlation" means $E[XY]$.

Q.39 MCQ · 2 MARKS

Networks (Electric Circuits) › AC, power & resonance

Q.39	In the given circuit, $L = 1$ μH and $C = 1$ μF. The phasor diagram for I_C and I_L is also shown. Assume that the phase $(\theta_1 + \theta_2)$ is 90° at a frequency of 159.15 kHz. Among the following options, what is the nearest integer value of $R_C \times R_L$? 
(A)	0
(B)	1
(C)	2
(D)	10

Official answer: **B**

SOLUTION

1. Angular frequency: $\omega = 2\pi f = 2\pi \times 159.15 \times 10^3$ Hz $\approx 9.9997 \times 10^5 \approx 10^6$ rad/s. So $X_L = \omega L = 10^6 \times 10^{-6} = 1\ \Omega$ and $X_C = \dfrac{1}{\omega C} = \dfrac{1}{10^6 \times 10^{-6}} = 1\ \Omega$.
2. RC branch: $I_C = \dfrac{V}{R_C - jX_C}$. The denominator has angle $-\tan^{-1}(X_C/R_C)$, so I_C leads V by θ_1 with $\tan\theta_1 = \dfrac{X_C}{R_C}$, where $0^\circ < \theta_1 < 90^\circ$.

3. RL branch: $I_L = \dfrac{V}{R_L + jX_L}$. The denominator has angle $+\tan^{-1}(X_L/R_L)$, so I_L lags V by θ_2 with $\tan\theta_2 = \dfrac{X_L}{R_L}$, where $0^\circ < \theta_2 < 90^\circ$.
4. Quadrature condition: $\theta_1 + \theta_2 = 90^\circ$ gives $\tan\theta_1 = \tan(90^\circ - \theta_2) = \cot\theta_2$, i.e. $\tan\theta_1 \tan\theta_2 = 1$.
5. Substitute: $\dfrac{X_C}{R_C} \cdot \dfrac{X_L}{R_L} = 1 \;\Rightarrow\; R_C R_L = X_L X_C = \omega L \cdot \dfrac{1}{\omega C} = \dfrac{L}{C}$.
6. $$R_C R_L = \frac{L}{C} = \frac{1 \times 10^{-6}\ \text{H}}{1 \times 10^{-6}\ \text{F}} = 1\ \Omega^2$$

   The frequency cancels, so the rounding of ω in step 1 does not affect the result: $R_C R_L = 1$ at any frequency. The nearest integer is 1, option (B).

KEY IDEA

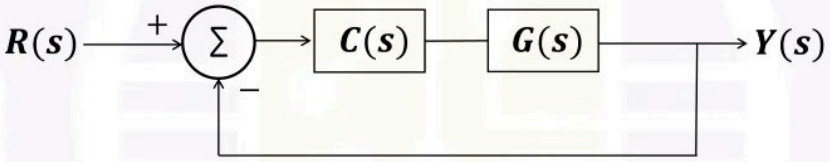
In an RC branch and an RL branch connected in parallel, the two branch currents are 90° apart exactly when $\tan\theta_1 \tan\theta_2 = 1$, i.e. $\dfrac{X_C X_L}{R_C R_L} = 1$. Since $X_L X_C = \omega L \cdot \dfrac{1}{\omega C} = \dfrac{L}{C}$, the condition becomes $R_C R_L = L/C$, and the frequency drops out.

WATCH OUT FOR

Setting $\theta_1 = \theta_2 = 45^\circ$ (taking $R_C = X_C$ and $R_L = X_L$ separately) is only one possible case. The question fixes only the sum of the angles, so only the product $R_C R_L$ is determined. Another slip is to write the quadrature condition as $\tan\theta_1 + \tan\theta_2 = 1$ or $\theta_1 = \theta_2$, when the correct relation is $\tan\theta_1 \tan\theta_2 = 1$. Some students also spend time computing ω exactly, even though it cancels.

Q.40 MCQ · 2 MARKS

Control Systems › Compensators & PID

Q.40	For the control system shown in the Figure, the transfer function of a plant, $G(s) = \frac{1}{(s+1)(s+2)}$ is connected in cascade with a compensator $C(s) = K\,(s+\alpha)$, where K and α are positive real valued constants. Which of the following pairs (K, α) represent the correct values for the closed loop system to have poles at $(-3 \pm j\sqrt{5})$? 
(A)	2, 3
(B)	3, 4
(C)	2, 4
(D)	3, 3

Official answer: **B**

SOLUTION

1. With unity negative feedback and $C(s)$ in cascade with $G(s)$, the open-loop transfer function is
$$C(s)G(s) = \frac{K(s+\alpha)}{(s+1)(s+2)}$$
2. The closed-loop poles are the roots of $1 + C(s)G(s) = 0$:
$$(s+1)(s+2) + K(s+\alpha) = 0 \;\Rightarrow\; s^2 + (3+K)\,s + (2+K\alpha) = 0$$

3. The characteristic polynomial the poles $-3 \pm j\sqrt{5}$ call for is
$$(s+3)^2 + \left(\sqrt{5}\right)^2 = s^2 + 6s + 14$$
4. Match the s coefficients: $3 + K = 6 \Rightarrow K = 3$.
5. Match the constant terms: $2 + K\alpha = 14 \Rightarrow 3\alpha = 12 \Rightarrow \alpha = 4$.
6. Check: $s^2 + 6s + 14 = 0$ gives $s = \dfrac{-6 \pm \sqrt{36 - 56}}{2} = -3 \pm j\sqrt{5}$, as required. Both K and α are positive, so $(K, \alpha) = (3, 4)$, which is option (B).

KEY IDEA

For a second-order closed loop, the complex poles $-\sigma \pm j\omega_d$ fix the characteristic polynomial exactly as $s^2 + 2\sigma s + (\sigma^2 + \omega_d^2)$. The compensator gives two free parameters, one for each coefficient. Here K alone sets the s coefficient $(3 + K)$, and once K is known, α sets the constant term $(2 + K\alpha)$.

WATCH OUT FOR

Taking the constant term of the desired polynomial as $3^2 - 5 = 4$ or as 9, when it is $3^2 + (\sqrt{5})^2 = 14$. The other mistake is forgetting that K multiplies α in the constant term, which gives $2 + \alpha = 14$. Every distractor is close. Option (D), $(3, 3)$, gets K right but gives $s^2 + 6s + 11$, whose poles are $-3 \pm j\sqrt{2}$. Options (A) and (C) have $K = 2$, which gives an s coefficient of 5 and a real part of -2.5.

Q.41 MCQ · 2 MARKS

Control Systems › State space

The state and output equations for a control system are:
$$\dot{x} = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1.5 & 0.625 \end{bmatrix} x$$

Which of the following expressions correctly represents the transfer function $\dfrac{Y(s)}{U(s)}$ of the system with zero initial conditions?

(A) $\dfrac{3s}{s^2 + 4s - 6}$

(B) $\dfrac{3s + 5}{s^2 + 4s - 6}$

(C) $\dfrac{3s + 5}{s^2 + 4s + 6}$

(D) $\dfrac{3s}{s^2 + 4s + 6}$

Official answer: **C**

SOLUTION

1. With no direct feed-through term ($D = 0$), the transfer function of a state-space model is
$$G(s) = \frac{Y(s)}{U(s)} = C\,(sI - A)^{-1}B = \frac{C\,\mathrm{adj}(sI - A)\,B}{\det(sI - A)}$$
where $A = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1.5 & 0.625 \end{bmatrix}$.
2. Form $sI - A$:
$$sI - A = \begin{bmatrix} s + 4 & 1.5 \\ -4 & s \end{bmatrix}$$
3. Characteristic polynomial:
$$\det(sI - A) = s(s + 4) - (1.5)(-4) = s^2 + 4s + 6$$
The constant term is $+6$, which rules out options (A) and (B).
4. Adjugate of a 2×2 matrix (swap the diagonal entries, negate the off-diagonal entries):

$$\operatorname{adj}(sI - A) = \begin{bmatrix} s & -1.5 \\ 4 & s+4 \end{bmatrix}$$

5. Multiply by B:

$$\operatorname{adj}(sI - A)\, B = \begin{bmatrix} s & -1.5 \\ 4 & s+4 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2s \\ 8 \end{bmatrix}$$

6. Multiply by C:

$$\begin{bmatrix} 1.5 & 0.625 \end{bmatrix} \begin{bmatrix} 2s \\ 8 \end{bmatrix} = 1.5(2s) + 0.625(8) = 3s + 5$$

   The numerator has a constant term 5, which rules out option (D).

7. Therefore

$$\frac{Y(s)}{U(s)} = \frac{3s+5}{s^2 + 4s + 6}$$

   which is option (C). Quick check: the DC gain $G(0) = -CA^{-1}B$ should equal $\frac{5}{6}$. Here $A^{-1} = \frac{1}{6}\begin{bmatrix} 0 & 1.5 \\ -4 & -4 \end{bmatrix}$, so $A^{-1}B = \frac{1}{6}\begin{bmatrix} 0 \\ -8 \end{bmatrix}$ and $-CA^{-1}B = -\frac{1}{6}(0.625)(-8) = \frac{5}{6}$, which agrees.

KEY IDEA

For a strictly proper state-space model, $G(s) = C\operatorname{adj}(sI - A)\,B \,/\, \det(sI - A)$. The denominator is the characteristic polynomial of A, and for a 2×2 system you can read it off directly as $s^2 - (\operatorname{trace} A)\, s + \det A = s^2 + 4s + 6$. The numerator needs only one column of the adjugate, because B has a single non-zero entry.

WATCH OUT FOR

Sign errors in $\det(sI - A)$. The off-diagonal product is $(1.5)(-4) = -6$, and subtracting it gives $+6$. Writing -6 leads to option (B). The other common slip is dropping the $0.625 \times 8 = 5$ term: the second state contributes to the output through C even though u does not drive it directly, and missing that term leads to option (D). Also watch the adjugate: swap the diagonal entries and negate the off-diagonal ones. Transposing without negating gives the wrong numerator.

Q.42 MCQ · 2 MARKS

Digital Electronics › Processors & memory

The address of the first location of a 256 kilo byte (KB) memory is $(2500)_H$.

Choose the correct address of the last location of the memory.

(A) $(2FFF)_H$

(B) $(124FF)_H$

(C) $(424FF)_H$

(D) $(324FF)_H$

Official answer: **C**

SOLUTION

1. Take the memory to be byte-addressable, so each address holds one byte. The number of locations is $256\ \text{KB} = 2^8 \times 2^{10} = 2^{18}$.
2. Write the size in hexadecimal. Since $2^{16} = (10000)_H$ and $2^{18} = 4 \times 2^{16}$, we get $2^{18} = (40000)_H$.
3. A memory of N locations that starts at address A ends at $A + N - 1$. The offset of the last location from the first is $N - 1 = (40000)_H - 1 = (3FFFF)_H$.
4. Add this offset to the start address in hexadecimal. Write $(3FFFF)_H = (40000)_H - 1$:

$$(2500)_H + (3FFFF)_H = (2500)_H + (40000)_H - 1 = (42500)_H - 1 = (424FF)_H$$

5. Check: $(424FF)_H - (2500)_H + 1 = (3FFFF)_H + 1 = (40000)_H = 2^{18}$ bytes $= 256$ KB. This is option (C).

KEY IDEA

A byte-addressable block of N locations that starts at address A occupies A to $A + N - 1$. Here $256\ \text{KB} = 2^{18} = (40000)_H$, so the last address is $(2500)_H + (3FFFF)_H = (424FF)_H$.

WATCH OUT FOR

One mistake is to add the full size, $(2500)_H + (40000)_H = (42500)_H$. That is the first address after the memory, so you still have to subtract 1. The other is to write $256\ \text{KB}$ as $(10000)_H$ or $(30000)_H$ instead of $(40000)_H$ (2^{16} is only 64 KB). That mistake leads to distractors such as (B) or (D).

Q.43 MCQ · 2 MARKS

Signals and Systems › Signals basics

Consider a real signal $x(t)$, $-\infty < t < \infty$, such that

$x(t) = 0$ for $t < 0$, $x(t) = 2$ for $0 \le t < 1$ and $x(t) = 0$ for $t \ge 1$.

Let $E[x(t)] = \int_{-\infty}^{\infty} [x(t)]^2\, dt$.

Which of the following options correctly represents the ratio,

$E[x(t)]/E[3\,x(-3t+5)]$?

(A) 3

(B) 1

(C) $1/3$

(D) $1/9$

Official answer: **C**

SOLUTION

1. Energy of the original pulse (height 2, width 1):
$$E[x(t)] = \int_0^1 2^2\, dt = 4$$
2. Amplitude scaling: $E[A\,y(t)] = \int_{-\infty}^{\infty} A^2 [y(t)]^2\, dt = A^2 E[y(t)]$, so the factor 3 multiplies the energy by $3^2 = 9$.
3. Time scaling, reversal and shifting, $y(t) = x(at + b)$ with $a \neq 0$: substitute $\tau = at + b$, so $dt = d\tau/a$. For $a < 0$ the limits also swap, and the two sign changes cancel, giving $dt \to d\tau/|a|$ in every case:
$$E[x(at+b)] = \frac{1}{|a|}\int_{-\infty}^{\infty} [x(\tau)]^2\, d\tau = \frac{1}{|a|} E[x(t)]$$
The shift b and the reversal (the sign of a) do not change the energy.
4. With $a = -3$: $E[x(-3t+5)] = \frac{1}{3} \cdot 4 = \frac{4}{3}$.
5. Combine the two rules: $E[3\,x(-3t+5)] = 9 \cdot \frac{4}{3} = 12$.
6. Direct check: $3\,x(-3t+5) = 3 \cdot 2 = 6$ where $0 \le -3t + 5 < 1$, i.e. $\frac{4}{3} < t \le \frac{5}{3}$, an interval of width $\frac{1}{3}$, and zero elsewhere. Energy $= 6^2 \cdot \frac{1}{3} = 12$, which agrees.
7. Ratio:
$$\frac{E[x(t)]}{E[3\,x(-3t+5)]} = \frac{4}{12} = \frac{1}{3}$$
which is option (C).

KEY IDEA

Energy scales with the square of the amplitude and inversely with the magnitude of the time-scale factor: $E[A\,x(at+b)] = \frac{A^2}{|a|}E[x(t)]$. A time shift or a time reversal leaves the energy unchanged.

WATCH OUT FOR

Three common slips: scaling energy by A instead of A^2 (this gives $E = 4$ and a ratio of 1, option B); multiplying by $|a|$ instead of dividing (compression by 3 makes the pulse narrower, so it has less energy, not more; this gives a ratio of 1/27, which is not an option); and thinking the shift $+5$ or the minus sign on t changes the energy.

Q.44 MCQ · 2 MARKS

Communications › Digital communication

A QPSK modulated signal from an additive white Gaussian noise (AWGN) channel is received with an $E_b/N_o = 8.4$ dB at the input of a coherent QPSK demodulator. A maximum-likelihood reception method is used in the demodulator.

Assume the complimentary error function $\text{erfc}(u) \cong [1/(u\sqrt{\pi})]\exp(-u^2)$.

Which is the nearest bit error rate (BER) at the output of the demodulator?

(A) 10^{-3}

(B) 10^{-4}

(C) 10^{-5}

(D) 10^{-6}

Official answer: **B**

SOLUTION

1. Convert the SNR to a ratio: $E_b/N_0 = 10^{8.4/10} = 10^{0.84} \approx 6.92$.
2. Coherent QPSK with Gray mapping and ML detection has the same bit error rate as BPSK, because the two quadrature bits are detected independently, each with energy E_b per bit: $P_b = Q\left(\sqrt{2E_b/N_0}\right) = \frac{1}{2}\,\text{erfc}\left(\sqrt{E_b/N_0}\right)$.
3. Set $u = \sqrt{E_b/N_0} = \sqrt{6.92} \approx 2.63$, so $u^2 \approx 6.92$.
4. Apply the given approximation $\text{erfc}(u) \cong \frac{e^{-u^2}}{u\sqrt{\pi}}$. Here $e^{-6.92} \approx 9.9 \times 10^{-4}$ and $u\sqrt{\pi} = 2.63 \times 1.7725 \approx 4.66$.
5. $\text{erfc}(u) \approx \frac{9.9 \times 10^{-4}}{4.66} \approx 2.12 \times 10^{-4}$.
6. $P_b = \frac{1}{2}\,\text{erfc}(u) \approx 1.06 \times 10^{-4}$. This is nearest to 10^{-4}, so the answer is option (B).

KEY IDEA

Coherent QPSK with Gray mapping has the same BER as BPSK at the same E_b/N_0: $P_b = \frac{1}{2}\,\text{erfc}\left(\sqrt{E_b/N_0}\right)$. Convert dB to a ratio first, then use the given erfc approximation.

WATCH OUT FOR

Using $E_s/N_0 = 2E_b/N_0$ inside the erfc argument without the matching change, or forgetting the factor $\frac{1}{2}$ in front of erfc. Forgetting the $\frac{1}{2}$ gives 2.1×10^{-4}, which is still nearest to 10^{-4}. Using $\sqrt{2E_b/N_0}$ as the erfc argument gives a far smaller value near 10^{-8}, which matches no option.

Q.45 MCQ · 2 MARKS

Digital Electronics › Number systems

What is the $10's$ complement of $(47)_{10}$?

(A) 52

(B) 53

(C) 54

(D) 55

Official answer: **B**

SOLUTION

1. For an n-digit number N in base r, the r's complement is $r^n - N$. Here $r = 10$, and $N = (47)_{10}$ has $n = 2$ digits.
2. Apply the formula directly: $10^2 - 47 = 100 - 47 = 53$.
3. Check it with the 9's complement. The $(r-1)$'s complement is $(r^n - 1) - N = 99 - 47 = 52$, which just replaces each digit d with $9 - d$ (so $4 \to 5$ and $7 \to 2$). Adding 1 gives the 10's complement: $52 + 1 = 53$.
4. Check the sum: $47 + 53 = 100$. Drop the end carry and 00 is left, which is what a number plus its r's complement must give. That is why the complement can stand for $-N$ in subtraction.
5. So the 10's complement of $(47)_{10}$ is 53, option (B).

KEY IDEA

In base r, the r's complement of an n-digit number N is $r^n - N$. It equals the $(r-1)$'s complement, $(r^n - 1) - N$, plus 1. For $N = 47$: $100 - 47 = 53$, or $99 - 47 = 52$ and then $52 + 1 = 53$.

WATCH OUT FOR

Stopping at the 9's complement (52, option A) and forgetting to add 1 is the usual slip. A second one is using the wrong power of the base, for example $10^3 - 47 = 953$. With $n = 2$ digits the power must be 10^2.

Q.46 MCQ · 2 MARKS

Signals and Systems › Fourier transform

Consider a real baseband signal $x(t) = e^{-2t}$, for t (in seconds) ≥ 0.

If 99% of energy of $x(t)$ lies within B Hz, then which of the following options is TRUE for the value of B?

(A) $B > 1$ kHz

(B) $63/\pi$ Hz $< B < 64/\pi$ Hz

(C) $126/\pi$ Hz $< B < 128/\pi$ Hz

(D) $B < 1$ Hz

Official answer: **B**

SOLUTION

1. **Total energy (time domain):** $E = \int_0^\infty \left(e^{-2t}\right)^2 dt = \int_0^\infty e^{-4t}\, dt = \frac{1}{4}$.
2. **Fourier transform in Hz:** $x(t) = e^{-2t}u(t) \;\Rightarrow\; X(f) = \frac{1}{2 + j2\pi f}$, so $|X(f)|^2 = \frac{1}{4 + 4\pi^2 f^2} = \frac{1}{4\left(1 + \pi^2 f^2\right)}$.
3. **Energy inside the band (Parseval / Rayleigh):** for a real baseband signal, a bandwidth of B Hz means the band $|f| \leq B$, i.e. both sides of the spectrum. Since $|X(f)|^2$ is even,
$$E_B = 2\int_0^B \frac{df}{4\left(1 + \pi^2 f^2\right)} = \frac{1}{2} \cdot \frac{1}{\pi} \tan^{-1}(\pi B) = \frac{1}{2\pi} \tan^{-1}(\pi B).$$
4. **Energy fraction:** $\frac{E_B}{E} = \frac{\tan^{-1}(\pi B)/(2\pi)}{1/4} = \frac{2}{\pi} \tan^{-1}(\pi B)$. As a check, this tends to 1 as $B \to \infty$, as it must.
5. **Set it equal to 0.99:** $\frac{2}{\pi} \tan^{-1}(\pi B) = 0.99 \Rightarrow \tan^{-1}(\pi B) = 0.495\pi$, so
$$\pi B = \tan(0.495\pi) = \cot(0.005\pi) = \cot(0.015708) \approx \frac{1}{0.015708} - \frac{0.015708}{3} \approx 63.66.$$

(For a small angle θ, $\cot\theta \approx \frac{1}{\theta} - \frac{\theta}{3}$; the correction here is only about 0.005, so πB lies well inside $63 < \pi B < 64$.)

6. **Bandwidth:** $B = \dfrac{63.66}{\pi}\ \text{Hz} \approx 20.3\ \text{Hz}$.
7. **Place it among the options:** $63 < 63.66 < 64$, so $\dfrac{63}{\pi}\ \text{Hz} < B < \dfrac{64}{\pi}\ \text{Hz}$, which is option (B). Option (A) is far too large and option (D) far too small. Cross-check in rad/s: $|X(\omega)|^2 = \dfrac{1}{4+\omega^2}$ gives $\dfrac{2}{\pi}\tan^{-1}\left(\dfrac{\omega_B}{2}\right) = 0.99$, so $\omega_B = 2 \times 63.66 = 127.3\ \text{rad/s}$ and $B = \dfrac{\omega_B}{2\pi} = \dfrac{63.66}{\pi}\ \text{Hz}$, the same result.

KEY IDEA

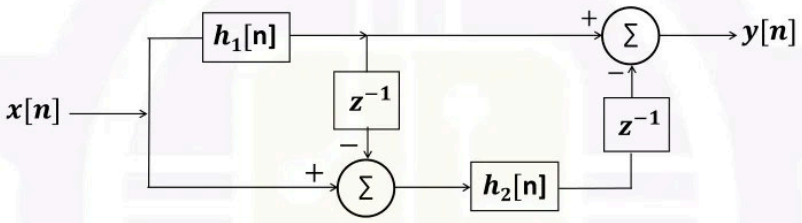
Use Rayleigh's (Parseval's) theorem. The total energy is $1/4$ and $|X(f)|^2 = \frac{1}{4(1+\pi^2 f^2)}$. Integrating this over the two-sided band $|f| \le B$ gives the energy fraction $\frac{2}{\pi}\tan^{-1}(\pi B)$. Setting that equal to 0.99 gives $\pi B = \cot(0.005\pi) \approx 63.66$.

WATCH OUT FOR

Working in rad/s, getting $\omega_B \approx 127.3\ \text{rad/s}$, then dividing by π instead of 2π. That gives $B \approx 127.3/\pi$ Hz, which is the trap option (C). A second slip is integrating only $0 \le f \le B$ without doubling for the negative frequencies. The fraction then tops out at 50% and can never reach 99%.

Q.47 MCQ · 2 MARKS

Signals and Systems › LTI & convolution

Q.47	Consider the discrete time system (S) with input $x[n]$ and output $y[n]$ as shown in the Figure. The two sub-systems represented by their impulse responses $h_1[n]$ and $h_2[n]$ are linear and time invariant. Which of the following statements is necessarily TRUE? 
(A)	S is causal.
(B)	S is linear and time invariant.
(C)	S is linear and time varying.
(D)	S is non-linear.

Official answer: **B**

SOLUTION

1. Name the internal signals. Let $w[n] = (h_1 * x)[n]$ be the output of h_1.
2. Lower summing junction: its '+' input is $x[n]$ and its '−' input is $w[n]$ passed through z^{-1}. So $v[n] = x[n] - w[n-1]$. This drives h_2, so $u[n] = (h_2 * v)[n]$.
3. Output summing junction: its '+' input is $w[n]$ and its '−' input is $u[n]$ passed through z^{-1}. So $y[n] = w[n] - u[n-1]$.

4. Every block (h_1, h_2, z^{-1}) is LTI, and summing junctions and pick-offs are LTI too. There is no feedback loop: signals only flow forward from x to y. Series, parallel and summing connections of LTI systems give an LTI system. To see it directly, set $x[n] = \delta[n]$: then $w[n] = h_1[n]$, $v[n] = \delta[n] - h_1[n-1]$ and $u[n] = h_2[n] - (h_1 * h_2)[n-1]$. The overall impulse response is
$$h[n] = h_1[n] - h_2[n-1] + (h_1 * h_2)[n-2]$$
5. In the z-domain (where the transforms exist), the same result is
$$Y(z) = H_1 X - z^{-1} H_2 \left(X - z^{-1} H_1 X\right)$$
$$H(z) = H_1(z) - z^{-1} H_2(z) + z^{-2} H_1(z) H_2(z)$$
So $y[n] = (h * x)[n]$ is a convolution with one fixed impulse response. That makes S linear and time invariant, which is option (B), and it rules out (C) and (D).
6. Causality: nothing says $h_1[n]$ or $h_2[n]$ is zero for $n < 0$. For example, take $h_1[n] = \delta[n+1]$, so $H_1(z) = z$. Then
$$H(z) = z - z^{-1} H_2(z) + z^{-2} \cdot z\, H_2(z) = z$$
This holds for every h_2, so $y[n] = x[n+1]$, which is non-causal. S is causal when both h_1 and h_2 are causal, but the question does not say they are. So (A) is not necessarily true.
7. Only (B) holds for every LTI h_1 and h_2.

KEY IDEA

Delays, adders, pick-offs and LTI blocks joined without a feedback loop always make an LTI system. Here the overall impulse response is fixed: $h[n] = h_1[n] - h_2[n-1] + (h_1 * h_2)[n-2]$, or $H(z) = H_1 - z^{-1} H_2 + z^{-2} H_1 H_2$. Whether the system is causal depends on whether h_1 and h_2 are causal, and the question does not tell us.

WATCH OUT FOR

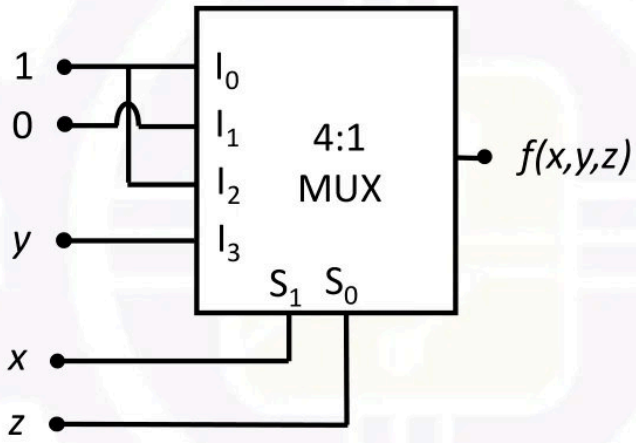
Picking (A) because the diagram contains only unit delays z^{-1}, which makes it look causal. A delay after an anti-causal block does not remove its anti-causality: with $h_1[n] = \delta[n+1]$ the whole system reduces to $y[n] = x[n+1]$ for any h_2. Another mistake is to read the cross-coupled branches as feedback that makes the system time-varying. There is no loop, and a fixed interconnection of LTI blocks stays LTI.

Q.48 MCQ · 2 MARKS

Digital Electronics › Combinational

Q.48 A Boolean function, $f(x,y,z)$ with x as MSB and z as LSB is realized by 4:1 multiplexer (MUX) with select lines, S_1 and S_0 (S_1 is MSB, S_0 is LSB) and inputs, I_0, I_1, I_2, I_3 as shown in the Figure.

Which of the following options is the correct expression of $f(x,y,z)$?



(A) $xz + y$

(B) $x\bar{y} + z$

(C) $xy + \bar{z}$

(D) $\bar{x}y + \bar{z}$

Official answer: **C**

SOLUTION

1. Read the data inputs off the figure. A vertical wire leaves the 1 line at a T-junction and runs down to I_2, so it carries 1. The 0 line hops over that wire (the arc means no connection) and goes to I_1. So $I_0 = 1$, $I_1 = 0$, $I_2 = 1$, $I_3 = y$, and the select lines are $S_1 = x$, $S_0 = z$. Complement is written with ‾ throughout.
2. Output equation of a 4:1 MUX:

$$f = \overline{S_1}\,\overline{S_0}\,I_0 + \overline{S_1}S_0\,I_1 + S_1\overline{S_0}\,I_2 + S_1S_0\,I_3$$

3. Substitute $S_1 = x$, $S_0 = z$ and the inputs:
$$f = \overline{x}\,\overline{z} \cdot 1 + \overline{x}z \cdot 0 + x\overline{z} \cdot 1 + xz \cdot y = \overline{x}\,\overline{z} + x\overline{z} + xyz$$
4. Combine the first two terms: $\overline{x}\,\overline{z} + x\overline{z} = \overline{z}(\overline{x} + x) = \overline{z}$. This leaves $f = \overline{z} + xyz$.
5. Apply the absorption rule $A + \overline{A}B = A + B$ with $A = \overline{z}$ (so $\overline{A} = z$) and $B = xy$:
$$f = \overline{z} + z\,(xy) = \overline{z} + xy$$
6. Check with the truth table. When $z = 0$ the MUX selects $I_0 = 1$ or $I_2 = 1$, so $f = 1$ (minterms 0, 2, 4, 6). When $z = 1$ it selects $I_1 = 0$ (for $x = 0$) or $I_3 = y$ (for $x = 1$), so the only 1 is at $xyz = 111$ (minterm 7). So $f = \sum m(0, 2, 4, 6, 7) = xy + \overline{z}$, which is option (C). The other options fail: (A) and (B) give $f = 0$ at 000, and (D) gives $f = 1$ at 011 and $f = 0$ at 111.

KEY IDEA

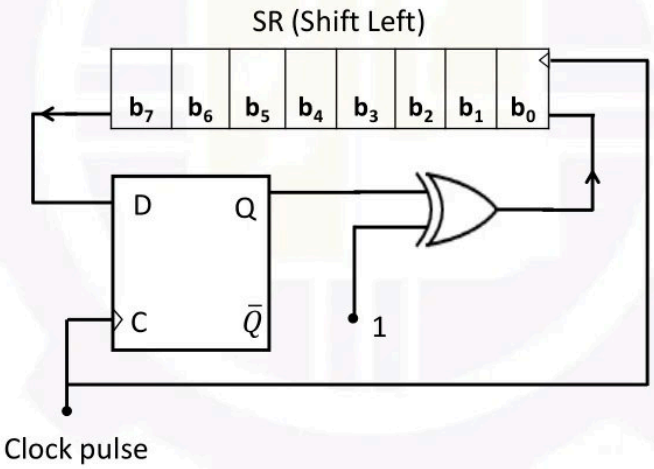
A 4:1 MUX outputs the data input chosen by its select lines, $f = \sum_k m_k(S_1, S_0)\, I_k$. Put in $S_1 = x$, $S_0 = z$ and the inputs read from the figure ($I_0 = 1$, $I_1 = 0$, $I_2 = 1$, $I_3 = y$), then simplify. The two constant-1 inputs together cover every case with $z = 0$.

WATCH OUT FOR

Most errors come from misreading the figure. If you treat the hop on the 0 line as a connection, you tie 0 to the vertical wire and get $I_2 = 0$ (and a 1-to-0 short). Mistaking which variable drives which select line is the other trap: x drives S_1 and z drives S_0, so y is not a select input at all. Some students also take the select order as (x, y) just because x is the MSB. That pairs the terms wrongly and leads to options such as (D).

Q.49 MCQ · 2 MARKS

Digital Electronics › Sequential

Q.49	A shift-left Shift Register (SR) and a D flip-flop are connected to a synchronized clock as shown in the Figure. Assume that the SR and D flip-flops are initially cleared and the XOR gate has no propagation delay. Which of the following options gives the correct binary representation ($b_7b_6b_5b_4b_3b_2b_1b_0$) of the content of the shift register immediately after the 5^{th} clock transition (positive edge)? 
(A)	00011111
(B)	10111111
(C)	00111111
(D)	11000011

Official answer: **A**

SOLUTION

1. **Read the connections.** The serial input of the shift-left register is the XOR output: $b_0^+ = Q \oplus 1 = \overline{Q}$ (overline = complement). The D flip-flop samples the bit being shifted out: $Q^+ = D = b_7$. On each positive edge, $b_7 \leftarrow b_6,\ b_6 \leftarrow b_5,\ \ldots,\ b_1 \leftarrow b_0,\ b_0 \leftarrow \overline{Q}$.
2. **Synchronous update rule.** The register and the flip-flop share one clock edge, so every new value is computed from the values present just before that edge. The XOR has no delay, so it simply keeps $\overline{Q}$ at the serial input.

3. **Initial state:** $SR = (00000000)_2$ and $Q = 0$, so the serial input is $\overline{Q} = 1$.
4. **Clock 1:** $b_0 \leftarrow 1$ and $Q \leftarrow b_7 = 0$, giving $SR = (00000001)_2$.
5. **Clocks 2 to 4:** before each of these edges $b_7 = 0$, so Q stays 0 and another 1 enters at b_0 each time: $(00000011)_2 \rightarrow (00000111)_2 \rightarrow (00001111)_2$.
6. **Clock 5:** just before the edge $b_7 = 0$ and $Q = 0$, so another 1 enters: $SR = (00011111)_2$.
7. **Why the feedback never acts here.** The first 1 enters b_0 at edge 1 and does not reach b_7 until edge 8, so Q cannot become 1 before edge 9. The first five edges therefore just shift in five 1s from the right, and the content is $(00011111)_2$: option (A).

KEY IDEA

XOR with a constant 1 is an inverter, so the serial input is $\overline{Q}$. Q samples b_7, which stays 0 until a 1 has travelled all 8 cells, so for the first 8 edges $Q = 0$ and a 1 shifts in at b_0 on every edge.

WATCH OUT FOR

Taking the serial input as Q instead of $Q \oplus 1 = \overline{Q}$ leaves the register at all zeros, which is not an option. Filling from the b_7 end, as if it were a shift-right register, gives $(11111000)_2$, also not an option. Counting one edge too many (for example, treating the initial clear as a clock) gives six 1s, $(00111111)_2$, which is the trap option (C).

Q.50 MCQ · 2 MARKS

Analog Electronics › Amplifiers

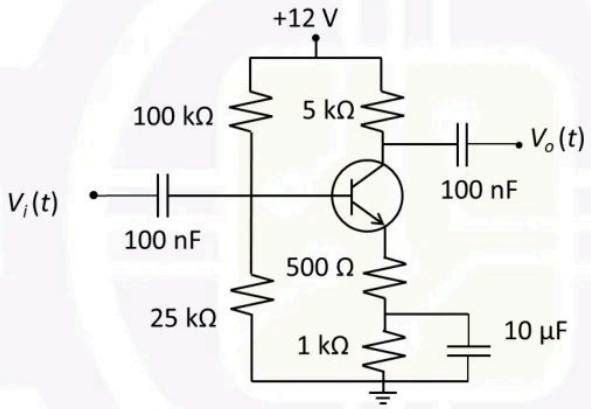
Q.50

A small signal source, $V_i(t) = A\cos(10^5 t) + B\sin(10^7 t)$ is applied to a BJT circuit as shown in the Figure.

Assume zero source resistance, $V_{BE} = 0.7$ V, $\beta_{dc} = 99$, Early voltage $= 100$ V and

Thermal voltage $= 25$ mV. Effect of internal parasitic capacitances of the BJT may be neglected.

Which expression is the best approximation of the output voltage $V_o(t)$?



(A)	$-9.1[A\cos(10^5 t) + B\sin(10^7 t)]$
(B)	$9.1[A\cos(10^5 t) - B\sin(10^7 t)]$
(C)	$-190.4[A\cos(10^5 t) + B\sin(10^7 t)]$
(D)	$190.4[A\cos(10^5 t) - B\sin(10^7 t)]$

Official answer: **A**

SOLUTION

1. **DC bias (all capacitors open).** Replace the base divider with its Thevenin equivalent: $V_{TH} = 12 \cdot \dfrac{25}{100+25} = 2.4$ V and $R_{TH} = 100\,\text{k}\Omega \parallel 25\,\text{k}\Omega = 20\,\text{k}\Omega$. With the 10 μF capacitor open, the DC current flows through both emitter resistors, so $R_{E,dc} = 500 + 1000 = 1.5\,\text{k}\Omega$.
2. **KVL around the base-emitter loop:**

$$I_B = \frac{2.4 - 0.7}{20\,\text{k} + (99+1)(1.5\,\text{k})} = \frac{1.7}{170\,\text{k}} = 10\ \mu\text{A}$$

so $I_C = \beta_{dc} I_B = 0.99\ \text{mA} \approx 1\ \text{mA}$ (the Early effect is ignored for the bias point) and $I_E = (\beta_{dc}+1)I_B = 1\ \text{mA}$. Check the region: $V_C = 12 - 0.99 \times 5 = 7.05\ \text{V}$ and $V_E = 1 \times 1.5 = 1.5\ \text{V}$, so $V_{CE} \approx 5.55\ \text{V}$. The transistor is in the active region.

3. **Small-signal parameters (hybrid-π model, taking $I_C \approx 1\ \text{mA}$):** $g_m = I_C/V_T = 1\,\text{mA}/25\,\text{mV} = 40\ \text{mS}$, $r_\pi = \beta/g_m \approx 99/0.04 \approx 2.5\ \text{k}\Omega$, $r_o = V_A/I_C \approx 100\ \text{k}\Omega$. In the model, r_π sits between base and emitter, and the source $g_m v_\pi$ and r_o both run from collector to emitter. The BJT's parasitic capacitances are left out, as the question says.
4. **Capacitor reactances at the lower tone, $\omega = 10^5\ \text{rad/s}$:** $X_{100\,\text{nF}} = \dfrac{1}{10^5 \cdot 10^{-7}} = 100\ \Omega$ and $X_{10\,\mu\text{F}} = \dfrac{1}{10^5 \cdot 10^{-5}} = 1\ \Omega$. At $\omega = 10^7\ \text{rad/s}$ both are 100 times smaller: $1\ \Omega$ and $0.01\ \Omega$.
5. **Both tones are in the mid-band.** *Input:* $R_{in} = R_{TH} \parallel [r_\pi + (\beta+1)(500)] = 20\,\text{k} \parallel 52.5\,\text{k} \approx 14.5\ \text{k}\Omega$. The source resistance is zero, so the input capacitor and R_{in} form a divider: $|V_b/V_i| = 1/\sqrt{1 + (100/14\,500)^2} > 0.9999$, with a phase lead of $\tan^{-1}(100/14\,500) \approx 0.4^\circ$ at $10^5\ \text{rad/s}$ and less at $10^7\ \text{rad/s}$. *Output:* there is no load, so no signal current flows through the output capacitor and $V_o = V_c$ at both frequencies. *Emitter:* the 1 kΩ in parallel with 10 µF has an impedance of about $1\ \Omega$ at $10^5\ \text{rad/s}$ ($0.01\ \Omega$ at 10^7). That is negligible next to the 500 Ω in series with it, so the AC emitter resistance is $R_E = 500\ \Omega$. *High frequencies:* the parasitic capacitances are neglected, so the gain does not roll off. Both tones therefore see the same real, inverting gain.
6. **Mid-band gain of a CE stage with an unbypassed $R_E = 500\ \Omega$.** The standard approximation takes $I_E \approx I_C$ and treats r_o as if it were in parallel with R_C:

$$A_v \approx -\frac{g_m(R_C \parallel r_o)}{1 + g_m R_E} = -\frac{0.04 \times (5\,\text{k} \parallel 100\,\text{k})}{1 + 0.04 \times 500} = -\frac{190.5}{21} \approx -9.07 \approx -9.1$$

Cross-check without those approximations. With r_o neglected, the model gives exactly $A_v = -\dfrac{\beta R_C}{r_\pi + (\beta+1)R_E} = -\dfrac{99 \times 5000}{2500 + 100 \times 500} \approx -9.43$. Putting r_o back where it really is (collector to emitter) moves this only to about -9.40: the emitter degeneration raises the resistance seen looking into the collector to about $r_o[1 + g_m(R_E \parallel r_\pi)] \approx 1.8\ \text{M}\Omega$, far above R_C. Every model gives a gain of about -9. None comes near 190.

7. **Output:** the same real gain multiplies both components, so

$$V_o(t) \approx -9.1\left[A\cos(10^5 t) + B\sin(10^7 t)\right]$$

Option (A) is the only option with a magnitude near 9 and the same inverting sign on both terms.

8. **Why not the others:** $g_m(R_C \parallel r_o) = 0.04 \times 4.76\,\text{k} \approx 190.5$ (printed as 190.4 in (C) and (D)) would be the gain only if the whole emitter resistance were bypassed. Here the 500 Ω is not bypassed, so (C) is out. Options (B) and (D) make the 10^5 term non-inverting and flip the sign of only the 10^7 term. A CE stage inverts, and a 180° phase difference between two tones that both sit in the flat mid-band is impossible.

KEY IDEA

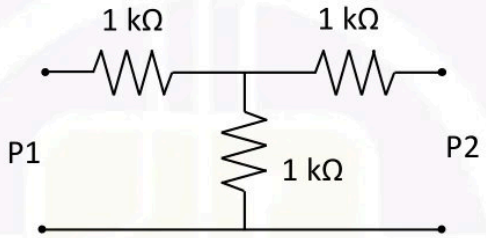
Bias with the full 1.5 kΩ in the emitter ($I_C \approx 1$ mA, so $g_m = 40$ mS). Then check that both tones are in the mid-band: at 10^5 rad/s the 100 nF capacitors are 100 Ω, which is tiny next to a 14.5 kΩ input and an open output, and the 10 µF is 1 Ω, which leaves only the 500 Ω unbypassed. With the parasitics neglected, both tones see one real gain, $-g_m R_C' / (1 + g_m R_E) \approx -9$.

WATCH OUT FOR

Treating the emitter as fully bypassed and taking the gain as $-g_m(R_C \parallel r_o) \approx -190$ (option C). The 10 µF bypasses only the 1 kΩ, so the 500 Ω still degenerates the gain by a factor of $1 + g_m R_E = 21$. Another trap is assuming the 10^7 rad/s tone picks up a different phase and flips sign (options B and D). With the parasitics neglected and the coupling capacitors already short at 10^5 rad/s, both tones sit in the flat mid-band. Also watch the bias: the DC emitter resistance is the full 1.5 kΩ, not 500 Ω.

Q.51 MCQ · 2 MARKS

Networks (Electric Circuits) › Two-port

Q.51	Consider the two-port network as shown in the Figure. Which of the following options provides the correct set of values of A, B, C and D parameters? 
(A)	A = 1/2, B = $3\times10^{3}\,\Omega$, C = $10^{-3}\,\Omega^{-1}$, D = 1/2
(B)	A = 2, B = 3 Ω, C = 1 Ω^{-1}, D = 2
(C)	A = 2, B = $6\times10^{3}\,\Omega$, C = $2\times10^{-3}\,\Omega^{-1}$, D = 2
(D)	A = 2, B = $3\times10^{3}\,\Omega$, C = $10^{-3}\,\Omega^{-1}$, D = 2

Official answer: **D**

SOLUTION

1. Definition of the transmission (ABCD) parameters, with I_2 taken as entering port 2: $V_1 = AV_2 - BI_2$ and $I_1 = CV_2 - DI_2$. Call the left series arm Z_1, the right series arm Z_2 and the shunt arm Z_3. Here $Z_1 = Z_2 = Z_3 = 1\,\text{k}\Omega$.
2. Port 2 open ($I_2 = 0$): I_1 flows through Z_1 and then Z_3, and no current flows in Z_2. So $V_1 = I_1(Z_1 + Z_3) = (2\,\text{k}\Omega)\,I_1$ and $V_2 = I_1 Z_3 = (1\,\text{k}\Omega)\,I_1$.

3. Hence $A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{Z_1+Z_3}{Z_3} = \frac{2}{1} = 2$ and $C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{1}{Z_3} = \frac{1}{10^3\,\Omega} = 10^{-3}\ \Omega^{-1}$.
4. Port 2 short-circuited ($V_2 = 0$): Z_2 now sits in parallel with Z_3, so the input sees $Z_1 + (Z_2 \parallel Z_3) = 1 + 0.5 = 1.5\,\text{k}\Omega$. So $V_1 = (1.5\,\text{k}\Omega)\, I_1$.
5. Current divider at the central node: the current through the short is $-I_2 = I_1 \frac{Z_3}{Z_2+Z_3} = I_1 \cdot \frac{1}{1+1} = \frac{I_1}{2}$.
6. Hence $D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 2$ and $B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = (1.5\,\text{k}\Omega) \times 2 = 3 \times 10^3\ \Omega$. The standard T-network result agrees: $B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3} = 1 + 1 + 1 = 3\,\text{k}\Omega$.
7. Checks: the network is made only of resistors, so it is reciprocal and $AD - BC$ must be 1. Here $AD - BC = 2 \cdot 2 - (3 \times 10^3)(10^{-3}) = 4 - 3 = 1$. It is also symmetrical, so $A = D$ must hold, and it does. The set $A = 2$, $B = 3 \times 10^3\ \Omega$, $C = 10^{-3}\ \Omega^{-1}$, $D = 2$ is option (D).

KEY IDEA

Find the ABCD parameters from two port-2 tests. With port 2 open ($I_2 = 0$) you get $A = V_1/V_2$ and $C = I_1/V_2$. With port 2 shorted ($V_2 = 0$) you get $B = V_1/(-I_2)$ and $D = I_1/(-I_2)$. For a T-network with series arms Z_1, Z_2 and shunt arm Z_3 this gives $A = 1 + Z_1/Z_3$, $B = Z_1 + Z_2 + Z_1 Z_2/Z_3$, $C = 1/Z_3$ and $D = 1 + Z_2/Z_3$. Then check with $AD - BC = 1$ (reciprocal) and $A = D$ (symmetrical).

WATCH OUT FOR

Doing the whole calculation in kΩ and never converting back. That gives $B = 3$ and $C = 1$, and writing these as $3\ \Omega$ and $1\ \Omega^{-1}$ lands on option (B). A second slip is to find V_2/V_1 in place of V_1/V_2, which gives $A = D = 1/2$ as in option (A). Option (A) also fails the reciprocity check, since $AD - BC = 1/4 - 3 \neq 1$.

Q.52 MCQ · 2 MARKS

Electromagnetics › Transmission lines

A complex load (in Ω) is represented as $\Gamma_L = 0.5\angle 30^o$ on the Smith chart.

A co-axial cable with a characteristic impedance of $50\ \Omega$ is connected to the load.

The new input impedance of the load now moves to a diametrically opposite point on the same Γ circle on the Smith chart.

Which option is the nearest input impedance of the cable connected load (in Ω)?

(A) $20.7 - j5.1$

(B) $17.7 - j11.8$

(C) $97.5 - j65.0$

(D) $97.5 + j65.0$

Official answer: **B**

SOLUTION

1. **Diametrically opposite point.** On a constant-$|\Gamma|$ circle, the diametrically opposite point has the same magnitude and a phase 180° away. So $\Gamma_{in} = -\Gamma_L = 0.5\angle(30^\circ + 180^\circ) = 0.5\angle 210^\circ = 0.5\angle -150^\circ$. One full turn of the Smith chart is $\lambda/2$ of line, so a half turn means the lossless cable is $\lambda/4$ long.
2. **Rectangular form.** $\Gamma_{in} = 0.5\,[\cos(-150^\circ) + j\sin(-150^\circ)] = 0.5(-0.866 - j0.5) = -0.433 - j0.25$.
3. **Normalised input impedance** from $z = \frac{1+\Gamma}{1-\Gamma}$:
$$z_{in} = \frac{0.567 - j0.25}{1.433 + j0.25} = \frac{(0.567 - j0.25)(1.433 - j0.25)}{1.433^2 + 0.25^2} = \frac{0.75 - j0.5}{2.116} = 0.354 - j0.236$$
4. **De-normalise** with $Z_0 = 50\ \Omega$: $Z_{in} = 50\, z_{in} \approx 17.7 - j11.8\ \Omega$.

5. **Cross-check with the quarter-wave rule** $Z_{in} = Z_0^2/Z_L$. The load is $z_L = \dfrac{1+\Gamma_L}{1-\Gamma_L} = \dfrac{1.433 + j0.25}{0.567 - j0.25} = \dfrac{0.75 + j0.5}{0.384} = 1.953 + j1.302$, so $Z_L \approx 97.7 + j65.1\ \Omega$. Then
$$Z_{in} = \frac{2500}{97.7 + j65.1} = \frac{2500\,(97.7 - j65.1)}{97.7^2 + 65.1^2} = \frac{2500\,(97.7 - j65.1)}{13775} \approx 17.7 - j11.8\ \Omega,$$
which agrees. (Equivalently, $\Gamma \to -\Gamma$ turns z into $1/z$: $z_{in} = 1/z_L$.)
6. The nearest option is (B), $17.7 - j11.8\ \Omega$.

KEY IDEA

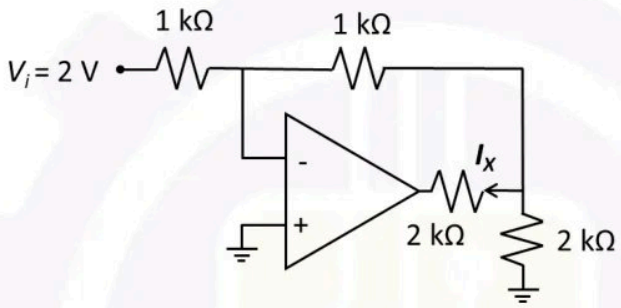
A diametrically opposite point on the same $|\Gamma|$ circle means $\Gamma_{in} = -\Gamma_L$. That is a 180° turn of the chart, which is a $\lambda/4$ lossless line, so the normalised impedance inverts: $z_{in} = 1/z_L$, or $Z_{in} = Z_0^2/Z_L$.

WATCH OUT FOR

Option (D), $97.5 + j65.0\ \Omega$, is the load impedance itself. You get it by converting Γ_L and forgetting to move to the opposite point. Option (C) is its conjugate, which you get by reflecting the point across the real axis (flipping the sign of the phase) instead of turning it through 180°. A further slip is adding 180° and then taking $\cos 210^\circ$ and $\sin 210^\circ$ with the wrong signs. Both parts of Γ_{in} are negative.

Q.53 MCQ · 2 MARKS

Analog Electronics › Op-amps

Q.53	A circuit using an ideal OP-AMP is shown in the Figure. Which of the following options gives the correct value of the current I_X? 
(A)	2.0 mA
(B)	1.5 mA
(C)	3.0 mA
(D)	0 mA

Official answer: **C**

SOLUTION

1. Assume an ideal op-amp with negative feedback. The output drives node X through the $2\ \text{k}\Omega$ resistor, and node X feeds back to the inverting input through the $1\ \text{k}\Omega$ resistor. If V_o rises, V_X rises and so does V_-, which pulls V_o back down, so the feedback is negative. That gives a virtual short, $V_- = V_+ = 0\ \text{V}$, and no current flows into either op-amp input.

2. Current in the input resistor (Ohm's law): $I_1 = \dfrac{V_i - V_-}{1\ \text{k}\Omega} = \dfrac{2-0}{1\ \text{k}\Omega} = 2\ \text{mA}$, flowing toward the inverting node.
3. No current enters the op-amp input, so the same $2\ \text{mA}$ flows on through the feedback $1\ \text{k}\Omega$ resistor, from the inverting node into node X. Therefore $V_X = V_- - (2\ \text{mA})(1\ \text{k}\Omega) = 0 - 2 = -2\ \text{V}$.
4. Current in the grounded $2\ \text{k}\Omega$ resistor: $\dfrac{0 - V_X}{2\ \text{k}\Omega} = \dfrac{0-(-2)}{2\ \text{k}\Omega} = 1\ \text{mA}$, flowing from ground into node X.
5. KCL at node X. The arrow shows I_X leaving X toward the op-amp output, so the two currents entering X must both leave as I_X:
$$I_X = 2\ \text{mA} + 1\ \text{mA} = 3\ \text{mA}$$
6. Check: $V_o = V_X - I_X(2\ \text{k}\Omega) = -2 - (3\ \text{mA})(2\ \text{k}\Omega) = -8\ \text{V}$. That is a finite output, so the ideal op-amp stays in its linear region and sinks $3\ \text{mA}$. The answer is (C).

KEY IDEA

With a virtual ground at the inverting input, the input current $V_i/1\ \text{k}\Omega = 2\ \text{mA}$ has to pass through the feedback $1\ \text{k}\Omega$ resistor, which puts node X at $-2\ \text{V}$. The op-amp output then has to sink this $2\ \text{mA}$ as well as the $1\ \text{mA}$ drawn from ground through the $2\ \text{k}\Omega$ load (KCL at X).

WATCH OUT FOR

Picking 2.0 mA by assuming the output resistor carries only the feedback current, which ignores the $1\ \text{mA}$ in the grounded $2\ \text{k}\Omega$ resistor at node X. Another mistake is to treat the circuit as a plain inverting amplifier with gain -1, so that $V_X = -2\ \text{V}$ is taken to be the output voltage. The op-amp output is actually on the far side of the $2\ \text{k}\Omega$ resistor, at $-8\ \text{V}$.

Q.54 MSQ · 2 MARKS

Electronic Devices › Optoelectronics

Consider an LED based on a direct bandgap semiconductor material with energy bandgap 1.3 eV.

Given: Plank's constant, $h = 6.63 \times 10^{-34}$ J s and speed of light in free space is $3 \times 10^8\ \text{m s}^{-1}$.

In which of the following wavelength ranges the LED will NOT emit?

(A) 1410 ± 20 nm

(B) 1090 ± 20 nm

(C) 950 ± 20 nm

(D) 510 ± 20 nm

Official answer: **A, B, D**

SOLUTION

1. An LED based on a direct-bandgap semiconductor emits photons whose energy is essentially equal to the material's bandgap energy: $E_\gamma \approx E_g$. So the emission is concentrated at a single wavelength $\lambda_g = \dfrac{hc}{E_g}$, not spread across an arbitrary range.
2. Compute hc: $hc = 6.63 \times 10^{-34} \times 3 \times 10^8 = 1.989 \times 10^{-25}\ \text{J m}$.
3. Convert to eV·nm using $q = 1.6 \times 10^{-19}$ C: $\dfrac{hc}{q} = \dfrac{1.989 \times 10^{-25}}{1.6 \times 10^{-19}} = 1.24313 \times 10^{-6}\ \text{eV m} = 1243.13\ \text{eV nm}$.
4. Find the emission wavelength: $\lambda_g = \dfrac{1243.13\ \text{eV nm}}{1.3\ \text{eV}} \approx 956\ \text{nm}$.
5. Check each option's window against $\lambda_g \approx 956$ nm: (A) 1390–1430 nm does not contain 956 nm. (B) 1070–1110 nm does not contain 956 nm. (C) 930–970 nm DOES contain 956 nm. (D) 490–530 nm does not contain 956 nm.
6. Since the LED emits only near $\lambda_g \approx 956$ nm, it will NOT emit in the ranges that exclude this value: (A), (B) and (D). It WILL emit within range (C), so C is not part of the answer.

KEY IDEA

A direct-bandgap LED emits photons with energy essentially equal to the bandgap energy, so its emission is concentrated at a single wavelength $\lambda_g = hc/E_g$, not spread over the full visible-to-IR spectrum; the LED "does not emit" in any stated window that does not bracket λ_g.

WATCH OUT FOR

Trying to match the bandgap to a "range" of wavelengths rather than computing a single sharp emission wavelength $\lambda_g = hc/E_g$ and checking which option windows exclude it; also common arithmetic slip: using q in the wrong power of ten when converting J to eV, giving a λ_g that falls in the wrong window.

Q.55 MSQ · 2 MARKS

Communications › Information theory

Let the relevant bandwidth (B) of a digital communication system be 1 MHz and $kT = -174$ dBm/Hz, where k is Boltzmann's constant and 'T' is equivalent noise temperature of the receiver. The power (S) of signal received through an additive Gaussian channel is -80 dBm. Which of the following options is/are TRUE about Shannon capacity (C) of the channel?

(A) $C = B$

(B) $C = 2B$

(C) $C > 3B$

(D) $C < B$

Official answer: **C**

SOLUTION

1. Take the given $kT = -174\ \text{dBm/Hz}$ as the one-sided noise power spectral density N_0 (the question gives kT directly, so no extra factor of 2 is applied). Then the noise power in the bandwidth $B = 1\ \text{MHz} = 10^6\ \text{Hz}$ is $N = kT\,B$.
2. In decibels: $N_{\text{dBm}} = -174 + 10\log_{10}(10^6) = -174 + 60 = -114\ \text{dBm}$.
3. The received signal power is $S = -80\ \text{dBm}$, so $\text{SNR}_{\text{dB}} = -80 - (-114) = 34\ \text{dB}$.
4. Convert to a ratio: $\dfrac{S}{N} = 10^{3.4} \approx 2512$.
5. Shannon capacity of the band-limited AWGN channel: $C = B\log_2\left(1 + \dfrac{S}{N}\right) = B\log_2(2513)$.
6. $\log_2(2513) = \dfrac{\ln 2513}{\ln 2} \approx \dfrac{7.829}{0.693} \approx 11.3$, so $C \approx 11.3\,B \approx 11.3\ \text{Mbit/s}$.
7. Check the options. (A) $C = B$: false, since $C \approx 11.3B$. (B) $C = 2B$: false. (C) $C > 3B$: true, since $11.3B > 3B$. (D) $C < B$: false.
8. Even if kT were read as $N_0/2$ (so $N_0 = 2kT$), the SNR would drop by 3 dB to 31 dB and $C \approx 10.3B$, still above $3B$. The result does not depend on this convention.

KEY IDEA

Convert the noise PSD in dBm/Hz to noise power in the band by adding $10\log_{10} B$, subtract in dB to get the SNR (34 dB), then apply $C = B\log_2(1 + \text{SNR})$. At this high SNR the capacity is many times B.

WATCH OUT FOR

Forgetting the bandwidth term $10\log_{10}(10^6) = 60$ dB, which gives an SNR of 94 dB instead of 34 dB. Another slip is treating the SNR in dB as a linear ratio, or taking $C = B\log_2(\text{SNR})$ and then confusing dB with ratio. Another is reading "$C > 3B$" as needing a precise value, when any SNR above 7 (8.5 dB) is enough.

Q.56 MSQ · 2 MARKS

Digital Electronics › Boolean logic

Consider the four-variable Boolean function,

$f(w,x,y,z)=\sum m(0,2,5,7,8,10,13,14,15)$ with 'w' as MSB and 'z' as LSB.

Which of the following expressions is/are the valid form(s) of $f(w,x,y,z)$?

(Complement is printed as an overbar and is written here with \overline{}.)

(A) $\overline{x}\,\overline{z}+xz+wxy$

(B) $xz+wxy+w\overline{x}\,\overline{z}+\overline{w}xy\overline{z}$

(C) $\overline{x}\,\overline{z}+wxy+w\overline{x}\,\overline{z}+\overline{w}xy\overline{z}$

(D) $\overline{x}\,\overline{z}+xz+wy\overline{z}$

Official answer: **A, D**

SOLUTION

1. Write each minterm as $wxyz$ (with w as MSB): $0=0000,\ 2=0010,\ 5=0101,\ 7=0111,\ 8=1000,\ 10=1010,\ 13=1101,\ 14=1110,\ 15=1111$. So $f=0$ at $1,3,4,6,9,11,12$. Complement is written here with $\overline{}$.
2. K-map grouping of the 1s, the four corners: $x=0,\ z=0$ gives $\{0,2,8,10\}$. This is a quad, so the term is $\overline{x}\,\overline{z}$.
3. K-map grouping of the 1s, the centre quad: $x=1,\ z=1$ gives $\{5,7,13,15\}$, so the term is xz.
4. Only $m_{14}=1110$ is still uncovered. It can pair with $m_{15}=1111$ (common literals $w=1,x=1,y=1$), giving wxy, or with $m_{10}=1010$ (common literals $w=1,y=1,z=0$), giving $wy\overline{z}$. Both pairs contain only 1s, so there are two equally minimal SOP forms:
$$f=\overline{x}\,\overline{z}+xz+wxy \quad \text{or} \quad f=\overline{x}\,\overline{z}+xz+wy\overline{z}$$
5. Check (A): $\overline{x}\,\overline{z}\to\{0,2,8,10\}$, $xz\to\{5,7,13,15\}$, $wxy\to\{14,15\}$. The union is exactly the nine given minterms and nothing else, so (A) is valid.
6. Check (D): $wy\overline{z}\to\{10,14\}$. Together with the two quads the union is again exactly $\{0,2,5,7,8,10,13,14,15\}$, so (D) is valid.
7. Check (B) and (C): both contain $\overline{w}xy\overline{z}$, which is $0110=m_6$, and $f(6)=0$. So both expressions output 1 where f is 0. In addition, (B) misses m_0 and m_2 (its $w\overline{x}\,\overline{z}$ covers only $8,10$), and (C) has no term covering m_5,m_7,m_{13}. Neither is valid.
8. Valid forms: (A) and (D).

KEY IDEA

Draw the K-map. The four corners give $\overline{x}\,\overline{z}$ and the centre quad gives xz. After that, m_{14} is the only 1 left, and it has two adjacent 1s (m_{15} and m_{10}). That gives two equally minimal covers, wxy or $wy\overline{z}$. A function can have more than one minimal SOP. To test any option, list the minterms its terms cover and compare that set with the given one.

WATCH OUT FOR

Stopping at the first minimal cover you find ($\overline{x}\,\overline{z}+xz+wxy$) and marking only (A), without noticing that m_{14} can also pair with m_{10}, which makes (D) valid too. Another mistake is to judge (B) and (C) only by whether they cover every 1. Both also include $\overline{w}xy\overline{z}=m_6$, where $f=0$.

Q.57 MSQ · 2 MARKS

Communications › Analog modulation

Consider a real, narrowband signal $x(t)=A(t)\cos[2\pi f_c t+\theta(t)]$ where the maximum frequency components of $A(t)$ and $\theta(t)$ are f_M and f_c $(=1000f_M)$, respectively. Which of the following statements is/are correct for $-\infty<t<\infty$?

(A) $x(t)$ represents a PSK modulated signal for suitable choices of $A(t)$ and $\theta(t)$.

(B) $x(t)$ represents an amplitude modulated signal for suitable choices of $A(t)$ and $\theta(t)$.

(C) $x(t)$ represents a band-limited Gaussian noise process.

(D) $x(t)$ never represents a narrowband FM signal.

Official answer: **A, B, C**

Q.58 MSQ · 2 MARKS

Signals and Systems › Fourier series

Let $x_1(t) = \cos(2\pi nt)$ and $x_2(t) = 2\sin(4\pi nt)$ represent two sinusoids for a positive integer n and $-\infty < t < \infty$.

Which of the following statements about $x_1(t)$ and $x_2(t)$ is/are valid?

(A) $x_1(t)$ and $x_2(t)$ are orthogonal to each other over $0 \le t < 1/n$.

(B) $x_1(t)$ and $x_2(t)$ are orthonormal to each other over $0 \le t < 1/n$.

(C) $x_2(t)$ is a harmonic of $x_1(t)$.

(D) $x_1(t)$ and $x_2(t)$ are non-orthogonal to each other over $0 \le t < 1/(2n)$.

Official answer: **A, C, D**

SOLUTION

1. **Frequencies.** x_1 has frequency n (period $1/n$). x_2 has frequency $2n$ (period $1/(2n)$). For real signals, two signals are orthogonal over an interval when their inner product $\int x_1(t)\,x_2(t)\,dt$ over that interval is zero.
2. **Product-to-sum.** Using $2\cos A \sin B = \sin(B+A) + \sin(B-A)$ with $A = 2\pi nt$ and $B = 4\pi nt$:
$$x_1 x_2 = 2\cos(2\pi nt)\sin(4\pi nt) = \sin(6\pi nt) + \sin(2\pi nt).$$
3. **(A) Interval $[0, 1/n)$.** Over this interval $\sin(2\pi nt)$ completes exactly 1 cycle and $\sin(6\pi nt)$ completes exactly 3 cycles, so each integrates to zero:
$$\int_0^{1/n} x_1 x_2\,dt = 0.$$
The signals are orthogonal over this interval, so (A) is valid.
4. **(B) Orthonormal also needs unit energy.**
$$\int_0^{1/n} \cos^2(2\pi nt)\,dt = \frac{1}{2n}, \qquad \int_0^{1/n} 4\sin^2(4\pi nt)\,dt = 4\cdot\frac{1}{2n} = \frac{2}{n}.$$
Both would have to equal 1, which needs $n = \frac{1}{2}$ for x_1 and $n = 2$ for x_2 at the same time. No positive integer n satisfies both (the first condition alone already fails for every positive integer), so (B) is not valid.
5. **(C) Harmonic relation.** The frequency of x_2 is $2n = 2\times$ (the frequency of x_1), an integer multiple. So x_2 is the second harmonic of x_1; its amplitude (2) and its phase (sine rather than cosine) do not change that. (C) is valid.
6. **(D) Interval $[0, 1/(2n))$.** This is only half a period of x_1: $\sin(2\pi nt)$ completes half a cycle and $\sin(6\pi nt)$ completes one and a half cycles, so neither term integrates to zero:
$$\int_0^{1/(2n)} \sin(2\pi nt)\,dt = \frac{1-\cos\pi}{2\pi n} = \frac{1}{\pi n}, \qquad \int_0^{1/(2n)} \sin(6\pi nt)\,dt = \frac{1-\cos 3\pi}{6\pi n} = \frac{1}{3\pi n}.$$
7. Adding the two terms gives
$$\int_0^{1/(2n)} x_1 x_2\,dt = \frac{1}{\pi n} + \frac{1}{3\pi n} = \frac{4}{3\pi n} \neq 0.$$
The signals are not orthogonal over this interval, so (D) is valid.
8. **Answer:** (A), (C) and (D) are valid.

KEY IDEA

Orthogonality depends on the interval. Harmonically related sinusoids are orthogonal over any whole period of the fundamental, because every product-to-sum term then completes a whole number of cycles. Over a fraction of that period the inner product is generally non-zero. Orthonormal is stricter still: each signal must also have unit energy over the interval.

WATCH OUT FOR

Assuming a sine and a cosine are always orthogonal and so rejecting (D). That is true only over a whole number of periods; over half a period of x_1 the inner product is $4/(3\pi n) \neq 0$. Other traps: marking (B) by treating orthogonal and orthonormal as the same thing (the energies are $1/(2n)$ and $2/n$, not 1), and rejecting (C) because x_2 is a sine with amplitude 2. A harmonic is defined by its frequency being an integer multiple of the fundamental, whatever its amplitude and phase.

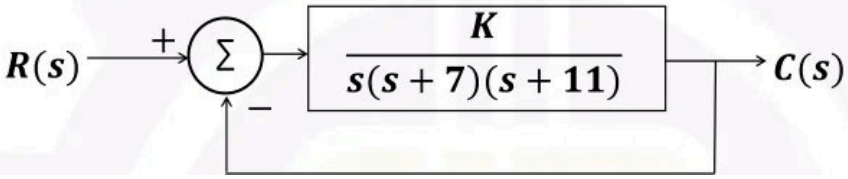
Q.59 NAT · 2 MARKS

Control Systems › Routh–Hurwitz

Q.59 Consider the unity negative feedback control system shown in the Figure.

The value of gain K (>0) at which the given system will remain marginally stable is __.

(Answer in integer)



Official answer: **1386**

SOLUTION

1. With unity negative feedback the closed-loop characteristic equation is $1 + G(s) = 0$. Since $(s+7)(s+11) = s^2 + 18s + 77$:
$$s(s+7)(s+11) + K = 0 \;\Rightarrow\; s^3 + 18s^2 + 77s + K = 0$$
2. Routh array:
$$\begin{aligned} s^3 &: \; 1 \quad 77 \\ s^2 &: \; 18 \quad K \\ s^1 &: \; \frac{18 \times 77 - K}{18} = \frac{1386 - K}{18} \\ s^0 &: \; K \end{aligned}$$
3. For stability every first-column entry must be positive: $K > 0$ and $1386 - K > 0$, so the system is stable for $0 < K < 1386$.
4. At the upper limit the s^1 row becomes all zeros: $1386 - K = 0 \Rightarrow K = 1386$. That is the marginal-stability gain.
5. Check with the auxiliary equation from the s^2 row: $18s^2 + 1386 = 0 \Rightarrow s^2 = -77 \Rightarrow s = \pm j\sqrt{77} \approx \pm j8.77 \text{ rad/s}$. Replacing the zero row by the derivative $36s$ gives first column $1,\ 18,\ 36,\ 1386$: no sign change, so no root lies in the right half-plane. Factorising confirms it: $s^3 + 18s^2 + 77s + 1386 = (s + 18)(s^2 + 77)$, which gives one real root at $s = -18$ and a non-repeated pair on the $j\omega$ axis. The system oscillates with constant amplitude at $\omega = \sqrt{77} \text{ rad/s}$, so it is marginally stable.
6. Shortcut for a cubic $a_0 s^3 + a_1 s^2 + a_2 s + a_3$ with all coefficients positive: it is marginally stable when $a_1 a_2 = a_0 a_3$. Here $18 \times 77 = 1 \times K$, so $K = 1386$.

KEY IDEA

Marginal stability happens at the gain that makes one row of the Routh array all zeros. The row above it then gives the auxiliary equation, and its roots are the pair on the $j\omega$ axis. For a cubic $a_0s^3 + a_1s^2 + a_2s + a_3$ this reduces to $a_1a_2 = a_0a_3$.

WATCH OUT FOR

Expanding $(s+7)(s+11)$ wrongly (for example as $s^2 + 18s + 18$, or leaving out the factor s) gives the wrong cubic. Another mistake is to report a range such as $0 < K < 1386$, or the oscillation frequency $\sqrt{77}$, when the question asks for the single marginal gain K.

Q.60 NAT · 2 MARKS

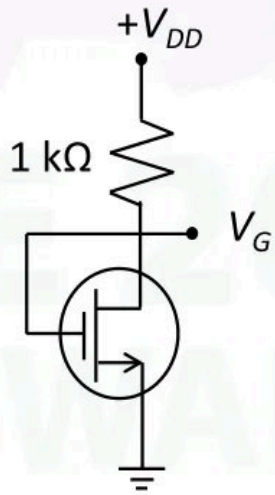
Analog Electronics › Biasing

Q.60 An n-channel MOSFET is connected as shown in the Figure.

Assume $V_{TH} = 1\text{ V}$, $V_{DD} = 5\text{ V}$, and $\mu C_{ox}\left(\frac{W}{L}\right) = 2\text{ mA V}^{-2}$ and neglect channel length modulation effects.

The gate voltage (V_G) of the n-channel MOSFET (in Volt) is __.

(rounded off to two decimal places)



Official answer: **2.50 to 2.60**

SOLUTION

1. **Circuit reading.** The gate is wired to the drain and the source is grounded, so $V_G = V_D = V_{GS} = V_{DS}$. This is a diode-connected MOSFET fed from V_{DD} through $R = 1\text{ k}\Omega$. No current flows into the gate, so all of the resistor current is the drain current I_D.

2. **Region of operation.** The device conducts only when $V_{GS} > V_{TH}$. Because $V_{TH} > 0$, it always holds that $V_{DS} = V_{GS} > V_{GS} - V_{TH}$, so the triode condition $V_{DS} < V_{GS} - V_{TH}$ can never be met. Once the device is on, it is in saturation.
3. **Square law in saturation ($\lambda = 0$, channel length modulation neglected):**
$$I_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2 = \frac{1}{2}(2\ \text{mA/V}^2)(V_G - 1)^2 = (V_G - 1)^2\ \text{mA}$$
4. **Ohm's law on the resistor:**
$$I_D = \frac{V_{DD} - V_G}{R} = \frac{5 - V_G}{1\ \text{k}\Omega} = (5 - V_G)\ \text{mA}$$
5. **Set the two equal.** Let $x = V_G - 1$, the overdrive voltage. Then $5 - V_G = 4 - x$, so
$$x^2 = 4 - x \ \Rightarrow\ x^2 + x - 4 = 0 \ \Rightarrow\ x = \frac{-1 \pm \sqrt{17}}{2}$$
6. **Choose the physical root.** $x = \dfrac{-1 + \sqrt{17}}{2} = \dfrac{-1 + 4.1231}{2} = 1.5616\ \text{V}$. The other root, $x = -2.56\ \text{V}$, would make $V_{GS} < V_{TH}$, which means the device is off. That contradicts the square-law equation it came from, so it is rejected. (If the device were off, $I_D = 0$ would give $V_G = 5\ \text{V} > V_{TH}$, so the device must be on.)
7. **Gate voltage:** $V_G = V_{TH} + x = 1 + 1.5616 = 2.5616\ \text{V} \approx 2.56\ \text{V}$.
8. **Check.** $I_D = (1.5616)^2\ \text{mA} = 2.438\ \text{mA}$. The drop across $1\ \text{k}\Omega$ is $2.438\ \text{V}$, so $V_G = 5 - 2.438 = 2.562\ \text{V}$, which agrees. Also $V_{DS} = 2.56\ \text{V} > V_{GS} - V_{TH} = 1.56\ \text{V}$, so the device is in saturation as assumed.

KEY IDEA

When the gate is tied to the drain, $V_{DS} = V_{GS}$, so an n-channel MOSFET that is on must be in saturation. Set the square-law current $\frac{1}{2}\mu C_{ox}\frac{W}{L}(V_G - V_{TH})^2$ equal to the resistor current $(V_{DD} - V_G)/R$. Solve the quadratic and keep the root with $V_{GS} > V_{TH}$.

WATCH OUT FOR

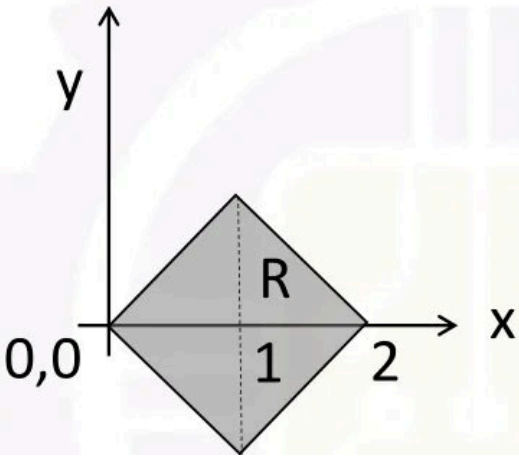
Writing $I_D = \mu C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2$ without the factor $\frac{1}{2}$. That gives $2x^2 + x - 4 = 0$, so $x = 1.19\ \text{V}$ and $V_G \approx 2.19\ \text{V}$, which is outside the key. The key uses the standard $\frac{1}{2}\mu C_{ox}\frac{W}{L}$ form. Other mistakes: keeping the negative root (it makes $V_G < V_{TH}$, so the device would be off), and checking for the triode region, which a diode-connected device can never enter.

Q.61 NAT · 2 MARKS

Engineering Mathematics › Calculus

Q.61 Consider the square region R in the X-Y plane as shown with the dark shading in the Figure. The value of $\iint_R (x^2 + y^2 - 1)dxdy$ is ____.

(rounded off to two decimal places)



Official answer: **0.60 to 0.70**

SOLUTION

1. Read the region. The figure puts the left and right corners of R on the x-axis at $(0, 0)$ and $(2, 0)$, so that diagonal has length 2 and midpoint $(1, 0)$. In a square the diagonals are equal and are perpendicular bisectors of each other, so the dotted vertical diagonal through $x = 1$ also has length 2. The other two corners are therefore $(1, 1)$ and $(1, -1)$. The side is $a = \sqrt{2}$, the area is $a^2 = 2$, and R is the set $|x - 1| + |y| \le 1$.
2. Move the origin to the centre with $u = x - 1$ (a translation, so the Jacobian is 1). Then $x^2 + y^2 - 1 = (1 + u)^2 + y^2 - 1 = u^2 + 2u + y^2$.
3. The region $|u| + |y| \le 1$ is symmetric under $u \to -u$ and $2u$ is odd in u, so $\iint_R 2u\, dA = 0$.
4. That leaves $I = \iint_R (u^2 + y^2)\, dA$ over the diamond $|u| + |y| \le 1$. For a fixed y with $|y| \le 1$, u runs from $-(1 - |y|)$ to $1 - |y|$, a width of $2(1 - |y|)$.
5. $$\iint_R y^2\, dA = \int_{-1}^{1} y^2 \cdot 2(1 - |y|)\, dy = 4\int_0^1 (y^2 - y^3)\, dy = 4\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{1}{3}$$
6. The diamond is unchanged when u and y are swapped, so $\iint_R u^2\, dA = \frac{1}{3}$ as well.
7. $$I = \frac{1}{3} + \frac{1}{3} = \frac{2}{3} \approx 0.67$$
8. Check: $\iint (u^2 + y^2)\, dA$ is the polar moment of area of a square about its centre, which is $\dfrac{a^4}{6}$ and does not change when the square is rotated. Here $\dfrac{(\sqrt{2})^4}{6} = \dfrac{4}{6} = \dfrac{2}{3}$, which agrees.

KEY IDEA

Put the origin at the centre of the square ($u = x - 1$). The integrand becomes $u^2 + 2u + y^2$. The odd term $2u$ integrates to zero by symmetry, and the -1 cancels against the $+1$ from $(1+u)^2$. What remains, $\iint (u^2 + y^2)\, dA$, is the polar moment of a square about its centre, $a^4/6$ with $a = \sqrt{2}$, which gives $2/3$.

WATCH OUT FOR

Taking the region as an axis-aligned square such as $[0,2] \times [-1,1]$ or $[0,1] \times [0,1]$. The figure shows a square standing on a corner, $|x-1| + |y| \le 1$, with side $\sqrt{2}$ and area 2, not side 2. A second slip is subtracting only 1 for the constant term instead of $1 \times \text{area} = 2$. That gives $\frac{8}{3} - 1 = \frac{5}{3}$ instead of $\frac{8}{3} - 2 = \frac{2}{3}$.

Q.62 NAT · 2 MARKS

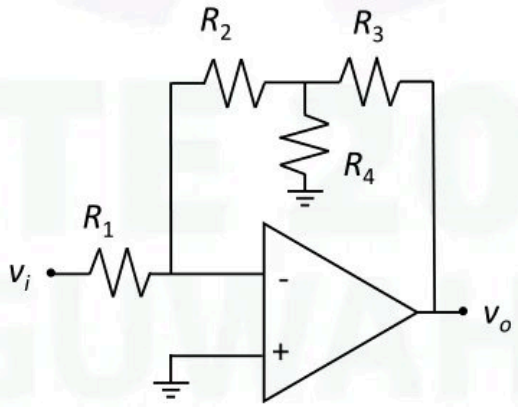
Analog Electronics › Op-amps

Q.62 Consider an ideal OP-AMP circuit as shown in the Figure.

The resistances $R_1 = R_2 = R_3 = R_4 = 50\ \text{k}\Omega$.

The magnitude of the closed loop gain is ___.

(rounded off to two decimal places)



Official answer: **2.90 to 3.10**

SOLUTION

1. Assume an ideal op-amp: infinite open-loop gain and no input current. The (+) input is grounded, so negative feedback holds the (–) input at virtual ground, $v_- = 0$.
2. KCL at the inverting input: no current enters the op-amp, so all of the current through R_1 continues through R_2 towards the T-node X:

$$\frac{v_i - 0}{R_1} = \frac{0 - v_X}{R_2} \;\Rightarrow\; v_X = -\frac{R_2}{R_1} v_i = -v_i$$

3. KCL at the T-node X. The current arriving through R_2 leaves through R_4 to ground and through R_3 to the output:

$$\frac{0 - v_X}{R_2} = \frac{v_X}{R_4} + \frac{v_X - v_o}{R_3}$$

4. Solve for v_o:

$$\frac{v_o}{R_3} = v_X\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right) \Rightarrow v_o = v_X\,R_3\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right) = v_X\cdot 50\,\text{k}\Omega\cdot\frac{3}{50\,\text{k}\Omega} = 3\,v_X$$

5. Substitute $v_X = -v_i$ to get $v_o = -3\,v_i$. Check against the standard T-network result:

$$\frac{v_o}{v_i} = -\frac{R_2+R_3+\dfrac{R_2R_3}{R_4}}{R_1} = -\frac{50+50+\dfrac{50\cdot 50}{50}}{50} = -\frac{150}{50} = -3$$

6. The closed-loop gain is -3, so its magnitude is $|v_o/v_i| = 3.00$.

KEY IDEA

This is an inverting amplifier whose feedback resistor has been replaced by a T-network. Virtual ground at the (–) input fixes the voltage at the T-node, $v_X = -(R_2/R_1)v_i$. KCL at the T-node then gives v_o. The general result is $v_o/v_i = -\left(R_2+R_3+R_2R_3/R_4\right)/R_1$: the shunt resistor R_4 adds the term R_2R_3/R_4, which raises the gain above what R_2+R_3 alone would give.

WATCH OUT FOR

One mistake is to treat the feedback as a simple series resistor R_2+R_3 and ignore R_4, which gives a gain of magnitude 2. Another is to treat R_4 as a load that can be dropped. R_4 takes current away from the T-node, so the output has to swing further to push the same current through R_2. It adds the term R_2R_3/R_4 and brings the gain to 3. A third error is to take the voltage at the (–) input as v_i, when with the (+) input grounded it is at virtual ground (0 V).

Q.63 NAT · 2 MARKS

Communications › Error control

The average bit error rate at the input of a (7, 4, 1) Hamming decoder is 0.10. The probability that the decoder will fail to decode a received word correctly is __. (rounded off to two decimal places)

Official answer: **0.10 to 0.20**

SOLUTION

1. A $(7, 4, 1)$ Hamming code has block length $n = 7$, $k = 4$ message bits and error-correcting capability $t = 1$ (minimum distance $d_{min} = 3$). It decodes a received word correctly whenever the word has at most one bit in error.
2. Take the bit errors as independent with probability $p = 0.10$ at the decoder input, so the number of errors in a 7-bit word is binomial with $n = 7$, $p = 0.1$.
3. The decoder fails when the word has 2 or more errors, so $P_{\text{fail}} = 1 - P(0) - P(1)$.
4. $$P(0) = (0.9)^7 \approx 0.4783, \qquad P(1) = 7(0.1)(0.9)^6 = 7(0.1)(0.5314) \approx 0.3720$$
5. $$P_{\text{fail}} = 1 - (0.4783 + 0.3720) = 1 - 0.8503 \approx 0.1497$$
6. Rounded to two decimal places, $P_{\text{fail}} \approx 0.15$.

KEY IDEA

A single-error-correcting code fails exactly when a word has two or more errors. With independent bit errors, use the binomial law and subtract the probabilities of 0 and 1 errors from 1.

WATCH OUT FOR

Using the bit error rate of 0.10 as the failure probability, or counting only exactly 2 errors and ignoring words with 3 or more errors. Another slip is forgetting the factor 7 in $P(1)$.

Q.64 NAT · 2 MARKS

Electronic Devices › Carrier transport

Consider that the concentration of electrons in a semiconductor bar varies linearly from $2 \times 10^{17}\ \text{cm}^{-3}$ at $x = 1\ \mu\text{m}$ to $1 \times 10^{16}\ \text{cm}^{-3}$ at $x = 4\ \mu\text{m}$ along the x-direction. Assume that the concentration of electrons is not varying along other directions (that is along y- and z-directions).

[Given: the mobility of electron is $1400\ \text{cm}^2\text{V}^{-1}\text{s}^{-1}$, thermal voltage is $25\ \text{mV}$ and electronic charge is 1.6×10^{-19} Coulomb.]

The density of electron diffusion current (in $\mathrm{A/mm^2}$) is ___.

(rounded off to two decimal places)

Official answer: **-36.50 to -34.50, or 34.50 to 36.50**

SOLUTION

1. Concentration gradient: $\frac{dn}{dx} = \frac{(1\times10^{16} - 2\times10^{17})\ \mathrm{cm^{-3}}}{(4-1)\ \mu\mathrm{m}} = \frac{-1.9\times10^{17}\ \mathrm{cm^{-3}}}{3\times10^{-4}\ \mathrm{cm}} = -6.3333\times10^{20}\ \mathrm{cm^{-4}}$
2. Diffusion coefficient via Einstein relation: $D_n = \mu_n V_T = 1400 \times 0.025 = 35\ \mathrm{cm^2/s}$
3. Electron diffusion current density: $J_{n,\mathrm{diff}} = qD_n\frac{dn}{dx}$
4. Magnitude: $|J_{n,\mathrm{diff}}| = (1.6\times10^{-19})(35)(6.3333\times10^{20}) = 3546.67\ \mathrm{A/cm^2}$
5. Convert $\mathrm{cm^2} \to \mathrm{mm^2}$ (since $1\ \mathrm{cm^2} = 100\ \mathrm{mm^2}$): $J_{n,\mathrm{diff}} = \frac{3546.67}{100} = 35.47\ \mathrm{A/mm^2}$

KEY IDEA

Diffusion current density is $J_{n,\mathrm{diff}} = qD_n\frac{dn}{dx}$, with D_n obtained from the mobility via the Einstein relation $D_n = \mu_n V_T$; only the magnitude and unit conversion (cm² to mm²) need care here.

WATCH OUT FOR

Forgetting that $1\ \mathrm{cm^2} = 100\ \mathrm{mm^2}$, so converting A/cm² to A/mm² requires DIVIDING by 100 (not multiplying), and mixing up the sign convention or the direction of the gradient (which point's concentration is subtracted from which).

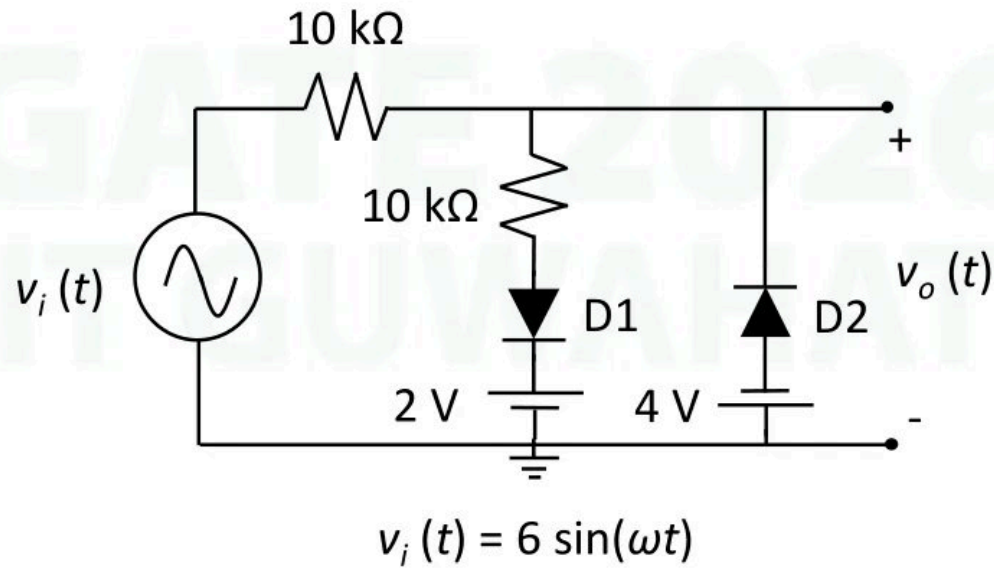
Q.65 NAT · 2 MARKS

Q.65

Consider the ideal diodes D1 and D2 as shown in the Figure with cut-in voltage $V_\gamma = 0$ Volt and $v_i(t)$ is in Volt.

The maximum voltage (Volt) of the output $v_o(t)$ is __.

(rounded off to two decimal places)



Official answer: **3.70 to 4.30**

SOLUTION

1. **Read the battery polarities.** The 2 V battery has its long (+) plate on top, so D1's cathode is held at $+2$ V. The 4 V battery has its short (–) plate on top, so D2's anode is held at -4 V.
2. **Find when each ideal diode conducts ($V_\gamma = 0$).** D1 (anode towards the output through 10 kΩ, cathode at $+2$ V) can conduct only when $v_o > 2$ V. D2 (cathode at the output, anode at -4 V) conducts only when $v_o < -4$ V. So on the positive half-cycle only D1 matters, and D2 stays reverse-biased.
3. **Region $-4\text{ V} \le v_i \le 2\text{ V}$ (both diodes off):** neither shunt branch carries current, so no current flows in the series 10 kΩ and $v_o = v_i$. (For $v_i < -4$ V, D2 conducts and holds v_o at -4 V; this affects only the minimum, not the maximum.)
4. **Region $v_i > 2$ V (D1 on, D2 off):** one loop current flows from the source, through the series 10 kΩ and the 10 kΩ in D1's branch, into the 2 V battery. By KVL,
$$i = \frac{v_i - 2}{10\text{ k}\Omega + 10\text{ k}\Omega} = \frac{v_i - 2}{20\text{ k}\Omega}$$
5. **Output voltage:**
$$v_o = v_i - (10\text{ k}\Omega)\, i = v_i - \frac{v_i - 2}{2} = \frac{v_i + 2}{2}$$
Above 2 V the output therefore keeps rising, with slope $1/2$. D1 does not clamp it flat, because the 10 kΩ in D1's branch sits between the output and the battery: the two 10 kΩ resistors form a divider between v_i and 2 V.

6. **Put in the peak $v_i = 6$ V.** Since v_o increases with v_i in this region, the maximum output occurs at the input peak:

$$v_{o,\max} = \frac{6 + 2}{2} = 4.00 \text{ V}$$

7. **Check the assumed diode states at the peak:** D1 current is $i = (6 - 2)/20\ \text{k}\Omega = 0.2\ \text{mA} > 0$, so D1 is on. D2 has $V_{AK} = -4 - 4 = -8\ \text{V} < 0$, so D2 is off. Both assumptions hold, and $v_{o,\max} = 4.00$ V.

KEY IDEA

A diode clipper with a resistor in series with the diode and its reference battery does not clip flat. Once D1 turns on, the series 10 kΩ and the branch 10 kΩ form a voltage divider between v_i and the 2 V battery, so above 2 V the output follows $v_o = (v_i + 2)/2$ and reaches 4 V at the 6 V input peak. D2, with its anode at −4 V, affects only the negative half-cycle.

WATCH OUT FOR

Treating D1 as a hard clamp and answering 2 V, which ignores the 10 kΩ resistor in D1's own branch. Another mistake is misreading the battery symbols (long plate = +). Taking D1's cathode at −2 V gives the wrong divider result, and taking D2's anode at +4 V wrongly brings D2 into the positive half-cycle.

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