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GATE EC · 2025

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GATE EC 2025 Solved Paper

INSIDE

- All 65 questions, in the order of the paper
- The official answer to every question
- A step-by-step solution to every question
- The answer key on one page

Question paper and official answer key: IIT Roorkee

Online, with the questions linked to their topic notes: **iaeacademy.in/study/papers/ec-2025**

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About this paper

65 questions, 100 marks. Question types: 39 MCQ · 14 MSQ · 12 NAT.

Official answer key: IIT Roorkee · https://gate2025.iitr.ac.in/doc/2025/2025_Key/EC_Keys.pdf

We placed each question under a subject and topic ourselves; the exam paper does not label them.

This paper online, with the questions linked to their topic notes:
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Answer key

Q	TYPE	MARKS	ANSWER
Q.1	MCQ	1	**A**
Q.2	MCQ	1	**D**
Q.3	MCQ	1	**B**
Q.4	MCQ	1	**B**
Q.5	MCQ	1	**A**
Q.6	MCQ	2	**B**
Q.7	MCQ	2	**D**
Q.8	MCQ	2	**Marks to all**
Q.9	MCQ	2	**C**
Q.10	MCQ	2	**B**
Q.11	MCQ	1	**A**
Q.12	MCQ	1	**B**
Q.13	MCQ	1	**A**
Q.14	MCQ	1	**A**
Q.15	MCQ	1	**A**
Q.16	MCQ	1	**D**
Q.17	MCQ	1	**D**
Q.18	MCQ	1	**A**
Q.19	MCQ	1	**A**
Q.20	MCQ	1	**A**
Q.21	MCQ	1	**A**
Q.22	MCQ	1	**C**
Q.23	MCQ	1	**A**
Q.24	MCQ	1	**B**
Q.25	MSQ	1	**A, B**
Q.26	MSQ	1	**C**
Q.27	MSQ	1	**C, D**
Q.28	MSQ	1	**A, D**
Q.29	MSQ	1	**A, C**
Q.30	MSQ	1	**B, D**
Q.31	MSQ	1	**A, D**
Q.32	NAT	1	**0.25**
Q.33	NAT	1	**3**
Q.34	NAT	1	**50**
Q.35	NAT	1	**250**
Q.36	MCQ	2	**D**
Q.37	MCQ	2	**A**
Q.38	MCQ	2	**A**
Q.39	MCQ	2	**A**
Q.40	MCQ	2	**A**
Q.41	MCQ	2	**D**
Q.42	MCQ	2	**A**
Q.43	MCQ	2	**D**
Q.44	MCQ	2	**A**
Q.45	MCQ	2	**A**
Q.46	MCQ	2	**B**
Q.47	MCQ	2	**A**
Q.48	MCQ	2	**B**
Q.49	MCQ	2	**C**
Q.50	MCQ	2	**A**
Q.51	MSQ	2	**A**
Q.52	MSQ	2	**A, B**
Q.53	MSQ	2	**B**
Q.54	MSQ	2	**B, C**
Q.55	MSQ	2	**A, D**
Q.56	MSQ	2	**B**
Q.57	MSQ	2	**A, D**
Q.58	NAT	2	**6.95 to 7.05**
Q.59	NAT	2	**2.95 to 3.05**
Q.60	NAT	2	**0.0**
Q.61	NAT	2	**13.32 to 13.34**
Q.62	NAT	2	**200**
Q.63	NAT	2	**0.23 to 0.24**
Q.64	NAT	2	**6.3 to 6.5**
Q.65	NAT	2	**0.62 to 0.63**

General Aptitude Q.1 to Q.10

Q.1 MCQ · 1 MARK

General Aptitude › Grammar & vocabulary

Here are two analogous groups, Group-I and Group-II, that list words in their decreasing order of intensity. Identify the missing word in Group-II.

Group-I: Abuse → Insult → Ridicule

Group-II: _____ → Praise → Appreciate

(A) Extol

(B) Prize

(C) Appropriate

(D) Espouse

Official answer: **A**

SOLUTION

1. Read the pattern in Group-I. Abuse → Insult → Ridicule are all ways of speaking *against* someone, ordered from the harshest (abuse) to the mildest (ridicule, i.e. mocking).
2. Group-II is the positive counterpart. Praise → Appreciate are ways of speaking *in favour of* someone, and praise is stronger than appreciate. So the blank needs a word with the same meaning that is stronger than *praise*.
3. Extol (A): to praise highly and with enthusiasm. That is a stronger form of praise, so it fits at the top of the chain.
4. Prize (B): as a verb, to value something highly. It describes how much you value a thing. It is not a way of expressing approval that is stronger than praise, so it breaks the chain.
5. Appropriate (C): as a verb it means to take something for one's own use, and as an adjective it means suitable. Neither meaning has anything to do with praise.
6. Espouse (D): to adopt or support a cause or belief. It has nothing to do with praising a person.
7. Only *extol* gives an order from strongest to mildest, Extol → Praise → Appreciate, just as Abuse → Insult → Ridicule does. The answer is (A).

KEY IDEA

Both chains list the same kind of action in decreasing intensity: Group-I is disapproval (abuse > insult > ridicule) and Group-II is approval. The missing word must be a stronger form of praise, which is extol.

WATCH OUT FOR

Picking a word because of how it looks or sounds rather than what it means. Prize resembles praise and Appropriate resembles appreciate, but neither expresses approval more strongly than praise. Another mistake is reading the chain in the wrong direction and looking for something milder than appreciate.

Q.2 MCQ · 1 MARK

General Aptitude › Grammar & vocabulary

Had I learnt acting as a child, I _____ a famous film star.

Select the most appropriate option to complete the above sentence.

(A) will be

(B) can be

(C) am going to be

(D) could have been

Official answer: **D**

SOLUTION

1. Spot the structure. "Had I learnt ..." is an inverted conditional: drop *if* and move *had* to the front of the clause. It means the same as "If I had learnt acting as a child".
2. Rule for the third conditional: the if-clause uses the past perfect (*had* + past participle) and describes an unreal past, something that did not happen. The main clause must then be *would / could / might have* + past participle.
3. The time is fixed in the past ("as a child"), and the condition did not happen: the speaker did not learn acting. So the result must also be an unreal past outcome.
4. (A) "will be" and (C) "am going to be" refer to the future. They belong to a first conditional ("If I learn ..., I will be ...") and cannot follow a past perfect if-clause.
5. (B) "can be" is a present-tense modal. It cannot express a hypothetical result in the past.
6. (D) "could have been" is modal + *have* + past participle, which is exactly the result clause of a third conditional: "Had I learnt acting as a child, I could have been a famous film star." The answer is (D).

KEY IDEA
A clause that starts with "Had + subject + past participle" is an inverted third conditional: an unreal past condition. Its result clause must be would / could / might + have + past participle. No option with a future or present verb can complete it.

WATCH OUT FOR
Missing the inversion. With no "if" in sight, some candidates read "Had I learnt" as a plain past statement, or match it to a first-conditional result like "will be". Another trap is to pick "can be" because it expresses ability, but a present modal cannot give the result of a condition in the past.

Q.3 MCQ · 1 MARK

General Aptitude › Numerical

The 12 musical notes are given as $C, C^{\#}, D, D^{\#}, E, F, F^{\#}, G, G^{\#}, A, A^{\#}$, and B. Frequency of each note is $\sqrt[12]{2}$ times the frequency of the previous note. If the frequency of the note C is 130.8 Hz, then the ratio of frequencies of notes $F^{\#}$ and C is:

(A) $\sqrt[6]{2}$

(B) $\sqrt{2}$

(C) $\sqrt[4]{2}$

(D) 2

Official answer: **B**

SOLUTION

1. Each note is $\sqrt[12]{2} = 2^{1/12}$ times the note before it, so the frequencies form a geometric progression with common ratio $r = 2^{1/12}$.
2. In a geometric progression, the term n places after a given term is $f_n = f_0\, r^n$. So the ratio of two notes depends only on how many steps apart they are: $\dfrac{f_n}{f_0} = r^n$.
3. Count the steps from C to $F^{\#}$ in the printed order: $C \to C^{\#} \to D \to D^{\#} \to E \to F \to F^{\#}$. That is $n = 6$ steps. Count the arrows, not the notes: there are 7 notes but only 6 steps.
4. $$\frac{f_{F^{\#}}}{f_C} = \left(2^{1/12}\right)^6 = 2^{6/12} = 2^{1/2} = \sqrt{2}$$
5. The given frequency of C (130.8 Hz) cancels in the ratio, so you do not need it.
6. The ratio $F^{\#} : C$ is $\sqrt{2}$, which is option (B).

KEY IDEA

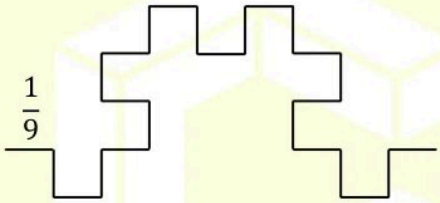
The notes form a geometric progression with common ratio $2^{1/12}$. Two notes that are n steps apart have a frequency ratio of $2^{n/12}$. $F^{\#}$ is 6 steps above C (half of the 12-step octave), so the ratio is $2^{6/12} = \sqrt{2}$. The actual frequency of C does not matter.

WATCH OUT FOR

Counting notes instead of steps. C to $F^{\#}$ covers 7 notes but only 6 multiplications, and $2^{7/12}$ is not among the options. Other slips: stopping at F (5 steps, giving $2^{5/12}$); treating the progression as arithmetic; or working out $130.8\sqrt{2} \approx 185\ \text{Hz}$ when the question asks for a ratio.

Q.4 MCQ · 1 MARK

General Aptitude › Numerical

Q.4	The following figures show three curves generated using an iterative algorithm. The total length of the curve generated after 'Iteration n' is: Note: The figures shown are representative.
	Iteration 0:  Iteration 1:  Length of each segment: $\frac{1}{3}$ Iteration 2:  Length of each segment: $\frac{1}{9}$
(A)	$\left(\frac{5}{3}\right)^{\frac{n}{2}}$
(B)	$\left(\frac{5}{3}\right)^{n}$
(C)	$\left(\frac{5}{3}\right)^{2n}$
(D)	$\left(\frac{5}{3}\right)^{n(2n-1)}$

Official answer: **B**

SOLUTION

1. Read the rule from Iteration 1. The straight segment is cut into thirds, and the middle third is replaced by three sides of a square (up, across, down), each side $\frac{1}{3}$ long. So **one segment becomes 5 segments, each $\frac{1}{3}$ as long**.
2. Iteration 0: 1 segment of length 1, so $L_0 = 1$.
3. Iteration 1: 5 segments of length $\frac{1}{3}$, so $L_1 = 5 \times \frac{1}{3} = \frac{5}{3}$.

4. Iteration 2: each of those 5 segments is replaced by the same motif at one-third scale. That gives $5^2 = 25$ segments of length $\frac{1}{9}$, which you can count in the figure: 5 on each baseline piece, 5 on each vertical side and 5 on the top. So $L_2 = 25 \times \frac{1}{9} = \frac{25}{9} = \left(\frac{5}{3}\right)^2$. The direction each small bump points in does not change the length. This is why the paper says the figures are representative.
5. General term: each iteration multiplies the number of segments by 5 and divides the length of each segment by 3. So the total length is multiplied by $\frac{5}{3}$ every time:
$$L_n = 5^n \times \left(\frac{1}{3}\right)^n = \left(\frac{5}{3}\right)^n$$
6. Check the options against $L_0 = 1$, $L_1 = \frac{5}{3}$ and $L_2 = \frac{25}{9}$. Option (B) gives 1, $\frac{5}{3}$ and $\frac{25}{9}$, so all three match. Option (A) gives $\sqrt{5/3}$ at $n = 1$ and option (C) gives $\left(\frac{5}{3}\right)^2$ at $n = 1$, so both fail. Option (D) gives $\left(\frac{5}{3}\right)^1 = \frac{5}{3}$ at $n = 1$, which matches, but at $n = 2$ it gives $\left(\frac{5}{3}\right)^{2\times 3} = \left(\frac{5}{3}\right)^6$, which does not. Only (B) fits every iteration shown.
7. **Answer: (B)** $\left(\frac{5}{3}\right)^n$

KEY IDEA

When every segment is replaced by k copies, each scaled by a factor r, the total length is multiplied by k times r at every iteration. The length therefore grows geometrically, L_n = (kr)^n × L_0. Here k = 5 and r = 1/3, so L_n = (5/3)^n.

WATCH OUT FOR

Checking only n = 1. Option (D) also gives 5/3 at n = 1, because n(2n-1) = 1 there. Only n = 2 (25/9, against (5/3)^6 for D) separates it from (B). A second mistake is counting the segments of Iteration 2 as 5 × 3 = 15 instead of 5 × 5 = 25, or being put off by bumps that point in different directions. Orientation does not change length.

Q.5 MCQ · 1 MARK

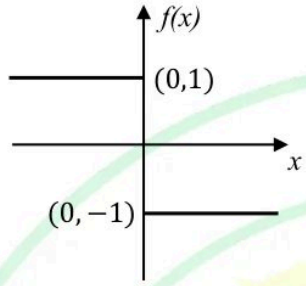
General Aptitude › Numerical

Q.5

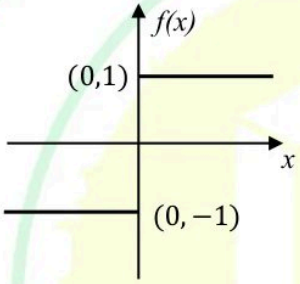
Which one of the following plots represents $f(x) = -\frac{|x|}{x}$, where x is a non-zero real number?

Note: The figures shown are representative.

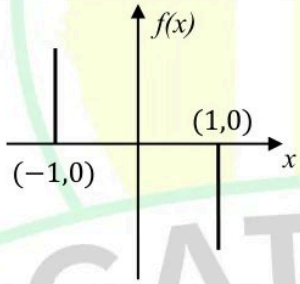
(A)



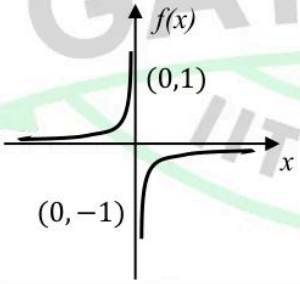
(B)



(C)



(D)



Official answer: **A**

SOLUTION

1. By the definition of absolute value, $|x| = x$ when $x > 0$ and $|x| = -x$ when $x < 0$. The function is not defined at $x = 0$, which is why the question says x is non-zero.
2. For $x > 0$: $f(x) = -\frac{x}{x} = -1$, a constant. Check: $x = 2$ gives $-\frac{2}{2} = -1$.
3. For $x < 0$: $f(x) = -\frac{-x}{x} = +1$, a constant. Check: $x = -3$ gives $-\frac{3}{-3} = +1$.

4. So $f(x) = -\operatorname{sgn}(x)$. Its graph is a horizontal line at height $+1$ to the left of the vertical axis and a horizontal line at height -1 to the right of it, with no point at $x = 0$. The labels $(0, 1)$ and $(0, -1)$ only mark the open ends of the two segments; the note says the figures are representative.
5. Now eliminate the other options. (B) has $+1$ for $x > 0$ and -1 for $x < 0$, which is $+\operatorname{sgn}(x)$, the sign-flipped function. (C) is made of vertical segments at $x = \pm 1$, which cannot be the graph of any function because one x would have many values. (D) is a pair of curves whose height changes with x and approaches 0 as $|x|$ grows, but f is constant on each side.
6. Only (A) shows $f(x) = 1$ for $x < 0$ and $f(x) = -1$ for $x > 0$. The answer is (A).

KEY IDEA

For any non-zero x, $\dfrac{|x|}{x}$ is the sign of x: it is $+1$ when $x > 0$ and -1 when $x < 0$. So $f(x) = -\dfrac{|x|}{x}$ is a step function. It is constant on each side of the origin, undefined at $x = 0$, and the leading minus sign puts the $+1$ level on the negative side.

WATCH OUT FOR

Dropping the leading minus sign and choosing (B), which is the graph of $\dfrac{|x|}{x}$ and not of $-\dfrac{|x|}{x}$. Another error is treating $\dfrac{|x|}{x}$ like $\dfrac{1}{x}$ and choosing the hyperbola-like curves in (D). Test one positive and one negative value of x to rule out both.

Q.6 MCQ · 2 MARKS

General Aptitude › Reading

Identify the option that has the most appropriate sequence such that a coherent paragraph is formed:

P. Over time, such adaptations lead to significant evolutionary changes with the potential to shape the development of new species.

Q. In natural world, organisms constantly adapt to their environments in response to challenges and opportunities.

R. This process of adaptation is driven by the principle of natural selection, where favorable traits increase an organism's chances of survival and reproduction.

S. As environments change, organisms that can adapt their behavior, structure and physiology to such changes are more likely to survive.

(A) P → Q → R → S

(B) Q → S → R → P

(C) R → S → Q → P

(D) S → P → R → Q

Official answer: **B**

SOLUTION

1. Look for words that point back to an earlier sentence. P begins "Over time, such adaptations", so the adaptations must already have been described. R begins "This process of adaptation", so the process must already have been described. S ("As environments change, organisms that can adapt … to such changes") refers back only to the start of its own sentence, so it is grammatically complete. But it takes up one particular case (environments that change) of a broader idea. Q ("In natural world, organisms constantly adapt to their environments in response to challenges and opportunities.") points back to nothing and states the paragraph's general theme, so Q is the natural opening sentence.
2. Rule out the other options. (A) opens with P, and "such adaptations" has nothing to refer to. (C) opens with R, and "This process of adaptation" has nothing to refer to. (D) S → P → R → Q ends with Q, a broad introductory statement that cannot close a paragraph that has already covered the mechanism and the long-term outcome. It also places R ("This process of adaptation") after P has already moved on to evolutionary change and new species, which is a step backward. Only (B) is left.

3. Check Q → S. Q says organisms adapt in response to challenges. S makes this concrete: when environments change, the organisms that can adapt their behavior, structure and physiology are the ones more likely to survive.
4. Check S → R. "This process of adaptation" in R refers to the adapting described in S. R then names the principle that drives it, natural selection, and its "chances of survival" picks up S's "more likely to survive".
5. Check R → P. "Such adaptations" in P refers to the adaptations through favorable traits described in R. "Over time" moves on to the long-term result, significant evolutionary change and new species. A statement of consequence like this is a fitting close.
6. The paragraph therefore runs general statement → a concrete case of survival → the underlying principle → the long-term outcome, which is Q → S → R → P, option (B).

KEY IDEA
Look for connecting words. "Such adaptations" (P) and "This process of adaptation" (R) both need an earlier sentence to refer to, so neither can open. Q is the only sentence that states the general theme with no back-reference, so it opens. P's "Over time" gives the long-term consequence, so it closes.

WATCH OUT FOR
Opening with S because it reads smoothly on its own ("such changes" refers back to "As environments change" in the same sentence). S describes one particular case of a general claim, so it belongs after Q. Starting with S, as option (D) does, leaves the general opening statement Q stranded at the end.

Q.7 MCQ · 2 MARKS

General Aptitude › Geometry

Q.7	A stick of length one meter is broken at two locations at distances of b_1 and b_2 from the origin (0), as shown in the figure. Note that $0 < b_1 < b_2 < 1$. Which one of the following is NOT a necessary condition for forming a triangle using the three pieces? Note: All lengths are in meter. The figure shown is representative.
	
(A)	$b_1 < 0.5$
(B)	$b_2 > 0.5$
(C)	$b_2 < b_1 + 0.5$
(D)	$b_1 + b_2 < 1$

Official answer: **D**

SOLUTION

1. The three pieces have lengths b_1, $b_2 - b_1$ and $1 - b_2$. All three are positive because $0 < b_1 < b_2 < 1$, and they add up to $1\ \mathrm{m}$.
2. Triangle inequality: each side must be shorter than the sum of the other two. The three sides add up to 1, so the other two sides of any side x add up to $1 - x$. The condition is then $x < 1 - x$, which means $x < 0.5$. So a triangle can be formed exactly when every piece is shorter than $0.5\ \mathrm{m}$. Each of these three conditions is necessary, and together they are sufficient.
3. First piece: $b_1 < 0.5$. This is option (A), so (A) is necessary.
4. Second piece: $b_2 - b_1 < 0.5 \Rightarrow b_2 < b_1 + 0.5$. This is option (C), so (C) is necessary.
5. Third piece: $1 - b_2 < 0.5 \Rightarrow b_2 > 0.5$. This is option (B), so (B) is necessary.
6. Option (D) is not one of these conditions, and one counter-example shows it is not necessary. Take $b_1 = 0.45$, $b_2 = 0.7$. The pieces are 0.45, 0.25 and $0.30\ \mathrm{m}$, all shorter than $0.5\ \mathrm{m}$, so a triangle forms. Yet $b_1 + b_2 = 1.15 > 1$, so (D) fails even though the triangle exists.
7. Therefore the condition that is NOT necessary is (D) $b_1 + b_2 < 1$.

KEY IDEA

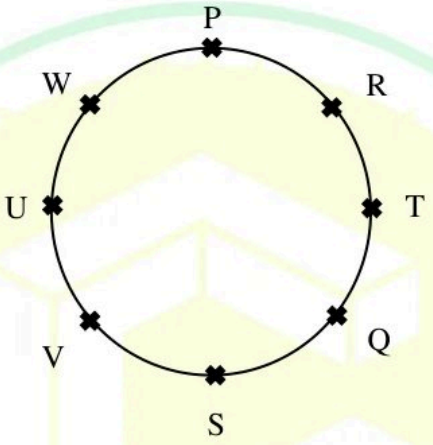
When three lengths add up to L, the triangle inequality reduces to one simple test: every side must be shorter than $L/2$. Here $L = 1$, so each of b_1, $b_2 - b_1$ and $1 - b_2$ must be less than 0.5. Write out these three inequalities and match them to the options. To show that a condition is not necessary, one counter-example is enough: a valid triangle that breaks that condition.

WATCH OUT FOR

Confusing 'necessary' with 'sufficient'. Each of (A), (B) and (C) alone is not enough to guarantee a triangle, so students sometimes wrongly mark one of them as the answer. The question asks which condition a triangle does not REQUIRE. Another trap is assuming $b_1 + b_2 < 1$ from the figure, or from a symmetry argument that puts both cuts near the middle. The figure is only representative. Cuts at 0.45 and 0.7 give a valid triangle with $b_1 + b_2 > 1$.

Q.8 MCQ · 2 MARKS

General Aptitude › Reasoning

Q.8	Eight students (P, Q, R, S, T, U, V, and W) are playing musical chairs. The figure indicates their order of position at the start of the game. They play the game by moving forward in a circle in the clockwise direction. After the 1st round, 4th student behind P leaves the game. After 2nd round, 5th student behind Q leaves the game. After 3rd round, 3rd student behind V leaves the game. After 4th round, 4th student behind U leaves the game. Who all are left in the game after the 4th round? Note: The figure shown is representative.
	
(A)	P; T; Q; S
(B)	V; P; T; Q
(C)	W; R; Q; V
(D)	Q; T; V; W

Official answer: **Marks to all**

SOLUTION

1. **Read the circle.** Going clockwise from the top, the order is P, R, T, Q, S, V, U, W, then back to P. The students move clockwise, so the student **behind** someone is the next one **anticlockwise** from them. For example, W is directly behind P. In musical chairs everyone walks round the circle together, so the students' order never changes. Only who is still playing matters, not which chair they are in.
2. **Round 1: the 4th student behind P.** Counting anticlockwise from P: W (1), U (2), V (3), S (4). **S leaves.** Still playing, in clockwise order: P, R, T, Q, V, U, W.

3. **Round 2: the 5th student behind Q.** Counting anticlockwise from Q: T (1), R (2), P (3), W (4), U (5). **U leaves.** Still playing: P, R, T, Q, V, W.
4. **Round 3: the 3rd student behind V.** Counting anticlockwise from V (S has gone, so the next student is Q): Q (1), T (2), R (3). **R leaves.** Still playing: P, T, Q, V, W.
5. **Check the options after three rounds.** The final four must come from $\{P, T, Q, V, W\}$. Option (A) contains S and option (C) contains R, so both are out. That leaves (B) and (D). Round 4 decides between them, because it must remove either W or P.
6. **Round 4: the question is defective as printed.** It asks for the 4th student behind U, but U left in round 2, so there is no U to count from. This flaw is why the official key gave marks to every candidate (MTA). The most natural repair is to count from the place where U was standing, between V and W. Counting anticlockwise from there, over the students still playing: V (1), Q (2), T (3), P (4). **P leaves.** Counting from W, the student just ahead of U's old place, gives the same result: V (1), Q (2), T (3), P (4).
7. **Check the direction.** If 'behind' meant clockwise instead, round 1 would remove S, round 2 R and round 3 P. U would still be playing in round 4, which would remove V. That leaves T, Q, U, W, which matches no option, so the anticlockwise reading is the intended one.
8. **Left after the 4th round: Q, T, V, W.** This is option (D). It rests on counting round 4 from U's vacated place. The official key is MTA.

KEY IDEA

The students move clockwise, so 'behind' means counting anticlockwise. Musical chairs never changes the students' order round the circle, so each round just removes one name from a cyclic list, and you count only the students still playing. Rounds 1 to 3 remove S, U and R. Once U and R are gone, options (A) and (C) are ruled out before round 4.

WATCH OUT FOR

The first common mistake is counting 'behind' clockwise, in the direction of motion. That removes S, R, P, V and leaves T, Q, U, W, which is not an option. The second is counting the empty chairs of students who have left as if someone were still sitting there. A third is not noticing that U has already left when round 4 asks for the student behind U. The question is flawed as printed, and that is why the official key is MTA.

Q.9 MCQ · 2 MARKS

General Aptitude › Data interpretation

Q.9 The table lists the top 5 nations according to the number of gold medals won in a tournament; also included are the number of silver and the bronze medals won by them. Based only on the data provided in the table, which one of the following statements is INCORRECT?

Nation	Gold	Silver	Bronze
USA	40	44	41
Canada	39	27	24
Japan	20	12	13
Australia	17	19	16
France	16	26	22

(A) France will occupy the third place if the list were made on the basis of the total number of medals won.

(B) The order of the top two nations will not change even if the list is made on the basis of the total number of medals won.

(C) USA and Canada together have less than 50% of the medals awarded to the nations in the above table.

(D) Canada has won twice as many total medals as Japan.

Official answer: **C**

SOLUTION

1. Total medals per nation (Gold + Silver + Bronze): USA $40 + 44 + 41 = 125$, Canada $39 + 27 + 24 = 90$, Japan $20 + 12 + 13 = 45$, Australia $17 + 19 + 16 = 52$, France $16 + 26 + 22 = 64$.
2. Rank by total: USA (125) > Canada (90) > France (64) > Australia (52) > Japan (45).
3. (A) France is third by total, so the statement is correct.
4. (B) By gold the top two are USA (40) and Canada (39). By total they are still USA (125) and Canada (90), in the same order. Correct.
5. (C) Grand total $= 125 + 90 + 45 + 52 + 64 = 376$, so half of it is 188. USA + Canada $= 125 + 90 = 215 > 188$. In percentage terms, $\frac{215}{376} \times 100 \approx 57.2\%$, which is more than 50%. The

claim of "less than 50%" is therefore false. INCORRECT.

6. (D) Canada $90 = 2 \times 45$, which is twice Japan's total. Correct.
7. The only incorrect statement is (C).

KEY IDEA

Add each row across to get every nation's total medal count, then test each statement against those totals. For a "less than 50%" claim, compare the part with half the grand total (here 188). That settles it without working out an exact percentage.

WATCH OUT FOR

Using only the Gold column, or reading the question as "which statement is correct". On gold alone, USA + Canada have 79 of 132 golds, which is also more than 50%, but options (A) and (D) need the totals across all three columns. Candidates also make arithmetic slips in the grand total (376); a slip there can make (C) look true.

Q.10 MCQ · 2 MARKS

General Aptitude › Numerical

An organization allows its employees to work independently on consultancy projects but charges an overhead on the consulting fee. The overhead is 20% of the consulting fee, if the fee is up to Rs. 5,00,000. For higher fees, the overhead is Rs. 1,00,000 plus 10% of the amount by which the fee exceeds Rs. 5,00,000. The government charges a Goods and Services Tax of 18% on the total amount (the consulting fee plus the overhead). An employee of the organization charges this entire amount, i.e., the consulting fee, overhead, and tax, to the client. If the client cannot pay more than Rs. 10,00,000, what is the maximum consulting fee that the employee can charge?

(A) Rs. 7,01,438

(B) Rs. 7,24,961

(C) Rs. 7,51,232

(D) Rs. 7,75,784

Official answer: **B**

SOLUTION

1. Decide which slab applies. If the fee $F \le 5{,}00{,}000$, the overhead is $0.2F$, so the client pays $1.18 \times 1.2F = 1.416F \le 1.416 \times 5{,}00{,}000 = 7{,}08{,}000$. That is well under the Rs. $10{,}00{,}000$ limit, so the maximum fee must be above Rs. $5{,}00{,}000$.
2. Overhead in the upper slab (the fixed part plus 10% of the excess): $1{,}00{,}000 + 0.1(F - 5{,}00{,}000) = 0.1F + 50{,}000$.
3. Fee plus overhead: $F + 0.1F + 50{,}000 = 1.1F + 50{,}000$.
4. GST of 18% multiplies this pre-tax amount by 1.18, so the client pays $1.18\,(1.1F + 50{,}000)$, and this must satisfy $1.18\,(1.1F + 50{,}000) \le 10{,}00{,}000$.
5. Remove the tax first: $1.1F + 50{,}000 \le \dfrac{10{,}00{,}000}{1.18} = 8{,}47{,}457.63$.
6. Solve for the fee: $1.1F \le 7{,}97{,}457.63 \Rightarrow F \le \dfrac{7{,}97{,}457.63}{1.1} = 7{,}24{,}961.48$.
7. Check with $F = 7{,}24{,}961$: overhead $= 1{,}00{,}000 + 0.1 \times 2{,}24{,}961 = 1{,}22{,}496.1$, pre-tax total $= 8{,}47{,}457.1$, and $8{,}47{,}457.1 \times 1.18 = 9{,}99{,}999.38 \le 10{,}00{,}000$. Of the options, (A) is affordable but is not the maximum, while (C) and (D) take the bill past Rs. $10{,}00{,}000$ (Rs. $10{,}34{,}099$ and Rs. $10{,}65{,}968$). So the maximum fee is Rs. $7{,}24{,}961$, option (B).

KEY IDEA

First work out which slab applies. Even at the top of the first slab (fee Rs. 5,00,000) the client pays only Rs. 7,08,000, so the fee must be in the upper slab. There the overhead is 0.1F + 50,000, so the client pays 1.18(1.1F + 50,000). Set that equal to Rs. 10,00,000 and work backwards: take out the 18% GST first, then the overhead.

WATCH OUT FOR

One mistake is to charge the 10% on the whole fee instead of only on the part above Rs. 5,00,000, which gives an overhead of 1,00,000 + 0.1F. Another is to use the 20% slab rate for the whole fee. A third is to take 18% off the Rs. 10,00,000 (multiplying by 0.82) instead of dividing by 1.18. Each of these moves the answer away from (B) and can land near one of the other options.

Electronics and Communication Engineering Q.11 to Q.65

Q.11 MCQ · 1 MARK

Engineering Mathematics › Linear algebra

Consider the matrix A below:

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 6 & 7 & 8 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & 0 & \gamma \end{bmatrix}$$

For which of the following combinations of α, β, and γ, is the rank of A at least three?

(i) $\alpha = 0$ and $\beta = \gamma \neq 0$.

(ii) $\alpha = \beta = \gamma = 0$.

(iii) $\beta = \gamma = 0$ and $\alpha \neq 0$.

(iv) $\alpha = \beta = \gamma \neq 0$.

(A) Only (i), (iii), and (iv)

(B) Only (iv)

(C) Only (ii)

(D) Only (i) and (iii)

Official answer: **A**

SOLUTION

1. Rows 1 and 2 are always independent, because the leading 2×2 minor is $\begin{vmatrix} 2 & 3 \\ 0 & 6 \end{vmatrix} = 12 \neq 0$. So $\rho(A) \geq 2$ for every choice, and $\rho(A) \geq 3$ exactly when some 3×3 minor is non-zero.
2. A is upper triangular, so $\det A = 2 \cdot 6 \cdot \alpha \cdot \gamma = 12\alpha\gamma$.
3. (i) $\alpha = 0,\ \beta = \gamma \neq 0$: rows 3 and 4 are both $[0\ 0\ 0\ \beta]$. The minor from rows 1, 2, 3 and columns 1, 2, 4 is $\begin{vmatrix} 2 & 3 & 5 \\ 0 & 6 & 8 \\ 0 & 0 & \beta \end{vmatrix} = 12\beta \neq 0$, so $\rho(A) = 3$. It is not 4, because $\det A = 12\alpha\gamma = 0$. The rank is at least 3.
4. (ii) $\alpha = \beta = \gamma = 0$: rows 3 and 4 are zero rows, so only the two independent rows 1 and 2 are left and $\rho(A) = 2$. This case fails.
5. (iii) $\beta = \gamma = 0,\ \alpha \neq 0$: row 3 is $[0\ 0\ \alpha\ 0]$ and row 4 is zero. The leading 3×3 minor is $2 \cdot 6 \cdot \alpha = 12\alpha \neq 0$, so $\rho(A) = 3$. The rank is at least 3.
6. (iv) $\alpha = \beta = \gamma \neq 0$: $\det A = 12\alpha^2 \neq 0$, so $\rho(A) = 4$. The rank is at least 3.
7. So (i), (iii) and (iv) give rank at least three, and only (ii) fails. That is option (A).

KEY IDEA

$\rho(A) \geq r$ exactly when some $r \times r$ minor is non-zero. Here the first two rows always give rank at least 2, so you only need to find one non-zero 3×3 minor for each case. For an upper triangular matrix, $\det A$ is the product of the diagonal entries, $12\alpha\gamma$. That settles rank 4 straight away, and a zero diagonal entry does not by itself push the rank below 3.

WATCH OUT FOR

Counting non-zero diagonal entries and taking that as the rank. In case (i) the diagonal is $2, 6, 0, \gamma$, which looks like rank 3 for the wrong reason, and a student can easily talk themselves into "$\alpha = 0$ breaks rank 3" and reject (i), which leads to option (D). Rank comes from independent rows or non-zero minors, not from the diagonal. In (i) the minor on columns 1, 2, 4 is $12\beta \neq 0$, so the rank is 3. Rows 3 and 4 are identical, so it cannot be 4.

Q.12 MCQ · 1 MARK

Engineering Mathematics › Calculus

Consider the following series:

(i) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

(ii) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

(iii) $\sum_{n=1}^{\infty} \frac{1}{n!}$

Choose the correct option.

(A) Only (ii) converges

(B) Only (ii) and (iii) converge

(C) Only (iii) converges

(D) All three converge

Official answer: **B**

SOLUTION

1. **Series (i), p-series test:** $\sum \frac{1}{n^p}$ converges only when $p > 1$. Here $\frac{1}{\sqrt{n}} = \frac{1}{n^{1/2}}$, so $p = \frac{1}{2} \le 1$ and **(i) diverges**. Its terms do go to zero, but that alone does not make a series converge.
2. **Series (ii), partial fractions:** $\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$.
3. **Telescoping partial sum:**
$$S_N = \left(1 - \tfrac{1}{2}\right) + \left(\tfrac{1}{2} - \tfrac{1}{3}\right) + \cdots + \left(\tfrac{1}{N} - \tfrac{1}{N+1}\right) = 1 - \frac{1}{N+1} \to 1$$
so **(ii) converges** (to 1). The limit comparison test with $\sum \frac{1}{n^2}$ ($p = 2 > 1$) gives the same result, since $\frac{1/(n(n+1))}{1/n^2} = \frac{n}{n+1} \to 1$.
4. **Series (iii), ratio test:** $\frac{a_{n+1}}{a_n} = \frac{n!}{(n+1)!} = \frac{1}{n+1} \to 0 < 1$, so **(iii) converges**. From the series for e^x at $x = 1$ (with the $n = 0$ term, 1, left out), its sum is $e - 1$.
5. Only (ii) and (iii) converge, which is **option (B)**.

KEY IDEA

Use a standard test for each series. For $\sum 1/n^p$, check whether $p > 1$. For $\sum 1/(n(n+1))$, split into partial fractions so the sum telescopes. For $\sum 1/n!$, the ratio test works well because consecutive factorials cancel down to $1/(n+1)$.

WATCH OUT FOR

Deciding that (i) converges because $1/\sqrt{n} \to 0$. A term going to zero is necessary for convergence but not sufficient. $\sum 1/\sqrt{n}$ is a p-series with $p = \frac{1}{2} \le 1$, so it diverges, and choosing option (D) this way is wrong. A second slip is to confuse $p = \frac{1}{2}$ with the convergent case and treat every series whose terms shrink as convergent.

Q.13 MCQ · 1 MARK

Engineering Mathematics › Probability & statistics

A pot contains two red balls and two blue balls. Two balls are drawn from this pot randomly without replacement.

What is the probability that the two balls drawn have different colours?

(A) $2/3$

(B) $1/3$

(C) $1/2$

(D) 1

Official answer: **A**

SOLUTION

1. Sample space: choosing any 2 of the 4 balls, with order ignored, gives $\binom{4}{2} = 6$ equally likely pairs.
2. Favourable outcomes: one red ball and one blue ball. By the multiplication rule there are $\binom{2}{1}\binom{2}{1} = 2 \times 2 = 4$ such pairs.
3. Classical probability: $P(\text{different}) = \dfrac{4}{6} = \dfrac{2}{3}$.
4. Check with the complement: $P(\text{same}) = \dfrac{\binom{2}{2} + \binom{2}{2}}{6} = \dfrac{2}{6} = \dfrac{1}{3}$, so $P(\text{different}) = 1 - \dfrac{1}{3} = \dfrac{2}{3}$.
5. Check by drawing one ball at a time: whatever colour the first ball is, 2 of the 3 balls left are the other colour, so $P = \dfrac{2}{3}$. This matches option (A).

KEY IDEA

Without replacement, the first ball can be anything. What matters is the second draw: 3 balls are left and 2 of them are the other colour, so the probability is $\frac{2}{3}$. You get the same answer by counting unordered pairs, $\frac{2 \cdot 2}{\binom{4}{2}} = \frac{4}{6}$.

WATCH OUT FOR

One mistake is to treat the draws as if they were with replacement, which gives $2 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$, option (C). Another is to use the ordered count (4 ordered red-blue pairs, only one colour order) over the unordered total, or to leave out the factor 2 for the two orders in the sequential method ($\frac{2}{4} \cdot \frac{2}{3} = \frac{1}{3}$, option (B)).

Q.14 MCQ · 1 MARK

Communications › Analog modulation

Consider a frequency-modulated (FM) signal $f(t) = A_c \cos(2\pi f_c t + 3\sin(2\pi f_1 t) + 4\sin(6\pi f_1 t))$, where A_c and f_c are, respectively, the amplitude and frequency (in Hz) of the carrier waveform. The frequency f_1 is in Hz, and assume that $f_c > 100 f_1$. The peak frequency deviation of the FM signal in Hz is _____.

(A) $15 f_1$

(B) $12 f_1$

(C) $4 f_1$

(D) $2 f_1$

Official answer: **A**

SOLUTION

1. Write the total phase: $\theta(t) = 2\pi f_c t + 3\sin(2\pi f_1 t) + 4\sin(6\pi f_1 t)$.
2. The instantaneous frequency in Hz is $f_i(t) = \dfrac{1}{2\pi}\dfrac{d\theta}{dt}$.
3. Differentiate: $\dfrac{d\theta}{dt} = 2\pi f_c + 3(2\pi f_1)\cos(2\pi f_1 t) + 4(6\pi f_1)\cos(6\pi f_1 t)$.
4. Divide by 2π: $f_i(t) = f_c + 3f_1\cos(2\pi f_1 t) + 12 f_1 \cos(6\pi f_1 t)$.
5. The frequency deviation is $\Delta f(t) = f_i(t) - f_c = 3f_1\cos(2\pi f_1 t) + 12 f_1 \cos(2\pi (3f_1) t)$.
6. The deviation is largest when both cosines equal 1. This happens together at $t = 0$, since both arguments are zero there. So $\Delta f_{\max} = 3f_1 + 12 f_1 = 15 f_1$. No other time can give a larger value, because each term is bounded by its amplitude.
7. The condition $f_c > 100 f_1$ only keeps the instantaneous frequency positive and the carrier well above the message tones. It does not change the deviation. Answer: (A) $15 f_1$.

KEY IDEA

Peak frequency deviation is the maximum of $\frac{1}{2\pi}\frac{d}{dt}$ of the phase excess over the carrier. Each sinusoidal phase term $\beta\sin(2\pi f_m t)$ adds $\beta f_m\cos(2\pi f_m t)$ to the frequency, so its amplitude is βf_m. The terms peak together at $t=0$ and add.

WATCH OUT FOR

Reading the phase amplitudes 3 and 4 as the deviations, or taking only the larger tone. The second term has frequency $3f_1$, so its deviation is $4\times 3f_1 = 12f_1$, not $4f_1$. Choosing $12f_1$ ignores the first tone, which adds $3f_1$.

Q.15 MCQ · 1 MARK

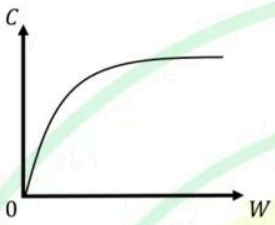
Communications › Information theory

Q.15

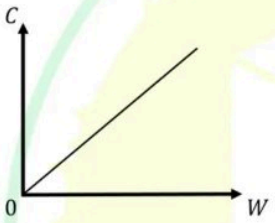
Consider an additive white Gaussian noise (AWGN) channel with bandwidth W and noise power spectral density $\frac{N_0}{2}$. Let P_{av} denote the average transmit power constraint.

Which one of the following plots illustrates the dependence of the channel capacity C on the bandwidth W (keeping P_{av} and N_0 fixed)?

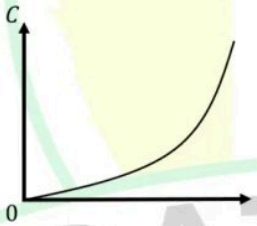
(A)



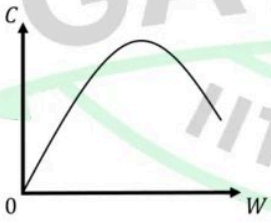
(B)



(C)



(D)



Official answer: **A**

SOLUTION

1. Use the two-sided noise PSD $\frac{N_0}{2}$. In a bandwidth W (band $-W$ to W in the baseband picture, total width $2W$) the noise power is $N = \frac{N_0}{2} \times 2W = N_0 W$.
2. Shannon capacity of the AWGN channel:

$$C = W \log_2\left(1 + \frac{P_{av}}{N_0 W}\right) \text{ bits/s}.$$

Write $a = \dfrac{P_{av}}{N_0}$ (fixed), so $C = \dfrac{1}{\ln 2}\, W \ln\!\left(1 + \dfrac{a}{W}\right)$.

3. Small W: $\dfrac{a}{W} \gg 1$, so $C \approx W \log_2 \dfrac{a}{W}$. This is 0 at $W = 0$ and rises steeply (slightly less than linearly).
4. Large W: $x = \dfrac{a}{W} \ll 1$ and $\ln(1 + x) \approx x$, so
$$C \to \frac{W}{\ln 2} \cdot \frac{a}{W} = \frac{P_{av}}{N_0} \log_2 e \approx 1.44\, \frac{P_{av}}{N_0},$$
a finite limit. Capacity does not grow without bound.
5. Slope:
$$\frac{dC}{dW} = \frac{1}{\ln 2}\left[\ln\!\left(1 + \frac{a}{W}\right) - \frac{a}{W + a}\right] > 0,$$
because $\ln(1 + x) > \dfrac{x}{1 + x}$ for $x > 0$. So C increases with W (no peak).
6. Curvature:
$$\frac{d^2C}{dW^2} = \frac{a}{\ln 2\,(W + a)}\left[\frac{1}{W + a} - \frac{1}{W}\right] < 0,$$
so C is concave in W.
7. So C starts at 0, increases, is concave, and flattens to the horizontal asymptote $1.44\, P_{av}/N_0$. A straight line (B), unbounded convex growth (C) and a curve with a peak (D) are all ruled out. This is plot (A).

KEY IDEA

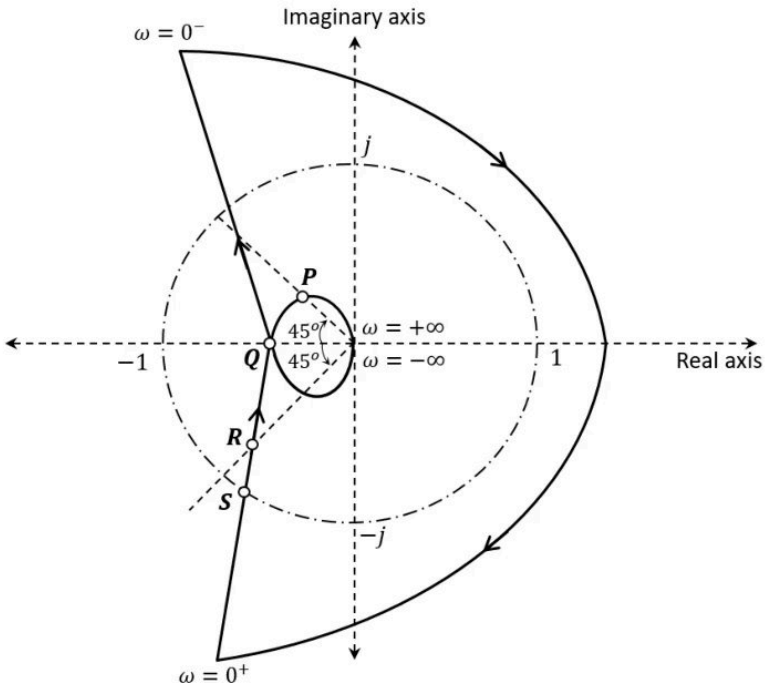
With fixed P_{av} the noise power grows with bandwidth ($N = N_0 W$), so $C = W \log_2(1 + P_{av}/N_0 W)$ is increasing and concave and tends to the finite limit $1.44\, P_{av}/N_0$: a bandwidth-limited regime at small W, a power-limited one at large W.

WATCH OUT FOR

Reading $C = W \log_2(1 + \mathrm{SNR})$ as linear in W and choosing the straight line (B), forgetting that the SNR itself falls as $1/W$ because noise power is $N_0 W$.

Q.16 MCQ · 1 MARK

Control Systems › Bode & margins

Q.16	The Nyquist plot of a system is given in the figure below. Let $\omega_P, \omega_Q, \omega_R$, and ω_S be the positive frequencies at the points $\boldsymbol{P}$, $\boldsymbol{Q}$, $\boldsymbol{R}$, and $\boldsymbol{S}$, respectively. Which one of the following statements is TRUE?
	
(A)	ω_S is the gain crossover frequency and ω_P is the phase crossover frequency
(B)	ω_Q is the gain crossover frequency and ω_R is the phase crossover frequency
(C)	ω_Q is the gain crossover frequency and ω_S is the phase crossover frequency
(D)	ω_S is the gain crossover frequency and ω_Q is the phase crossover frequency

Official answer: **D**

SOLUTION

1. **Definitions.** The gain crossover frequency ω_{gc} is where $|G(j\omega)H(j\omega)| = 1$. On a Nyquist plot, that is where the positive-frequency locus crosses the unit circle. The phase crossover frequency ω_{pc} is where $\angle G(j\omega)H(j\omega) = -180^\circ$, i.e. where the locus crosses the negative real axis.
2. **Find the positive-frequency branch.** Follow the arrows from $\omega = 0^+$. The locus rises through S, then R, then Q, then goes round the upper half of the small loop through P and ends at the origin at $\omega = +\infty$. The large arc on the right (from $\omega = 0^-$ to $\omega = 0^+$) and the branch from the origin ($\omega = -\infty$)

through the lower half of the loop and Q up to $\omega = 0^-$ are not positive frequencies. Along the positive branch the phase keeps falling: about -125° at S, -135° at R, -180° at Q and -225° at P.

3. **Point S gives the gain crossover.** S lies on the dash-dot unit circle, so $|G(j\omega_S)H(j\omega_S)| = 1$ and $\omega_S = \omega_{gc}$. The positive branch meets the unit circle only once: it is outside the circle before S and stays inside after it. So this is the only gain crossover. S is **not** on the -135° ray. That ray crosses the locus at R, so the phase at S is not -135°. It is less lagging, roughly -125° as drawn.
4. **Point Q gives the phase crossover.** Q lies on the negative real axis, so $\angle G(j\omega_Q)H(j\omega_Q) = -180^\circ$ and $\omega_Q = \omega_{pc}$. The positive branch crosses the negative real axis only here, because the upper half of the loop stays above the axis until it reaches the origin. At Q the magnitude is about $0.45 \neq 1$, so Q is not a gain crossover.
5. **Rule out R and P.** R is on the -135° ray inside the circle: $|GH| < 1$ and $\angle GH = -135^\circ$. P is on the 135° ray inside the circle. The phase has already passed -180° at Q and is still falling, so that angle means $\angle GH = -225^\circ$, and again $|GH| < 1$. Neither point meets either crossover condition. That rules out (A), which makes ω_P the phase crossover, and (B), which makes ω_R the phase crossover. (B) and (C) also fail because they make ω_Q the gain crossover, and (C) makes ω_S the phase crossover.
6. **Answer.** ω_S is the gain crossover frequency and ω_Q is the phase crossover frequency, which is option **(D)**. As a check, the gain margin is $1/|G(j\omega_Q)H(j\omega_Q)| \approx 1/0.45 \approx 2.2 > 1$. The phase margin is $180^\circ + \angle G(j\omega_S)H(j\omega_S)$, roughly 55° as drawn. The 45° marks are **not** the phase margin. They mark the rays through R and P, and those points are distractors.

KEY IDEA

On a Nyquist plot, read each crossover from where the positive-frequency locus is, not from the angle marks. It crosses the unit circle at the gain crossover ($|GH| = 1$) and the negative real axis at the phase crossover ($\angle GH = -180^\circ$).

WATCH OUT FOR

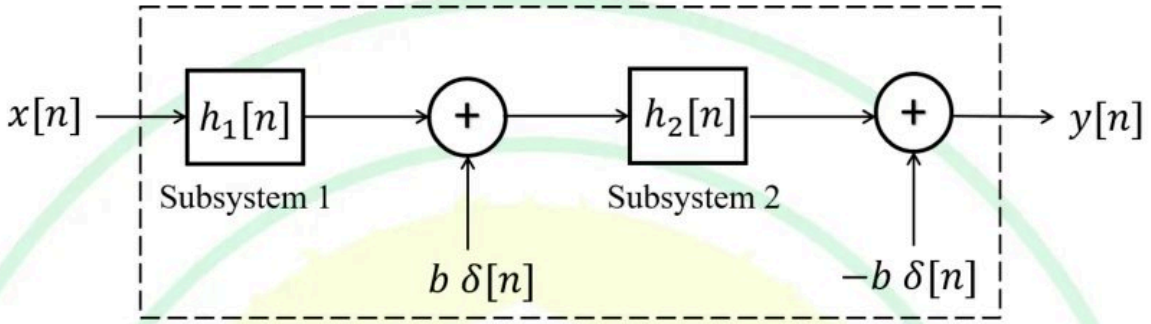
Swapping the two definitions, which gives (C). Another error is to treat R as special because it sits on the marked 45° ray. It looks like a 45° phase margin, but R has $|GH| < 1$, so it is neither crossover. P is on the other 45° ray at phase -225°, not -180°. Some students also assume S is on the -135° ray and write the phase margin as 45°. In the figure S is below that ray, so its phase is less lagging than -135°.

Q.17 MCQ · 1 MARK

Signals and Systems › LTI & convolution

Q.17 Consider the discrete-time system below with input $x[n]$ and output $y[n]$. In the figure, $h_1[n]$ and $h_2[n]$ denote the impulse responses of LTI Subsystems 1 and 2, respectively. Also, $\delta[n]$ is the unit impulse, and $b > 0$.

Assuming $h_2[n] \neq \delta[n]$, the overall system (denoted by the dashed box) is _____.



(A) linear and time invariant

(B) linear and time variant

(C) nonlinear and time invariant

(D) nonlinear and time variant

Official answer: **D**

SOLUTION

1. **Input–output relation.** The output of Subsystem 1 is $x[n] * h_1[n]$. After the first adder the signal is $x[n] * h_1[n] + b\,\delta[n]$. Subsystem 2 is LTI, so its response to a sum is the sum of its responses, and $\delta[n] * h_2[n] = h_2[n]$:
$$y[n] = x[n] * h_1[n] * h_2[n] + b\,h_2[n] - b\,\delta[n]$$
2. Write $h[n] = h_1[n] * h_2[n]$ and $g[n] = b\,(h_2[n] - \delta[n])$, so
$$y[n] = x[n] * h[n] + g[n]$$

   $g[n]$ does not depend on the input. Since $b > 0$ and $h_2[n] \neq \delta[n]$, $g[n]$ is not the zero signal. The $+b\,\delta[n]$ and $-b\,\delta[n]$ do not cancel, because the first one passes through Subsystem 2 and comes out as $b\,h_2[n]$.
3. **Linearity (zero in, zero out).** Put $x[n] = 0$. Then $y[n] = g[n] \neq 0$. A linear system must give zero output for zero input (homogeneity with scale factor 0), so the system is **nonlinear**. Superposition gives the

same result: input $x_1 + x_2$ produces $x_1 * h + x_2 * h + g$, while $y_1 + y_2 = x_1 * h + x_2 * h + 2g$, and these differ because $g \neq 0$.

4. **Time invariance (shift test).** Apply $x[n - n_0]$. The LTI part delays its output by the same amount, so the response is $x[n - n_0] * h[n] + g[n]$. The original output delayed by n_0 is $y[n - n_0] = x[n - n_0] * h[n] + g[n - n_0]$.
5. The two match for every input and every n_0 only if $g[n - n_0] = g[n]$ for all n and n_0. That means $g[n]$ has to be a constant sequence. A constant added offset would **not** break time invariance, so the question to settle is whether $g[n]$ is constant.
6. $g[n]$ **is not constant.** Suppose $g[n] = K$ for every n. Since $g \neq 0$, $K \neq 0$, and then $h_2[n] = \delta[n] + \frac{K}{b}$. That impulse response keeps the same nonzero value at every n, forever in both directions: it never dies out, it is not absolutely summable, and it is nonzero for $n < 0$. This degenerate case is the only way out, and it is not what the question means by an LTI subsystem. For any $h_2[n]$ whose samples die out as $|n| \to \infty$, $g[n] \to 0$ while g is nonzero somewhere, so g cannot be constant. In other words, $g[n]$ is tied to the time origin through $\delta[n]$ and $h_2[n]$ and does not move when the input is delayed. So for some n_0, $y[n - n_0]$ differs from the response to $x[n - n_0]$, and the system is **time variant**.
7. The system is nonlinear and time variant, so the answer is option (D).

KEY IDEA

Reduce the block diagram to $y[n] = x[n] * h_1[n] * h_2[n] + b\,(h_2[n] - \delta[n])$. That is an LTI response plus a fixed, nonzero signal that does not depend on the input. Because the added signal is nonzero, zero input does not give zero output, so the system is nonlinear. Because the added signal is tied to $n = 0$ and is not a constant, it does not move when the input is delayed, so the system is time variant.

WATCH OUT FOR

Three traps. (1) Choosing (A) because every block is LTI, without noticing that the $\delta[n]$ sources inside the dashed box are not part of the input. (2) Thinking $+b\,\delta[n]$ and $-b\,\delta[n]$ cancel. They do not: the first one passes through $h_2[n]$ and becomes $b\,h_2[n]$, so they cancel only when $h_2[n] = \delta[n]$, which the question rules out. (3) Choosing (C) by treating the added term like a constant bias. A constant offset keeps a system time invariant, but $b(h_2[n] - \delta[n])$ is a signal fixed to $n = 0$, not a constant.

Q.18 MCQ · 1 MARK

Signals and Systems › Fourier transform

Consider a continuous-time, real-valued signal $f(t)$ whose Fourier transform $F(\omega) = \int_{-\infty}^{\infty} f(t) \exp(-j\,\omega t)\,dt$ exists.

Which one of the following statements is always TRUE?

(A) $|F(\omega)| \leq \int_{-\infty}^{\infty} |f(t)|\,dt$
(B) $|F(\omega)| > \int_{-\infty}^{\infty} |f(t)|\,dt$
(C) $|F(\omega)| \leq \int_{-\infty}^{\infty} f(t)\,dt$
(D) $|F(\omega)| \geq \int_{-\infty}^{\infty} f(t)\,dt$

Official answer: **A**

SOLUTION

1. **Triangle inequality for integrals.** The magnitude of an integral is never more than the integral of the magnitude:
$$|F(\omega)| = \left| \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt \right| \leq \int_{-\infty}^{\infty} |f(t)|\, \left| e^{-j\omega t} \right| dt$$
2. **Unit magnitude of the kernel.** For real ω and t, $\left| e^{-j\omega t} \right| = 1$, so
$$|F(\omega)| \leq \int_{-\infty}^{\infty} |f(t)|\, dt \quad \text{for every } \omega$$

This is statement (A). It holds for any $f(t)$ whose transform exists: if $f(t)$ is not absolutely integrable, the right-hand side is infinite and the inequality is trivially true.

3. **(B) is ruled out** because it is the exact negation of the bound just proved, so it is false for every signal at every ω. For example, for any $f(t) \geq 0$, at $\omega = 0$ we get $|F(0)| = \int |f(t)|\, dt$ (equality, not $>$).
4. **(C) fails for a signal that integrates to zero.** With $\text{rect}(t) = 1$ for $|t| < \frac{1}{2}$ and 0 otherwise, take $f(t) = \text{rect}(t - \frac{1}{2}) - \text{rect}(t + \frac{1}{2})$: a $+1$ pulse on $(0, 1)$ and a -1 pulse on $(-1, 0)$. Then $\int f\, dt = F(0) = 0$, while
$$F(\omega) = \frac{1 - e^{-j\omega}}{j\omega} - \frac{e^{j\omega} - 1}{j\omega} = \frac{2 - 2\cos\omega}{j\omega} = \frac{4\sin^2(\omega/2)}{j\omega}$$
At $\omega = \pi$, $|F(\pi)| = \dfrac{4}{\pi} > 0$, so $|F(\omega)| \leq 0$ is false.
5. **(D) fails at a spectral null.** Take $f(t) = \text{rect}(t)$: $\int f\, dt = 1$, while $F(\omega) = \dfrac{\sin(\omega/2)}{\omega/2}$ is 0 at $\omega = 2\pi$. Then $|F(2\pi)| = 0 < 1$.
6. Only (A) is always true.

KEY IDEA

$|F(\omega)| = \left|\int f(t) e^{-j\omega t}\, dt\right| \leq \int |f(t)|\, |e^{-j\omega t}|\, dt = \int |f(t)|\, dt$. The area under $|f(t)|$ bounds the spectrum's magnitude at every frequency. The bound is reached at $\omega = 0$ when $f(t) \geq 0$, which is why the strict inequality in (B) can never hold.

WATCH OUT FOR

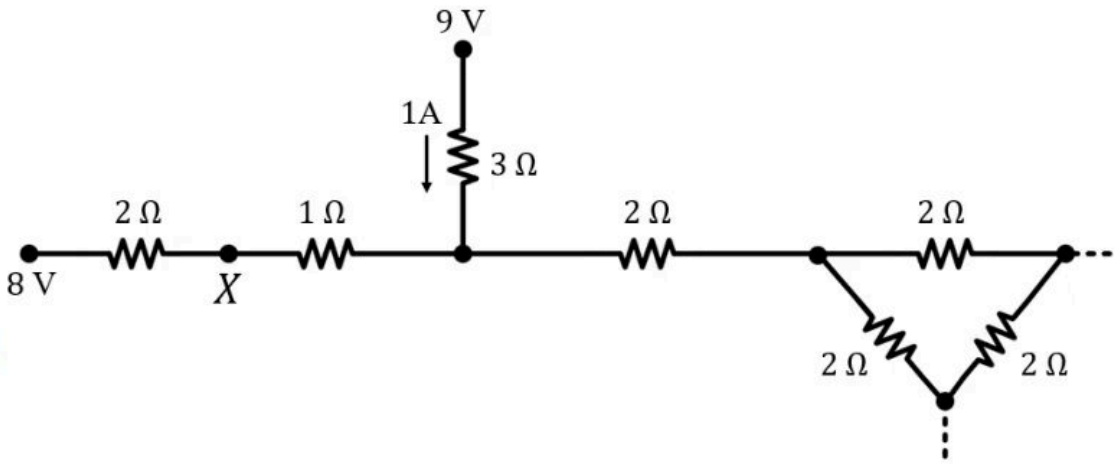
Choosing (C) or (D) because $F(0) = \int f(t)\, dt$. The signed area only gives the spectrum's value at $\omega = 0$. It bounds $|F(\omega)|$ in neither direction: it can be zero or negative while $|F(\omega)| > 0$ elsewhere, which breaks (C), and $|F(\omega)|$ can fall to zero at a spectral null, which breaks (D). A bound that holds at every frequency needs the absolute value inside the integral.

Q.19 MCQ · 1 MARK

Networks (Electric Circuits) › Laws & node/mesh

Q.19 Consider a part of an electrical network as shown below. Some node voltages, and the current flowing through the 3Ω resistor are as indicated.

The voltage (in Volts) at node X is _____.



(A) $20/3$

(B) $32/3$

(C) $22/3$

(D) $2/3$

Official answer: **A**

SOLUTION

1. Call the junction where the $1\,\Omega$, the $3\,\Omega$ and the right-hand $2\,\Omega$ meet node N. You do not need to solve the rest of the network (the delta and the dotted part), because the given current already fixes V_N.
2. Ohm's law on the $3\,\Omega$ resistor. 1 A flows downward, from the 9 V node into N, so the voltage falls by IR in that direction:
$$V_N = 9 - (1)(3) = 6\ \text{V}$$
3. Node X has only two branches: the $2\,\Omega$ to the 8 V node and the $1\,\Omega$ to N. By KCL at X, the current arriving through one branch must leave through the other:
$$\frac{8 - V_X}{2} = \frac{V_X - 6}{1}$$

4. Solve: $8 - V_X = 2V_X - 12 \;\Rightarrow\; 3V_X = 20 \;\Rightarrow\; V_X = \dfrac{20}{3}\ \text{V} \approx 6.67\ \text{V}$.
5. Check with the voltage-divider view. The $2\,\Omega$ and $1\,\Omega$ are in series across $8 - 6 = 2\ \text{V}$, so the current through them is $\dfrac{2}{3}\ \text{A}$. The $1\,\Omega$ drops $\dfrac{2}{3}\ \text{V}$, so $V_X = 6 + \dfrac{2}{3} = \dfrac{20}{3}\ \text{V}$. Going the other way, the $2\,\Omega$ drops $\dfrac{4}{3}\ \text{V}$ and $8 - \dfrac{4}{3} = \dfrac{20}{3}\ \text{V}$. The value lies between 6 V and 8 V, as it must. This is option (A).

KEY IDEA

You only need the part of the network next to X. Ohm's law on the $3\,\Omega$ resistor gives the junction voltage, $V_N = 9 - 1 \times 3 = 6\ \text{V}$. Node X has just two branches, so the $2\,\Omega$ and the $1\,\Omega$ carry the same current and act as a voltage divider between 8 V and 6 V. The delta and the dotted part never enter the calculation.

WATCH OUT FOR

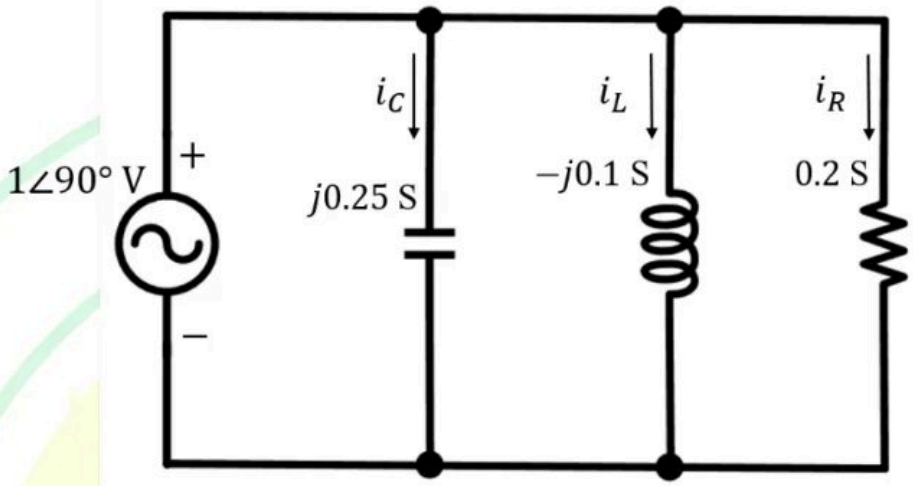
Each wrong option matches a specific slip. (B) $32/3$: taking the 1 A as flowing up, out of N, which gives $V_N = 9 + 3 = 12\ \text{V}$ instead of 6 V. (C) $22/3$: swapping the divider weights, i.e. putting the larger drop across the $1\,\Omega$ instead of the $2\,\Omega$. (D) $2/3$: giving the series current, $2/3\ \text{A}$, instead of the node voltage. Some students also waste time trying to reduce the delta, which the given 1 A makes unnecessary.

Q.20 MCQ · 1 MARK

Networks (Electric Circuits) › AC, power & resonance

Q.20 Let i_C, i_L, and i_R be the currents flowing through the capacitor, inductor, and resistor, respectively, in the circuit given below. The AC admittances are given in Siemens (S).

Which one of the following is TRUE?



(A) $i_C = 0.25\angle 180°$ A, $i_L = 0.1\angle 0°$ A, $i_R = 0.2\angle 90°$ A

(B) $i_C = 4\angle 180°$ A, $i_L = 10\angle 0°$ A, $i_R = 5\angle 90°$ A

(C) $i_C = 0.25\angle 270°$ A, $i_L = 0.1\angle 90°$ A, $i_R = 0.2\angle 90°$ A

(D) $i_C = 4\angle 90°$ A, $i_L = 10\angle 270°$ A, $i_R = 5\angle 0°$ A

Official answer: **A**

SOLUTION

1. All three elements are connected directly across the source. Each current is arrowed from the + rail down to the − rail, so each element sees the full source voltage in the same direction as its current: $V = 1\angle 90°\ \text{V} = j\ \text{V}$.
2. The values given are admittances, so Ohm's law in phasor form is $I = YV$: multiply by the admittance, do not divide. In polar form the magnitudes multiply and the angles add. Here $|V| = 1$ and $\angle V = 90°$.
3. Capacitor, $Y_C = j0.25 = 0.25\angle 90°$ S:
$$i_C = (j0.25)(j) = -0.25 = 0.25\angle 180°\ \text{A}$$
It leads V by $90°$, as a capacitor current should.

4. Inductor, $Y_L = -j0.1 = 0.1\angle -90^\circ$ S:
$$i_L = (-j0.1)(j) = 0.1 = 0.1\angle 0^\circ \text{ A}$$
It lags V by 90°, as an inductor current should.
5. Resistor, $Y_R = 0.2\angle 0^\circ$ S:
$$i_R = (0.2)(j) = 0.2\angle 90^\circ \text{ A}$$
It is in phase with V.
6. Only option (A) matches all three currents. Options (B) and (D) have magnitudes 4, 10 and 5, which you get by dividing by the admittance ($1/0.25$, $1/0.1$, $1/0.2$). Option (C) has the right magnitudes but the wrong phases for i_C and i_L.

KEY IDEA

In a parallel circuit every branch has the same voltage. When branch values are given as admittances, each branch current is $I = YV$. In polar form the magnitudes multiply and the angles add. The source angle of 90° gives $90^\circ + 90^\circ = 180^\circ$ for the capacitor, $90^\circ - 90^\circ = 0^\circ$ for the inductor and $90^\circ + 0^\circ = 90^\circ$ for the resistor.

WATCH OUT FOR

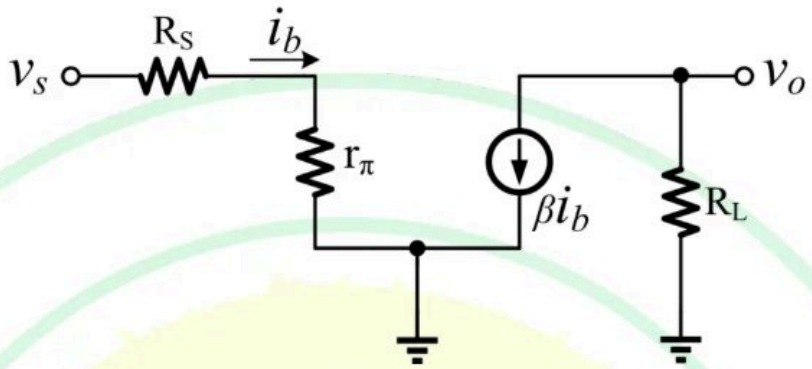
Treating the siemens values as impedances and computing $I = V/Y$, which gives the magnitudes 4, 10 and 5 in options (B) and (D). A second slip is to take the source as the 0° reference and forget the 90° source angle, or to add the angles with the wrong sign. That produces phase sets like the ones in (C) and (D).

Q.21 MCQ · 1 MARK

Analog Electronics › Amplifiers

Q.21 A simplified small-signal equivalent circuit of a BJT-based amplifier is given below.

The small-signal voltage gain v_o/v_s (in V/V) is _____.



(A) $\dfrac{-\beta R_L}{R_S + r_\pi}$

(B) $\dfrac{+\beta R_L}{R_S}$

(C) $\dfrac{-\beta R_L}{R_S}$

(D) $\dfrac{+\beta R_L}{R_S + r_\pi}$

Official answer: **A**

SOLUTION

1. Model as drawn: the transistor appears as its hybrid-pi input resistance r_π and a dependent current source βi_b. No output resistance r_o is drawn, so it is taken as infinite (open circuit). The input and output halves are joined only through the common ground.
2. KVL around the input loop ($v_s \to R_S \to r_\pi \to$ ground). The same current i_b flows through R_S and r_π: $v_s = i_b(R_S + r_\pi)$, so

$$i_b = \frac{v_s}{R_S + r_\pi}$$

3. Direction of the dependent source: its arrow points down, so the current βi_b enters the source's top terminal from the v_o node and leaves at the bottom terminal into ground. In other words, the source pulls βi_b out of the output node.
4. KCL at the output node, summing currents leaving it:
$$\frac{v_o}{R_L} + \beta i_b = 0 \;\Rightarrow\; v_o = -\beta i_b R_L$$
5. Substitute i_b from step 2:
$$\frac{v_o}{v_s} = \frac{-\beta R_L}{R_S + r_\pi}\ \mathrm{V/V}$$
6. The gain is negative, because current pulled out of the load node inverts the phase as in a common-emitter stage. Its magnitude has $R_S + r_\pi$ in the denominator. That is option (A).

KEY IDEA

The source resistance R_S and r_π are in series, so $i_b = v_s/(R_S + r_\pi)$. The dependent source forces βi_b through R_L, which gives $|v_o| = \beta i_b R_L$. The source arrow points away from the output node, so the output voltage is negative.

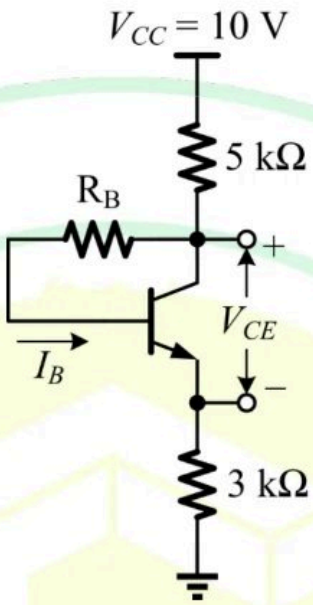
WATCH OUT FOR

One mistake is to drop r_π from the input loop and write $i_b = v_s/R_S$, which leads to option (C). The other is to misread the source's direction and lose the minus sign, which leads to (D). The downward arrow means the source pulls current OUT of the v_o node through R_L from ground, so v_o is negative.

Q.22 MCQ · 1 MARK

Q.22 The ideal BJT in the circuit given below is biased in the active region with a β of 100.

If I_B is 10 μA, then V_{CE} (in Volts, *rounded off to two decimal places*) is _____ .



(A)	4.95
(B)	3.03
(C)	1.92
(D)	3.73

Official answer: **C**

SOLUTION

1. Active region, so $I_C = \beta I_B = 100 \times 10\ \mu\text{A} = 1\ \text{mA}$ and $I_E = (\beta + 1) I_B = 101 \times 10\ \mu\text{A} = 1.01\ \text{mA}$.
2. KCL at the collector node: R_B runs from the collector node to the base, so the current I_B in R_B also leaves this node. The $5\ \text{k}\Omega$ resistor therefore carries both currents: $I_{5\text{k}} = I_C + I_B = I_E = 1.01\ \text{mA}$.
3. Collector voltage (Ohm's law across the $5\ \text{k}\Omega$ resistor): $V_C = 10 - (5\ \text{k}\Omega)(1.01\ \text{mA}) = 10 - 5.05 = 4.95\ \text{V}$.
4. Emitter voltage (Ohm's law across the $3\ \text{k}\Omega$ resistor, which carries I_E): $V_E = (3\ \text{k}\Omega)(1.01\ \text{mA}) = 3.03\ \text{V}$.

5. Hence
$$V_{CE} = V_C - V_E = V_{CC} - I_E\,(5\ \text{k}\Omega + 3\ \text{k}\Omega) = 10 - 1.01 \times 8 = 1.92\ \text{V}$$
6. Check: $V_{CE} = 1.92$ V is well above $V_{CE,\text{sat}} \approx 0.2$ V, and with collector-to-base feedback $V_{CB} = I_B R_B > 0$, so the collector-base junction is reverse biased and the active-region assumption holds. Neither R_B nor V_{BE} is needed, because I_B is given. Answer: (C).

KEY IDEA

This is collector-to-base feedback bias. The base current is drawn from the collector node through R_B, so the collector resistor carries $I_C + I_B = I_E$, the same current as the emitter resistor. That gives $V_{CE} = V_{CC} - I_E(R_C + R_E)$ directly, and once I_B is given you do not need R_B or V_{BE}.

WATCH OUT FOR

The wrong options are the other node voltages in this circuit: 4.95 V is V_C, 3.03 V is V_E, and 3.73 V is $V_B = V_E + 0.7$. Picking one of them means you answered for a node voltage instead of the difference $V_C - V_E$. A second trap is putting only $I_C = 1$ mA through the $5\ \text{k}\Omega$ resistor and forgetting the base current that also leaves the collector node through R_B. That gives 1.97 V, or 2.00 V if I_C is used for both resistors, and neither is an option.

Q.23 MCQ · 1 MARK

Digital Electronics › Boolean logic

A 3-input majority logic gate has inputs X, Y, and Z. The output F of the gate is logic '1' if two or more of the inputs are logic '1'. The output F is logic '0' if two or more of the inputs are logic '0'.

Which one of the following options is a Boolean expression of the output F?

(A) $XY + YZ + ZX$

(B) $X \oplus Y \oplus Z$

(C) $X + Y + Z$

(D) XYZ

Official answer: **A**

SOLUTION

1. **Truth table from the definition.** $F = 1$ exactly when at least two of the three inputs are 1. With the minterm index read as XYZ in binary, that happens in rows $XYZ = 011, 101, 110, 111$. So $F = \sum m(3, 5, 6, 7)$. In every other row at least two inputs are 0, so $F = 0$ there.
2. **K-map grouping of the 1s.** Minterm 7 ($XYZ = 111$) is adjacent to each of the other three 1s. Each pair differs in exactly one variable, and that variable drops out: $m_3 + m_7 = YZ$ (X drops), $m_5 + m_7 = XZ$ (Y drops), $m_6 + m_7 = XY$ (Z drops). The three pairs cover every 1 and nothing else, so
$$F = XY + YZ + ZX.$$
This is option (A). No complemented literals appear anywhere in this question.
3. **Check (B) at a single-1 input.** At $XYZ = 100$: $X \oplus Y \oplus Z = 1 \oplus 0 \oplus 0 = 1$. But only one input is 1, so F must be 0. (B) is the odd-parity function (the sum output of a full adder), not the majority function.
4. **Check (C) at the same input.** At $XYZ = 100$: $X + Y + Z = 1$, but F must be 0. OR needs only one input high.
5. **Check (D) at a two-1 input.** At $XYZ = 110$: $XYZ = 1 \cdot 1 \cdot 0 = 0$, but two inputs are 1, so F must be 1. AND needs all three inputs high.
6. Each of (B), (C) and (D) gives the wrong output in at least one row. (A) was derived from the full truth table, so it is correct in all 8 rows. **Answer: (A).**

KEY IDEA

A 3-input majority function is 1 when at least two inputs are 1, so it is $\sum m(3,5,6,7)$. Each 1 in that set pairs with $m_7 = XYZ$, so the minimal sum of products is one AND term per pair of inputs: $F = XY + YZ + ZX$. This is also the carry output of a full adder.

WATCH OUT FOR

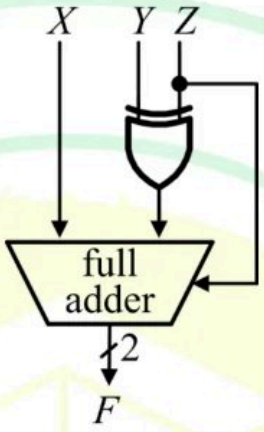
Picking $X \oplus Y \oplus Z$ because it also comes from the full adder. That expression is the SUM output (odd parity), which is 1 when exactly one input is 1. The majority function is the CARRY output, $XY + YZ + ZX$. Testing any single-1 input such as $XYZ = 100$ rules out both XOR and OR at once.

Q.24 MCQ · 1 MARK

Digital Electronics › Combinational

Q.24 A full adder and an XOR gate are used to design a digital circuit with inputs X, Y, and Z, and output F, as shown below. The input Z is connected to the carry-in input of the full adder.

If the input Z is set to logic '1', then the circuit functions as _____ with X and Y as inputs.



(A) an adder

(B) a subtractor

(C) a multiplier

(D) a binary to Gray code converter

Official answer: **B**

SOLUTION

1. Read the circuit: the full adder's operands are $A = X$ and $B = Y \oplus Z$, and its carry-in is $C_{in} = Z$.

2. XOR acts as a controlled inverter: $Y \oplus 1 = \overline{Y}$ (an overline means complement). So with $Z = 1$ we get $B = \overline{Y}$ and $C_{in} = 1$.
3. The full adder now computes $X + \overline{Y} + 1$. Since $\overline{Y} + 1$ is the 2's complement of Y, this is $X - Y$, which is subtraction by 2's complement.
4. Check the sum bit: $S = X \oplus \overline{Y} \oplus 1 = X \oplus Y$. This is exactly the difference bit D of a subtractor.
5. Check the carry bit using $C_{out} = AB + C_{in}(A \oplus B)$: $C_{out} = X\overline{Y} + (X \oplus \overline{Y}) \cdot 1 = X\overline{Y} + XY + \overline{X}\,\overline{Y} = X + \overline{Y}$. Its complement is $\overline{C_{out}} = \overline{X}Y$, which is the borrow of $X - Y$.
6. Truth table for $(X, Y) \to (C_{out}, S)$: $00 \to 10$, $01 \to 01$, $10 \to 11$, $11 \to 10$. So $S = X \oplus Y$, and the borrow $= \overline{C_{out}}$ is 1 only for $X = 0, Y = 1$. That is the subtractor's truth table.
7. (When $Z = 0$, $B = Y$ and $C_{in} = 0$, so the same circuit is an adder. Z is the add/subtract control.) With $Z = 1$ the circuit is a subtractor, so the answer is (B).

KEY IDEA

An XOR gate with one input tied to a control line M passes Y through when $M = 0$ and inverts it when $M = 1$. If the same M also drives the carry-in, the full adder computes $X + Y$ for $M = 0$ and $X + \overline{Y} + 1 = X - Y$ (2's complement) for $M = 1$. This is the standard adder-subtractor, and the difference bit is $X \oplus Y$ while the borrow is $\overline{C_{out}}$.

WATCH OUT FOR

Answering "an adder" because the block is labelled full adder, without noticing that the XOR gate complements Y when $Z = 1$ and that Z also supplies the carry-in of 1. A second slip is reading C_{out} itself as the borrow. In 2's-complement subtraction the borrow is the complement of the carry-out: here $C_{out} = X + \overline{Y}$, and borrow $= \overline{X}Y$.

Q.25 MSQ · 1 MARK

Engineering Mathematics › Calculus

Consider the function $f : \mathbb{R} \to \mathbb{R}$, defined as

$$f(x) = 2x^3 - 3x^2 - 12x + 1.$$

Which of the following statements is/are correct?

(Here, $\mathbb{R}$ is the set of real numbers.)

(A) f has no global maximizer

(B) f has no global minimizer

(C) $x = -1$ is a local minimizer of f

(D) $x = 2$ is a local maximizer of f

Official answer: **A, B**

SOLUTION

1. Find the stationary points. $f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x - 2)(x + 1)$, so the critical points are $x = -1$ and $x = 2$.
2. Second derivative: $f''(x) = 12x - 6$.
3. At $x = -1$: $f''(-1) = -18 < 0$, so $x = -1$ is a local **maximizer**, with $f(-1) = -2 - 3 + 12 + 1 = 8$. So (C) is false.
4. At $x = 2$: $f''(2) = 18 > 0$, so $x = 2$ is a local **minimizer**, with $f(2) = 16 - 12 - 24 + 1 = -19$. So (D) is false.
5. Behaviour on all of $\mathbb{R}$: f is a cubic with positive leading coefficient, so $f(x) \to +\infty$ as $x \to +\infty$ and $f(x) \to -\infty$ as $x \to -\infty$.
6. f is unbounded above, so no point is a global maximizer. For example $f(4) = 128 - 48 - 48 + 1 = 33 > 8$, so the local maximum at $x = -1$ is not global. So (A) is true.

7. f is unbounded below, so no point is a global minimizer. For example $f(-3) = -54 - 27 + 36 + 1 = -44 < -19$, so the local minimum at $x = 2$ is not global. So (B) is true.
8. Correct options: (A) and (B).

KEY IDEA

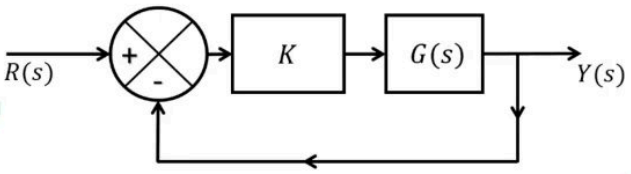
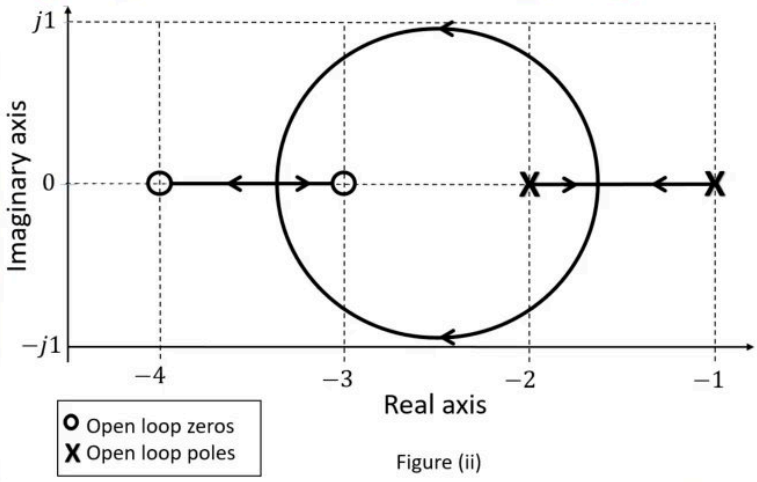
A local extremum is global only if the function never goes past it anywhere else in the domain. A polynomial of odd degree on all of $\mathbb{R}$ goes to $+\infty$ at one end and $-\infty$ at the other, so it never has a global maximizer or a global minimizer, whatever its local extrema are. To classify the stationary points of $f'(x) = 6(x-2)(x+1)$, use the sign of $f''(x) = 12x - 6$.

WATCH OUT FOR

Swapping the local classifications. For a cubic with positive leading coefficient the smaller critical point ($x = -1$) is the local maximum and the larger one ($x = 2$) is the local minimum. Another mistake is to call a local extremum global without checking the ends of the domain: on a closed interval the cubic would have global extrema, but on $\mathbb{R}$ it has none.

Q.26 MSQ · 1 MARK

Control Systems › Root locus

Q.26	Consider the unity-negative-feedback system shown in Figure (i) below, where gain $K \geq 0$. The root locus of this system is shown in Figure (ii) below. For what value(s) of K will the system in Figure (i) have a pole at $-1 + j1$?
	 Figure (i)  Figure (ii)
(A)	$K = 5$
(B)	$K = {}^{1}/_{5}$
(C)	For no positive value of K
(D)	For all positive values of K

Official answer: **C**

SOLUTION

1. Read the plot. The open-loop poles are at $-1, -2$ and the zeros are at $-3, -4$, so $G(s) = \dfrac{(s+3)(s+4)}{(s+1)(s+2)}$. Any positive constant multiplying this can be absorbed into K. The real-axis segments $[-2, -1]$ and $[-4, -3]$ are the ones with an odd number of poles and zeros to their right, which confirms that this is the ordinary $K \geq 0$ (180°) locus.

2. A point s_0 is a closed-loop pole for some $K \geq 0$ only if it lies on the root locus. That requires the **angle condition**, $\angle G(s_0) = \pm 180^\circ$ (an odd multiple of 180°). The magnitude condition $K = 1/|G(s_0)|$ gives the gain only after the angle condition holds.
3. Test $s_0 = -1 + j1$. Zero vectors: $s_0 + 3 = 2 + j1$ has angle $\tan^{-1}(1/2) = 26.57^\circ$, and $s_0 + 4 = 3 + j1$ has angle $\tan^{-1}(1/3) = 18.43^\circ$.
4. Pole vectors: $s_0 + 1 = j1$ has angle 90°, and $s_0 + 2 = 1 + j1$ has angle 45°.
5. $$\angle G(-1 + j1) = (26.57^\circ + 18.43^\circ) - (90^\circ + 45^\circ) = 45^\circ - 135^\circ = -90^\circ \neq \pm 180^\circ$$
   The angle condition fails, so $-1 + j1$ is **not** on the locus. It is not on the $K < 0$ locus either, which would need 0°. So no real K puts a pole there.
6. Cross-check with the geometry. The characteristic equation is $(s+1)(s+2) + K(s+3)(s+4) = 0$, so $K = -\dfrac{(s+1)(s+2)}{(s+3)(s+4)}$. Setting $\dfrac{dK}{ds} = 0$ gives $2s^2 + 10s + 11 = 0$, so $s = -2.5 \pm 0.866$: breakaway at -1.634 and break-in at -3.366.
7. For the complex roots, write the equation as $(1+K)s^2 + (3+7K)s + (2+12K) = 0$. Then $\mathrm{Re}(s) = -\dfrac{3+7K}{2(1+K)}$ and $|s|^2 = \dfrac{2+12K}{1+K}$. Substituting these gives $|s + 2.5|^2 = |s|^2 + 5\,\mathrm{Re}(s) + 6.25 = \dfrac{3}{4}$ for every K. The complex part of the locus is therefore the circle centred at -2.5 with radius $\sqrt{3}/2 = 0.866$.
8. The distance from $-1 + j1$ to the centre is $\sqrt{(1.5)^2 + 1^2} = \sqrt{3.25} = 1.80 \neq 0.866$, so the point is off the circle. No $K \geq 0$ puts a closed-loop pole there.
9. Direct confirmation, which also shows where the wrong options come from. Put $s = -1 + j1$ into the characteristic equation: $(s+1)(s+2) = j(1+j) = -1 + j$ and $(s+3)(s+4) = (2+j)(3+j) = 5 + 5j$. So
   $$K = -\frac{-1+j}{5+5j} = \frac{1-j}{5(1+j)} = \frac{(1-j)^2}{10} = -\frac{j}{5}$$
   The gain needed is imaginary, so no real gain works. Its magnitude is $\frac{1}{5}$, the same value the magnitude condition gives on its own: $|G(-1+j1)| = \dfrac{\sqrt{5}\,\sqrt{10}}{1 \cdot \sqrt{2}} = 5$, so $K = \frac{1}{5}$. That is option (B), and it means nothing here because the angle condition has already failed. Option (A), $K = 5$, is $|G|$ itself, taken wrongly as the gain. Option (D) is ruled out because the point is never on the locus. Only (C) is correct.

KEY IDEA

A point is a closed-loop pole for some $K \geq 0$ only if it lies on the root locus. That is decided by the angle condition, $\angle G(s) = \pm 180^\circ$, not by the magnitude condition. A quicker check is to substitute the point into $1 + KG(s) = 0$ and solve for K. The point is reachable only if K comes out real and non-negative; here $K = -j/5$. For two real poles and two real zeros, the complex part of the locus is a circle, here centred at -2.5 with radius 0.866, and $-1 + j1$ is 1.80 from that centre.

WATCH OUT FOR

Using only the magnitude condition. $|G(-1+j1)| = 5$ suggests $K = 1/5$, which is option (B), or $K = 5$ if the ratio is inverted, which is option (A). Both skip the angle test, which gives -90°. Another trap is judging from the drawing: the stretched vertical scale makes the circle look as if it reaches $j1$, but its true radius is 0.866 and it never gets near $-1 + j1$.

Q.27 MSQ · 1 MARK

Signals and Systems › z-transform

Let $x[n]$ be a discrete-time signal whose z-transform is $X(z)$.

Which of the following statements is/are TRUE?

(A) The discrete-time Fourier transform (DTFT) of $x[n]$ always exists

(B) The region of convergence (RoC) of $X(z)$ contains neither poles nor zeros

(C) The discrete-time Fourier transform (DTFT) exists if the region of convergence (RoC) contains the unit circle

(D) If $x[n] = \alpha\delta[n]$, where $\delta[n]$ is the unit impulse and α is a scalar, then the region of convergence (RoC) is the entire z-plane

Official answer: **C, D**

SOLUTION

1. **How the two transforms are linked.** The DTFT is the z-transform evaluated on the unit circle:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} = X(z)\big|_{z=e^{j\omega}}$$

   The RoC is the set of z for which the series converges **absolutely**, $\sum_n |x[n]|\,|z|^{-n} < \infty$. On $|z| = 1$ that sum is exactly $\sum_n |x[n]|$. So the DTFT sum converges absolutely exactly when the unit circle lies in the RoC.

2. **(A) False.** Counter-example: $x[n] = 2^n u[n]$ has $X(z) = \dfrac{1}{1 - 2z^{-1}}$ with RoC $|z| > 2$, so its z-transform exists. Its DTFT sum $\sum_{n\geq 0} 2^n e^{-j\omega n}$ has terms of magnitude $2^n \to \infty$, so it diverges at every ω. Having a z-transform does not guarantee a DTFT.

3. **(B) False.** The RoC can never contain a pole. Inside the RoC the series converges to a finite value, but $|X(z)| \to \infty$ at a pole. The RoC **can** contain zeros, though. Counter-example: $x[n] = \delta[n] - \delta[n-1]$ gives $X(z) = 1 - z^{-1} = \dfrac{z-1}{z}$. It has a pole at $z = 0$, a zero at $z = 1$, and RoC $|z| > 0$. That RoC contains the zero at $z = 1$, so the statement "neither poles nor zeros" is false.

4. **(C) True.** If the RoC contains $|z| = 1$, then by the definition of the RoC in Step 1, $\sum_n |x[n]| < \infty$. The signal is absolutely summable, so the DTFT sum converges for every ω and equals $X(e^{j\omega})$.

5. **(D) True.** $X(z) = \sum_n \alpha\,\delta[n]\, z^{-n} = \alpha$, a constant. There is no negative power of z, which would blow up at $z = 0$, and no positive power of z, which would blow up as $z \to \infty$. $X(z)$ is finite everywhere, so the RoC is the entire z-plane, including $z = 0$ and $z = \infty$.

6. **Correct options:** (C) and (D).

KEY IDEA

The DTFT is $X(z)$ evaluated on $|z| = 1$. The RoC is defined by absolute convergence, so a unit circle inside the RoC means $\sum_n |x[n]| < \infty$, and then the DTFT exists. The RoC can never contain a pole, but it can contain zeros. A signal that is nonzero only at $n = 0$ has a constant $X(z)$, so its RoC is the whole z-plane with no exception at 0 or ∞.

WATCH OUT FOR

Marking (B) true by misremembering the rule. The RoC contains no poles, but zeros can lie inside it. Marking (A) true on the assumption that any signal with a z-transform also has a DTFT; a growing signal such as $2^n u[n]$ has one and not the other. In (D), excluding $z = 0$ or $z = \infty$ by habit. Those exclusions apply to a shifted impulse $\delta[n-k]$ with $k \neq 0$, not to $\delta[n]$.

Q.28 MSQ · 1 MARK

Communications › Analog modulation

Consider a message signal $m(t)$ which is bandlimited to $[-W, W]$, where W is in Hz. Consider the following two modulation schemes for the message signal:
- Double sideband-suppressed carrier (DSB-SC): $f_{DSB}(t) = A_c\, m(t)\cos(2\pi f_c t)$
- Amplitude modulation (AM): $f_{AM}(t) = A_c\,(1 + \mu\, m(t))\cos(2\pi f_c t)$

Here, A_c and f_c are the amplitude and frequency (in Hz) of the carrier, respectively. In the case of AM, μ denotes the modulation index.
Consider the following statements:
(i) An envelope detector can be used for demodulation in the DSB-SC scheme if $m(t) > 0$ for all t.
(ii) An envelope detector can be used for demodulation in the AM scheme only if $m(t) > 0$ for all t.
Which of the following options is/are correct?

(A) (i) is TRUE

(B) (i) is FALSE

(C) (ii) is TRUE

(D) (ii) is FALSE

Official answer: **A, D**

SOLUTION

1. An envelope detector (with $f_c \gg W$) outputs the magnitude of the slowly varying amplitude factor: for $x(t) = a(t)\cos(2\pi f_c t)$ the output is $|a(t)|$.
2. DSB-SC: $a(t) = A_c\, m(t)$, so the output is $A_c|m(t)|$. If $m(t) > 0$ for all t, then $|m(t)| = m(t)$, so the output is $A_c\, m(t)$ and the message is recovered. Statement (i) is TRUE, so (A) is correct and (B) is wrong.
3. AM: $a(t) = A_c\,(1 + \mu\, m(t))$, so the output is $A_c\,|1 + \mu\, m(t)|$. This equals $A_c(1 + \mu\, m(t))$, which is the message plus a DC term, exactly when $1 + \mu\, m(t) \geq 0$ for all t. That is the no-overmodulation condition $\mu\,|m(t)|_{\max} \leq 1$ for a message with negative values.
4. This condition does not need $m(t) > 0$. Take $m(t) = \cos(2\pi W t)$ and $\mu \leq 1$. Then $1 + \mu\, m(t) \geq 1 - \mu \geq 0$, so the envelope detector works, although $m(t)$ is negative for part of each cycle.
5. So the claim that an envelope detector works in AM only if $m(t) > 0$ for all t is not necessary. Statement (ii) is FALSE, so (D) is correct and (C) is wrong.
6. Correct options: (A) and (D).

KEY IDEA
An envelope detector returns |a(t)|, the magnitude of the amplitude factor of the carrier. It recovers the message whenever that amplitude factor never changes sign. For DSB-SC this means m(t) must stay positive. For AM it means 1 + μ m(t) must stay non-negative, which allows a message with negative values.

WATCH OUT FOR
Believing an envelope detector needs a positive message in AM as well. What it needs there is 1 + μ m(t) ≥ 0, that is no overmodulation. The message itself may go negative.

Q.29 MSQ · 1 MARK

Analog Electronics › Op-amps

Which of the following statements is/are TRUE with respect to an ideal opamp?

(A) It has an infinite input resistance

(B) It has an infinite output resistance

(C) It has an infinite open-loop differential gain

(D) It has an infinite open-loop common-mode gain

Official answer: **A, C**

SOLUTION

1. **Ideal op-amp model.** Treat the ideal op-amp as a voltage-controlled voltage source $v_o = A_d(v_+ - v_-) + A_{cm}\,\dfrac{v_+ + v_-}{2}$ with input resistance $R_i \to \infty$, output resistance $R_o = 0$, differential gain $A_d \to \infty$, common-mode gain $A_{cm} = 0$ (so $\mathrm{CMRR} = A_d/A_{cm} \to \infty$) and infinite bandwidth.
2. **(A) Input resistance.** $R_i \to \infty$, so the input terminals draw no current ($i_+ = i_- = 0$). That is why KCL at the inverting node can be written with zero input current. TRUE.
3. **(B) Output resistance.** An ideal voltage source has zero internal resistance, so $R_o = 0$ and v_o does not depend on the load. An infinite R_o describes an ideal current source, not a voltage amplifier. FALSE.

4. **(C) Open-loop differential gain.** $A_d \to \infty$. With negative feedback and a finite v_o, this gives $v_+ - v_- = v_o/A_d \to 0$, which is the virtual short. TRUE.
5. **(D) Open-loop common-mode gain.** An ideal op-amp rejects any signal common to both inputs completely, so $A_{cm} = 0$ and $\mathrm{CMRR} \to \infty$. An infinite common-mode gain would be the worst possible amplifier, not the ideal one. FALSE.
6. The true statements are (A) and (C).

KEY IDEA
The ideal op-amp is an ideal voltage-controlled voltage source that responds only to the difference between its inputs. Its input resistance and differential gain are infinite; its output resistance and common-mode gain are zero, so CMRR is infinite.

WATCH OUT FOR
Marking every option with the word "infinite". Output resistance is zero, not infinite: an ideal voltage source must not depend on the load. Common-mode gain is zero, not infinite: it is the CMRR that is infinite. Students often mix up A_{cm} and CMRR.

Q.30 MSQ · 1 MARK

Analog Electronics › Amplifiers

Which of the following statements is/are TRUE with respect to ideal MOSFET-based DC-coupled single-stage amplifiers having finite load resistors?

(A) The common-gate amplifier has an infinite input resistance

(B) The common-source amplifier has an infinite input resistance

(C) The input and output voltages of the common-source amplifier are in phase

(D) The input and output voltages of the common-drain amplifier are in phase

Official answer: **B, D**

SOLUTION

1. **Model and assumptions.** The MOSFET is ideal, so the gate draws no current ($i_g = 0$), $r_o \to \infty$ and body effect is neglected. Small-signal model: a controlled current source $g_m v_{gs}$ from drain to source, with the gate an open circuit. The load resistor (R_D at the drain, or R_S at the source for the follower) is finite. The amplifier is DC-coupled, so there is no bias divider at the gate.
2. **(A) Common gate.** The gate is at signal ground ($v_g = 0$) and the signal drives the source, so $v_{gs} = -v_{in}$. The channel current $g_m v_{gs}$ leaves the transistor at the source, so KCL at the source node gives $i_{in} = -g_m v_{gs} = g_m v_{in}$. Hence
$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{1}{g_m}$$
This is small and finite (with a source resistor it becomes $R_S \parallel \frac{1}{g_m}$, still finite). (A) is FALSE.
3. **(B) Common source.** The signal drives the gate. Since $i_g = 0$ and a DC-coupled stage has no gate bias resistors,
$$R_{in} = \frac{v_{in}}{i_g} \to \infty$$
(B) is TRUE.
4. **(C) Common-source phase.** With the source grounded, $v_{gs} = v_{in}$. The drain current $g_m v_{in}$ flows from the drain terminal through the transistor to ground, so it is drawn out of the output node through R_D:
$$v_{out} = -g_m R_D\, v_{in}, \qquad A_v = -g_m R_D$$
The minus sign is a 180° phase shift: the common-source stage inverts. (C) is FALSE.
5. **(D) Common-drain (source follower) phase.** The drain is at signal ground and the output is taken across R_S at the source: $v_{out} = g_m v_{gs} R_S$ with $v_{gs} = v_{in} - v_{out}$. Solving,
$$v_{out}(1 + g_m R_S) = g_m R_S\, v_{in} \quad \Rightarrow \quad A_v = \frac{g_m R_S}{1 + g_m R_S}$$

This gain is positive (and just under 1), so input and output are in phase. (D) is TRUE.

6. **Result.** The true statements are (B) and (D).

KEY IDEA

Look at which terminal the signal enters. If it enters at the gate (common source or common drain), the input resistance is infinite because an ideal gate draws no current. If it enters at the source (common gate), the input resistance is only $1/g_m$. For phase, look at the sign of the gain: common source gives $-g_m R_D$ (inverting), while common drain gives $+\frac{g_m R_S}{1+g_m R_S}$ and common gate gives $+g_m R_D$ (both non-inverting).

WATCH OUT FOR

Assuming every MOSFET amplifier has infinite input resistance because the gate is insulated. In the common-gate stage the signal enters at the source, not the gate, and the whole channel current flows through the input, so $R_{in} = 1/g_m$. A second slip is treating the common-source stage as non-inverting. Its gain $-g_m R_D$ carries a 180° phase shift.

Q.31 MSQ · 1 MARK

Electronic Devices › Semiconductors

Which of the following can be used as an n-type dopant for silicon? Select the correct option(s).

(A) Arsenic

(B) Boron

(C) Gallium

(D) Phosphorous

Official answer: **A, D**

SOLUTION

1. Silicon is a Group IV (tetravalent) semiconductor: each Si atom shares its 4 valence electrons in covalent bonds with 4 neighboring atoms in the crystal lattice.
2. n-type doping requires donor impurity atoms with 5 valence electrons (Group V of the periodic table). Four of the five valence electrons satisfy the covalent bonding requirement with neighboring Si atoms; the fifth electron is only loosely bound to its parent atom and easily ionizes at room temperature, donating a free electron to the conduction band.
3. p-type doping requires acceptor impurity atoms with 3 valence electrons (Group III). Such an atom can form only 3 covalent bonds, leaving one bond incomplete; this incomplete bond acts as a hole that can accept an electron from a neighboring Si-Si bond, creating a mobile hole in the valence band.
4. Classify each option by its periodic table group: Arsenic (As): Group V, hence an n-type donor for Si. Boron (B): Group III, hence a p-type acceptor for Si. Gallium (Ga): Group III, hence a p-type acceptor for Si. Phosphorous (P): Group V, hence an n-type donor for Si (the most commonly used n-type dopant in practice).
5. Therefore, the valid n-type dopants for silicon among the given options are Arsenic and Phosphorous, i.e., options (A) and (D).

KEY IDEA

n-type doping of a Group IV semiconductor (silicon) requires pentavalent (Group V) donor impurities, which contribute a loosely bound extra electron to the conduction band; p-type doping requires trivalent (Group III) acceptor impurities.

WATCH OUT FOR

Confusing donor and acceptor group numbers: assuming Group III elements donate electrons, or forgetting that silicon (Group IV) needs a 5-valence-electron dopant to supply the extra "donor" electron for n-type conduction.

Q.32 NAT · 1 MARK

Engineering Mathematics › Differential equations

The function $y(t)$ satisfies

$$t^2y''(t) - 2ty'(t) + 2y(t) = 0,$$

where $y'(t)$ and $y''(t)$ denote the first and second derivatives of $y(t)$, respectively.

Given $y'(0) = 1$ and $y'(1) = -1$, the maximum value of $y(t)$ over $[0, 1]$ is _____ (rounded off to two decimal places).

Official answer: **0.25**

SOLUTION

1. The equation $t^2y'' - 2ty' + 2y = 0$ is a Cauchy-Euler equation: each derivative $y^{(k)}$ is multiplied by t^k. So try $y = t^m$, which gives $y' = mt^{m-1}$ and $y'' = m(m-1)t^{m-2}$.
2. Substituting and dividing by t^m gives the auxiliary equation $m(m-1) - 2m + 2 = 0 \Rightarrow m^2 - 3m + 2 = 0 \Rightarrow (m-1)(m-2) = 0$, so $m = 1$ or $m = 2$.
3. The roots are real and distinct, so the general solution is $y(t) = c_1t + c_2t^2$, with $y'(t) = c_1 + 2c_2t$.
4. Apply the given conditions, which are on the derivative: $y'(0) = c_1 = 1$, and $y'(1) = c_1 + 2c_2 = -1 \Rightarrow 2c_2 = -2 \Rightarrow c_2 = -1$.
5. So $y(t) = t - t^2$. It is a polynomial, so it is smooth on all of $[0, 1]$, including the singular point $t = 0$, and it satisfies the equation there too: $0 - 0 + 0 = 0$.
6. For the maximum, set $y'(t) = 1 - 2t = 0 \Rightarrow t = \frac{1}{2}$, which lies inside $[0, 1]$. Since $y''(t) = -2 < 0$, this point is a maximum.
7. $$y\left(\tfrac{1}{2}\right) = \tfrac{1}{2} - \tfrac{1}{4} = 0.25$$
   At the endpoints $y(0) = y(1) = 0$, so the maximum of $y(t)$ over $[0, 1]$ is 0.25.

KEY IDEA

This is a Cauchy-Euler equation, where each derivative $y^{(k)}$ is multiplied by t^k. Substituting $y = t^m$ turns it into the algebraic equation $m(m-1) - 2m + 2 = 0$. Its roots $m = 1, 2$ give the polynomial solution $y = c_1t + c_2t^2$, and the two conditions on y' fix c_1 and c_2. What is left is an ordinary maximisation over a closed interval: check the stationary point and both endpoints.

WATCH OUT FOR

The most common slip is reading the conditions as $y(0) = 1$ and $y(1) = -1$ when they are on y'. This cannot even work, because every solution has $y(0) = 0$. The second is writing the auxiliary equation as $m^2 - 2m + 2 = 0$, which drops the $-m$ from $m(m-1)$ and gives false complex roots. The third is forgetting to compare the stationary value with the endpoint values $y(0)$ and $y(1)$.

Q.33 NAT · 1 MARK

Communications › Error control

The generator matrix of a (6,3) binary linear block code is given by G = [[1,0,0,1,0,1],[0,1,0,0,1,1],[0,0,1,1,1,0]]. The minimum Hamming distance d_min between codewords equals ____ (answer in integer).

Official answer: **3**

SOLUTION

1. For a linear block code the minimum Hamming distance equals the minimum weight of a nonzero codeword, because the distance between two codewords is the weight of their sum, and that sum is itself a codeword. The code has $2^3 = 8$ codewords, so there are 7 nonzero ones to check.
2. The rows of G, with all arithmetic over GF(2), are $g_1 = 100101$, $g_2 = 010011$, $g_3 = 001110$.
3. Single-row codewords: $w(g_1) = 3$, $w(g_2) = 3$, $w(g_3) = 3$.
4. Sums of two rows: $g_1 \oplus g_2 = 110110$ with weight 4; $g_1 \oplus g_3 = 101011$ with weight 4; $g_2 \oplus g_3 = 011101$ with weight 4.
5. Sum of all three rows: $g_1 \oplus g_2 \oplus g_3 = 110110 \oplus 001110 = 111000$ with weight 3.
6. The weights of the seven nonzero codewords are $3, 3, 3, 4, 4, 4, 3$, so the smallest is 3.
7. Hence $d_{min} = 3$.

KEY IDEA

In a linear block code, d_{min} is the minimum weight among the nonzero codewords. Enumerate the $2^k - 1$ nonzero combinations of the rows of G over GF(2).

WATCH OUT FOR

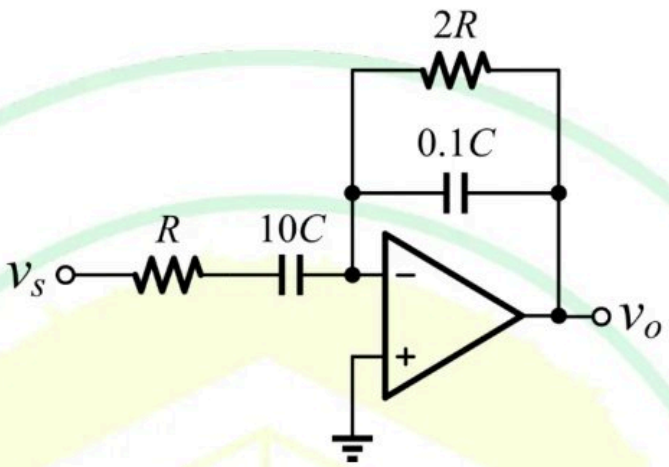
Checking only the rows of G (all weight 3) and not the sums of rows. The sums can have a lower weight, so every combination must be checked. Here the three-row sum happens to have weight 3 again, so the answer stays 3. A second mistake is adding rows with ordinary arithmetic instead of mod 2.

Q.34 NAT · 1 MARK

Analog Electronics › Filters & waveforms

Q.34 All the components in the bandpass filter given below are ideal. The lower -3 dB frequency of the filter is 1 MHz.

The upper -3 dB frequency (in MHz, *rounded off to the nearest integer*) is _____.



Official answer: **50**

SOLUTION

1. **Assume an ideal op-amp.** The non-inverting input is grounded, so the virtual short holds the inverting input at virtual ground and no current enters the op-amp. The circuit is an inverting amplifier: $H(s) = -\dfrac{Z_f}{Z_i}$, with $Z_i = R + \dfrac{1}{s(10C)}$ and $Z_f = 2R \parallel \dfrac{1}{s(0.1C)}$.
2. **Substitute the impedances.** $Z_i = \dfrac{1 + s\,10RC}{s\,10C}$ and $Z_f = \dfrac{2R}{1 + s\,0.2RC}$, so
$$H(s) = -\frac{s\,(20RC)}{(1 + s\,10RC)\,(1 + s\,0.2RC)}$$
The input network (R, $10C$) is a high-pass section with corner $\omega_1 = \dfrac{1}{10RC}$. The feedback network ($2R$, $0.1C$) is a low-pass section with corner $\omega_2 = \dfrac{1}{0.2RC} = 50\,\omega_1$.
3. **Midband gain.** Between the two corners, $10C$ acts as a short and $0.1C$ acts as an open, so $H = -\dfrac{2R}{R} = -2$ and the midband gain is $A_M = 2$. The lower and upper -3 dB frequencies are where the gain falls 3 dB below A_M, which means $|H(j\omega)| = \dfrac{A_M}{\sqrt{2}} = \sqrt{2}$.

4. **Normalise the frequency.** Put $x = \omega/\omega_1 = 10RC\,\omega$. Then $0.2RC\,\omega = x/50$ and $20RC\,\omega = 2x$, so
$$|H(j\omega)| = \frac{2x}{\sqrt{(1+x^2)\left(1+\dfrac{x^2}{2500}\right)}}$$
5. **Apply the -3 dB condition $|H|^2 = 2$:**
$$4x^2 = 2\left(1+x^2\right)\left(1+\frac{x^2}{2500}\right) \;\Rightarrow\; 5000x^2 = 2500 + 2501x^2 + x^4 \;\Rightarrow\; x^4 - 2499x^2 + 2500 = 0$$
6. **Solve the quadratic in x^2:**
$$x^2 = \frac{2499 \pm \sqrt{2499^2 - 4(2500)}}{2} = \frac{2499 \pm 2496.9984}{2}$$

   This gives $x_L^2 = 1.0008$ and $x_H^2 = 2497.9992$, so $x_L = 1.0004$ and $x_H = 49.980$. The product of the roots is $x_L^2 x_H^2 = 2500$, so $x_L x_H = 50$.
7. **Ratio of the band edges.** Since $x_L = 50/x_H$,
$$\frac{f_H}{f_L} = \frac{x_H}{x_L} = \frac{x_H^2}{50} = \frac{2497.9992}{50} = 49.96$$

   The lower edge sits at $1.0004\,f_1$, almost exactly on the lower corner, and the upper edge sits at $49.98\,f_1$, almost exactly on the upper corner, because the two corners are a factor of 50 apart.
8. **Put in the data.** With $f_L = 1$ MHz, $f_H = 49.96 \times 1$ MHz $= 49.96$ MHz. Rounded to the nearest integer, $f_H = 50$ MHz.
9. **Note on the reference level.** Because the corners are only a factor of 50 apart, the gain actually peaks at $\omega = \sqrt{\omega_1\omega_2}$ with value $\dfrac{2\cdot 50}{51} \approx 1.96$, a little below $A_M = 2$. If -3 dB is measured from that peak instead of from the midband gain, the edges spread out to $f_H \approx 54$ MHz. The official key, 50, measures from the midband gain $A_M = 2R/R = 2$, which is the usual reference for an amplifier or filter whose band is set by separate low- and high-frequency corner networks.

KEY IDEA

The input network (R in series with 10C) is a high-pass section with corner 1/(2π·10RC). The feedback network (2R in parallel with 0.1C) is a low-pass section with corner 1/(2π·0.2RC). The midband gain is -2R/R = -2. The -3 dB points, taken 3 dB below that midband gain, fall at 1.0004 and 49.98 times the lower corner. That makes f_H/f_L = 49.96, which is essentially the corner ratio 10RC/0.2RC = 50, so f_H is about 50 MHz.

WATCH OUT FOR

Measuring -3 dB from the actual peak gain (about 1.96, at the geometric-mean frequency) instead of from the midband gain -2R/R = 2. That moves the band edges apart and gives 54 MHz, which is wrong. A second mistake is pairing the wrong components: using R with 0.1C, or 2R with 10C, for the corners gives the wrong ratio (100 or 25 instead of 50).

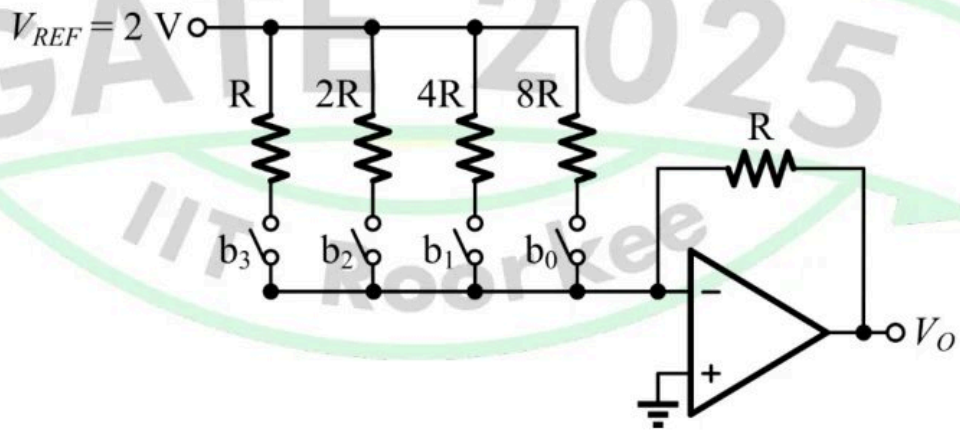
Q.35 NAT · 1 MARK

Digital Electronics › ADC & DAC

Q.35

A 4-bit weighted-resistor DAC with inputs b_3, b_2, b_1, and b_0 (MSB to LSB) is designed using an ideal opamp, as shown below. The switches are closed when the corresponding input bits are logic *'1'* and open otherwise.

When the input $b_3b_2b_1b_0$ changes from 1110 to 1101, the magnitude of the change in the output voltage V_O (in mV, *rounded off to the nearest integer*) is _____.



Official answer: **250**

SOLUTION

1. The op-amp is ideal and its + input is grounded, so the – input is a **virtual ground** (0 V). A closed switch on bit b_i carries $I_i = \dfrac{V_{REF}}{R_i}$ into that node. An open switch carries no current.
2. Total current into the summing node:
$$I = V_{REF}\left(\frac{b_3}{R} + \frac{b_2}{2R} + \frac{b_1}{4R} + \frac{b_0}{8R}\right)$$
3. No current enters an ideal op-amp input, so all of I flows through the feedback resistor $R_f = R$. This is an **inverting summing amplifier**:
$$V_O = -IR_f = -V_{REF}\left(b_3 + \frac{b_2}{2} + \frac{b_1}{4} + \frac{b_0}{8}\right) = -2\left(b_3 + \frac{b_2}{2} + \frac{b_1}{4} + \frac{b_0}{8}\right)\ \text{V}$$
4. For $b_3b_2b_1b_0 = 1110$: $V_O = -2\,(1 + 0.5 + 0.25) = -3.5$ V.
5. For $b_3b_2b_1b_0 = 1101$: $V_O = -2\,(1 + 0.5 + 0.125) = -3.25$ V.
6. Magnitude of the change: $|\Delta V_O| = |-3.25 - (-3.5)| = 0.25\ \text{V} = 250\ \text{mV}$.
7. Check using the step size. One LSB is $V_{REF} \cdot \dfrac{R_f}{8R} = \dfrac{2}{8} = 0.25$ V. The input code drops from $(1110)_2 = 14$ to $(1101)_2 = 13$, which is exactly one LSB, so $|\Delta V_O| = 1 \times 250\ \text{mV} = 250\ \text{mV}$.

KEY IDEA

In a weighted-resistor DAC with a virtual-ground summing node, $V_O = -V_{REF}\dfrac{R_f}{R}\left(b_3 + \dfrac{b_2}{2} + \dfrac{b_1}{4} + \dfrac{b_0}{8}\right)$. The output is proportional to the binary value of the input code, so any change in output equals (change in code) $\times$ (one LSB step). Here one LSB is $V_{REF}\,R_f/(8R) = 0.25\ \text{V}$, and $14 \to 13$ is a change of one step.

WATCH OUT FOR

Two bits flip (b_1: $1 \to 0$, b_0: $0 \to 1$), and a common error is to add the two contributions as magnitudes ($0.5 + 0.25 = 0.75\ \text{V}$) when they have opposite signs and partly cancel, leaving a net $0.25\ \text{V}$. Two other slips: pairing the LSB b_0 with the smallest resistor R instead of $8R$, which reverses the bit weights, and taking the step as $V_{REF}/2^4 = 0.125\ \text{V}$ as if this were an R-2R ladder with full-scale V_{REF}. In this circuit the feedback resistor equals the MSB resistor, so the MSB alone gives V_{REF} and the LSB step is $V_{REF}/8$.

Q.36 MCQ · 2 MARKS

Control Systems › Compensators & PID

Q.36

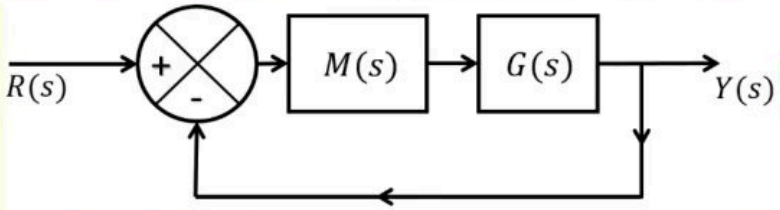
Let $G(s) = \frac{1}{10s^2}$ be the transfer function of a second-order system. A controller $M(s)$ is connected to the system $G(s)$ in the configuration shown below.

Consider the following statements.

(i) There exists no controller of the form $M(s) = \frac{K_I}{s}$, where K_I is a positive real number, such that the closed loop system is stable.

(ii) There exists at least one controller of the form $M(s) = K_P + sK_D$, where K_P and K_D are positive real numbers, such that the closed loop system is stable.

Which one of the following options is correct?



(A) (i) is TRUE and (ii) is FALSE

(B) (i) is FALSE and (ii) is TRUE

(C) Both (i) and (ii) are FALSE

(D) Both (i) and (ii) are TRUE

Official answer: **D**

SOLUTION

1. The loop has unity negative feedback with $M(s)$ and $G(s)$ in cascade, so the closed-loop characteristic equation is $1 + M(s)G(s) = 0$, where $G(s) = \frac{1}{10s^2}$ is a double integrator.
2. **Statement (i), integral controller.** With $M(s) = \frac{K_I}{s}$:

$$1 + \frac{K_I}{s} \cdot \frac{1}{10s^2} = 0 \;\Rightarrow\; 10s^3 + K_I = 0$$

There is no pole-zero cancellation, so this cubic is the full characteristic polynomial.

3. The necessary condition for stability fails: every coefficient of a stable polynomial must be present and share one sign, but here the s^2 and s^1 coefficients are zero. So the loop cannot be stable for any $K_I > 0$.
4. The roots confirm it. From $s^3 = -\dfrac{K_I}{10}$, the three roots have magnitude $\sqrt[3]{K_I/10}$ and angles 180° and $\pm 60^\circ$. The two roots at $\pm 60^\circ$ have positive real part, so they lie in the right half-plane.
5. The Routh array gives the same result. Row s^3: $10,\ 0$. Row s^2: $0,\ K_I$. Its first element is zero, so replace it with a small $\varepsilon > 0$. Then row s^1 is $\dfrac{\varepsilon \cdot 0 - 10K_I}{\varepsilon} = -\dfrac{10K_I}{\varepsilon}$, which is large and negative, and row s^0 is K_I. The first column $10,\ \varepsilon,\ -\dfrac{10K_I}{\varepsilon},\ K_I$ changes sign twice, so there are two right-half-plane roots.
6. So no controller $M(s) = \dfrac{K_I}{s}$ with $K_I > 0$ stabilises the loop, and (i) is TRUE. In physical terms, the integrator turns the double integrator into a triple integrator, which only adds phase lag.
7. **Statement (ii), PD controller.** With $M(s) = K_P + sK_D$:

$$1 + (K_P + sK_D) \cdot \frac{1}{10s^2} = 0 \ \Rightarrow\ 10s^2 + K_D s + K_P = 0$$

   There is no cancellation, because the zero at $s = -K_P/K_D$ is not at the origin.
8. For a second-order polynomial, all coefficients present and positive is both necessary and sufficient for stability. With $K_P, K_D > 0$ this always holds. In standard form, $\omega_n = \sqrt{K_P/10}$ and $\zeta = \dfrac{K_D}{2\sqrt{10K_P}} > 0$. For example, $K_P = 10,\ K_D = 10$ gives $10s^2 + 10s + 10 = 0$, that is $s^2 + s + 1 = 0$, with roots $s = -\dfrac{1}{2} \pm j\dfrac{\sqrt{3}}{2}$, both in the left half-plane.
9. In fact every PD controller with positive gains stabilises the loop, so 'at least one exists' is TRUE. The derivative term supplies the damping term $K_D s$ that the double integrator lacks. Both statements are TRUE, so the answer is option (D).

KEY IDEA

A double integrator $\dfrac{1}{10s^2}$ has no s^1 term in its characteristic polynomial, so it has no damping. Stabilising it needs a controller that adds that term. A PD controller adds $K_D s$ and gives $10s^2 + K_D s + K_P$, which is stable for all positive gains. An integral controller raises the order to $10s^3 + K_I$ and still leaves the s^2 and s coefficients at zero. That fails the necessary coefficient condition, whatever the gain.

WATCH OUT FOR

One mistake is misreading the wording of the statements. Statement (i) is a claim that no stabilising controller exists. The integral loop is unstable, so (i) is TRUE, not FALSE. A second mistake is stopping the Routh test for $10s^3 + K_I$ at the zero in the first column of row s^2 and calling the system marginally stable. Completing the array with ε shows two sign changes, so two roots are in the right half-plane and the loop is unstable. A third is thinking that (ii) needs a particular tuning of K_P and K_D. For a second-order polynomial, positive coefficients alone guarantee stability.

Q.37 MCQ · 2 MARKS

Control Systems › Routh–Hurwitz

Consider the polynomial $p(s) = s^5 + 7s^4 + 3s^3 - 33s^2 + 2s - 40$. Let (L, I, R) be defined as follows.

L is the number of roots of $p(s)$ with negative real parts.

I is the number of roots of $p(s)$ that are purely imaginary.

R is the number of roots of $p(s)$ with positive real parts.

Which one of the following options is correct?

(A) $L = 2,\ I = 2$, and $R = 1$

(B) $L = 3,\ I = 2$, and $R = 0$

(C) $L = 1,\ I = 2$, and $R = 2$

(D) $L = 0,\ I = 4$, and $R = 1$

Official answer: **A**

SOLUTION

1. **Routh array, first two rows** (coefficients taken alternately): $s^5:\ 1,\ 3,\ 2$ and $s^4:\ 7,\ -33,\ -40$.
2. **Row s^3**: $\frac{7(3) - 1(-33)}{7} = \frac{54}{7}$ and $\frac{7(2) - 1(-40)}{7} = \frac{54}{7}$. Dividing a row by a positive number does not change any sign, so divide by $\frac{54}{7}$ to get $1,\ 1$.
3. **Row s^2** (using the scaled s^3 row): $\frac{1(-33) - 7(1)}{1} = -40$ and $\frac{1(-40) - 7(0)}{1} = -40$, so the row is $-40,\ -40$.
4. **Row s^1**: $\frac{(-40)(1) - (1)(-40)}{-40} = 0$. The whole row is zero. This is the special case where $p(s)$ has roots placed symmetrically about the origin.
5. **Auxiliary polynomial** from the s^2 row: $A(s) = -40s^2 - 40 = -40(s^2 + 1)$. Its roots are $s = \pm j$, and they are also roots of $p(s)$. Replace the zero row with the coefficients of $\frac{dA}{ds} = -80s$, so the s^1 row is -80. The s^0 row is then -40.
6. **First column**: $1,\ 7,\ \frac{54}{7}\ (\text{or } 1 \text{ after scaling}),\ -40,\ -80,\ -40$. There is exactly one sign change, between the s^3 and s^2 entries, so $p(s)$ has one root with positive real part: $R = 1$.
7. **From the auxiliary row down** (the entries $-40,\ -80,\ -40$) there is no sign change, so $A(s)$ has no root in the right half-plane. Its roots are symmetric about the origin, so it has none in the left half-plane either, and both lie on the $j\omega$ axis: $I = 2$. The remaining roots have negative real parts: $L = 5 - 1 - 2 = 2$.
8. **Check by factoring**: $p(s) = (s^2 + 1)(s^3 + 7s^2 + 2s - 40) = (s^2 + 1)(s - 2)(s + 4)(s + 5)$. The roots are $\pm j,\ 2,\ -4,\ -5$, which gives $(L, I, R) = (2, 2, 1)$. This is option (A).

KEY IDEA

A whole row of zeros in the Routh array means $p(s)$ has roots placed symmetrically about the origin, and those roots are the roots of the auxiliary polynomial built from the row above. After replacing the zero row with dA/ds, the total number of sign changes in the first column counts all right-half-plane roots. The sign changes from the auxiliary row downwards count only the auxiliary polynomial's right-half-plane roots. Here that count is zero, so both roots of $A(s) = -40(s^2 + 1)$ lie on the $j\omega$ axis.

WATCH OUT FOR

Stopping at the zero row, or putting a small ϵ in place of the whole zero row (the ϵ method is only for a single zero in the first column). That hides the $\pm j$ pair and leads to option (B) or (C). A second trap is reading the one sign change as the only non-left-half-plane root and forgetting to count the imaginary pair: that gives $L = 4$, which matches no option.

Q.38 MCQ · 2 MARKS

Signals and Systems › Sampling

Consider a continuous-time finite-energy signal $f(t)$ whose Fourier transform vanishes outside the frequency interval $[-\omega_c, \omega_c]$, where ω_c is in rad/sec.

The signal $f(t)$ is uniformly sampled to obtain $y(t) = f(t)\,p(t)$. Here,

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - \tau - nT_s),$$

with $\delta(t)$ being the Dirac impulse, $T_s > 0$, and $\tau > 0$. The sampled signal $y(t)$ is passed through an ideal

lowpass filter $h(t) = \omega_c T_s \dfrac{\sin(\omega_c t)}{\pi \omega_c t}$ with cutoff frequency ω_c and passband gain T_s.

The output of the filter is given by _____.

(A) $f(t)$ if $T_s < \pi/\omega_c$

(B) $f(t-\tau)$ if $T_s < \pi/\omega_c$

(C) $f(t-\tau)$ if $T_s < 2\pi/\omega_c$

(D) $T_s f(t)$ if $T_s < 2\pi/\omega_c$

Official answer: **A**

SOLUTION

1. **What the sampler produces.** By the sifting property, $y(t) = \sum_n f(\tau + nT_s)\,\delta(t - \tau - nT_s)$. Each impulse carries the true value of f at the instant where it sits, so the offset τ only moves the sampling grid. It does not delay the signal.
2. **Spectrum of the shifted impulse train.** Let $\omega_s = 2\pi/T_s$. The unshifted train $\sum_n \delta(t - nT_s)$ has transform $\omega_s \sum_k \delta(\omega - k\omega_s)$. A shift by τ multiplies this by $e^{-j\omega\tau}$, and at each impulse $\omega = k\omega_s$, so
$$P(\omega) = \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} e^{-jk\omega_s\tau}\,\delta(\omega - k\omega_s).$$
3. **Multiplication theorem** ($Y = \frac{1}{2\pi} F * P$):
$$Y(\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} e^{-jk\omega_s\tau}\,F(\omega - k\omega_s).$$
The $k = 0$ copy is $\frac{1}{T_s}F(\omega)$ and has **no** phase factor, because $e^0 = 1$. The offset τ only rotates the phase of the shifted copies ($k \neq 0$).
4. **The filter.** $h(t) = \omega_c T_s \dfrac{\sin(\omega_c t)}{\pi\omega_c t} = T_s \dfrac{\sin(\omega_c t)}{\pi t}$. Since $\dfrac{\sin(\omega_c t)}{\pi t}$ transforms to 1 for $|\omega| < \omega_c$ and 0 elsewhere, $H(\omega) = T_s$ for $|\omega| < \omega_c$ and 0 elsewhere. This is an ideal LPF with gain T_s.
5. **No-aliasing condition (Nyquist).** The $k = \pm 1$ copies occupy $[\omega_s - \omega_c,\ \omega_s + \omega_c]$ and its mirror image. The copies with $|k| \geq 2$ sit even farther out. For every shifted copy to stay clear of the passband $|\omega| < \omega_c$ we need $\omega_s - \omega_c > \omega_c$, which gives
$$\omega_s > 2\omega_c \ \Rightarrow\ \frac{2\pi}{T_s} > 2\omega_c \ \Rightarrow\ T_s < \frac{\pi}{\omega_c}.$$
6. **Output.** Under that condition only the $k = 0$ term passes:
$$X_{out}(\omega) = T_s \cdot \frac{1}{T_s} F(\omega) = F(\omega) \ \Rightarrow\ x_{out}(t) = f(t).$$
The gain T_s cancels the $1/T_s$, and there is no delay.
7. **Answer.** The output is $f(t)$ if $T_s < \pi/\omega_c$, which is option (A). (B) wrongly adds a delay τ. (C) and (D) use $T_s < 2\pi/\omega_c$, which only gives $\omega_s > \omega_c$ and lets the $k = \pm 1$ copies alias into the passband. (D) also keeps a stray gain T_s.

KEY IDEA

Shifting the sampling grid by τ multiplies the k-th spectral copy by $e^{-jk\omega_s\tau}$. The baseband copy ($k = 0$) gets $e^0 = 1$, so the offset has no effect on what the lowpass filter recovers. The sampled spectrum carries a factor $1/T_s$, which the passband gain T_s cancels. Nyquist ($\omega_s > 2\omega_c$) gives $T_s < \pi/\omega_c$.

WATCH OUT FOR

Picking (B) on the belief that the offset τ delays the output. Here f is multiplied by the shifted train, so each impulse carries f's true value at its own instant, and the only τ-dependent phase lands on the $k \neq 0$ copies, which the filter removes. Other traps: writing the Nyquist condition as $\omega_s > \omega_c$, which gives $2\pi/\omega_c$ as in (C) and (D), and forgetting that the gain T_s cancels the $1/T_s$ from sampling, as in (D).

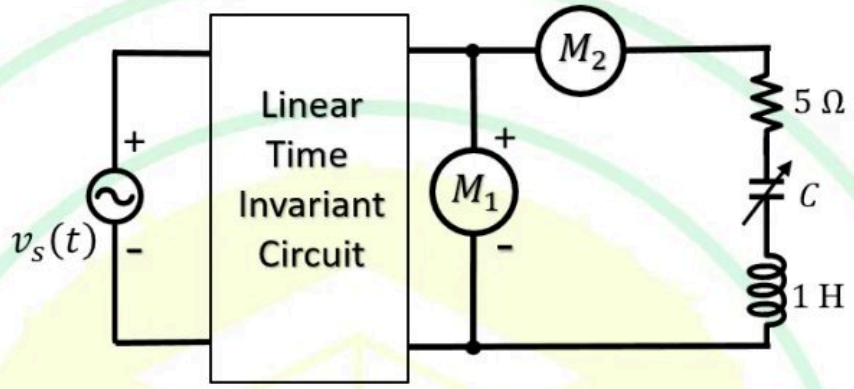
Q.39 MCQ · 2 MARKS

Networks (Electric Circuits) › AC, power & resonance

Q.39

In the circuit below, M_1 is an ideal AC voltmeter and M_2 is an ideal AC ammeter. The source voltage (in Volts) is $v_s(t) = 100\cos(200t)$.

What should be the value of the variable capacitor C such that the RMS readings on M_1 and M_2 are 25 V and 5 A, respectively?



(A) 25 μF

(B) 4 μF

(C) 0.25 μF

(D) Insufficient information to find C

Official answer: **A**

SOLUTION

1. The box is linear and time-invariant, so in sinusoidal steady state every voltage and current at its output is at the source frequency, $\omega = 200\ \text{rad/s}$. We never need to know what is inside the box: the two meter readings describe the load branch completely.
2. The ideal ammeter M_2 has zero impedance and drops no voltage, so the ideal voltmeter M_1 reads the voltage across the whole series branch of $5\ \Omega$, C and $1\ \text{H}$. M_2 reads the current through that same branch.
3. Ohm's law in rms magnitudes: $|Z| = \dfrac{|V|}{|I|} = \dfrac{25\ \text{V}}{5\ \text{A}} = 5\ \Omega$.

4. Series branch impedance: $Z = 5 + j\left(\omega L - \dfrac{1}{\omega C}\right)\ \Omega$, so $|Z| = \sqrt{5^2 + X^2}$ with $X = \omega L - \dfrac{1}{\omega C}$.

5. $\sqrt{25 + X^2} = 5 \Rightarrow X = 0$. The branch must be at series resonance: $\omega L = \dfrac{1}{\omega C}$.

6. $$C = \frac{1}{\omega^2 L} = \frac{1}{(200)^2 \times 1}\ \mathrm{F} = \frac{1}{40000}\ \mathrm{F} = 25\ \mu\mathrm{F}$$

7. Check: $X_L = 200 \times 1 = 200\ \Omega$ and $X_C = \dfrac{1}{200 \times 25 \times 10^{-6}} = 200\ \Omega$. They cancel, so $|Z| = 5\ \Omega$, and $25\ \mathrm{V}/5\ \Omega = 5\ \mathrm{A}$ matches both readings. Answer: (A).

KEY IDEA

The meters measure the load branch directly, so the unknown LTI box does not matter. The only thing the box sets is the frequency, which stays at $\omega = 200\ \mathrm{rad/s}$. The readings give $|Z| = V_{\mathrm{rms}}/I_{\mathrm{rms}} = 5\ \Omega$, which equals the branch resistance, so the net reactance must be zero (series resonance) and $C = 1/(\omega^2 L)$.

WATCH OUT FOR

Picking (D) because the contents of the box are unknown. The box only has to be LTI, which fixes the frequency at 200 rad/s, and the voltmeter and ammeter already give the branch's voltage and current. A second mistake is to use the 100 V source amplitude, or to mix peak and rms values, instead of the ratio of the two meter readings. A third is to use $f = 200\ \mathrm{Hz}$ instead of $\omega = 200\ \mathrm{rad/s}$, which gives the wrong capacitance.

Q.40 MCQ · 2 MARKS

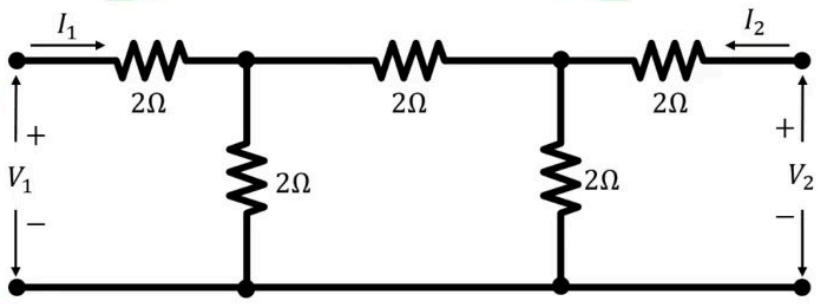
Networks (Electric Circuits) › Two-port

Q.40

The Z-parameter matrix of a two port network relates the port voltages and port currents as follows:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = Z \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

The Z-parameter matrix (with each entry in Ohms) of the network shown below is _____.



(A) $\begin{bmatrix} 10/3 & 2/3 \\ 2/3 & 10/3 \end{bmatrix}$

(B) $\begin{bmatrix} 2/3 & 10/3 \\ 10/3 & 2/3 \end{bmatrix}$

(C) $\begin{bmatrix} 10 & 2 \\ 2 & 10 \end{bmatrix}$

(D) $\begin{bmatrix} 10/3 & 1/3 \\ 1/3 & 10/3 \end{bmatrix}$

Official answer: **A**

SOLUTION

1. **Definition of the Z-parameters.** With port 2 open ($I_2 = 0$): $z_{11} = V_1/I_1$ and $z_{21} = V_2/I_1$. With port 1 open ($I_1 = 0$): $z_{22} = V_2/I_2$ and $z_{12} = V_1/I_2$. Call the node after the first series resistor A and the node after the second series resistor B.
2. z_{11} **(port 2 open).** With $I_2 = 0$ no current flows in the right-hand series $2\,\Omega$, so the branch leaving node A to the right is just the middle series $2\,\Omega$ plus the shunt $2\,\Omega$ at B, i.e. $4\,\Omega$. Seen from port 1:

$$z_{11} = 2 + (2 \parallel 4) = 2 + \frac{2 \cdot 4}{2+4} = 2 + \frac{4}{3} = \frac{10}{3}\,\Omega$$

3. z_{21} **(port 2 open), current divider at node A.** I_1 splits between the shunt $2\,\Omega$ at A and the $4\,\Omega$ path. The current in the $4\,\Omega$ path (through the shunt at B) is
$$I_{AB} = I_1 \cdot \frac{2}{2+4} = \frac{I_1}{3}$$
4. Because $I_2 = 0$, there is no drop across the right-hand series resistor, so V_2 equals the voltage across the shunt at B:
$$V_2 = V_B = 2 \cdot \frac{I_1}{3} = \frac{2}{3} I_1 \quad \Rightarrow \quad z_{21} = \frac{2}{3}\,\Omega$$
(Check by nodal analysis: at B, $(V_A - V_B)/2 = V_B/2$ gives $V_A = 2V_B$; at A, $I_1 = V_A/2 + (V_A - V_B)/2 = \frac{3}{2}V_B$, so $V_B = \frac{2}{3}I_1$ and $V_1 = 2I_1 + V_A = \frac{10}{3}I_1$.)
5. **Reciprocity and symmetry.** The network contains only resistors, so it is reciprocal: $z_{12} = z_{21} = \frac{2}{3}\,\Omega$. It is also a mirror image of itself about its centre (series 2, shunt 2, series 2, shunt 2, series 2), so it is symmetric: $z_{22} = z_{11} = \frac{10}{3}\,\Omega$.
6. **Result.**
$$Z = \begin{bmatrix} 10/3 & 2/3 \\ 2/3 & 10/3 \end{bmatrix} \Omega$$
This is option (A).

KEY IDEA

Open-circuit one port at a time. With a port open, its series resistor carries no current, so it drops out of the input impedance and adds no drop to the output voltage. That makes z_{11} a series-parallel reduction and z_{21} a single current division. Reciprocity (the network has only resistors) and mirror symmetry then give $z_{12} = z_{21}$ and $z_{22} = z_{11}$ with no extra work.

WATCH OUT FOR

Each wrong option matches a specific slip. Adding all five resistors in series, as if the shunts were series elements, gives 10 and 2 (option C). Writing the divided current $I_1/3$ as z_{21} and forgetting to multiply by the $2\,\Omega$ shunt at B gives 1/3 (option D). Swapping the driving-point and transfer entries gives option B: z_{11} must be the larger one, since V_1 includes the drop across the first series resistor. Another trap is counting the open-circuited right-hand series resistor in z_{21}. With $I_2 = 0$ it has no drop across it.

Q.41 MCQ · 2 MARKS

Communications › Digital communication

A source transmits symbol S that takes values uniformly at random from the set $\{-2, 0, 2\}$. The receiver obtains $Y = S + N$, where N is a zero-mean Gaussian random variable independent of S. The receiver uses the maximum likelihood decoder to estimate the transmitted symbol S. Suppose the probability of symbol estimation error P_e is expressed as $P_e = \alpha\, P(N > 1)$, where $P(N > 1)$ denotes the probability that N exceeds 1. What is the value of α?

(A) $1/3$

(B) 1

(C) $2/3$

(D) $4/3$

Official answer: **D**

SOLUTION

1. The three symbols are equally likely and the noise is zero-mean Gaussian with the same variance for every symbol. The ML decoder is therefore the minimum-distance decoder: it picks the symbol closest to Y.

2. The decision thresholds sit midway between neighbouring symbols, at $Y = -1$ and $Y = +1$. Decide -2 if $Y < -1$, decide 0 if $-1 < Y < 1$, and decide 2 if $Y > 1$.
3. Let $p = P(N > 1)$. Since N is zero-mean Gaussian, its density is symmetric, so $P(N < -1) = p$ as well.
4. Given $S = -2$: $Y = -2 + N$. The decoder goes wrong only if $Y > -1$, that is $N > 1$. So $P(\text{error} \mid S = -2) = p$.
5. Given $S = 2$: $Y = 2 + N$. The decoder goes wrong only if $Y < 1$, that is $N < -1$. So $P(\text{error} \mid S = 2) = p$.
6. Given $S = 0$: $Y = N$. The decoder goes wrong if $Y > 1$ or $Y < -1$, that is $|N| > 1$. So $P(\text{error} \mid S = 0) = P(N > 1) + P(N < -1) = 2p$.
7. Average over the three equally likely symbols:
$$P_e = \frac{1}{3}\,(p + 2p + p) = \frac{4}{3}\,p = \frac{4}{3}\,P(N > 1).$$
8. Comparing with $P_e = \alpha\, P(N > 1)$ gives $\alpha = \dfrac{4}{3}$, which is option (D).

KEY IDEA

With equal priors and equal-variance Gaussian noise, ML decoding is nearest-symbol decoding, so the thresholds lie midway between symbols. The two outer symbols have only one neighbouring threshold and the middle symbol has two. Average the error probabilities over the symbols.

WATCH OUT FOR

Giving the middle symbol only one tail, which gives $P_e = p$ and $\alpha = 1$. The symbol 0 is wrong when the noise exceeds 1 in either direction, so its error probability is $2p$. Another slip is to multiply every symbol's error by 2 and get $2p$ overall.

Q.42 MCQ · 2 MARKS

Communications › Random processes

Consider a real-valued random process f(t) = sum_{n=1}^{N} a_n p(t - nT), where T > 0 and N is a positive integer. Here, p(t) = 1 for t in [0, 0.5T] and 0 otherwise. The coefficients a_n are pairwise independent, zero-mean unit-variance random variables. Read the following statements about the random process and choose the correct option. (i) The mean of the process f(t) is independent of time t. (ii) The autocorrelation function E[f(t)f(t+tau)] is independent of time t for all tau. (Here, E[.] is the expectation operation.)

(A) (i) is TRUE and (ii) is FALSE

(B) Both (i) and (ii) are TRUE

(C) Both (i) and (ii) are FALSE

(D) (i) is FALSE and (ii) is TRUE

Official answer: **A**

SOLUTION

1. Mean: $E[f(t)] = \sum_{n=1}^{N} E[a_n]\, p(t - nT) = 0$ because every a_n has zero mean. This is a constant, so it does not depend on t. Statement (i) is TRUE.
2. Autocorrelation: $E[f(t)f(t+\tau)] = \sum_{n=1}^{N}\sum_{m=1}^{N} E[a_n a_m]\, p(t - nT)\, p(t + \tau - mT)$.
3. For $n \neq m$, independence and zero mean give $E[a_n a_m] = E[a_n]E[a_m] = 0$. For $n = m$, unit variance and zero mean give $E[a_n^2] = 1$. So $E[a_n a_m] = \delta_{nm}$ and
$$R(t, t+\tau) = \sum_{n=1}^{N} p(t - nT)\, p(t + \tau - nT).$$
4. Take $\tau = 0$: $R(t,t) = \sum_{n=1}^{N} p^2(t - nT)$. Each pulse is nonzero only on $[nT,\, nT + 0.5T]$, and these intervals do not overlap because $0.5T < T$. So $R(t,t) = 1$ for $t \in [nT,\, nT + 0.5T]$ and $R(t,t) = 0$ for $t \in (nT + 0.5T,\, (n+1)T)$.

5. For example, $R(T, T) = 1$ (the $n = 1$ pulse is on), but $R(1.75T, 1.75T) = 0$, since $0.75T \notin [0, 0.5T]$ and $-0.25T \notin [0, 0.5T]$ and no other pulse is on. So the value changes with t at $\tau = 0$.
6. The autocorrelation therefore depends on t (the process is cyclostationary with period T, not wide-sense stationary), so it is not independent of t for all τ. Statement (ii) is FALSE.
7. (i) TRUE and (ii) FALSE is option (A).

KEY IDEA

The mean is zero, so it is constant. The autocorrelation at $\tau = 0$ is $\sum p^2(t - nT)$, which switches between 1 and 0 as t moves through each period. A time-dependent autocorrelation means the process is not wide-sense stationary.

WATCH OUT FOR

Seeing a constant mean and a zero-mean, uncorrelated set of coefficients and assuming the process is stationary. Stationarity also needs the autocorrelation to be free of t, and a pulse train with a fixed timing grid fails that test.

Q.43 MCQ · 2 MARKS

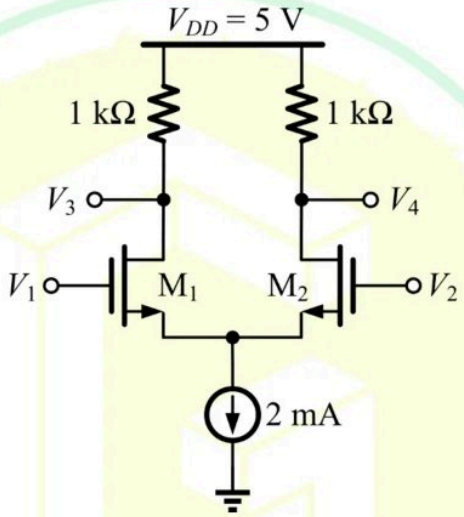
Q.43 The identical MOSFETs M_1 and M_2 in the circuit given below are ideal and biased in the saturation region. M_1 and M_2 have a transconductance g_m of 5 mS.

The input signals (in Volts) are:

$$V_1 = 2.5 + 0.01 \sin \omega t$$

$$V_2 = 2.5 - 0.01 \sin \omega t$$

The output signal V_3 (in Volts) is _____.



(A) $3 + 0.05 \sin \omega t$

(B) $3 - 0.1 \sin \omega t$

(C) $4 + 0.1 \sin \omega t$

(D) $4 - 0.05 \sin \omega t$

Official answer: **D**

SOLUTION

1. **Assumptions.** Both MOSFETs are ideal and in saturation ($r_o \to \infty$, so channel-length modulation is neglected). The 2 mA tail source is ideal, the devices are matched with $g_m = 5\ \text{mS}$, and both gates sit at the same DC level of $2.5\ \text{V}$.
2. **DC bias (symmetry).** The gate voltages are equal and the devices are matched, so the tail current splits equally: $I_{D1} = I_{D2} = \dfrac{2\ \text{mA}}{2} = 1\ \text{mA}$.

3. **DC output (KVL through the drain resistor).** $V_{3,\mathrm{DC}} = V_{DD} - I_{D1}R_D = 5 - (1\ \mathrm{mA})(1\ \mathrm{k\Omega}) = 4\ \mathrm{V}$.
4. **Split the input signals.** The differential part is $v_d = v_1 - v_2 = 0.02\sin\omega t\ \mathrm{V}$ and the common-mode part is $v_{cm} = \dfrac{v_1 + v_2}{2} = 0$. The signal input is purely differential.
5. **Common source node is an AC ground.** With a purely differential input the common source node does not move (half-circuit concept), so $v_{gs1} = +\dfrac{v_d}{2} = 0.01\sin\omega t\ \mathrm{V}$ and $v_{gs2} = -0.01\sin\omega t\ \mathrm{V}$. This is small enough for the small-signal model: the overdrive voltage is $V_{ov} = \dfrac{2I_D}{g_m} = \dfrac{2(1\ \mathrm{mA})}{5\ \mathrm{mS}} = 0.4\ \mathrm{V}$, and $10\ \mathrm{mV} \ll 0.4\ \mathrm{V}$.
6. **Small-signal drain current ($i_d = g_m v_{gs}$).** $i_{d1} = g_m\dfrac{v_d}{2} = (5\ \mathrm{mS})(0.01\sin\omega t\ \mathrm{V}) = 0.05\sin\omega t\ \mathrm{mA}$, flowing down into the drain of M_1.
7. **AC output (inverting at the drain).** A rise in i_{d1} increases the drop across the 1 kΩ drain resistor, so the drain voltage falls: $v_3 = -i_{d1}R_D = -(0.05\sin\omega t\ \mathrm{mA})(1\ \mathrm{k\Omega}) = -0.05\sin\omega t\ \mathrm{V}$. The single-ended gain is $\dfrac{v_3}{v_d} = -\dfrac{g_m R_D}{2} = -2.5$, which gives the same result.
8. **Superpose DC and AC.**

$$V_3 = 4 - 0.05\sin\omega t\ \mathrm{V}$$

This is option (D).

KEY IDEA

In a symmetric differential pair driven by a purely differential signal, the tail current splits equally at DC and the common source node acts as an AC ground. Each drain therefore behaves like a common-source stage driven by half the differential input. At drain 1 the output is $V_{DD} - \frac{I_{SS}}{2}R_D$ at DC, plus an AC part of $-g_m\frac{v_d}{2}R_D$.

WATCH OUT FOR

One mistake is to use the full differential input $v_d = 0.02\sin\omega t$ rather than $v_d/2$ at each gate, which gives $0.1\sin\omega t$ (options B, C). Another is to drop the inversion at the drain of M_1: V_1 rising makes V_3 fall. A third is to put the whole 2 mA tail current through one resistor, which gives a DC level of 3 V instead of 4 V.

Q.44 MCQ · 2 MARKS

Digital Electronics › ADC & DAC

A 10-bit analog-to-digital converter (ADC) has a sampling frequency of 1 MHz and a full scale voltage of 3.3 V.

For an input sinusoidal signal with frequency 500 kHz, the maximum SNR (in dB, *rounded off to two decimal places*) and the data rate (in Mbps) at the output of the ADC are _____, respectively.

(A) 61.96 and 10

(B) 61.96 and 5

(C) 33.36 and 10

(D) 33.36 and 5

Official answer: **A**

SOLUTION

1. **Maximum SNR of an ideal n-bit ADC.** The largest signal is a full-scale sinusoid, with amplitude $\dfrac{V_{FS}}{2}$, so its power is $\dfrac{(V_{FS}/2)^2}{2} = \dfrac{V_{FS}^2}{8}$. The step size is $\Delta = \dfrac{V_{FS}}{2^n}$, and the quantisation noise power is $\dfrac{\Delta^2}{12} = \dfrac{V_{FS}^2}{12\cdot 2^{2n}}$. Dividing the first by the second gives $\mathrm{SNR} = \dfrac{12\cdot 2^{2n}}{8} = \dfrac{3}{2}\cdot 2^{2n}$.
2. In decibels, $\mathrm{SNR}_{\max} = 20\log_{10} 2^n + 10\log_{10} 1.5$, which is the standard rule

$$\mathrm{SNR}_{\max} = 6.02\,n + 1.76\ \mathrm{dB}$$

The result depends only on n. The full-scale voltage (3.3 V) cancels in the ratio, and the input frequency does not appear at all.

3. With $n = 10$: $\text{SNR}_{\text{max}} = 6.02(10) + 1.76 = 60.20 + 1.76 = 61.96$ dB. (If you work it exactly, $20\log_{10} 2^{10} + 10\log_{10} 1.5 = 60.206 + 1.761 \approx 61.97$ dB. The options use the $6.02n + 1.76$ form, so 61.96 is the intended value. The other SNR value offered, 33.36 dB, is nowhere near either figure.)
4. **Output data rate.** Each sample produces one n-bit output word, so the bit rate is $R_b = nf_s$.
5. $R_b = 10\ \text{bits/sample} \times 10^6\ \text{samples/s} = 10\ \text{Mbps}$. The sampling frequency sets the data rate. The 500 kHz input frequency does not.
6. The pair is 61.96 dB and 10 Mbps, which is option (A).

KEY IDEA

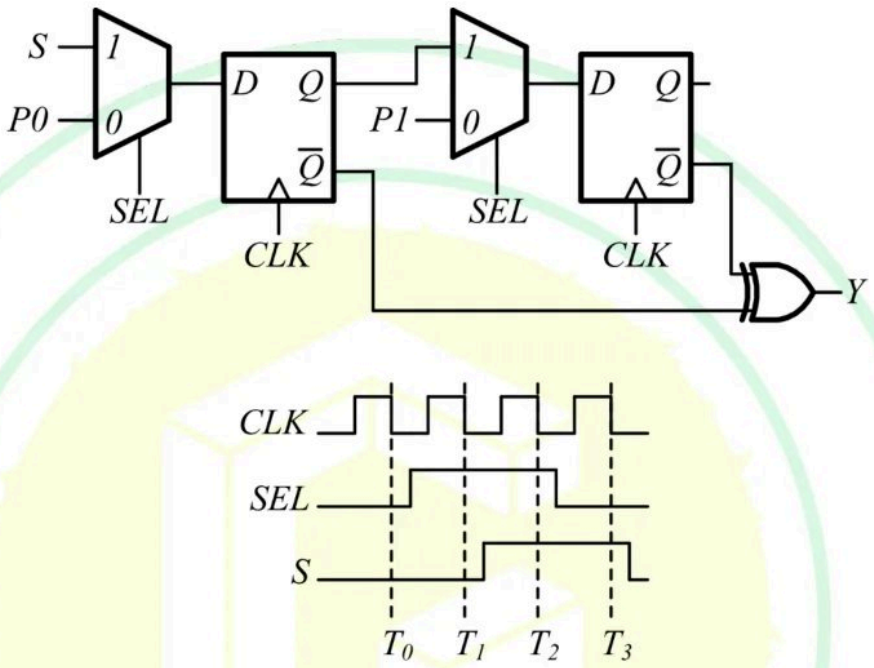
An ideal n-bit ADC driven by a full-scale sinusoid has $\text{SNR}_{\text{max}} = \frac{3}{2} \cdot 2^{2n}$, which is $6.02n + 1.76$ dB. This depends only on the resolution, not on V_{FS} or the signal frequency. The output bit rate is resolution times sampling rate, $R_b = nf_s$.

WATCH OUT FOR

Using the 500 kHz input frequency instead of the 1 MHz sampling frequency for the data rate gives $10 \times 0.5 = 5$ Mbps (options B and D). Another mistake is thinking the 3.3 V full-scale voltage changes the SNR. It cancels, because the signal and the quantisation step both scale with V_{FS}. Some students also drop the $+1.76$ dB term, or use $10\log_{10}$ where $20\log_{10}$ belongs.

Q.45 MCQ · 2 MARKS

Digital Electronics › Sequential

Q.45	A positive-edge-triggered sequential circuit is shown below. There are no timing violations in the circuit. Input $P0$ is set to logic '0' and $P1$ is set to logic '1' at all times. The timing diagram of the inputs SEL and S are also shown below. The sequence of output Y from time T_0 to T_3 is _____.
	
(A)	1011
(B)	0100
(C)	0010
(D)	1101

Official answer: **A**

SOLUTION

1. **Read the circuit.** A 2:1 MUX passes input 1 when its select line is 1 and input 0 when it is 0. With $P0 = 0$ and $P1 = 1$:

$$D_1 = \begin{cases} S, & SEL = 1 \\ P0 = 0, & SEL = 0 \end{cases} = SEL \cdot S, \qquad D_2 = \begin{cases} Q_1, & SEL = 1 \\ P1 = 1, & SEL = 0 \end{cases}$$

2. **Find the output.** The XOR takes the complemented outputs $\overline{Q_1}$ and $\overline{Q_2}$ (overline means complement). Complementing both inputs of an XOR does not change its output, because $\overline{A} \oplus \overline{B} = A \oplus B$. So $Y = Q_1 \oplus Q_2$.
3. **Find where the flip-flops change.** The dashed lines T_0 to T_3 sit on the falling edges of CLK. The flip-flops change only on rising edges, and exactly one rising edge comes just before each T_k. So $Y(T_k)$ is set by the state loaded at the rising edge before T_k. SEL and S change only between a falling edge and the next rising edge, so each has one clear value at every rising edge. Both flip-flops sample at the same edge. So D_2 sees the value Q_1 held just before the edge, not the value FF1 is about to load (D flip-flop rule: $Q^+ = D$).
4. **Rising edge before T_0:** $SEL = 0$ and $S = 0$, so $D_1 = 0$ and $D_2 = P1 = 1$. Then $Q_1Q_2 = 01$ and $Y(T_0) = 0 \oplus 1 = 1$. No initial state is needed, because with $SEL = 0$ neither D input depends on the earlier state.
5. **Rising edge before T_1:** $SEL = 1$ and $S = 0$, so $D_1 = S = 0$ and $D_2 = Q_1 = 0$. Then $Q_1Q_2 = 00$ and $Y(T_1) = 0 \oplus 0 = 0$.
6. **Rising edge before T_2:** $SEL = 1$ and $S = 1$, so $D_1 = 1$ and $D_2 = Q_1 = 0$ (the old Q_1). Then $Q_1Q_2 = 10$ and $Y(T_2) = 1 \oplus 0 = 1$.
7. **Rising edge before T_3:** SEL has already gone back to 0, while $S = 1$. So $D_1 = P0 = 0$ and $D_2 = P1 = 1$. Then $Q_1Q_2 = 01$ and $Y(T_3) = 0 \oplus 1 = 1$.
8. **Result.** From T_0 to T_3, $Y = 1, 0, 1, 1$, so the sequence is 1011, which is option (A).

KEY IDEA

The XOR is fed by both complemented outputs, so $Y = \overline{Q_1} \oplus \overline{Q_2} = Q_1 \oplus Q_2$. At each rising edge, read SEL and S. When $SEL = 1$ the two flip-flops act as a 2-bit shift register ($Q_1 \leftarrow S$, $Q_2 \leftarrow$ old Q_1). When $SEL = 0$ they load the constants $Q_1Q_2 = 01$, whatever their earlier state. Each T_k is a falling edge, so it shows the state loaded at the rising edge just before it.

WATCH OUT FOR

One mistake is feeding FF1's new value into FF2 in the same clock edge, as if the flip-flops rippled. At the third edge this gives $Q_2 = 1$ and $Y(T_2) = 0$, so the sequence becomes 1001, which is not an option. The other is reading SEL at the dashed line T_2, where it is still 1, instead of at the next rising edge, where it is already 0. That gives $Q_1Q_2 = 11$ at T_3 and the sequence 1010. Taking the XOR inputs as Q instead of $\overline{Q}$ does not change the answer, because complementing both inputs leaves the XOR unchanged.

Q.46 MCQ · 2 MARKS

Electronic Devices › Carrier transport

The intrinsic carrier concentration of a semiconductor is $2.5 \times 10^{16}\ \mathrm{m}^{-3}$ at 300 K. If the electron and hole mobilities are $0.15\ \mathrm{m}^2/\mathrm{Vs}$ and $0.05\ \mathrm{m}^2/\mathrm{Vs}$, respectively, then the intrinsic resistivity of the semiconductor (in kΩ·m) at 300 K is _____. (Charge of an electron $e = 1.6 \times 10^{-19}\ \mathrm{C}$.)

(A) 1.65

(B) 1.25

(C) 0.85

(D) 1.95

Official answer: **B**

SOLUTION

1. For an intrinsic semiconductor, the electron concentration equals the hole concentration, both equal to n_i. Both carrier types contribute to conduction, so the intrinsic conductivity is

$$\sigma_i = e\, n_i (\mu_n + \mu_p)$$

2. Substitute the given values: $e = 1.6 \times 10^{-19}\ \text{C}$, $n_i = 2.5 \times 10^{16}\ \text{m}^{-3}$, and $\mu_n + \mu_p = 0.15 + 0.05 = 0.20\ \text{m}^2/\text{Vs}$.
3. $$\sigma_i = (1.6 \times 10^{-19})(2.5 \times 10^{16})(0.20) = 8 \times 10^{-4}\ \text{S/m}$$
4. The resistivity is the reciprocal of conductivity:
$$\rho_i = \frac{1}{\sigma_i} = \frac{1}{8 \times 10^{-4}} = 1250\ \Omega \cdot \text{m}$$
5. Converting to $\text{k}\Omega \cdot \text{m}$:
$$\rho_i = 1.25\ \text{k}\Omega \cdot \text{m}$$
which matches option (B).

KEY IDEA
In an intrinsic semiconductor, electrons and holes exist in equal concentration n_i, so both carrier types contribute to conductivity: $\sigma_i = e\, n_i(\mu_n + \mu_p)$, and resistivity is simply the reciprocal, $\rho_i = 1/\sigma_i$.

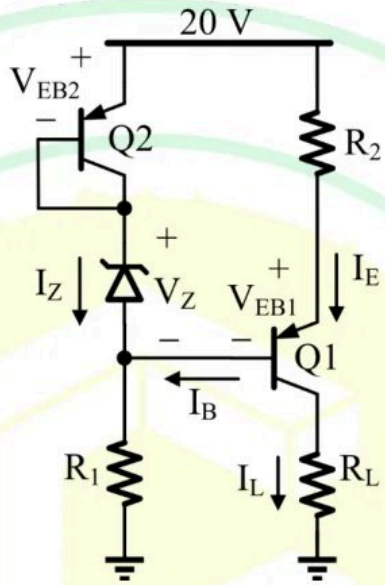
WATCH OUT FOR
Using only the electron mobility (or only the hole mobility) instead of the sum $\mu_n + \mu_p$, or forgetting to convert the final answer from $\Omega \cdot \text{m}$ to $\text{k}\Omega \cdot \text{m}$ (giving 1250 instead of 1.25).

Q.47 MCQ · 2 MARKS

Analog Electronics › Biasing

Q.47 In the circuit shown, the identical transistors Q1 and Q2 are biased in the active region with $\beta = 120$. The Zener diode is in the breakdown region with $V_Z = 5$ V and $I_Z = 25$ mA.

If $I_L = 12$ mA and $V_{EB1} = V_{EB2} = 0.7$ V, then the values of R_1 and R_2 (in kΩ, *rounded off to one decimal place*) are _____, respectively.



(A) 0.6 and 0.4

(B) 1.4 and 2.5

(C) 14.0 and 25.0

(D) 6.0 and 4.0

Official answer: **A**

SOLUTION

1. Assumptions, as given: both PNP transistors are in the active region with $V_{EB1} = V_{EB2} = 0.7$ V and $\beta = 120$, and the Zener is in breakdown with $V_Z = 5$ V. Q2 has its base tied to its collector (diode-connected), and its collector/base node connects only to the Zener cathode, so Q2's whole emitter current is the Zener current $I_Z = 25$ mA. Q2 simply drops $V_{EB2} = 0.7$ V.
2. Node voltages from the 20 V rail downward (KVL through Q2 and then the Zener): $V_{C2} = 20 - 0.7 = 19.3$ V, so the Zener anode, which is also Q1's base, sits at $V_{B1} = 19.3 - V_Z = 19.3 - 5 = 14.3$ V.

3. Emitter of Q1 (KVL across its emitter-base junction): $V_{E1} = V_{B1} + V_{EB1} = 14.3 + 0.7 = 15.0$ V. The voltage across R_2 is therefore $20 - 15 = 5$ V.
4. Q1's collector current flows through R_L, so $I_C = I_L = 12$ mA. Active-region relations: $I_B = \frac{I_C}{\beta} = \frac{12}{120} = 0.1$ mA and $I_E = I_C + I_B = 12.1$ mA.
5. Ohm's law for R_2, which carries I_E:
$$R_2 = \frac{5\text{ V}}{12.1\text{ mA}} = 0.413\text{ k}\Omega \approx 0.4\text{ k}\Omega$$
6. KCL at the node joining the Zener anode, Q1's base and the top of R_1: the Zener current enters from above, and the PNP base current I_B flows out of Q1's base into this node, as the figure's arrow shows. Both leave through R_1, so $I_{R1} = I_Z + I_B = 25 + 0.1 = 25.1$ mA.
7. Ohm's law for R_1 (top node at 14.3 V, bottom at ground):
$$R_1 = \frac{14.3\text{ V}}{25.1\text{ mA}} = 0.570\text{ k}\Omega \approx 0.6\text{ k}\Omega$$
8. So $R_1 \approx 0.6\text{ k}\Omega$ and $R_2 \approx 0.4\text{ k}\Omega$, which is option (A).

KEY IDEA

The diode-connected Q2 and the Zener form a fixed voltage stack below the 20 V rail. That stack pins Q1's base at $20 - 0.7 - 5 = 14.3$ V, and so its emitter at 15 V. R_2 then sets the emitter current $I_E = (1 + \beta)I_L/\beta$ across a fixed 5 V drop. R_1 carries the Zener current plus the PNP base current that flows out of Q1's base.

WATCH OUT FOR

A common slip is measuring the Zener voltage from ground and putting Q1's base at 5 V. The Zener actually sits under Q2, so what R_1 sees is $20 - 0.7 - 5 = 14.3$ V. The other trap is units: volts divided by milliamps gives kilo-ohms directly, and a slip of a factor of 10 lands on option (D). The base current (0.1 mA) barely changes either value at one decimal place, but its direction matters. For a PNP it flows out of the base, so it adds to the Zener current in R_1.

Q.48 MCQ · 2 MARKS

Electronic Devices › Carrier transport

The electron mobility μ_n in a non-degenerate germanium semiconductor at 300 K is 0.38 m²/Vs.

The electron diffusivity D_n at 300 K (in cm²/s, rounded off to the nearest integer) is _____.

(Consider the Boltzmann constant $k_B = 1.38 \times 10^{-23}$ J/K and the charge of an electron $e = 1.6 \times 10^{-19}$ C.)

(A) 26

(B) 98

(C) 38

(D) 10

Official answer: **B**

SOLUTION

1. Apply the Einstein relation: $\frac{D_n}{\mu_n} = V_T = \frac{k_B T}{q}$
2. Compute the thermal voltage using the given constants: $V_T = \frac{(1.38 \times 10^{-23})(300)}{1.6 \times 10^{-19}} = 0.02588$ V
3. Compute diffusivity in SI units: $D_n = \mu_n V_T = 0.38 \times 0.02588 = 9.83 \times 10^{-3}\ \text{m}^2/\text{s}$
4. Convert to cm^2/s (1 m² = 10^4 cm²): $D_n = 9.83 \times 10^{-3} \times 10^4 = 98.3\ \text{cm}^2/\text{s}$
5. Round off to the nearest integer: $D_n \approx 98\ \text{cm}^2/\text{s}$, matching option (B)

KEY IDEA

The Einstein relation links diffusivity and mobility: $D_n/\mu_n = V_T = k_B T/q$, the thermal voltage.

WATCH OUT FOR

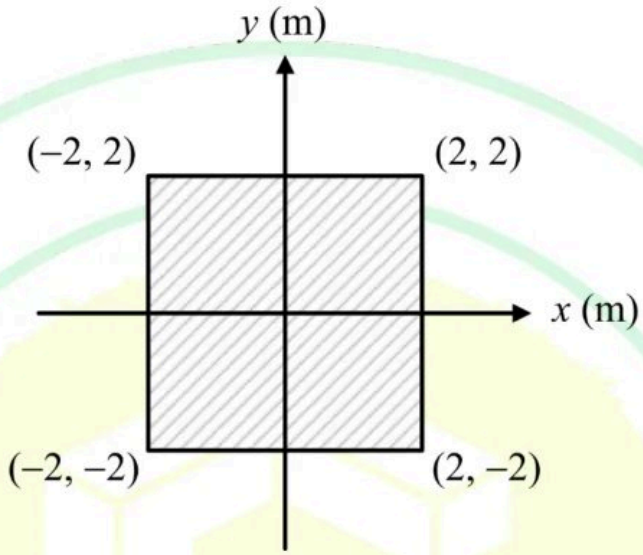
Forgetting to convert from m²/s to cm²/s (multiply by 10^4), which would give an answer near 0.0098 and no matching option, or misreading μn's units.

Q.49 MCQ · 2 MARKS

Electromagnetics › Electrostatics

Q.49 A square metal sheet of 4 m × 4 m is placed on the x-y plane as shown in the figure below.

If the surface charge density (in μC/m^2) on the sheet is $\rho_s(x, y) = 4|y|$, then the total charge (in μC, *rounded off to the nearest integer*) on the sheet is _____.



(A) 16

(B) 85

(C) 64

(D) 256

Official answer: **C**

SOLUTION

1. **Total charge on a surface.** Integrate the surface charge density over the area: $Q = \iint_S \rho_s \, dS$. The sheet lies in the x-y plane, so $dS = dx\, dy$.
2. **Limits from the figure.** The corners are at $(\pm 2, \pm 2)$, so the sheet covers $-2 \le x \le 2$ and $-2 \le y \le 2$ (in m). That is a $4\ \text{m} \times 4\ \text{m}$ square, as the question says.
3. **Separate the integral.** $\rho_s = 4|y|$ does not depend on x, so the double integral splits into a product:

$$Q = \int_{-2}^{2}\int_{-2}^{2} 4|y|\,dx\,dy = 4\left(\int_{-2}^{2} dx\right)\left(\int_{-2}^{2} |y|\,dy\right)$$

4. **Integrate $|y|$ using symmetry.** $|y|$ is an even function, so $\int_{-2}^{2}|y|\,dy = 2\int_{0}^{2} y\,dy = 2\cdot\dfrac{2^2}{2} = 4\ \text{m}^2$. The x-integral is just the width of the sheet: $\int_{-2}^{2} dx = 4\ \text{m}$.
5. **Combine.** With ρ_s in $\mu\text{C/m}^2$:

$$Q = 4 \times 4 \times 4 = 64\ \mu\text{C}$$

This is already an integer, and it is option (C).

KEY IDEA

Total charge is the surface integral of ρ_s over the sheet. Here $\rho_s = 4|y|$ does not depend on x, so the integral splits into (width in x) $\times \int |y|\,dy$. Because $|y|$ is even, $\int_{-2}^{2}|y|\,dy$ is twice the integral from 0 to 2.

WATCH OUT FOR

Dropping the modulus and integrating $4y$ from -2 to 2 gives $Q = 0$. The absolute value makes the density positive on both halves, so the two halves add instead of cancelling. The other common slip is to treat ρ_s as uniform, using its edge value $4 \times 2 = 8\ \mu\text{C/m}^2$ times the area $16\ \text{m}^2$, which gives 128. The density rises linearly from zero at $y = 0$, so its average over the sheet is only $4\ \mu\text{C/m}^2$.

Q.50 MCQ · 2 MARKS

Electromagnetics › Electrostatics

An electric field of 0.01 V/m is applied along the length of a copper wire of circular cross-section with diameter 1 mm. Copper has a conductivity of 5.8×10^7 S/m.

The current (in Amperes, *rounded off to two decimal places*) flowing through the wire is _____.

(A) 0.46

(B) 1.82

(C) 0.58

(D) 1.12

Official answer: **A**

SOLUTION

1. Ohm's law in point form: the conduction current density is $\mathbf{J} = \sigma\mathbf{E}$. It points along the wire, in the same direction as $\mathbf{E}$.
2. $$J = \sigma E = (5.8 \times 10^7\ \text{S/m})(0.01\ \text{V/m}) = 5.8 \times 10^5\ \text{A/m}^2$$
3. Cross-section: the diameter is $1\ \text{mm}$, so the radius is $r = 0.5\ \text{mm} = 5 \times 10^{-4}\ \text{m}$ and $$A = \pi r^2 = \pi(5 \times 10^{-4})^2 = 7.854 \times 10^{-7}\ \text{m}^2$$
4. The field is steady (DC), so there is no skin effect and $\mathbf{J}$ is uniform over the cross-section. The current is therefore $I = \int_S \mathbf{J}\cdot d\mathbf{S} = JA$.
5. $$I = (5.8 \times 10^5)(7.854 \times 10^{-7}) = 0.4555\ \text{A} \approx 0.46\ \text{A}$$
6. This is option (A).

KEY IDEA

Use Ohm's law in point form, $\mathbf{J} = \sigma\mathbf{E}$. With a steady field the current density is uniform across the wire, so the current is just J times the cross-sectional area: $I = \sigma E \pi r^2$.

WATCH OUT FOR

Putting the diameter in place of the radius gives $A = \pi(10^{-3})^2$ and a current four times too large, $1.82\ \text{A}$, which is option (B). Forgetting the factor π gives $0.145\ \text{A}$, and taking the area as d^2 gives $0.58\ \text{A}$, which is option (C). Make sure mm is converted to m before you square it.

Q.51 MSQ · 2 MARKS

Engineering Mathematics › Calculus

Consider a non-negative function $f(x)$ which is continuous and bounded over the interval [2, 8]. Let M and m denote, respectively, the maximum and the minimum values of $f(x)$ over the interval.

Among the combinations of α and β given below, choose the one(s) for which the inequality

$$\beta \leq \int_2^8 f(x)\,dx \leq \alpha$$

is guaranteed to hold.

(A) $\beta = 5m,\ \alpha = 7M$

(B) $\beta = 6m,\ \alpha = 5M$

(C) $\beta = 7m,\ \alpha = 6M$

(D) $\beta = 7m,\ \alpha = 5M$

Official answer: **A**

SOLUTION

1. Bounds on the integrand: f is continuous on the closed interval $[2, 8]$, so its maximum M and minimum m exist, and $m \leq f(x) \leq M$ for every x in the interval.
2. Integrate over $[2, 8]$, whose length is $8 - 2 = 6$. Integration preserves inequalities, so
$$6m \leq \int_2^8 f(x)\,dx \leq 6M.$$
3. Both bounds are attained: for a constant function $f(x) = c \geq 0$ we have $m = M = c$, and the integral is exactly $6c$. So a combination $\beta = pm,\ \alpha = qM$ works for every admissible f only if $p \leq 6$ and $q \geq 6$.
4. Non-negativity matters: $f \geq 0$ gives $m \geq 0$ and $M \geq 0$, so $5m \leq 6m \leq 7m$ and $5M \leq 6M \leq 7M$. (If m could be negative, $5m$ would be larger than $6m$ and the argument below would fail.)
5. (A) $\beta = 5m \leq 6m \leq \int_2^8 f(x)\,dx \leq 6M \leq 7M = \alpha$. The inequality holds for every admissible f, so it is guaranteed.
6. (B) $\alpha = 5M$: take $f(x) = 1$, so $M = 1$. The integral is 6, but $\alpha = 5 < 6$. Fails.
7. (C) $\beta = 7m$: take $f(x) = 1$, so $m = 1$. The integral is 6, but $\beta = 7 > 6$. Fails.
8. (D) With $f(x) = 1$, both $\beta = 7 > 6$ and $\alpha = 5 < 6$. Fails. Only (A) is guaranteed.

KEY IDEA

Bound the integral using the interval length: $(b - a)\,m \leq \int_a^b f(x)\,dx \leq (b - a)\,M$, which here is $6m \leq \int_2^8 f \leq 6M$. A constant function reaches both bounds, so a lower bound pm needs $p \leq 6$ and an upper bound qM needs $q \geq 6$. The step from $6m$ down to $5m$ and from $6M$ up to $7M$ is valid only because $f \geq 0$.

WATCH OUT FOR

Using the upper limit 8 (or the lower limit 2) as the multiplier instead of the interval length $8 - 2 = 6$. Another mistake is dropping the non-negativity condition: if f could be negative, then $5m > 6m$ when $m < 0$, and even (A) fails. For example, $f(x) = -1$ gives an integral of -6, which is less than $5m = -5$. A third mistake is marking (C) because $\alpha = 6M$ is a valid upper bound, while missing that $\beta = 7m$ breaks the lower bound. Both halves must hold.

Q.52 MSQ · 2 MARKS

Engineering Mathematics › Complex analysis

Which of the following statements involving contour integrals (evaluated counter-clockwise) on the unit circle C in the complex plane is/are TRUE?

(A) $\oint_C e^z\,dz = 0$

(B) $\oint_C z^n\,dz = 0$, where n is an even integer

(C) $\oint_C \cos z\, dz \neq 0$

(D) $\oint_C \sec z\, dz \neq 0$

Official answer: **A, B**

SOLUTION

1. **Tool: Cauchy's integral theorem.** If $f(z)$ is analytic on and inside the closed contour C (here $|z| = 1$), then $\oint_C f(z)\, dz = 0$. So for each option we look for singularities inside $|z| < 1$.
2. **(A)** e^z is entire (it has no singularity anywhere), so by Cauchy's theorem $\oint_C e^z\, dz = 0$. **TRUE.**
3. **(B)** Cauchy's theorem does not settle negative n, because then z^n has a pole at $z = 0$. So evaluate directly. Parametrise $z = e^{j\theta}$, $dz = je^{j\theta}\, d\theta$, $0 \le \theta \le 2\pi$:
$$\oint_C z^n\, dz = j\int_0^{2\pi} e^{j(n+1)\theta}\, d\theta = \begin{cases} 2\pi j, & n = -1 \\ 0, & n \neq -1 \end{cases}$$
For $n \neq -1$ the antiderivative $\dfrac{e^{j(n+1)\theta}}{j(n+1)}$ takes the same value at $\theta = 0$ and $\theta = 2\pi$, so the integral is 0.
4. The only nonzero case is $n = -1$, which is odd. So for every even n the integral is 0. That includes $n = 0$ and negative even values such as $n = -2$, where z^{-2} has a double pole at 0 with residue 0. **TRUE.**
5. **(C)** $\cos z$ is entire, so $\oint_C \cos z\, dz = 0$. The statement "$\neq 0$" is **FALSE.**
6. **(D)** $\sec z = \dfrac{1}{\cos z}$ is singular only where $\cos z = 0$. Write $z = x + jy$:
$$\cos z = \cos x \cosh y - j \sin x \sinh y$$
Setting the real part to zero gives $\cos x = 0$ (since $\cosh y \ge 1$). Then $\sin x = \pm 1$, and setting the imaginary part to zero gives $\sinh y = 0$, so $y = 0$. So every zero of $\cos z$ is real: $z = \pm\dfrac{\pi}{2}, \pm\dfrac{3\pi}{2}, \ldots$
7. The poles nearest the origin have $|z| = \dfrac{\pi}{2} \approx 1.571 > 1$, so every pole lies outside C. That makes $\sec z$ analytic on and inside $|z| = 1$, and Cauchy's theorem gives $\oint_C \sec z\, dz = 0$. The statement "$\neq 0$" is **FALSE.**
8. Correct options: **(A) and (B)**.

KEY IDEA

By Cauchy's theorem, a contour integral is zero whenever the integrand is analytic on and inside the contour. So first find where the singularities are. For powers of z, use the standard result $\oint_{|z|=1} z^n\, dz = 2\pi j$ if $n = -1$ and 0 for every other integer n. A pole inside the contour does not by itself make the integral nonzero; only the residue counts.

WATCH OUT FOR

Two traps. First, treating (B) as false because a negative even power such as z^{-2} has a pole inside C: that pole's residue is 0, so the integral is still 0. Second, marking (D) true because $\sec z$ has poles at all: they sit at $\pm\pi/2, \pm 3\pi/2, \ldots$ (all real, since $\cos z$ has no non-real zeros), which is outside the unit circle. Also watch the $\neq$ sign in (C) and (D): both integrals are 0, so those statements are false.

Q.53 MSQ · 2 MARKS

Control Systems › State space

Consider a system where $x_1(t)$, $x_2(t)$, and $x_3(t)$ are three internal state signals and $u(t)$ is the input signal. The differential equations governing the system are given by
$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u(t).$$
Which of the following statements is/are TRUE?

(A) The signals $x_1(t)$, $x_2(t)$, and $x_3(t)$ are bounded for all bounded inputs

(B) There exists a bounded input such that at least one of the signals $x_1(t),\ x_2(t)$, and $x_3(t)$ is unbounded

(C) There exists a bounded input such that the signals $x_1(t),\ x_2(t)$, and $x_3(t)$ are unbounded

(D) The signals $x_1(t),\ x_2(t)$, and $x_3(t)$ are unbounded for all bounded inputs

Official answer: **B**

SOLUTION

1. A is diagonal, so the three states are decoupled: $\dot{x}_1 = 2x_1 + u, \quad \dot{x}_2 = -2x_2 + u, \quad \dot{x}_3 = u$. The modes are $\lambda = 2,\ -2,\ 0$.
2. Each state is a first-order system driven by u. x_1 sees $\dfrac{1}{s-2}$ (pole in the right half-plane), x_2 sees $\dfrac{1}{s+2}$ (pole in the left half-plane) and x_3 sees $\dfrac{1}{s}$ (pole at the origin).
3. Take the bounded input $u(t) = 1$. Then $x_3(t) = x_3(0) + t \to \infty$ whatever the initial state is. With zero initial state, $x_1(t) = \dfrac{e^{2t} - 1}{2}$ also goes to ∞. So the states are not bounded for every bounded input: (A) is false and (B) is true.
4. x_2 is BIBO stable. If $|u| \le M$, then $|x_2(t)| \le |x_2(0)|e^{-2t} + \displaystyle\int_0^t e^{-2(t-\tau)}|u(\tau)|\,d\tau \le |x_2(0)| + \frac{M}{2}$. So x_2 stays bounded for **every** bounded input and every initial state.
5. Since x_2 can never be unbounded, no bounded input makes all three states unbounded. (C) is false.
6. For the same reason it is not true that the states are unbounded for all bounded inputs, because x_2 is always bounded. (A concrete case: $u(t) = 0$ with zero initial state gives $x_1 = x_2 = x_3 = 0$.) (D) is false.
7. The only true statement is (B).

KEY IDEA

With a diagonal A, the states decouple into first-order systems $\dfrac{1}{s-\lambda_i}$. Every $b_i = 1$, so every mode is driven by the input, and each state's boundedness is decided by its own eigenvalue alone. $\lambda = 2$ (right half-plane) and $\lambda = 0$ (a pole at the origin, whose step response is a ramp) can be made unbounded by a bounded input. $\lambda = -2$ cannot. So the option you need is "at least one state is unbounded", not "all states are unbounded".

WATCH OUT FOR

Treating the whole system as one unit and choosing (C) because it is unstable, without noticing that x_2 ($\lambda = -2$) stays bounded for every bounded input. Another mistake is to count the $\lambda = 0$ state as marginally stable and so bounded: under a constant input, a pole at the origin integrates the input into a ramp.

Q.54 MSQ · 2 MARKS

Communications › Information theory

The random variable X takes values in $\{-1, 0, 1\}$ with probabilities $P(X = -1) = P(X = 1) = \alpha$ and $P(X = 0) = 1 - 2\alpha$, where $0 < \alpha < 1/2$. Let $g(\alpha)$ denote the entropy of X (in bits), parameterized by α. Which of the following statements is/are TRUE?

(A) $g(0.4) > g(0.3)$

(B) $g(0.3) > g(0.4)$

(C) $g(0.3) > g(0.25)$

(D) $g(0.25) > g(0.3)$

Official answer: **B, C**

SOLUTION

1. Entropy in bits:

$$g(\alpha) = -2\alpha \log_2 \alpha - (1 - 2\alpha)\log_2(1 - 2\alpha).$$

2. A three-symbol source has maximum entropy when all probabilities are equal, so g peaks where $\alpha = 1 - 2\alpha$, i.e. $\alpha = 1/3$. The function g is concave in α, so it increases for $\alpha < 1/3$ and decreases for $\alpha > 1/3$.
3. $g(0.3)$: probabilities $(0.3,\ 0.4,\ 0.3)$.
$$g(0.3) = -0.6\log_2 0.3 - 0.4\log_2 0.4 = 0.6(1.737) + 0.4(1.322) \approx 1.571 \text{ bits.}$$
4. $g(0.4)$: probabilities $(0.4,\ 0.2,\ 0.4)$.
$$g(0.4) = -0.8\log_2 0.4 - 0.2\log_2 0.2 = 0.8(1.322) + 0.2(2.322) \approx 1.522 \text{ bits.}$$
5. $g(0.25)$: probabilities $(0.25,\ 0.5,\ 0.25)$.
$$g(0.25) = 0.5(2) + 0.5(1) = 1.5 \text{ bits.}$$
6. Compare: $g(0.3) \approx 1.571 > g(0.4) \approx 1.522$, so (B) is true and (A) is false. Also $g(0.3) \approx 1.571 > g(0.25) = 1.5$, so (C) is true and (D) is false.
7. Check with the peak: 0.3 is 0.033 from $1/3$, while 0.4 is 0.067 away and 0.25 is 0.083 away. Because g is concave with its maximum at $1/3$, 0.3 is closest to the peak and gives the largest entropy of the three. Hence (B) and (C) are true.

KEY IDEA

Entropy of the distribution $(\alpha, 1-2\alpha, \alpha)$ is concave in α with its maximum ($\log_2 3$ bits) at the uniform point $\alpha = 1/3$. The closer α is to $1/3$, the larger the entropy, so evaluate each value in $-\sum p \log_2 p$.

WATCH OUT FOR

Assuming entropy increases with α (so picking A), or forgetting the probability of the middle symbol is $1 - 2\alpha$. Another slip is to treat this as a single-answer question and stop after finding one true statement, when MSQ needs every true option.

Q.55 MSQ · 2 MARKS

Signals and Systems › Fourier series

Let $f(t)$ be a periodic signal with fundamental period $T_0 > 0$. Consider the signal $y(t) = f(\alpha t)$, where $\alpha > 1$.

The Fourier series expansions of $f(t)$ and $y(t)$ are given by

$$f(t) = \sum_{k=-\infty}^{\infty} c_k\, e^{j\frac{2\pi}{T_0}kt} \quad \text{and} \quad y(t) = \sum_{k=-\infty}^{\infty} d_k\, e^{j\frac{2\pi}{T_0}\alpha kt}.$$

Which of the following statements is/are TRUE?

(A) $c_k = d_k$ for all k

(B) $y(t)$ is periodic with a fundamental period αT_0

(C) $c_k = d_k/\alpha$ for all k

(D) $y(t)$ is periodic with a fundamental period T_0/α

Official answer: **A, D**

SOLUTION

1. **Fundamental period of $y(t)$ (time scaling).** $y(t+T) = f(\alpha t + \alpha T)$, so $y(t+T) = y(t)$ for every t exactly when αT is a period of f. The smallest positive period of f is T_0, so the smallest positive T is given by $\alpha T = T_0$, that is $T = T_0/\alpha$. Compressing a signal in time ($\alpha > 1$) shortens its period; it does not lengthen it. So (D) is TRUE and (B) is FALSE.
2. **Substitute αt into the series for f.** Replace t by αt in $f(t) = \sum_k c_k e^{j\frac{2\pi}{T_0}kt}$:
$$y(t) = f(\alpha t) = \sum_{k=-\infty}^{\infty} c_k\, e^{j\frac{2\pi}{T_0}k(\alpha t)} = \sum_{k=-\infty}^{\infty} c_k\, e^{j\frac{2\pi}{T_0}\alpha kt}.$$

3. **Compare with the given expansion of $y(t)$.** The problem writes $y(t) = \sum_k d_k e^{j\frac{2\pi}{T_0}\alpha k t}$. This uses the same basis functions $e^{jk\omega_y t}$, with $\omega_y = \dfrac{2\pi\alpha}{T_0} = \dfrac{2\pi}{T_0/\alpha}$, which is the fundamental frequency of y from Step 1. These basis functions are orthogonal over one period T_0/α, so the Fourier series of y is unique. Therefore $d_k = c_k$ for every k, and (A) is TRUE.
4. **Check with the analysis formula.** $d_k = \dfrac{\alpha}{T_0}\displaystyle\int_0^{T_0/\alpha} f(\alpha t)\, e^{-j\frac{2\pi}{T_0}\alpha k t}\, dt$. Put $\tau = \alpha t$, so $dt = d\tau/\alpha$ and the limits become 0 to T_0:
$$d_k = \frac{1}{T_0}\int_0^{T_0} f(\tau)\, e^{-j\frac{2\pi}{T_0}k\tau}\, d\tau = c_k.$$
The factor $1/\alpha$ from dt cancels the α in $1/(T_0/\alpha)$, so no α appears in the coefficients.
5. **Why (C) is false.** If (C) held as well, then $c_k = d_k/\alpha = c_k/\alpha$, so $c_k(1 - 1/\alpha) = 0$. Since $\alpha > 1$, every c_k would be 0 and $f(t) \equiv 0$. The zero signal has no fundamental period, so it cannot be the given $f(t)$. (C) is FALSE.
6. **Result.** Time scaling by α leaves the Fourier series coefficients unchanged, multiplies the fundamental frequency by α and divides the fundamental period by α. The correct options are (A) and (D).

KEY IDEA

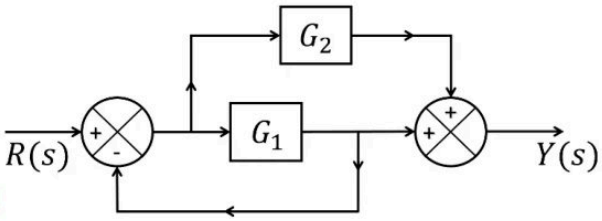
Time scaling $y(t) = f(\alpha t)$ compresses the period to T_0/α and raises the fundamental frequency to $\alpha\omega_0$, but the Fourier series coefficients stay the same. The same harmonic weights now sit at harmonics of a higher fundamental frequency. In the analysis integral, the $1/\alpha$ from the change of variable cancels the α in $1/(T_0/\alpha)$.

WATCH OUT FOR

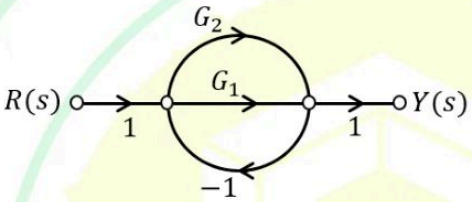
Carrying over the Fourier transform scaling rule $f(\alpha t) \leftrightarrow \frac{1}{|\alpha|}F(\omega/\alpha)$ and picking (C). The $1/\alpha$ appears in the transform because the transform integrates over all time. The series averages over one period, and the shorter period cancels it. A second slip is thinking compression lengthens the period and choosing (B).

Q.56 MSQ · 2 MARKS

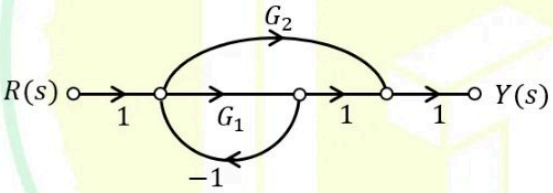
Q.56 Consider a system represented by the block diagram shown below. Which of the following signal flow graphs represent(s) this system? Choose the correct option(s).



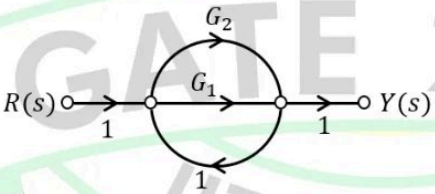
(A)



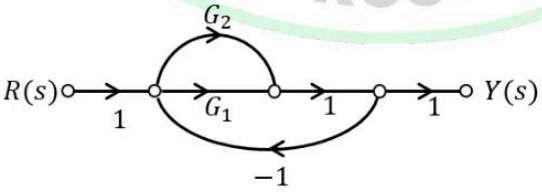
(B)



(C)



(D)



Official answer: **B**

SOLUTION

1. **Block diagram.** Let E be the output of the first summer. The feedback path starts after G_1 and before the second summer, so it carries only G_1E. That gives $E = R - G_1E$, so $E = \dfrac{R}{1+G_1}$.
2. The second summer adds both branches: $Y = G_1E + G_2E = (G_1 + G_2)E$.
3. So the target transfer function is

$$\frac{Y(s)}{R(s)} = \frac{G_1 + G_2}{1 + G_1}$$

The system has one loop, with gain $-G_1$. The G_2 branch is a feedforward path outside that loop.

4. **(A)** The forward paths are $P_1 = G_1$ and $P_2 = G_2$. The -1 branch leaves n_2, where G_1 and G_2 have already merged. That makes two loops, $L_1 = -G_1$ and $L_2 = -G_2$. They share n_1 and n_2, so they touch, and $\Delta = 1 + G_1 + G_2$. Both forward paths touch both loops, so $\Delta_1 = \Delta_2 = 1$ and $T_A = \frac{G_1 + G_2}{1 + G_1 + G_2}$. A is wrong.
5. **(B)** The -1 branch leaves n_2, which carries only $G_1 n_1$. G_2 joins later, at n_3, and nothing leads from n_3 back into the graph, so the only loop is $L_1 = -G_1$ and $\Delta = 1 + G_1$. The forward paths $P_1 = G_1$ (through n_1, n_2, n_3) and $P_2 = G_2$ (through n_1, n_3) both pass through n_1, so both touch the loop and $\Delta_1 = \Delta_2 = 1$. That gives $T_B = \frac{G_1 + G_2}{1 + G_1}$, which matches. Node equations give the same result: $n_1 = R - G_1 n_1$, so $n_1 = \frac{R}{1 + G_1}$, and $Y = n_3 = (G_1 + G_2) n_1$.
6. **(C)** The paths merge before the feedback node, as in A, but the feedback gain is $+1$. The loops are $+G_1$ and $+G_2$, so $\Delta = 1 - G_1 - G_2$ and $T_C = \frac{G_1 + G_2}{1 - G_1 - G_2}$. C is wrong.
7. **(D)** The feedback leaves n_3, and $n_3 = n_2 = (G_1 + G_2) n_1$, so the fed-back signal again includes G_2's output. The loops are $-G_1$ and $-G_2$, as in A, so $T_D = \frac{G_1 + G_2}{1 + G_1 + G_2}$. D is wrong.
8. Only graph B represents the block diagram. The answer is **B**.

KEY IDEA

In a signal flow graph, a feedback branch must start at a node that carries exactly the signal the block diagram feeds back. Here the feedback takes G_1's output before G_2's output is added. The graph therefore needs a node that holds $G_1 E$ alone, separate from a later node where $G_2 E$ joins. Only graph B has one, so its only loop is $-G_1$ and G_2 stays a feedforward path.

WATCH OUT FOR

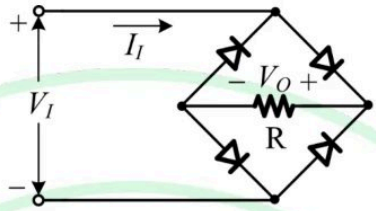
Treating the take-off point after G_1 and the second summing junction as one node. Graphs A and D do this. The fed-back signal then becomes $(G_1 + G_2)E$, which adds a second loop $-G_2$ and gives $\Delta = 1 + G_1 + G_2$. A second slip is missing the sign change in C: a $+1$ feedback branch turns the minus signs in Δ into plus signs.

Q.57 MSQ · 2 MARKS

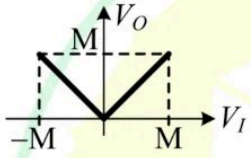
Analog Electronics › Diode circuits

Q.57 All the diodes in the circuit given below are ideal.

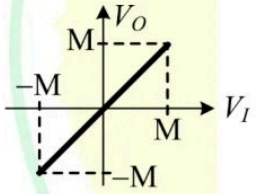
Which of the following plots is/are correct when V_I (in Volts) is swept from −M to M ?



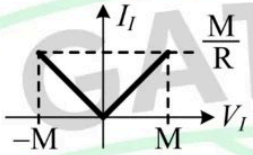
(A)



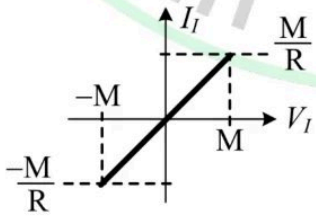
(B)



(C)



(D)



Official answer: **A, D**

SOLUTION

1. Read the circuit from the diode directions. Call the top node T, the bottom node B, the left end of R P (the $-$ of V_O) and the right end Q (the $+$ of V_O). The diodes $T \to Q$ and $B \to Q$ both have their cathodes at Q, and the diodes $P \to T$ and $P \to B$ both have their anodes at P. This is a full-wave bridge rectifier: the input is across T–B, the output is across P–Q, and $V_O = V_Q - V_P$. Ideal diode means zero drop when conducting and an open circuit when reverse biased.

2. Case $V_I > 0$ (T positive with respect to B). Take D_{TQ} and D_{PB} as conducting (short circuits). Then $V_Q = V_T$ and $V_P = V_B$. Check the other two: D_{PT} has anode at V_B and cathode at V_T, and D_{BQ} has anode at V_B and cathode at $V_Q = V_T$, so each is reverse biased by V_I. The assumption is consistent. Current flows $T \to Q \to R \to P \to B$, and
$$V_O = V_T - V_B = V_I, \qquad I_I = \frac{V_I}{R} > 0.$$
3. Case $V_I < 0$ (B positive with respect to T). Now D_{BQ} and D_{PT} conduct, and D_{TQ} and D_{PB} are each reverse biased by $|V_I|$. Current flows $B \to Q \to R \to P \to T$ and returns to the source's $+$ terminal along the top wire. Now $V_Q = V_B$ and $V_P = V_T$, so
$$V_O = V_B - V_T = -V_I = |V_I|.$$
4. Direction of the source current for $V_I < 0$: the current leaves the bridge at T and flows back into the source's $+$ terminal, which is opposite to the marked I_I arrow. Its size is $\frac{|V_I|}{R}$, so
$$I_I = -\frac{|V_I|}{R} = \frac{V_I}{R} < 0.$$
5. Output voltage: in both cases $V_O = |V_I|$. This is a V-shape through the origin that reaches M at both $V_I = -M$ and $V_I = M$. So (A) is correct and (B) is wrong: (B) would need V_O to go negative, but the current through R always flows from Q to P, so $V_O \geq 0$.
6. Input current: $I_I = \frac{V_I}{R}$ for every V_I. The bridge reverses the connection of R when V_I changes sign, but the source still sees a plain resistance R, so the plot is a straight line from $\left(-M, -\frac{M}{R}\right)$ to $\left(M, \frac{M}{R}\right)$. (D) is correct and (C) is wrong.
7. Correct options: (A) and (D).

KEY IDEA

A full-wave bridge rectifies what the LOAD sees, not what the SOURCE sees. The current through R always flows the same way ($Q \to P$), so $V_O = |V_I|$. The current at the source terminals still reverses with V_I. With ideal diodes the source sees the resistance R connected one way round or the other, so $I_I = V_I/R$ is linear and odd.

WATCH OUT FOR

Picking (C) by assuming that since the output is rectified, the input current is rectified too. The load current $V_O/R = |V_I|/R$ is always positive, but for $V_I < 0$ the source current flows against the marked I_I arrow, so I_I is negative. Picking (B) is the other slip: it treats the bridge as a straight-through connection and forgets that the two diodes conducting for $V_I < 0$ swap which end of R connects to which terminal.

Q.58 NAT · 2 MARKS

Engineering Mathematics › Probability & statistics

Two fair dice (with faces labeled 1, 2, 3, 4, 5, and 6) are rolled. Let the random variable X denote the sum of the outcomes obtained.

The expectation of X is _____ (rounded off to two decimal places).

Official answer: **6.95 to 7.05**

SOLUTION

1. Write $X = X_1 + X_2$, where X_1 and X_2 are the faces shown by the first and second die.
2. Each face of a fair die has probability $\frac{1}{6}$, so $E[X_1] = \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = 3.5$. The same holds for X_2.
3. By linearity of expectation, which holds whether or not the dice are independent, $E[X] = E[X_1] + E[X_2] = 3.5 + 3.5 = 7$.
4. Check with the distribution of the sum. Of the 36 equally likely outcomes, $6 - |k-7|$ give the sum k, so $P(X = k) = \frac{6 - |k-7|}{36}$ for $k = 2, \ldots, 12$. These probabilities add to

$\frac{1+2+3+4+5+6+5+4+3+2+1}{36} = 1$. The distribution is symmetric about $k = 7$, so its mean is 7.

5. Rounded off to two decimal places: $E[X] = 7.00$.

KEY IDEA

Linearity of expectation: the mean of a sum is the sum of the means, $E[X_1 + X_2] = E[X_1] + E[X_2]$, so there is no need to build the distribution of the sum. One fair die has mean 3.5, so two dice have mean 7.

WATCH OUT FOR

Averaging only the 11 possible sums, 2 to 12, as if each were equally likely. The mean happens to come out as 7 here, but that is luck from symmetry: the sums are not equally likely ($P(X = 7) = \frac{6}{36}$ while $P(X = 2) = \frac{1}{36}$), and the same method fails for any non-symmetric case. A second mistake is thinking linearity needs the dice to be independent. It does not; independence matters for the variance, not the mean.

Q.59 NAT · 2 MARKS

Engineering Mathematics › Linear algebra

Consider the vectors

$$\boldsymbol{a} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad \boldsymbol{b} = \begin{bmatrix} 0 \\ 3\sqrt{2} \end{bmatrix}.$$

For real-valued scalar variable x, the value of

$$\min_{x} \|\boldsymbol{a}x - \boldsymbol{b}\|_2$$

is _____ (rounded off to two decimal places).

$\|\cdot\|_2$ denotes the Euclidean norm, i.e., for $\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, $\|\boldsymbol{y}\|_2 = \sqrt{y_1^2 + y_2^2}$.

Official answer: **2.95 to 3.05**

SOLUTION

1. Minimising $\|\boldsymbol{a}x - \boldsymbol{b}\|_2$ over the scalar x is a least-squares problem: the minimiser makes $\boldsymbol{a}x$ the orthogonal projection of $\boldsymbol{b}$ onto the line spanned by $\boldsymbol{a}$, so the residual $\boldsymbol{a}x - \boldsymbol{b}$ must be perpendicular to $\boldsymbol{a}$.
2. Normal equation $\boldsymbol{a}^T(\boldsymbol{a}x - \boldsymbol{b}) = 0$ gives
$$x^* = \frac{\boldsymbol{a}^T\boldsymbol{b}}{\boldsymbol{a}^T\boldsymbol{a}} = \frac{1 \cdot 0 + 1 \cdot 3\sqrt{2}}{1^2 + 1^2} = \frac{3\sqrt{2}}{2}.$$
(Equivalently, $\|\boldsymbol{a}x - \boldsymbol{b}\|_2^2 = x^2 + (x - 3\sqrt{2})^2 = 2x^2 - 6\sqrt{2}\,x + 18$, a parabola opening upward with vertex at $x = \frac{6\sqrt{2}}{4} = \frac{3\sqrt{2}}{2}$.)
3. Residual at the minimiser:
$$\boldsymbol{a}x^* - \boldsymbol{b} = \begin{bmatrix} \frac{3\sqrt{2}}{2} \\ \frac{3\sqrt{2}}{2} \end{bmatrix} - \begin{bmatrix} 0 \\ 3\sqrt{2} \end{bmatrix} = \begin{bmatrix} \frac{3\sqrt{2}}{2} \\ -\frac{3\sqrt{2}}{2} \end{bmatrix}.$$
Check: $\boldsymbol{a}^T(\boldsymbol{a}x^* - \boldsymbol{b}) = \frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2} = 0$, so the residual is indeed perpendicular to $\boldsymbol{a}$.
4. Norm of the residual:
$$\sqrt{\left(\tfrac{3\sqrt{2}}{2}\right)^2 + \left(\tfrac{3\sqrt{2}}{2}\right)^2} = \sqrt{\tfrac{9}{2} + \tfrac{9}{2}} = \sqrt{9} = 3.$$
5. Check by Pythagoras: $\|\boldsymbol{b}\|_2^2 - \frac{(\boldsymbol{a}^T\boldsymbol{b})^2}{\boldsymbol{a}^T\boldsymbol{a}} = 18 - \frac{18}{2} = 9$, so the minimum distance is $\sqrt{9} = 3.00$.

KEY IDEA

The smallest value of $\|\boldsymbol{a}x - \boldsymbol{b}\|_2$ is the distance from $\boldsymbol{b}$ to the line spanned by $\boldsymbol{a}$. It is reached at the orthogonal projection, $x^* = \dfrac{\boldsymbol{a}^T\boldsymbol{b}}{\boldsymbol{a}^T\boldsymbol{a}}$. The minimum distance squared is $\|\boldsymbol{b}\|_2^2 - \dfrac{(\boldsymbol{a}^T\boldsymbol{b})^2}{\boldsymbol{a}^T\boldsymbol{a}}$.

WATCH OUT FOR

Students often report the minimiser $x^* = \dfrac{3\sqrt{2}}{2} \approx 2.12$ when the question asks for the minimum value of the norm, which is 3. Another slip is to report the squared norm, 9. A third is to forget to divide by $\boldsymbol{a}^T\boldsymbol{a} = 2$, which gives $x^* = 3\sqrt{2}$ and a residual of norm $3\sqrt{2} \approx 4.24$.

Q.60 NAT · 2 MARKS

Communications › Information theory

X and Y are Bernoulli random variables taking values in $\{0, 1\}$. The joint probability mass function of the random variables is given by: $P(X = 0, Y = 0) = 0.06$, $P(X = 0, Y = 1) = 0.14$, $P(X = 1, Y = 0) = 0.24$, $P(X = 1, Y = 1) = 0.56$. The mutual information $I(X; Y)$ is _____ (rounded off to two decimal places).

Official answer: **0.0**

SOLUTION

1. Marginal of X: $P(X = 0) = 0.06 + 0.14 = 0.20$ and $P(X = 1) = 0.24 + 0.56 = 0.80$.
2. Marginal of Y: $P(Y = 0) = 0.06 + 0.24 = 0.30$ and $P(Y = 1) = 0.14 + 0.56 = 0.70$.
3. Mutual information is $I(X; Y) = \sum_{x,y} P(x, y) \log_2 \dfrac{P(x, y)}{P(x)P(y)}$. Each term is zero when $P(x, y) = P(x)P(y)$, and $I(X; Y) = 0$ exactly when this holds for every pair.
4. Check each cell: $0.2 \times 0.3 = 0.06$, $0.2 \times 0.7 = 0.14$, $0.8 \times 0.3 = 0.24$, $0.8 \times 0.7 = 0.56$. Every product equals the given joint probability.
5. So X and Y are independent and every log term is $\log_2 1 = 0$. Hence $I(X; Y) = 0$ bits, which is 0.00 to two decimal places.

KEY IDEA

Mutual information is zero exactly when the joint PMF equals the product of its marginals. Compute the marginals and test this before doing any entropy arithmetic.

WATCH OUT FOR

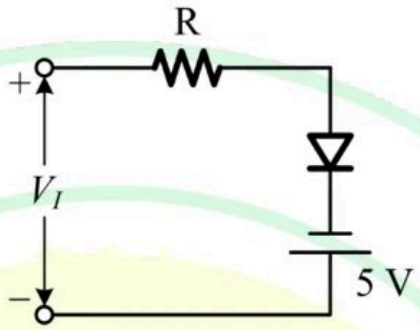
Computing $H(X)$, $H(Y)$ and $H(X, Y)$ by hand and letting rounding errors leave a small non-zero value. Another slip is to take the joint probabilities as a dependent pair without checking the products of the marginals.

Q.61 NAT · 2 MARKS

Analog Electronics › Diode circuits

Q.61 The diode in the circuit shown below is ideal. The input voltage (in Volts) is given by $V_I = 10 \sin 100\pi t$, where time t is in seconds.

The time duration (in ms, *rounded off to two decimal places*) for which the diode is forward biased during one period of the input is _____.



Official answer: **13.32 to 13.34**

SOLUTION

1. **Battery polarity.** In the figure the short plate (negative terminal) is on top and connects to the diode cathode. The long plate (positive terminal) is at the bottom and connects to the bottom rail, which is the $-$ terminal of V_I. Taking that rail as reference, the cathode sits at $V_K = -5$ V.
2. **Condition for the ideal diode to conduct.** The diode points downward, with its anode towards R and its cathode towards the battery. KVL around the loop gives $V_I = iR + v_D - 5$. If the diode is OFF, $i = 0$, so there is no drop across R and the anode is at V_I. The diode is forward biased when its anode is above its cathode, i.e. when $V_I > -5$ V. This is consistent with the ON state: with $v_D = 0$ the current $i = \dfrac{V_I + 5}{R}$ is positive only when $V_I > -5$ V.
3. **Condition in terms of phase.** Let $\theta = 100\pi t$. Then $10 \sin\theta > -5 \Rightarrow \sin\theta > -\dfrac{1}{2}$.
4. **Portion of the cycle.** Over one cycle $0 \le \theta < 2\pi$, $\sin\theta < -\frac{1}{2}$ only for $\dfrac{7\pi}{6} < \theta < \dfrac{11\pi}{6}$. That interval is $\dfrac{2\pi}{3}$ rad long. So the diode is forward biased for $2\pi - \dfrac{2\pi}{3} = \dfrac{4\pi}{3}$ rad, which is $\dfrac{2}{3}$ of each cycle.
5. **Period.** $\omega = 100\pi$ rad/s $\Rightarrow T = \dfrac{2\pi}{100\pi} = 0.02$ s $= 20$ ms.
6. **Conduction time.**

$$t_{ON} = \frac{2}{3} \times 20 \text{ ms} = \frac{40}{3} \text{ ms} \approx 13.33 \text{ ms}$$

As a check, the diode is OFF from $t = \dfrac{7}{600}$ s $= 11.67$ ms to $t = \dfrac{11}{600}$ s $= 18.33$ ms. That is 6.67 ms OFF, which leaves $20 - 6.67 = 13.33$ ms ON.

KEY IDEA

An ideal diode conducts whenever its anode is above its cathode. Read the battery polarity from the figure: the long plate is +, and here it is grounded. That puts the cathode at −5 V, so the diode conducts whenever $V_I > -5$ V. Turn this into a phase condition, $\sin\theta > -1/2$, which holds for $4\pi/3$ of every 2π rad. The diode therefore conducts for 2/3 of the 20 ms period.

WATCH OUT FOR

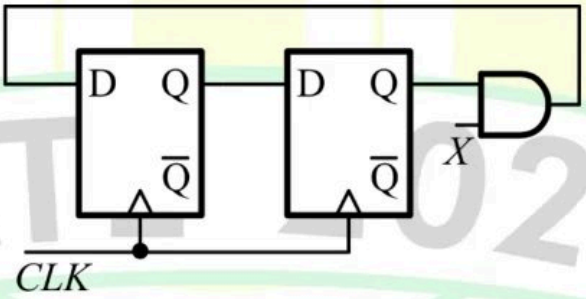
The usual slip is misreading the battery polarity and taking the cathode as +5 V. That leads to the condition $\sin\theta > 1/2$, conduction for only 1/3 of the cycle, and an answer of 6.67 ms. The other slip is dropping the battery altogether and treating the circuit as a plain half-wave rectifier, which gives 10 ms.

Q.62 NAT · 2 MARKS

Digital Electronics › Sequential

Q.62 In the circuit shown below, the AND gate has a propagation delay of 1 ns. The edge-triggered flip-flops have a set-up time of 2 ns, a hold-time of 0 ns, and a clock-to-Q delay of 2 ns.

The maximum clock frequency (in MHz, *rounded off to the nearest integer*) such that there are no setup violations is _____.



Official answer: **200**

SOLUTION

1. Both flip-flops are clocked by the same CLK edge, so there are two flip-flop-to-flip-flop paths to check: FF1 → FF2 (a direct wire from Q of FF1 to D of FF2), and FF2 → AND gate → FF1 (the feedback path from Q of FF2, through the AND gate, back to D of FF1).
2. For any register-to-register path, the data launched at one clock edge must settle at the next flip-flop's D input at least one set-up time before the following edge: $T_{clk} \geq t_{clk\text{-}Q} + t_{comb} + t_{setup}$.
3. Path FF1 → FF2 has no logic in between: $T_{clk} \geq 2 + 0 + 2 = 4$ ns.
4. Path FF2 → AND → FF1 passes through the AND gate: $T_{clk} \geq 2 + 1 + 2 = 5$ ns.
5. The clock has to satisfy the slower path, so $T_{min} = \max(4, 5) = 5$ ns.
6. $$f_{max} = \frac{1}{T_{min}} = \frac{1}{5\text{ ns}} = 200\text{ MHz}$$
7. The hold time (0 ns) only enters hold checks, not the set-up-limited maximum frequency. The external input X has no timing given, so it is taken to be stable in time and does not limit the clock.

KEY IDEA

Check every path from one flip-flop to the next with $T_{clk} \geq t_{clk\text{-}Q} + t_{comb} + t_{setup}$. The path with the most logic in it (here the feedback loop through the AND gate) sets the minimum clock period, and $f_{max} = 1/T_{min}$.

WATCH OUT FOR

One mistake is checking only the direct FF1 → FF2 wire, which gives 4 ns and 250 MHz. Another is adding the hold time into the set-up equation, or adding the delays of both paths together (2 + 2 + 1 + 2 + 2 = 9 ns) as if the data had to cross both flip-flops in one clock cycle.

Q.63 NAT · 2 MARKS

Electronic Devices › P-N junction

An ideal p-n junction germanium diode has a reverse saturation current of 10 μA at 300 K.

The voltage (in Volts, rounded off to two decimal places) to be applied across the junction to get a forward bias current of 100 mA at 300 K is _____.

(Consider the Boltzmann constant $k_B = 1.38 \times 10^{-23}\ \mathrm{J/K}$ and the charge of an electron $e = 1.6 \times 10^{-19}\ \mathrm{C}$.)

Official answer: **0.23 to 0.24**

SOLUTION

1. Diode law in forward bias: $I = I_S\left(e^{V/V_T} - 1\right)$, with $V_T = \dfrac{k_B T}{q}$.
2. Compute $V_T = \dfrac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} = 0.025875\ \mathrm{V}$.
3. Since $I = 100\ \mathrm{mA} \gg I_S = 10\ \mu\mathrm{A}$ (ratio 10^4), the -1 term is negligible: $I \approx I_S\, e^{V/V_T}$.
4. So $V = V_T \ln\left(\dfrac{I}{I_S}\right) = V_T \ln\left(\dfrac{100 \times 10^{-3}}{10 \times 10^{-6}}\right) = V_T \ln(10^4)$.
5. $\ln(10^4) = 4\ln(10) = 9.21034$.
6. $V = 0.025875 \times 9.21034 = 0.238317\ \mathrm{V}$.
7. Rounded to two decimal places: $V \approx 0.24\ \mathrm{V}$.

KEY IDEA

In strong forward bias, $I \gg I_S$, so the diode equation $I = I_S(e^{V/V_T} - 1)$ reduces to $I \approx I_S e^{V/V_T}$, giving $V = V_T \ln(I/I_S)$ with $V_T = k_B T/q$.

WATCH OUT FOR

Rounding $\ln(10^4)$ too early (e.g. using $\ln 10 \approx 2.3$ instead of 2.302585) or misplacing a decimal in V_T shifts the final answer's second decimal place, since the true value (0.2383 V) sits close to the 0.235 rounding boundary.

Q.64 NAT · 2 MARKS

Electromagnetics › Transmission lines

A $50\ \Omega$ lossless transmission line is terminated with a load Z_L of $(50 - j75)\ \Omega$.

If the average incident power on the line is $10\ \mathrm{mW}$, then the average power delivered to the load (in mW, *rounded off to one decimal place*) is _____.

Official answer: **6.3 to 6.5**

SOLUTION

1. Reflection coefficient at the load, with $Z_0 = 50\ \Omega$ and $Z_L = (50 - j75)\ \Omega$:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(50 - j75) - 50}{(50 - j75) + 50} = \frac{-j75}{100 - j75}$$

2. Power depends only on the magnitude of Γ_L:

$$|\Gamma_L|^2 = \frac{75^2}{100^2 + 75^2} = \frac{5625}{15625} = 0.36 \quad (|\Gamma_L| = 0.6)$$

3. On a lossless line the reflected average power is $P_{ref} = |\Gamma_L|^2 P_{inc} = 0.36 \times 10 = 3.6\ \text{mW}$.
4. The line absorbs nothing, so power balance gives the power delivered to the load:
$$P_L = P_{inc} - P_{ref} = P_{inc}\left(1 - |\Gamma_L|^2\right) = 10 \times (1 - 0.36) = 6.4\ \text{mW}$$
5. Check with the load resistance directly:
$$1 - |\Gamma_L|^2 = \frac{4R_L Z_0}{|Z_L + Z_0|^2} = \frac{4 \times 50 \times 50}{100^2 + 75^2} = \frac{10000}{15625} = 0.64$$
which gives the same $P_L = 6.4\ \text{mW}$.

KEY IDEA

On a lossless line the load receives the incident power minus the reflected power: $P_L = P_{inc}(1 - |\Gamma_L|^2)$. Only $|\Gamma_L|$ matters. Here $|\Gamma_L|^2 = 75^2/(100^2 + 75^2) = 0.36$, so the load takes 64% of the incident power. The phase of Γ_L does not enter.

WATCH OUT FOR

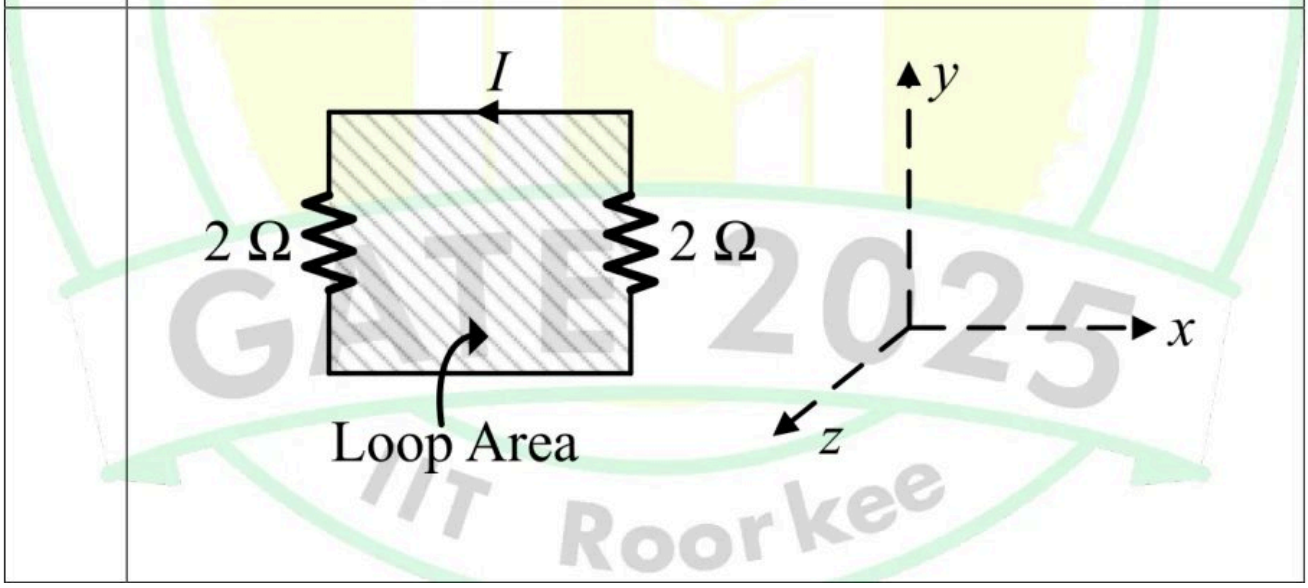
Using $|\Gamma_L|$ in place of $|\Gamma_L|^2$ gives $10(1 - 0.6) = 4\ \text{mW}$, which is wrong because power goes as the square of the voltage reflection coefficient. Other errors: reading the reflected power (3.6 mW) as the answer, or thinking the reactive part $-j75$ takes power. Only the resistive part of Z_L absorbs power, and the reactance only sets how much power is reflected.

Q.65 NAT · 2 MARKS

Electromagnetics › Maxwell's equations

Q.65 Two resistors are connected in a circuit loop of area 5 m^2, as shown in the figure below. The circuit loop is placed on the x-y plane.

When a time-varying magnetic flux, with flux-density $B(t) = 0.5t$ (in Tesla), is applied along the positive z-axis, the magnitude of current I (in Amperes, *rounded off to two decimal places*) in the loop is _____.



Official answer: **0.62 to 0.63**

SOLUTION

1. **Flux through the loop.** The field $\mathbf{B} = 0.5t\,\hat{a}_z\ \text{T}$ is uniform and the loop lies in the x-y plane, so $\mathbf{B}$ is normal to the loop over its whole area $A = 5\ \text{m}^2$:
$$\Phi(t) = B(t)\,A = 0.5t \times 5 = 2.5t\ \text{Wb}$$
2. **Faraday's law**, $\mathcal{E} = -\dfrac{d\Phi}{dt}$:

$$|\mathcal{E}| = \frac{d}{dt}(2.5t) = 2.5 \text{ V}$$

The emf does not change with time.

3. **Loop resistance.** Both $2\ \Omega$ resistors sit in the same single loop, and the connecting wires are ideal, so the resistors are in series: $R = 2 + 2 = 4\ \Omega$.
4. **Ohm's law around the loop:**
$$|I| = \frac{|\mathcal{E}|}{R} = \frac{2.5}{4} = 0.625 \text{ A} \approx 0.63 \text{ A}$$
5. **Direction (Lenz's law, for completeness).** With x to the right and y up, the right-handed z-axis points out of the page. The flux along $+\hat{a}_z$ is increasing, so the induced current sets up a field along $-\hat{a}_z$ (into the page). Seen from $+z$, that current circulates clockwise, so it runs left-to-right along the top wire. That is opposite to the arrow marked I, which is why the question asks only for the magnitude.

KEY IDEA

A uniform flux density that rises linearly with time gives a constant motional-free (transformer) emf, $|\mathcal{E}| = A\dfrac{dB}{dt}$. Every element in a single closed loop carries the same current, so the emf drives that current through the total series resistance.

WATCH OUT FOR

Treating the two $2\ \Omega$ resistors as parallel because they are drawn on opposite sides, which gives $R = 1\ \Omega$ and $I = 2.5$ A. There is only one loop, so they are in series. A second slip is using $B = 0.5t$ itself as the flux and forgetting to multiply by the $5\ \text{m}^2$ area, which gives 0.125 A.

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