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GATE EC · 2023

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GATE EC 2023 Solved Paper

INSIDE

- All 65 questions, in the order of the paper
- The official answer to every question
- Step-by-step worked solutions
- The answer key on one page

Question paper and official answer key: IIT Kanpur

Online, with the questions linked to their topic notes: **iaeacademy.in/study/papers/ec-2023**

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About this paper

65 questions, 100 marks. Question types: 45 MCQ · 4 MSQ · 16 NAT.

Official answer key: IIT Kanpur · https://gate.iitk.ac.in/GATE2023/doc/papers/2023/ANS/EC_ANS_GATE2023.pdf

We placed each question under a subject and topic ourselves; the exam paper does not label them.

This paper online, with the questions linked to their topic notes:
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Answer key

Q	TYPE	MARKS	ANSWER
Q.1	MCQ	1	**C**
Q.2	MCQ	1	**A**
Q.3	MCQ	1	**C**
Q.4	MCQ	1	**C**
Q.5	MCQ	1	**C**
Q.6	MCQ	2	**D**
Q.7	MCQ	2	**A**
Q.8	MCQ	2	**B**
Q.9	MCQ	2	**A**
Q.10	MCQ	2	**C**
Q.11	MCQ	1	**C**
Q.12	MCQ	1	**B**
Q.13	MCQ	1	**A**
Q.14	MCQ	1	**B**
Q.15	MCQ	1	**C**
Q.16	MCQ	1	**A**
Q.17	MCQ	1	**A**
Q.18	MCQ	1	**D**
Q.19	MCQ	1	**B**
Q.20	MCQ	1	**D**
Q.21	MCQ	1	**D**
Q.22	MCQ	1	**Marks to all**
Q.23	MCQ	1	**A**
Q.24	MCQ	1	**A**
Q.25	MCQ	1	**A**
Q.26	MCQ	1	**B**
Q.27	MCQ	1	**B**
Q.28	MCQ	1	**C**
Q.29	MCQ	1	**B**
Q.30	MSQ	1	**A, B**
Q.31	NAT	1	**10**
Q.32	NAT	1	**1.30 to 1.40**
Q.33	NAT	1	**80**
Q.34	NAT	1	**02**
Q.35	NAT	1	**04**
Q.36	MCQ	2	**B**
Q.37	MCQ	2	**B**
Q.38	MCQ	2	**A**
Q.39	MCQ	2	**A**
Q.40	MCQ	2	**A**
Q.41	MCQ	2	**A**
Q.42	MCQ	2	**A**
Q.43	MCQ	2	**A**
Q.44	MCQ	2	**B**
Q.45	MCQ	2	**B**
Q.46	MCQ	2	**Marks to all**
Q.47	MCQ	2	**A**
Q.48	MCQ	2	**C**
Q.49	MCQ	2	**A**
Q.50	MCQ	2	**D**
Q.51	MCQ	2	**A**
Q.52	MSQ	2	**B, C**
Q.53	MSQ	2	**A, B, D**
Q.54	MSQ	2	**B, C, D**
Q.55	NAT	2	**0**
Q.56	NAT	2	**2.20 to 2.30**
Q.57	NAT	2	**4**
Q.58	NAT	2	**15**
Q.59	NAT	2	**0.24 to 0.26**
Q.60	NAT	2	**0.12 to 0.14**
Q.61	NAT	2	**60.00 to 70.00**
Q.62	NAT	2	**8.30 to 8.34**
Q.63	NAT	2	**250, or 500**
Q.64	NAT	2	**2.00**
Q.65	NAT	2	**1.97 to 1.99**

General Aptitude Q.1 to Q.10

Q.1 MCQ · 1 MARK

General Aptitude › Grammar & vocabulary

"I cannot support this proposal. My _____ will not permit it."

(A) conscious

(B) consensus

(C) conscience

(D) consent

Official answer: **C**

SOLUTION

1. Start with the grammar. The blank follows the possessive *My* and is the subject of *will not permit*, so it must be a **noun**.
2. (A) *conscious* is an **adjective** meaning aware or awake, as in "conscious of the risk". An adjective cannot stand alone after *My* as the subject of a verb, so (A) is out.
3. (B) *consensus* is a noun, but it means general agreement **within a group**. One person cannot have a consensus, so *My consensus* is not idiomatic. (B) is out.
4. (D) *consent* is a noun meaning the permission that **I give**. Consent is the permission itself; it does not act as an agent that *permits* or forbids anything. The idiomatic forms are *I will not give my consent* or *it does not have my consent*, so *My consent will not permit it* is not natural English. (D) is out.
5. (C) *conscience* is a noun meaning a person's inner sense of right and wrong. *My conscience will not permit it* is a fixed English expression for refusing something on moral grounds.
6. Check the meaning. The speaker refuses to support the proposal and gives a personal, moral reason. Only *conscience* fits both the grammar and the meaning, so the answer is **(C) conscience**.

KEY IDEA

The four options look alike, so check two things: part of speech, then meaning. The blank must be a noun that belongs to one person and can act as the subject of "will not permit". Only *conscience*, a person's inner moral sense, does all of that, and "my conscience will not permit it" is a fixed idiom.

WATCH OUT FOR

Picking (A) *conscious* because it sounds like *conscience*, when it is an adjective. The other trap is (D) *consent*, because refusing permission seems to fit. But consent is the permission itself, which you give or withhold; it cannot "permit" anything. (B) *consensus* is always a group's agreement, never one person's.

Q.2 MCQ · 1 MARK

General Aptitude › Reasoning

Courts : _____ : : Parliament : Legislature

(By word meaning)

(A) Judiciary

(B) Executive

(C) Governmental

(D) Legal

Official answer: **A**

SOLUTION

1. Start with the right-hand pair, because it is complete. Parliament is the institution that makes up the **Legislature**, the organ of government that makes laws. So the relation is: institution : the organ of government it forms.

2. In an analogy the left-hand pair must stand in the same relation. So the blank must name the organ of government that the courts make up.
3. Government has three organs. The Legislature makes laws, through Parliament. The Executive carries out laws, through the ministers and the administration. The Judiciary interprets laws and decides disputes, through the courts.
4. The courts make up the **Judiciary**, so (A) fits: Courts : Judiciary : : Parliament : Legislature.
5. Why the others fail. (B) Executive is an organ of government, but it is the one that runs the administration, not the one the courts form. (C) Governmental and (D) Legal are adjectives, not names of organs of government, so they cannot stand in the place that the noun Legislature holds in the other pair.

KEY IDEA

Name the relation in the complete pair first. Parliament is the institution that forms the Legislature, so the blank must be the organ of government that the courts form, and that is the Judiciary. Of the three organs of government (Legislature, Executive, Judiciary), only the Judiciary works through the courts.

WATCH OUT FOR

Picking (D) Legal because courts deal with law. The link then goes from an institution to its subject matter, which is not the institution-to-organ link that Parliament : Legislature sets up, and 'Legal' is an adjective while 'Legislature' is a noun. Some candidates pick (B) Executive only because it is the other organ of government they remember. But the Executive carries out laws. It does not decide cases.

Q.3 MCQ · 1 MARK

General Aptitude › Numerical

What is the smallest number with distinct digits whose digits add up to 45?

(A) 123555789

(B) 123457869

(C) 123456789

(D) 99999

Official answer: **C**

SOLUTION

1. Check the options against the "distinct digits" condition first. (A) 123555789 uses the digit 5 three times and (D) 99999 uses 9 five times, so both are ruled out even though their digits do add up to 45.
2. Find the fewest digits that can work. The largest sum you can get from 8 distinct digits is $9+8+7+6+5+4+3+2=44<45$. So no number with 8 or fewer distinct digits reaches 45, and the number needs at least 9 digits.
3. The ten digits add up to $0+1+2+\cdots+9=\dfrac{9\times10}{2}=45$. If 9 distinct digits add up to 45, the one digit left out must be $45-45=0$. So the digits are exactly $1,2,\ldots,9$, each used once.
4. Using 0 as a tenth digit keeps the sum at 45, but the number then has 10 digits, and every 10-digit number is larger than every 9-digit number. So the smallest number has exactly 9 digits.
5. Once the number of digits is fixed, the smallest arrangement puts the digits in ascending order, so the smallest digit sits in the highest place value. That gives 123456789.
6. Check (B): 123457869 uses the same nine digits. It agrees with 123456789 in the first five digits, but its sixth digit is 7 where (C) has 6, so (B) is larger. The smallest such number is 123456789, which is option (C).

KEY IDEA

The ten digits 0 to 9 add up to exactly 45, and any 8 distinct digits add up to at most 44. So the number must use all of 1 to 9 exactly once. Leaving out 0 keeps it to 9 digits, and writing the digits in ascending order makes it as small as possible.

WATCH OUT FOR

Picking (D) 99999 because it is the shortest number whose digits add up to 45, without noticing that it breaks the distinct-digits condition. Two smaller slips: adding 0 as a tenth digit (the number gets longer, so it gets larger), and choosing (B) without comparing it digit by digit with (C).

Q.4 MCQ · 1 MARK

General Aptitude › Numerical

In a class of 100 students,

(i) there are 30 students who neither like romantic movies nor comedy movies,

(ii) the number of students who like romantic movies is twice the number of students who like comedy movies, and

(iii) the number of students who like both romantic movies and comedy movies is 20.

How many students in the class like romantic movies?

(A) 40

(B) 20

(C) 60

(D) 30

Official answer: **C**

SOLUTION

1. Let R be the set of students who like romantic movies and C the set of students who like comedy movies. Write $x = n(C)$. From (ii), $n(R) = 2x$. From (iii), $n(R \cap C) = 20$.
2. Complement rule: the students who like at least one of the two kinds of movie are all the students except the 30 who like neither (condition (i)), so $n(R \cup C) = 100 - 30 = 70$.
3. Inclusion–exclusion for two sets: $n(R \cup C) = n(R) + n(C) - n(R \cap C)$.
4. Substitute:
$$2x + x - 20 = 70 \;\Rightarrow\; 3x = 90 \;\Rightarrow\; x = 30$$
So 30 students like comedy movies.
5. Hence the number who like romantic movies is $n(R) = 2x = 60$.
6. Check: romantic only $= 60 - 20 = 40$, comedy only $= 30 - 20 = 10$, both $= 20$, neither $= 30$; total $= 40 + 10 + 20 + 30 = 100$, and $60 = 2 \times 30$ as required by (ii). The answer is 60, option (C).

KEY IDEA

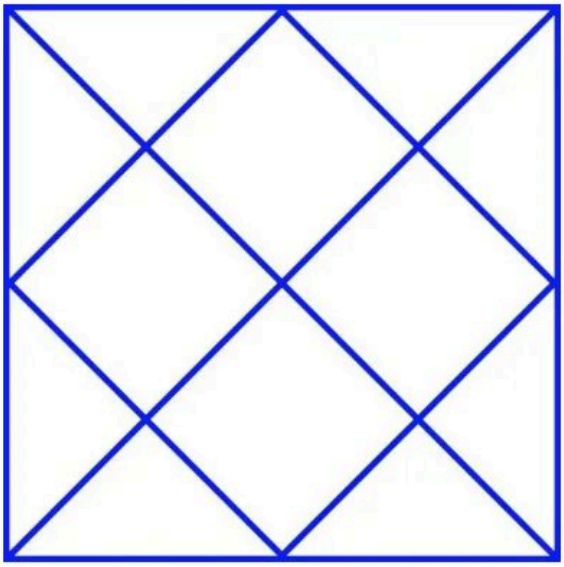
Two-set inclusion–exclusion: $n(R \cup C) = n(R) + n(C) - n(R \cap C)$. Get $n(R \cup C)$ by subtracting the 'neither' group from the total. Then the ratio condition leaves one unknown and one equation.

WATCH OUT FOR

One slip is to forget to subtract the overlap: $2x + x = 70$ gives $x = 23.33$, which is not a whole number. Another is to read 'twice' as applying to the students who like only romantic and only comedy movies. That gives $r_{\text{only}} = 2c_{\text{only}}$, so $3c_{\text{only}} = 50$, which is also not a whole number. Some candidates also mark 40, which is the number who like ONLY romantic movies. The question asks for everyone who likes romantic movies, so the overlap must be counted too.

Q.5 MCQ · 1 MARK

General Aptitude › Spatial

Q.5	How many rectangles are present in the given figure?
	
(A)	8
(B)	9
(C)	10
(D)	12

Official answer: **C**

SOLUTION

1. Sort the lines by direction. Horizontal and vertical: only the four sides of the outer square. Slanting one way (↗): the diagonal DB and the diamond sides SP and RQ. Slanting the other way (↘): the diagonal AC and the diamond sides SR and PQ.
2. A rectangle needs every corner to be a right angle. The horizontal and vertical lines meet each other at 90° but meet the slanting lines at 45°, so a rectangle cannot mix the two kinds of line. The only rectangle made from horizontal and vertical lines is the outer square $ABCD$: **1**.
3. The six slanting lines form a tilted grid: 3 parallel lines one way (SP, DB, RQ) and 3 parallel lines the other way (SR, AC, PQ), and the two families are perpendicular. Each line of one family crosses every line of the other family at a point that is actually drawn (the 9 crossings are S, P, Q, R, the centre O and the four midpoints of the diamond's sides). So the diamond is a 2×2 grid of small squares.

4. Rule: a grid formed by m parallel lines crossing n perpendicular parallel lines, with every crossing drawn, contains $\binom{m}{2}\binom{n}{2}$ rectangles, one for each choice of two lines from each family.
5. Tilted rectangles: $\binom{3}{2} \times \binom{3}{2} = 3 \times 3 = 9$. These are the 4 small squares, the 4 rectangles made of two small squares side by side (2 in each direction), and the diamond $PQRS$ itself: $4 + 4 + 1 = 9$.
6. Outside the diamond, the diagonals only cut the four corner triangles in two and have no parallel partner there, so they form no further rectangle.
7. Total $= 1 + 9 = 10$, which is option (C).

KEY IDEA

A rectangle's sides come from two pairs of perpendicular parallel lines. Sort the lines into perpendicular families. A grid of m lines crossing n lines contains $\binom{m}{2}\binom{n}{2}$ rectangles. Here that is $\binom{3}{2}\binom{3}{2} = 9$ for the tilted 2×2 grid, plus the outer square: 10.

WATCH OUT FOR

Counting only the 4 small squares, or forgetting that a square is also a rectangle, so the 2x1 rectangles, the diamond $PQRS$ or the outer square $ABCD$ get left out. Answers of 8 or 9 usually come from missing the outer square or the diamond. Another mistake is counting the corner triangles or treating a diagonal as the side of an extra rectangle outside the diamond.

Q.6 MCQ · 2 MARKS

General Aptitude › Reasoning

Forestland is a planet inhabited by different kinds of creatures. Among other creatures, it is populated by animals all of whom are ferocious. There are also creatures that have claws, and some that do not. All creatures that have claws are ferocious.

Based only on the information provided above, which one of the following options can be logically inferred with *certainty*?

(A) All creatures with claws are animals.

(B) Some creatures with claws are non-ferocious.

(C) Some non-ferocious creatures have claws.

(D) Some ferocious creatures are creatures with claws.

Official answer: **D**

SOLUTION

1. Write the facts as statements. (1) All animals are ferocious. (2) Creatures with claws exist, and so do creatures without claws. (3) All creatures that have claws are ferocious.
2. Draw it as sets. The set of animals lies inside the set of ferocious creatures, and so does the set of clawed creatures. Nothing is said about how animals and clawed creatures overlap.
3. (A) is not certain. Animals and clawed creatures both lie inside the ferocious set, but two subsets of one set can be disjoint, can overlap, or one can contain the other. Sharing the property 'ferocious' does not make a clawed creature an animal. This is the fallacy of the undistributed middle.
4. (B) is false. 'All creatures with claws are ferocious' and 'Some creatures with claws are non-ferocious' are contradictories: if one is true, the other must be false. Statement (3) is given as true, so (B) is false.
5. (C) is false for the same reason. 'Some non-ferocious creatures have claws' says the same thing as (B), because 'Some X are Y' and 'Some Y are X' are equivalent. Statement (3) rules it out.
6. (D) is certain. By statement (2), at least one clawed creature exists, and by statement (3) it is ferocious. That creature is both ferocious and clawed, so 'Some ferocious creatures are creatures with claws' follows. This converts 'All C are F' into 'Some F are C', which is valid only because the clawed set is known to be non-empty.
7. Only (D) follows with certainty.

KEY IDEA

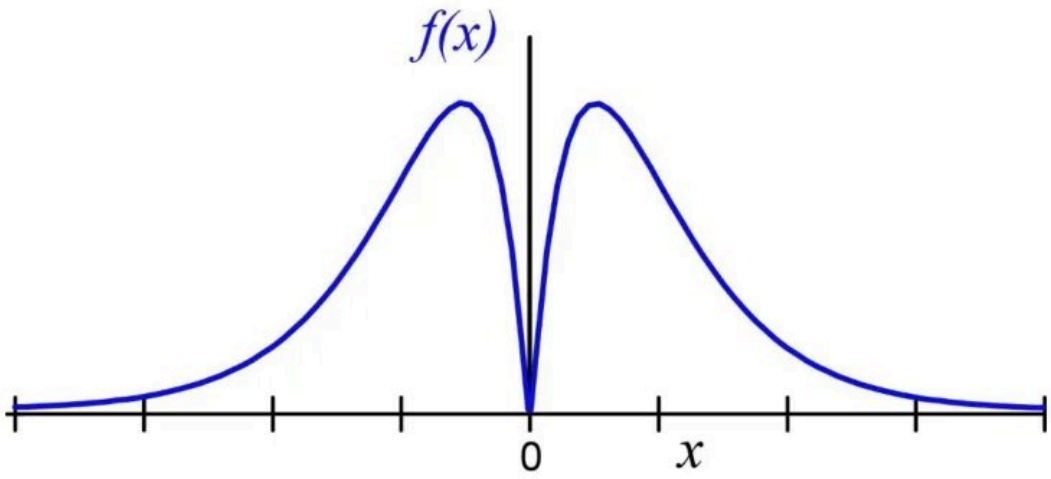
From 'All C are F' you can infer 'Some F are C' only if at least one C exists. The stem gives that directly ("There are also creatures that have claws"), so option (D) is guaranteed. Being inside the same larger set (ferocious) says nothing about how animals and clawed creatures relate to each other.

WATCH OUT FOR

Choosing (A) by reasoning 'animals are ferocious and clawed creatures are ferocious, so clawed creatures are animals'. That is the undistributed middle, because two groups that share a property need not overlap. A second trap is rejecting (D) on the grounds that a universal statement does not prove anything exists. The stem states outright that clawed creatures exist, so the existence condition is met.

Q.7 MCQ · 2 MARKS

General Aptitude › Numerical

Q.7	Which one of the following options represents the given graph?
	
(A)	$f(x) = x^2 2^{-\lvert x\rvert}$
(B)	$f(x) = x\, 2^{-\lvert x\rvert}$
(C)	$f(x) = \lvert x\rvert 2^{-x}$
(D)	$f(x) = x\, 2^{-x}$

Official answer: **A**

SOLUTION

1. Read the graph's features: it is symmetric about the $f(x)$-axis (an even function), $f(0) = 0$, $f(x) \geq 0$ for every x, and $f(x) \to 0$ both as $x \to +\infty$ and as $x \to -\infty$.
2. Test each option for evenness, i.e. $f(-x) = f(x)$. (B) $f(-x) = -x\, 2^{-\lvert -x\rvert} = -x\, 2^{-\lvert x\rvert} = -f(x)$, so it is odd, not even. (D) $f(-x) = -x\, 2^{x}$, which is not equal to $f(x)$. (C) $f(-x) = \lvert -x\rvert\, 2^{x} = \lvert x\rvert\, 2^{x} \neq \lvert x\rvert\, 2^{-x}$ for $x \neq 0$. Only (A) is even: $(-x)^2\, 2^{-\lvert -x\rvert} = x^2\, 2^{-\lvert x\rvert}$.
3. Check the sign at a negative point: at $x = -1$, (B) gives $-1 \cdot 2^{-1} = -0.5$ and (D) gives $-1 \cdot 2^{1} = -2$. Both go below the x-axis, but the graph never does.

4. Check the left tail: at $x = -4$, (C) gives $4 \cdot 2^4 = 64$ and (D) gives $-4 \cdot 2^4 = -64$. For negative x the factor 2^{-x} grows without bound, so neither (C) nor (D) can decay to 0 on the left the way the graph does.
5. Confirm that (A) has every feature: $f(0) = 0$; $x^2 \geq 0$ and $2^{-|x|} > 0$ make $f(x) \geq 0$; and exponential decay beats polynomial growth, so $x^2\, 2^{-|x|} \to 0$ as $|x| \to \infty$.
6. (A) also has the two humps. For $x > 0$, $f(x) = x^2\, 2^{-x}$ and $f'(x) = 2^{-x}\left(2x - x^2 \ln 2\right) = x\, 2^{-x}\,(2 - x \ln 2)$, which is zero at $x = \dfrac{2}{\ln 2} \approx 2.89$. By symmetry the other maximum is at $x \approx -2.89$, with $f(0) = 0$ as the minimum between them. The figure is a sketch, not drawn to scale: the axis ticks are unlabelled, so the peak positions cannot be checked against a scale, and near the origin $f(x) \approx x^2$, so the true minimum is flat ($f'(0) = 0$) rather than the sharp point drawn. The question is decided by symmetry, sign and decay, and only (A) has all three.
7. Hence the graph is $f(x) = x^2\, 2^{-|x|}$, option (A).

KEY IDEA

Match a sketch to a formula by its qualitative features, not by plotting. Here there are three: symmetry (an even function needs $f(-x) = f(x)$), sign (the curve is never negative) and end behaviour (it decays to 0 on both sides). Only $x^2\, 2^{-|x|}$ is even, never negative and decaying at both ends, because $2^{-|x|}$ decays whichever way x goes and beats the polynomial factor.

WATCH OUT FOR

Looking only at the right half of the graph, where all four options behave alike (zero at 0, a rise, then decay), and then picking (B) or (C). Test a negative value of x: (B) and (D) go negative there, and (C) and (D) blow up because 2^{-x} grows as $x \to -\infty$. A second trap is rejecting (A) because the sketch shows a sharp point at the origin while $x^2\, 2^{-|x|}$ is actually flat there. The figure is only a qualitative sketch.

Q.8 MCQ · 2 MARKS

General Aptitude › Reading

Which one of the following options can be inferred from the given passage alone?

When I was a kid, I was partial to stories about other worlds and interplanetary travel. I used to imagine that I could just gaze off into space and be whisked to another planet.

[Excerpt from *The Truth about Stories* by T. King]

(A) It is a child's description of what he or she likes.

(B) It is an adult's memory of what he or she liked as a child.

(C) The child in the passage read stories about interplanetary travel only in parts.

(D) It teaches us that stories are good for children.

Official answer: **B**

SOLUTION

1. Rule for 'inferred from the given passage alone': accept an option only if the passage's own words guarantee it. Reject anything that needs outside knowledge, or a moral that the passage does not state.
2. Look at the time frame. "When I was a kid" and "I used to imagine" are both in the past tense, and "used to" marks a habit that no longer holds. So the narrator is no longer a kid and is looking back on his or her childhood.
3. (A) says a child is describing what he or she likes now. The opening words "When I was a kid" rule this out: the speaker places childhood in the past, so the speaker is no longer a child, and the likings are reported in the past tense ("was partial to"). Rejected.
4. (B) says an adult is remembering what he or she liked as a child. This follows directly. The speaker is past childhood, so grown up, and is recalling a childhood preference ("was partial to", "used to imagine"). Accepted.

5. (C) comes from misreading the idiom. "Partial to" means fond of. It has nothing to do with reading a story "in parts", and the passage says nothing about how much of each story was read. Rejected.
6. (D) draws a moral. The passage only reports a personal liking and a daydream. It never claims that stories are good for children. Rejected.
7. Only (B) is guaranteed by the passage. Answer: (B).

KEY IDEA

Look at the tense and the time markers. "When I was a kid" and "used to imagine" place childhood in the past, so the speaker is a grown-up looking back. "Partial to" is an idiom meaning fond of.

WATCH OUT FOR

Taking "partial to" literally as "in part" and picking (C). Also common: picking (A) without noticing that "When I was a kid" means the speaker is no longer a child, or picking (D), a moral the passage never states.

Q.9 MCQ · 2 MARKS

General Aptitude › Reasoning

Out of 1000 individuals in a town, 100 unidentified individuals are covid positive. Due to lack of adequate covid-testing kits, the health authorities of the town devised a strategy to identify these covid-positive individuals. The strategy is to:

(i) Collect saliva samples from all 1000 individuals and randomly group them into sets of 5.

(ii) Mix the samples within each set and test the mixed sample for covid.

(iii) If the test done in (ii) gives a negative result, then declare all the 5 individuals to be covid negative.

(iv) If the test done in (ii) gives a positive result, then all the 5 individuals are separately tested for covid.

Given this strategy, no more than _____ testing kits will be required to identify all the 100 covid positive individuals irrespective of how they are grouped.

(A) 700

(B) 600

(C) 800

(D) 1000

Official answer: **A**

SOLUTION

1. Number of sets: $\frac{1000}{5} = 200$. Step (ii) uses one kit on the mixed sample of every set, so 200 kits are always used.
2. A set needs more kits only if its mixed sample tests positive. Step (iv) then tests all 5 members separately, which uses 5 more kits.
3. If k sets test positive, the total is $200 + 5k$ kits. So the worst case ("irrespective of how they are grouped") is the grouping with the most positive sets.
4. A set tests positive only if it holds at least one positive person. There are 100 positive people and 200 sets, so at most $\min(200,\ 100) = 100$ sets can be positive. That happens when every positive person is in a different set, which is possible because $100 \leq 200$.
5. Extra kits in the worst case: $100 \times 5 = 500$.
6. Most kits ever needed:
$$200 + 500 = 700$$
7. Check: any grouping that puts two or more positives in one set has fewer than 100 positive sets, so it uses fewer than 700 kits. The bound of 700 is therefore never exceeded, and it is actually reached, so the answer is 700, option (A).

KEY IDEA

Total kits = (number of sets) + 5 × (number of positive sets). The pooled tests always cost 200 kits, so the worst case is the grouping with the most positive sets. With 100 positives, that is 100 sets, one positive in each.

WATCH OUT FOR

One mistake is to count only the 100 positive people and answer 200 + 100 = 300, or to forget the 200 pooled tests and answer 500. Both miss that a positive pool means retesting all 5 members, and that the 200 pooled tests happen whatever the result. Another is to take the worst case as all 200 sets being positive (200 + 1000 = 1200), which is impossible because there are only 100 positive people.

Q.10 MCQ · 2 MARKS

General Aptitude › Spatial

A 100 cm $\times$ 32 cm rectangular sheet is folded 5 times. Each time the sheet is folded, the long edge aligns with its opposite side. Eventually, the folded sheet is a rectangle of dimensions 100 cm $\times$ 1 cm.

The total number of creases visible when the sheet is unfolded is _____.

(A) 32

(B) 5

(C) 31

(D) 63

Official answer: **C**

SOLUTION

1. Folding so that a long edge (100 cm) meets the opposite long edge makes the fold line parallel to the long edges. It halves the short side and leaves the 100 cm length unchanged. Over the 5 folds the width goes $32 \to 16 \to 8 \to 4 \to 2 \to 1$ cm, which gives the stated final size of $100\,\text{cm} \times 1\,\text{cm}$.
2. Each fold doubles the number of layers, so after n folds there are 2^n layers. Here $2^5 = 32$ layers, each $1\,\text{cm}$ wide, and $32 \times 1\,\text{cm} = 32\,\text{cm}$ is the original width, so the numbers agree.
3. When the sheet is unfolded, it is divided into 32 strips of $100\,\text{cm} \times 1\,\text{cm}$, all parallel to the long edge. Adjacent strips were joined at a fold, so every boundary between two adjacent strips is a crease.
4. Cutting a width into k equal parts takes $k - 1$ dividing lines (fence-post rule). That gives $32 - 1 = 31$ creases, one at every whole centimetre from $1\,\text{cm}$ to $31\,\text{cm}$ across the width.
5. Cross-check fold by fold. Before fold k there are 2^{k-1} layers, and the fold puts one new crease in each layer, each at a new position (the middle of each current strip). Total: $1 + 2 + 4 + 8 + 16 = 2^5 - 1 = 31$.
6. The answer is 31, option (C).

KEY IDEA

Each fold halves the 32 cm side and doubles the layers. After 5 folds the unfolded sheet is 32 strips 1 cm wide, and 32 strips have 31 boundaries between them. You can also count it as $1 + 2 + 4 + 8 + 16 = 2^5 - 1$, because each fold creases every layer that exists at that moment.

WATCH OUT FOR

Picking 32 counts the strips (layers), not the creases between them. Picking 5 counts one crease per fold and forgets that a fold creases every layer of the stack. Picking 63 comes from $2^6 - 1$, which is an off-by-one in the number of folds.

Electronics and Communication Engineering Q.11 to Q.65

Q.11 MCQ · 1 MARK

Engineering Mathematics › Linear algebra

Let $\boldsymbol{v_1} = \begin{bmatrix}1\\2\\0\end{bmatrix}$ and $\boldsymbol{v_2} = \begin{bmatrix}2\\1\\3\end{bmatrix}$ be two vectors. The value of the coefficient α in the expression $\boldsymbol{v_1} = \alpha \boldsymbol{v_2} + \boldsymbol{e}$, which minimizes the length of the error vector $\boldsymbol{e}$, is

(A) $\frac{7}{2}$

(B) $\frac{-2}{7}$

(C) $\frac{2}{7}$

(D) $\frac{-7}{2}$

Official answer: **C**

SOLUTION

1. The error vector is $\boldsymbol{e} = \boldsymbol{v_1} - \alpha \boldsymbol{v_2}$. Its length is smallest when $\alpha \boldsymbol{v_2}$ is the orthogonal projection of $\boldsymbol{v_1}$ onto $\boldsymbol{v_2}$, i.e. when $\boldsymbol{e} \perp \boldsymbol{v_2}$.
2. Orthogonality condition: $\boldsymbol{v_2}^T(\boldsymbol{v_1} - \alpha \boldsymbol{v_2}) = 0 \;\Rightarrow\; \alpha = \dfrac{\boldsymbol{v_1} \cdot \boldsymbol{v_2}}{\boldsymbol{v_2} \cdot \boldsymbol{v_2}}$.
3. The same result follows from calculus: $\|\boldsymbol{e}\|^2 = \|\boldsymbol{v_1}\|^2 - 2\alpha\,(\boldsymbol{v_1} \cdot \boldsymbol{v_2}) + \alpha^2 \|\boldsymbol{v_2}\|^2$. Setting $\dfrac{d}{d\alpha}\|\boldsymbol{e}\|^2 = -2(\boldsymbol{v_1} \cdot \boldsymbol{v_2}) + 2\alpha\|\boldsymbol{v_2}\|^2 = 0$ gives the same α, and the second derivative $2\|\boldsymbol{v_2}\|^2 > 0$ confirms it is a minimum.
4. Dot product: $\boldsymbol{v_1} \cdot \boldsymbol{v_2} = (1)(2) + (2)(1) + (0)(3) = 4$.
5. Squared norm: $\boldsymbol{v_2} \cdot \boldsymbol{v_2} = 2^2 + 1^2 + 3^2 = 14$.
6. Therefore $\alpha = \dfrac{4}{14} = \dfrac{2}{7}$, which is option (C).
7. Check: $\boldsymbol{e} = \begin{bmatrix}1\\2\\0\end{bmatrix} - \frac{2}{7}\begin{bmatrix}2\\1\\3\end{bmatrix} = \frac{1}{7}\begin{bmatrix}3\\12\\-6\end{bmatrix}$, and $\boldsymbol{e} \cdot \boldsymbol{v_2} = \frac{1}{7}(6 + 12 - 18) = 0$, so $\boldsymbol{e}$ is orthogonal to $\boldsymbol{v_2}$ as required.

KEY IDEA

The error is shortest when it is perpendicular to $\boldsymbol{v_2}$, which makes $\alpha \boldsymbol{v_2}$ the orthogonal projection of $\boldsymbol{v_1}$ onto $\boldsymbol{v_2}$. So $\alpha = \dfrac{\boldsymbol{v_1} \cdot \boldsymbol{v_2}}{\boldsymbol{v_2} \cdot \boldsymbol{v_2}}$ (least squares with a single unknown).

WATCH OUT FOR

Swapping the roles of the vectors and dividing by $\|\boldsymbol{v_1}\|^2 = 5$ (which gives $4/5$), or inverting the ratio to get $7/2$ (option A). The denominator is the squared length of the vector you are projecting onto, here $\boldsymbol{v_2}$, so it is 14. A sign slip in the dot product leads to the negative options B or D.

Q.12 MCQ · 1 MARK

Engineering Mathematics › Vector calculus

The rate of increase, of a scalar field $f(x, y, z) = xyz$, in the direction $\boldsymbol{v} = (2, 1, 2)$ at a point $(0, 2, 1)$ is

(A) $\frac{2}{3}$

(B) $\frac{4}{3}$

(C) 2

(D) 4

Official answer: **B**

SOLUTION

1. The rate of increase of f in a given direction is the directional derivative $D_{\hat{v}}f = \nabla f \cdot \hat{v}$, where $\hat{v}$ is the unit vector along $\boldsymbol{v}$.
2. Gradient of $f = xyz$: $\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) = (yz,\ xz,\ xy)$.
3. At the point $(0, 2, 1)$, where $x = 0$, $y = 2$, $z = 1$: $\nabla f = (2 \cdot 1,\ 0 \cdot 1,\ 0 \cdot 2) = (2,\ 0,\ 0)$.
4. Normalise the direction: $|\boldsymbol{v}| = \sqrt{2^2 + 1^2 + 2^2} = \sqrt{9} = 3$, so $\hat{v} = \left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$.
5. Directional derivative:
$$D_{\hat{v}}f = (2, 0, 0) \cdot \left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right) = 2 \cdot \frac{2}{3} + 0 + 0 = \frac{4}{3}$$
6. The rate of increase is $\frac{4}{3}$, which is option (B).

KEY IDEA

The rate of increase in a direction is the directional derivative, $\nabla f \cdot \hat{v}$. You must dot the gradient with the unit vector along $\boldsymbol{v}$, not with $\boldsymbol{v}$ as given.

WATCH OUT FOR

The usual slip is to dot with $\boldsymbol{v} = (2, 1, 2)$ without dividing by $|\boldsymbol{v}| = 3$. That gives 4, which is option (D). The other slip is to answer with $|\nabla f| = 2$ (option C), which is the largest rate of increase over all directions, not the rate along $\boldsymbol{v}$.

Q.13 MCQ · 1 MARK

Engineering Mathematics › Complex analysis

Let $w^4 = 16j$. Which of the following cannot be a value of w?

(A) $2e^{\frac{j2\pi}{8}}$

(B) $2e^{\frac{j\pi}{8}}$

(C) $2e^{\frac{j5\pi}{8}}$

(D) $2e^{\frac{j9\pi}{8}}$

Official answer: **A**

SOLUTION

1. Write the right-hand side in polar form: $16j = 16\,e^{j\pi/2}$. The argument is fixed only up to multiples of 2π, so in general $16j = 16\,e^{j(\pi/2 + 2k\pi)}$ for any integer k.
2. Take fourth roots (De Moivre's theorem for n-th roots):
$$w = 16^{1/4}\,e^{j\left(\frac{\pi/2 + 2k\pi}{4}\right)} = 2\,e^{j\left(\frac{\pi}{8} + \frac{k\pi}{2}\right)}, \quad k = 0, 1, 2, 3$$
3. List the four roots. They lie on the circle $|w| = 2$, spaced $\pi/2 = 4\pi/8$ apart:
$$w \in \left\{2e^{j\pi/8},\ 2e^{j5\pi/8},\ 2e^{j9\pi/8},\ 2e^{j13\pi/8}\right\}$$
4. Match the options: (B) $2e^{j\pi/8}$ is $k = 0$, (C) $2e^{j5\pi/8}$ is $k = 1$ and (D) $2e^{j9\pi/8}$ is $k = 2$. The argument $2\pi/8$ in (A) differs from $\pi/8$ by $\pi/8$, which is not a multiple of $\pi/2$, so (A) is not one of the roots.
5. Check (A) directly: $\left(2e^{j2\pi/8}\right)^4 = 16\,e^{j8\pi/8} = 16\,e^{j\pi} = -16 \neq 16j$. So (A) cannot be a value of w.

KEY IDEA

The n-th roots of $re^{j\theta}$ are $r^{1/n}e^{j(\theta+2k\pi)/n}$ for $k = 0, 1, \ldots, n-1$. They sit on a circle of radius $r^{1/n}$, spaced $2\pi/n$ apart. For $16j$ that means radius 2, starting angle $\pi/8$ and spacing $\pi/2$. Any option whose angle is not $\pi/8$ plus a multiple of $\pi/2$ is not a root, and raising it to the fourth power confirms this.

WATCH OUT FOR

A common mistake is to take the argument of $16j$ as $\pi/4$ instead of $\pi/2$, or to divide only the principal argument by 4 and forget the $2k\pi/4 = k\pi/2$ spacing. Either slip gives the wrong set of roots, and then (A), with angle $2\pi/8 = \pi/4$, looks like a valid root. Another slip is to read "cannot be" as "can be" and pick (B), the principal root.

Q.14 MCQ · 1 MARK

Engineering Mathematics › Complex analysis

The value of the contour integral, $\oint_C \left(\dfrac{z+2}{z^2+2z+2}\right) dz$, where the contour C is $\left\{z : \left|z+1-\frac{3}{2}j\right| = 1\right\}$, taken in the counter clockwise direction, is

(A) $-\pi(1+j)$

(B) $\pi(1+j)$

(C) $\pi(1-j)$

(D) $-\pi(1-j)$

Official answer: **B**

SOLUTION

1. Find the poles by solving $z^2+2z+2=0$: $z = \dfrac{-2 \pm \sqrt{4-8}}{2} = -1 \pm j$. They are distinct, so both are simple poles, and $z^2+2z+2 = \big(z-(-1+j)\big)\big(z-(-1-j)\big)$. The numerator $z+2$ is not zero at either of them.
2. Rewrite the contour as $\left|z-\left(-1+\frac{3}{2}j\right)\right| = 1$: a circle with centre $-1+\frac{3}{2}j$ and radius 1.
3. Check each pole against the circle. $\left|(-1+j)-\left(-1+\frac{3}{2}j\right)\right| = \left|-\frac{1}{2}j\right| = \frac{1}{2} < 1$, so $-1+j$ is inside. $\left|(-1-j)-\left(-1+\frac{3}{2}j\right)\right| = \left|-\frac{5}{2}j\right| = \frac{5}{2} > 1$, so $-1-j$ is outside. Neither pole lies on C.
4. Residue at the simple pole $z_0 = -1+j$: $\mathrm{Res} = \left.\dfrac{z+2}{z-(-1-j)}\right|_{z=-1+j} = \dfrac{(-1+j)+2}{(-1+j)+1+j} = \dfrac{1+j}{2j}$.
5. Apply the Cauchy residue theorem (counter-clockwise, one pole inside):
$$\oint_C \frac{z+2}{z^2+2z+2}\,dz = 2\pi j \cdot \frac{1+j}{2j} = \pi(1+j).$$
6. This is option (B).

KEY IDEA

Only the poles inside the contour count. The circle is centred at $-1+\frac{3}{2}j$ with radius 1, so it contains $-1+j$ (distance $\frac{1}{2}$) but not $-1-j$ (distance $\frac{5}{2}$). The integral is therefore $2\pi j$ times the single residue $\frac{1+j}{2j}$.

WATCH OUT FOR

Reading the centre as $1-\frac{3}{2}j$ instead of $-1+\frac{3}{2}j$. The condition $|z+1-\frac{3}{2}j| = 1$ means $|z-(-1+\frac{3}{2}j)| = 1$, and with the wrong sign the circle encloses neither pole, which gives a wrong value of 0. A second mistake is adding the residues at both poles, which gives $2\pi j$ (the full integral around a large circle) and is not one of the options.

Q.15 MCQ · 1 MARK

Engineering Mathematics › Linear algebra

Let the sets of eigenvalues and eigenvectors of a matrix B be $\{\lambda_k \mid 1 \le k \le n\}$ and $\{\boldsymbol{v}_k \mid 1 \le k \le n\}$, respectively. For any invertible matrix P, the sets of eigenvalues and eigenvectors of the matrix A, where $B = P^{-1}AP$, respectively, are

(A) $\{\lambda_k \det(A) \mid 1 \le k \le n\}$ and $\{P\boldsymbol{v}_k \mid 1 \le k \le n\}$

(B) $\{\lambda_k \mid 1 \le k \le n\}$ and $\{\boldsymbol{v}_k \mid 1 \le k \le n\}$

(C) $\{\lambda_k \mid 1 \le k \le n\}$ and $\{P\boldsymbol{v}_k \mid 1 \le k \le n\}$

(D) $\{\lambda_k \mid 1 \le k \le n\}$ and $\{P^{-1}\boldsymbol{v}_k \mid 1 \le k \le n\}$

Official answer: **C**

SOLUTION

1. Write A in terms of B. Since P is invertible, $B = P^{-1}AP \Rightarrow A = PBP^{-1}$.
2. A similarity transformation does not change eigenvalues. Using $\lambda I = P(\lambda I)P^{-1}$:
$$\det(A - \lambda I) = \det\left(P(B - \lambda I)P^{-1}\right) = \det(P)\,\det(B - \lambda I)\,\det(P^{-1}) = \det(B - \lambda I),$$
because $\det(P)\det(P^{-1}) = 1$. So A and B have the same characteristic polynomial, and the eigenvalues of A are exactly the λ_k. The extra factor $\det(A)$ in (A) is wrong: it would only give the right values in the special case $\det(A) = 1$.
3. For the eigenvectors, start from $B\boldsymbol{v}_k = \lambda_k \boldsymbol{v}_k$ and substitute $B = P^{-1}AP$:
$$P^{-1}AP\,\boldsymbol{v}_k = \lambda_k \boldsymbol{v}_k.$$
4. Premultiply both sides by P:
$$A\,(P\boldsymbol{v}_k) = \lambda_k\,(P\boldsymbol{v}_k).$$
5. $P\boldsymbol{v}_k \neq \boldsymbol{0}$, because P is invertible (its null space is only $\boldsymbol{0}$) and an eigenvector $\boldsymbol{v}_k$ is non-zero. So $P\boldsymbol{v}_k$ is an eigenvector of A for the eigenvalue λ_k. In general $\boldsymbol{v}_k$ itself (B) and $P^{-1}\boldsymbol{v}_k$ (D) are not: $P^{-1}\boldsymbol{v}_k$ would be an eigenvector of $P^{-1}BP$, not of $A = PBP^{-1}$.
6. So the eigenvalues of A are $\{\lambda_k\}$ and its eigenvectors are $\{P\boldsymbol{v}_k\}$. This is option (C).

KEY IDEA

Similar matrices have the same characteristic polynomial, so they have the same eigenvalues. Their eigenvectors are different, though: they are linked by the same matrix that links the two matrices. Write $A = PBP^{-1}$. If $B\boldsymbol{v} = \lambda\boldsymbol{v}$, then $A(P\boldsymbol{v}) = \lambda(P\boldsymbol{v})$, so each eigenvector of A is P times an eigenvector of B.

WATCH OUT FOR

The usual slip is getting the direction of P wrong. Students see $B = P^{-1}AP$ and pick $P^{-1}\boldsymbol{v}_k$ (D), when solving for A gives $A = PBP^{-1}$ and so the eigenvectors are $P\boldsymbol{v}_k$. The other slip is assuming similar matrices share eigenvectors as well as eigenvalues (B). To check the direction, substitute the answer back: $A(P\boldsymbol{v}_k) = PBP^{-1}P\boldsymbol{v}_k = PB\boldsymbol{v}_k = \lambda_k P\boldsymbol{v}_k$.

Q.16 MCQ · 1 MARK

Electronic Devices › Semiconductors

In a semiconductor, if the Fermi energy level lies in the conduction band, then the semiconductor is known as

(A) degenerate n-type.

(B) degenerate p-type.

(C) non-degenerate n-type.

(D) non-degenerate p-type.

Official answer: **A**

SOLUTION

1. A semiconductor is called degenerate when the doping concentration is so high that the Fermi level E_F moves out of the forbidden energy gap and into one of the allowed bands, rather than remaining within the gap as in a normal (non-degenerate) semiconductor.
2. If E_F lies inside the conduction band, this occurs specifically due to an extremely high concentration of donor (electron-donating) impurities; by definition, this is a degenerate n-type semiconductor.

3. For comparison: if E_F lies inside the valence band instead, the material is degenerate p-type, resulting from extremely high acceptor (hole-donating) doping.
4. Since the question states E_F lies in the conduction band, the semiconductor is degenerate n-type, i.e., option (A).

KEY IDEA

In a non-degenerate semiconductor, the Fermi level E_F lies within the forbidden energy gap. When the doping (donor or acceptor) concentration becomes very high (typically approaching or exceeding the effective density of states, $N_D \gtrsim N_C$ or $N_A \gtrsim N_V$), the semiconductor becomes degenerate and E_F moves out of the gap into the conduction band (for very heavy donor/n-type doping) or into the valence band (for very heavy acceptor/p-type doping).

WATCH OUT FOR

Confusing "degenerate" with mere heavy doping without connecting it to the Fermi level actually crossing into the band edge; or reversing n-type/p-type by forgetting that donor (electron) doping pushes E_F upward toward the conduction band while acceptor doping pushes it downward toward the valence band.

Q.17 MCQ · 1 MARK

Electronic Devices › Semiconductors

For an intrinsic semiconductor at temperature $T = 0\ K$, which of the following statement is true?

(A) All energy states in the valence band are filled with electrons and all energy states in the conduction band are empty of electrons.

(B) All energy states in the valence band are empty of electrons and all energy states in the conduction band are filled with electrons.

(C) All energy states in the valence and conduction band are filled with holes.

(D) All energy states in the valence and conduction band are filled with electrons.

Official answer: **A**

SOLUTION

1. An intrinsic semiconductor has no dopant atoms, so any electrons in the conduction band or holes in the valence band arise purely from thermal excitation of an electron across the bandgap E_g.
2. The probability that a state at energy E is occupied by an electron is given by the Fermi-Dirac distribution: $f(E) = \dfrac{1}{1 + e^{(E-E_F)/kT}}$, where E_F is the Fermi level (near mid-gap for an intrinsic semiconductor).
3. As $T \to 0\ \mathrm{K}$, $kT \to 0$. For any conduction-band state ($E > E_F$), the exponent $(E - E_F)/kT \to +\infty$, so $f(E) \to 0$: every conduction-band state becomes empty. For any valence-band state ($E < E_F$), the exponent $\to -\infty$, so $f(E) \to 1$: every valence-band state becomes fully occupied.
4. This matches the intrinsic carrier concentration relation $n_i = \sqrt{N_C N_V}\, e^{-E_g/(2kT)}$, which gives $n_i \to 0$ as $T \to 0\ \mathrm{K}$: no free electrons or holes exist at absolute zero.
5. Therefore, at $T = 0\ \mathrm{K}$: the valence band is completely filled with electrons and the conduction band is completely empty of electrons: option (A).
6. Option (B) describes an unphysical population-inverted state that never occurs in an ordinary semiconductor. Options (C) and (D) are physically meaningless: a band cannot be described as simultaneously filled with holes (absence of electrons) while also being called filled, and both bands cannot be entirely electron-filled without contradiction, since a hole is by definition an unoccupied electron state.

KEY IDEA

At absolute zero, an intrinsic semiconductor behaves like a perfect insulator: with no thermal energy available, no electron can be excited across the bandgap, so the valence band stays completely full and the conduction band stays completely empty.

WATCH OUT FOR

Confusing "intrinsic" with "no free carriers ever" versus the T = 0 K condition, or assuming some residual thermal population exists at absolute zero; also mixing up which band is which (some students reverse valence/conduction in option (B), thinking of a hypothetical population-inverted or superconducting-like state, which never occurs in an ordinary semiconductor).

Q.18 MCQ · 1 MARK

Networks (Electric Circuits) › AC, power & resonance

A series RLC circuit has a quality factor Q of 1000 at a center frequency of 10^6 rad/s. The possible values of R, L and C are

(A) $R = 1\ \Omega$, $L = 1\ \mu H$ and $C = 1\ \mu F$

(B) $R = 0.1\ \Omega$, $L = 1\ \mu H$ and $C = 1\ \mu F$

(C) $R = 0.01\ \Omega$, $L = 1\ \mu H$ and $C = 1\ \mu F$

(D) $R = 0.001\ \Omega$, $L = 1\ \mu H$ and $C = 1\ \mu F$

Official answer: **D**

SOLUTION

1. All four options use the same $L = 1\ \mu\text{H}$ and $C = 1\ \mu\text{F}$, so only R has to be found.
2. Resonant (center) frequency of a series RLC circuit: $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{10^{-6} \times 10^{-6}}} = 10^6\ \text{rad/s}$. This matches the given center frequency, so the L and C in the options are consistent with the question.
3. Quality factor of a series RLC circuit:
$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$
4. Here $\omega_0 L = 10^6 \times 10^{-6} = 1\ \Omega$ (equivalently $\sqrt{L/C} = \sqrt{10^{-6}/10^{-6}} = 1\ \Omega$), so $Q = \dfrac{1\ \Omega}{R}$.
5. Setting $Q = 1000$ gives $R = \dfrac{1\ \Omega}{1000} = 0.001\ \Omega$. Check against the other options: $R = 1,\ 0.1,\ 0.01\ \Omega$ give $Q = 1,\ 10,\ 100$ respectively. Only option (D) gives $Q = 1000$.

KEY IDEA

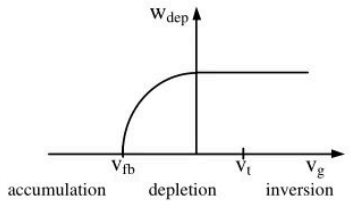
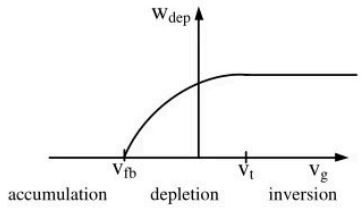
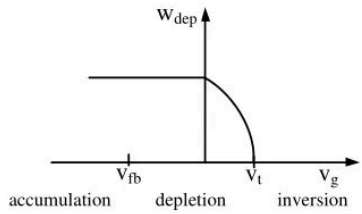
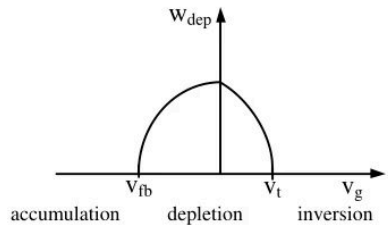
For a series RLC circuit, $\omega_0 = 1/\sqrt{LC}$ and $Q = \omega_0 L/R = \frac{1}{R}\sqrt{L/C}$. With $L = 1\ \mu\text{H}$ and $C = 1\ \mu\text{F}$, the characteristic impedance $\sqrt{L/C}$ is $1\ \Omega$, so $Q = 1/R$ and $Q = 1000$ needs $R = 1\ \text{m}\Omega$.

WATCH OUT FOR

Using the parallel RLC formula $Q = R\sqrt{C/L}$ (or $Q = \omega_0 RC$). In a series circuit a smaller R gives a larger Q. The parallel formula flips that and points to $R = 1000\ \Omega$, which is not an option, or pushes a student to guess (A).

Q.19 MCQ · 1 MARK

Electronic Devices › MOS devices

Q.19	For a MOS capacitor, V_{fb} and V_t are the flat-band voltage and the threshold voltage, respectively. The variation of the depletion width (W_{dep}) for varying gate voltage (V_g) is best represented by
(A)	
(B)	
(C)	
(D)	

Official answer: **B**

SOLUTION

1. Accumulation region ($V_g < V_{fb}$): majority carriers pile up at the oxide-semiconductor interface and screen the applied field, so no depletion region forms: $W_{dep} = 0$.
2. Depletion region ($V_{fb} < V_g < V_t$): as V_g increases past V_{fb}, the surface potential ϕ_s becomes positive and grows with V_g. The depletion width follows $W_{dep} = \sqrt{\frac{2\epsilon_s \phi_s}{qN_A}}$, which is zero exactly at $V_g = V_{fb}$ (where $\phi_s = 0$) and rises smoothly for $V_g > V_{fb}$.

3. Threshold ($V_g = V_t$): strong inversion begins once the surface potential reaches $\phi_s = 2\phi_F$, at which point the depletion width reaches its maximum value $W_{dep,max} = \sqrt{\dfrac{4\epsilon_s \phi_F}{qN_A}}$.
4. Inversion ($V_g > V_t$): once the inversion layer of minority carriers forms, it screens the semiconductor from further gate-field penetration (charge-control / strong-inversion approximation). Any further increase in V_g is dropped across the oxide and the inversion charge, not across the depletion region, so W_{dep} stays pinned at $W_{dep,max}$.
5. Putting the four regions together: the curve is flat at zero up to V_{fb}, rises smoothly starting exactly at V_{fb}, saturates to a constant plateau exactly at V_t, and stays flat for all $V_g > V_t$.
6. Comparing this shape against the four plots: only option (B) has its rise beginning exactly at the V_{fb} marker and its saturation reached exactly at the V_t marker, staying flat afterward; the other options either saturate before reaching V_t (A), decay to zero in inversion (C), or fall back to zero after a peak instead of saturating (D).

KEY IDEA

In a MOS capacitor, the depletion width under the gate is set by the surface potential ϕ_s: it is zero in accumulation, grows through depletion as ϕ_s increases with V_g (starting from zero exactly at the flat-band point V_{fb}), and saturates at its maximum value once strong inversion sets in at $V_g = V_t$; the inversion charge then screens the semiconductor, so W_{dep} stays pinned at $W_{dep,max}$ for all $V_g > V_t$.

WATCH OUT FOR

Picking a curve whose rise or saturation point is offset from the actual V_{fb}/V_t markers (option A), assuming W_{dep} continues to grow (or falls) in inversion instead of saturating (options C, D), or confusing the depletion-width-vs-gate-voltage curve with the C–V curve of the same device.

Q.20 MCQ · 1 MARK

Electromagnetics › Plane waves

Consider a narrow band signal, propagating in a lossless dielectric medium ($\varepsilon_r = 4, \mu_r = 1$), with phase velocity v_p and group velocity v_g. Which of the following statement is true? (c is the velocity of light in vacuum.)

(A) $v_p > c, v_g > c$

(B) $v_p < c, v_g > c$

(C) $v_p > c, v_g < c$

(D) $v_p < c, v_g < c$

Official answer: **D**

SOLUTION

1. For a uniform plane wave in an unbounded, lossless dielectric, the phase constant is
$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\mu_r\,\varepsilon_0\varepsilon_r} = \frac{\omega}{c}\sqrt{\mu_r\varepsilon_r},$$
where $c = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}} \approx 3\times10^8\ \text{m/s}$, with $\mu_0 = 4\pi\times10^{-7}\ \text{H/m}$ and $\varepsilon_0 \approx 8.854\times10^{-12}\ \text{F/m}$.
2. Phase velocity:
$$v_p = \frac{\omega}{\beta} = \frac{c}{\sqrt{\mu_r\varepsilon_r}} = \frac{c}{\sqrt{1\times4}} = \frac{c}{2} \approx 1.5\times10^8\ \text{m/s}.$$
3. Group velocity: $v_g = \left(\dfrac{d\beta}{d\omega}\right)^{-1}$. With ε_r and μ_r constant, β is directly proportional to ω, so
$$\frac{d\beta}{d\omega} = \frac{\sqrt{\mu_r\varepsilon_r}}{c} \quad\Rightarrow\quad v_g = \frac{c}{\sqrt{\mu_r\varepsilon_r}} = \frac{c}{2}.$$

4. The medium is non-dispersive (the ratio ω/β does not depend on frequency), so every frequency component of the narrow-band signal moves at the same speed and $v_p = v_g = \dfrac{c}{2}$. Their product is $v_p v_g = \dfrac{c^2}{4}$, not c^2. The result $v_p v_g = c^2$ belongs to a dispersive guided wave such as an air-filled hollow waveguide, where $v_p > c$ and $v_g < c$. It does not apply to an unbounded plane wave.
5. Both velocities are $\dfrac{c}{2}$, so $v_p < c$ and $v_g < c$. Option (D) is correct.

KEY IDEA

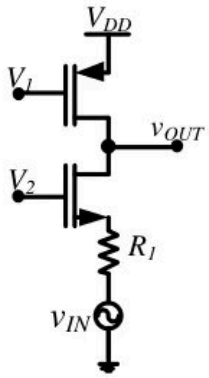
In an unbounded lossless dielectric with constant ε_r and μ_r, $\beta = \dfrac{\omega}{c}\sqrt{\mu_r \varepsilon_r}$ is linear in ω. The medium is therefore non-dispersive, and the phase and group velocities are equal: $v_p = v_g = \dfrac{c}{\sqrt{\mu_r \varepsilon_r}}$. With $\mu_r \varepsilon_r = 4 > 1$, both velocities equal $c/2$, which is less than c.

WATCH OUT FOR

Using the waveguide result $v_p v_g = c^2$ and so picking (C), $v_p > c$ and $v_g < c$. That relation comes from the dispersive cut-off behaviour of an air-filled hollow waveguide. A plane wave in an unbounded lossless dielectric has no cut-off and no dispersion, so $v_p = v_g = c/2$. The words 'narrow band' only make group velocity meaningful. They do not make the medium dispersive.

Q.21 MCQ · 1 MARK

Analog Electronics › Amplifiers

Q.21	In the circuit shown below, V_1 and V_2 are bias voltages. Based on input and output impedances, the circuit behaves as a
	
(A)	voltage controlled voltage source.
(B)	voltage controlled current source.
(C)	current controlled voltage source.
(D)	current controlled current source.

Official answer: **D**

SOLUTION

1. **Identify the devices.** The top PMOS has its source at V_{DD} and its gate at the DC bias V_1. Both are AC ground, so it has no small-signal drive. Seen from its drain, it is just its output resistance r_{op}, an active (current-source) load. The bottom NMOS has its gate at the DC bias V_2, so its gate is at AC ground. The signal enters at its source and leaves at its drain, so the NMOS is a **common-gate** stage. The source

v_{IN} in series with R_1 is the signal source together with its own resistance. Its Norton equivalent is a current v_{IN}/R_1 in parallel with R_1. So the amplifier's input port is the NMOS source and its output port is the drain node v_{OUT}.

2. **Small-signal model and assumptions.** Each MOSFET is modelled as $g_m v_{gs}$ with r_o from drain to source. Body effect is neglected, and we take $g_m r_{on} \gg 1$. The NMOS gate is at AC ground, so $v_{gs} = -v_s$.
3. **Input resistance, looking into the NMOS source.** Apply a test voltage v_x at the source and let i_x flow in. The gate draws no current, so i_x leaves through the drain into r_{op}, which gives $v_d = i_x r_{op}$. The current through the NMOS from source to drain is

$$i_x = g_m v_x + \frac{v_x - v_d}{r_{on}} = g_m v_x + \frac{v_x}{r_{on}} - \frac{i_x r_{op}}{r_{on}}$$

Solving for v_x/i_x:

$$R_{in} = \frac{r_{on} + r_{op}}{1 + g_m r_{on}} \approx \frac{1}{g_m}\left(1 + \frac{r_{op}}{r_{on}}\right)$$

With r_{op} comparable to r_{on}, $R_{in} \approx 2/g_m$. That is of the order of $1/g_m$, so the input impedance is **low**.

4. **Output resistance, looking into the drain node.** Set $v_{IN} = 0$, so the NMOS source goes to ground through R_1. Apply v_x at the drain and let i_x flow into the NMOS drain. Then $v_s = i_x R_1$ and $v_{gs} = -i_x R_1$, so

$$i_x = -g_m i_x R_1 + \frac{v_x - i_x R_1}{r_{on}} \;\Rightarrow\; R_{down} = r_{on} + R_1 + g_m r_{on} R_1 \approx r_{on}(1 + g_m R_1)$$

Looking up, you see the PMOS resistance r_{op}. So

$$R_{out} = r_{op} \parallel r_{on}(1 + g_m R_1)$$

This is of the order of r_o, so the output impedance is **high**.

5. **Transfer.** The gate draws no current, so the current entering the NMOS source is exactly the current leaving its drain: $i_d = i_s$. R_{in} is much smaller than the source resistance R_1, so almost all of the Norton current v_{IN}/R_1 goes into the source: $i_{in} \approx v_{IN}/R_1$. With the output shorted to AC ground, r_{op} carries no current and $i_{out} = i_{in}$. The short-circuit current gain is 1, so the stage is a current buffer.
6. **Classify.** Each ideal controlled source has fixed port impedances. VCVS: $Z_{in} \to \infty$, $Z_{out} \to 0$. VCCS: $Z_{in} \to \infty$, $Z_{out} \to \infty$. CCVS: $Z_{in} \to 0$, $Z_{out} \to 0$. CCCS: $Z_{in} \to 0$, $Z_{out} \to \infty$. This circuit has a low Z_{in} (about $1/g_m$) and a high Z_{out} (about r_o), which matches the **current-controlled current source**, option (D).

KEY IDEA

Classify an amplifier by its port impedances. A low input impedance means it takes a current in, and a high output impedance means it delivers a current. A common-gate MOSFET has $R_{in} \approx 1/g_m$ at the source, R_{out} of the order of r_o at the drain, and $i_d = i_s$, so it is a current buffer, which is a CCCS.

WATCH OUT FOR

Students often classify from the labels instead of the impedances. Because the input is drawn as a voltage v_{IN} and the output is labelled v_{OUT}, they pick VCVS, or VCCS by thinking of a transconductance stage. Others count R_1 as part of the amplifier's input impedance. R_1 belongs to the driving source (Norton current v_{IN}/R_1), and the amplifier's own input, the NMOS source, has an impedance of only about $1/g_m$. A third error is to read the NMOS as a source follower, which would give low output impedance. Its gate is at AC ground and the output is at its drain, so it is common-gate.

Q.22 MCQ · 1 MARK

Analog Electronics › Feedback & oscillators

A cascade of common-source amplifiers in a unity gain feedback configuration oscillates when

(A) the closed loop gain is less than 1 and the phase shift is less than 180°.

(B) the closed loop gain is greater than 1 and the phase shift is less than 180°.

(C) the closed loop gain is less than 1 and the phase shift is greater than 180°.

(D) the closed loop gain is greater than 1 and the phase shift is greater than 180°.

Official answer: **Marks to all**

SOLUTION

1. **What the options must mean.** Connect the cascade so that the feedback is negative at low frequency. For an odd number of inverting CS stages, that means tying the output straight back to the input. With unity feedback ($\beta = 1$), the loop gain is $L(j\omega) = \beta A(j\omega) = A(j\omega)$ and the closed-loop gain is $A_f = \dfrac{A}{1+L}$. Oscillation is decided by the denominator: $1 + L(j\omega) = 0$. At that point A_f is unbounded, and a stable unity-feedback amplifier with large $|A|$ has $A_f \approx 1$. So the *closed-loop* gain is not the quantity any oscillation criterion uses. The options only make sense if "closed loop gain" is read as the **loop gain** $|L(j\omega)|$, and "phase shift" as the phase lag of $L(j\omega)$, on top of the 180° inversion that makes the feedback negative. The official key awards marks to all candidates and gives no reason. The solution below answers the question as intended, under this reading.
2. **Model the cascade.** Assume N identical stages. Each CS stage has a low-frequency gain magnitude $A_0 = g_m R_D$ (r_o absorbed into R_D) and one pole from its output-node capacitance C_L, at $\omega_p = \dfrac{1}{R_D C_L}$. C_{gd} and its zero are neglected. Then
$$L(j\omega) = \frac{A_0^N}{(1 + j\omega/\omega_p)^N}, \qquad |L(j\omega)| = \frac{A_0^N}{\left(1 + \omega^2/\omega_p^2\right)^{N/2}}, \qquad \phi(\omega) = N \tan^{-1}\left(\frac{\omega}{\omega_p}\right)$$
where ϕ is the phase lag of the loop. As ω rises, $|L|$ falls monotonically and ϕ rises monotonically from 0 towards $N \times 90^\circ$.
3. **How many stages can reach 180°.** At every finite frequency, each pole contributes $\tan^{-1}(\omega/\omega_p) < 90^\circ$. So ϕ reaches 180° at a finite frequency only if $N \geq 3$. Call that frequency ω_{180}: $N \tan^{-1}(\omega_{180}/\omega_p) = 180^\circ$. With $N = 2$, ϕ approaches 180° only as $\omega \to \infty$, where $|L| \to 0$. A two-stage loop of this kind therefore cannot oscillate.
4. **Oscillation criterion.** A closed-loop pole on the $j\omega$ axis needs $1 + L(j\omega) = 0$, that is, $\angle L(j\omega) = -180^\circ$ and $|L(j\omega)| = 1$ (Barkhausen). The extra 180° of lag turns the negative feedback into positive feedback, and the signal comes back round the loop at its own amplitude. All open-loop poles are in the left half-plane and ϕ is monotonic, so the Nyquist plot of L crosses the negative real axis once, at $-|L(j\omega_{180})|$. If $|L(j\omega_{180})| > 1$, the plot encircles -1. The closed loop then has poles in the right half-plane, and the oscillation builds up until the transistors' nonlinearity limits its amplitude. If $|L(j\omega_{180})| < 1$, the loop is stable. The boundary case $|L(j\omega_{180})| = 1$ is the constant-amplitude Barkhausen condition.
5. **Restate it with the two crossover frequencies.** Let ω_{gc} be the gain crossover, where $|L(j\omega_{gc})| = 1$. Because $|L|$ falls and ϕ rises monotonically, $|L(j\omega_{180})| > 1 \iff \omega_{180} < \omega_{gc}$. So the loop oscillates exactly when $\omega_{180} < \omega_{gc}$. In that case every frequency between the two has $|L| > 1$ and $\phi > 180^\circ$, which is a negative phase margin: $180^\circ - \phi(\omega_{gc}) < 0$. The loop is stable exactly when $\omega_{gc} < \omega_{180}$. In that case every frequency between the two has $|L| < 1$ and $\phi < 180^\circ$.
6. **(D):** for $|L| > 1$ and $\phi > 180^\circ$ to hold at the same frequency, we need $\omega > \omega_{180}$ and $\omega < \omega_{gc}$. That is possible only when $\omega_{180} < \omega_{gc}$, which is exactly the oscillating case. Conversely, every oscillating loop has such a band of frequencies. So (D) is the oscillation condition.
7. **(A):** $|L| < 1$ together with $\phi < 180^\circ$ needs $\omega_{gc} < \omega < \omega_{180}$. That is possible only when $\omega_{gc} < \omega_{180}$, so this loop is stable, with positive gain and phase margins.

8. **(B) and (C):** $|L| > 1$ with $\phi < 180^\circ$ holds at low frequencies, below both ω_{gc} and ω_{180}, in every amplifier, stable or not. It is just the ordinary amplifying region. $|L| < 1$ with $\phi > 180^\circ$ holds at high frequencies, above both crossovers, in every loop whose phase passes 180°, stable or not. Neither pair tells an oscillating loop from a stable one. If (C) is read as describing the 180° point, it says $|L(j\omega_{180})| < 1$. That is a positive gain margin, so the loop is stable.
9. **Check with three stages (a ring oscillator).** For $N = 3$, $\phi = 180^\circ$ when each stage lags by 60°: $\tan^{-1}(\omega_{180}/\omega_p) = 60^\circ$, so $\omega_{180} = \sqrt{3}\omega_p$. At that frequency
$$|L(j\omega_{180})| = \frac{A_0^3}{(1+3)^{3/2}} = \frac{A_0^3}{8}$$
Sustained oscillation needs $A_0 = 2$. With $A_0 > 2$, the loop gain is still above 1 where the phase lag reaches 180°, so the oscillation starts and builds up. This is the (D) condition.
10. **Answer: (D)**, read as "the loop gain is greater than 1 while the phase shift exceeds 180°". As printed, "closed loop gain" is the wrong quantity (see the first step), and the official key awarded marks to all candidates.

KEY IDEA

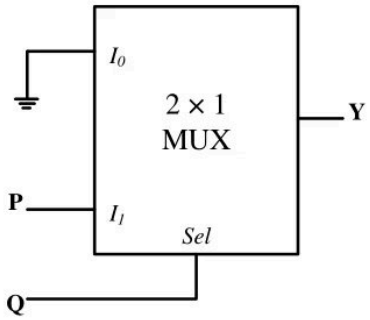
The loop gain $L = \beta A$ decides oscillation, not the closed-loop gain. A negative-feedback loop oscillates when $|L|$ is still above 1 at the frequency where the phase lag reaches 180°. Put another way, the phase crossover comes before the gain crossover, which is a negative phase margin. Each single-pole CS stage adds less than 90° of lag, so at least three stages are needed. A three-stage ring oscillates at $\omega_{180} = \sqrt{3}\omega_p$ once each stage's gain exceeds 2.

WATCH OUT FOR

Taking "closed loop gain" literally. A stable unity-feedback amplifier has a closed-loop gain of about 1, and at oscillation that gain is unbounded, so it is not the test. Also: (1) choosing (C) on the idea that more than 180° of phase shift is enough on its own, when the loop gain must still exceed 1 where the lag reaches 180°; (2) believing two single-pole stages can reach 180°, when they only approach it as $\omega \to \infty$, where the gain has gone to zero; (3) confusing the start-up condition ($|L| > 1$ at 180°) with the Barkhausen condition for a steady amplitude ($|L| = 1$ at exactly 180°).

Q.23 MCQ · 1 MARK

Digital Electronics › Combinational

Q.23	In the circuit shown below, P and Q are the inputs. The logical function realized by the circuit shown below is
	
(A)	$Y = PQ$
(B)	$Y = P + Q$
(C)	$Y = \overline{PQ}$
(D)	$Y = \overline{P + Q}$

Official answer: **A**

SOLUTION

1. Output equation of a 2×1 MUX with select line S (complement written with $\overline{\ }$): $S = 0$ passes I_0 and $S = 1$ passes I_1, so
$$Y = \overline{S}\, I_0 + S\, I_1$$
2. Read the connections off the figure: $S = Q$, $I_0 = 0$ (tied to ground), $I_1 = P$.
3. Substitute:

$$Y = \overline{Q} \cdot 0 + Q \cdot P = PQ$$

4. Check with the truth table: when $Q = 0$ the MUX passes $I_0 = 0$, so $Y = 0$ whatever P is. When $Q = 1$ it passes $I_1 = P$, so $Y = P$. $Y = 1$ only for $P = Q = 1$, which is the AND function.
5. Hence $Y = PQ$, option (A).

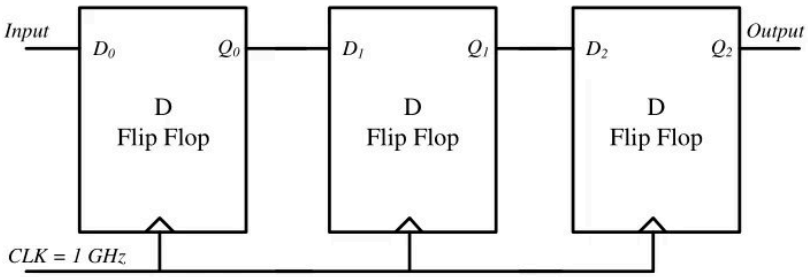
KEY IDEA

A 2×1 MUX computes $Y = \overline{S}\, I_0 + S\, I_1$. Tie one data input to a constant and feed a variable into the other, and it becomes a basic gate. With $I_0 = 0$ and $I_1 = P$, the output is $Y = SP$. That is an AND gate between the select signal and P.

WATCH OUT FOR

Getting the select convention backwards, as if $S = 1$ picked I_0. That gives $Y = \overline{Q}P$, which matches none of the options, and some students then guess OR or NAND. Another slip is reading the grounded input as logic 1 instead of 0.

Q.24 MCQ · 1 MARK

Q.24	The synchronous sequential circuit shown below works at a clock frequency of 1 GHz. The throughput, in Mbits/s, and the latency, in ns, respectively, are
	
(A)	1000, 3
(B)	333.33, 1
(C)	2000, 3
(D)	333.33, 3

Official answer: **A**

SOLUTION

1. The circuit is a 3-stage shift register, which works as a 3-stage pipeline. All three D flip-flops share one clock, so on every active clock edge each bit moves one stage to the right: $Q_0 \leftarrow \text{Input}$, $Q_1 \leftarrow Q_0$, $Q_2 \leftarrow Q_1$.
2. Clock period: $T_{clk} = \dfrac{1}{f_{clk}} = \dfrac{1}{1\ \text{GHz}} = 1\ \text{ns}$.

3. Throughput is the rate at which bits leave the output once the pipeline is full. On every clock edge a new bit enters at D_0 and a new bit appears at Q_2, so the circuit delivers one bit per clock cycle: throughput $= \dfrac{1\text{ bit}}{1\text{ ns}} = 10^9\text{ bits/s} = 1000\text{ Mbits/s}$.
4. Throughput does not depend on the number of stages. The three flip-flops hold three different bits at the same time, so they work in parallel, not one after another.
5. Latency is the time one particular bit takes to travel from Input to Output. The bit has to be clocked through all three flip-flops, one stage per edge, so it takes 3 clock cycles: latency $= 3 \times T_{clk} = 3 \times 1\text{ ns} = 3\text{ ns}$.
6. So the throughput is 1000 Mbits/s and the latency is 3 ns, which is option (A).

KEY IDEA

In a pipeline driven by one clock, throughput is set by the clock alone: one bit per cycle, so f_{clk} bits/s whatever the number of stages. Latency is set by the depth: N stages take N clock periods. Here that gives $1\text{ bit/ns} = 1000\text{ Mbits/s}$ and $3 \times 1\text{ ns} = 3\text{ ns}$.

WATCH OUT FOR

Dividing the clock rate by the number of stages (1000/3 = 333.33 Mbits/s, options B and D), as if each bit had to clear the whole register before the next could enter. In fact all three stages hold different bits at once, so a bit leaves on every clock. A second mistake is to take the latency as one clock period (1 ns, option B) when a bit has to pass through all three flip-flops.

Q.25 MCQ · 1 MARK

Control Systems › Bode & margins

The open loop transfer function of a unity negative feedback system is

$$G(s) = \frac{k}{s(1+sT_1)(1+sT_2)}$$

where k, T_1 and T_2 are positive constants. The phase cross-over frequency, in rad/s, is

(A) $\dfrac{1}{\sqrt{T_1T_2}}$
(B) $\dfrac{1}{T_1T_2}$
(C) $\dfrac{1}{T_1\sqrt{T_2}}$
(D) $\dfrac{1}{T_2\sqrt{T_1}}$

Official answer: **A**

SOLUTION

1. **Phase crossover definition:** the phase crossover frequency ω_{pc} is the frequency at which $\angle G(j\omega) = -180^\circ$. The gain $k > 0$ adds no phase, so it drops out and the answer cannot depend on k.
2. **Phase of each factor:** the pole at the origin contributes -90°, and each first-order pole $(1 + j\omega T)$ with $T > 0$ contributes $-\tan^{-1}(\omega T)$:

$$\angle G(j\omega) = -90^\circ - \tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$$

   As ω rises from 0 to ∞, this phase falls steadily from -90° to -270°, so it passes through -180° exactly once.
3. **Set the phase to -180°:**

$$\tan^{-1}(\omega T_1) + \tan^{-1}(\omega T_2) = 90^\circ$$

4. **Solve the arctangent equation:** write $a = \omega T_1 > 0$ and $b = \omega T_2 > 0$. Then $\tan^{-1} b = 90^\circ - \tan^{-1} a = \tan^{-1}\left(\dfrac{1}{a}\right)$, so $b = \dfrac{1}{a}$, which means $ab = 1$. (Equivalently, the denominator of $\tan^{-1}\dfrac{a+b}{1-ab}$ goes to

zero.)

$$\omega^2 T_1 T_2 = 1$$

5. **Solve for ω:**

$$\omega_{pc} = \frac{1}{\sqrt{T_1 T_2}} \text{ rad/s}$$

6. **Cross-check with real and imaginary parts:** the denominator of $G(j\omega)$ is

$$j\omega(1 + j\omega T_1)(1 + j\omega T_2) = -\omega^2(T_1 + T_2) + j\omega\left(1 - \omega^2 T_1 T_2\right)$$

At $\omega^2 T_1 T_2 = 1$ the imaginary part is zero and the real part $-\omega^2(T_1 + T_2)$ is negative. So $G(j\omega)$ is a negative real number there, which is a phase of -180°. This matches option (A).

KEY IDEA

Phase crossover is where $\angle G(j\omega) = -180^\circ$, and the gain k has no effect on it. The integrator already gives -90°, so the two lag angles must add up to 90°. That happens when $\omega T_1 \cdot \omega T_2 = 1$. So ω_{pc} is the geometric mean of the two corner frequencies $1/T_1$ and $1/T_2$.

WATCH OUT FOR

Picking (B), $1/(T_1 T_2)$, by solving $\omega T_1 T_2 = 1$ instead of $\omega^2 T_1 T_2 = 1$, or by multiplying the corner frequencies instead of taking their geometric mean. Another mistake is to forget the -90° from the pole at the origin and require the two lags to reach 180°, which they never do. Students also mix up phase crossover (phase $= -180^\circ$) with gain crossover ($|G| = 1$, which does depend on k).

Q.26 MCQ · 1 MARK

Signals and Systems › LTI & convolution

Consider a system with input $x(t)$ and output $y(t) = x(e^t)$. The system is

(A) Causal and time invariant.

(B) Non-causal and time varying.

(C) Causal and time varying.

(D) Non-causal and time invariant.

Official answer: **B**

SOLUTION

1. **Causality test:** a system is causal if $y(t)$ depends only on $x(\tau)$ for $\tau \le t$. Here $y(t)$ is the input read at the single instant $\tau = e^t$.
2. Check one instant: at $t = 1$, $y(1) = x(e) \approx x(2.718)$, a future value of the input. In fact $e^t > t$ for every real t (the function $e^t - t$ has its minimum value 1 at $t = 0$), so the output always depends on a future input value. The system is **non-causal**.
3. **Time-invariance test:** delay the input by t_0 to get $x_1(t) = x(t - t_0)$. Pass it through the system: $y_1(t) = x_1(e^t) = x(e^t - t_0)$.
4. Now delay the original output by t_0: $y(t - t_0) = x(e^{t-t_0}) = x\left(e^t e^{-t_0}\right)$.
5. Compare the two arguments: $e^t - t_0$ and e^{t-t_0} are different functions of t. For example, at $t = 0$ they are $1 - t_0$ and e^{-t_0}, and $e^{-t_0} > 1 - t_0$ for every $t_0 \neq 0$. A concrete input shows the outputs really differ: with $x(t) = t$ and $t_0 = 1$,

$$y_1(0) = e^0 - 1 = 0, \qquad y(0 - 1) = e^{-1} \approx 0.368$$

The time axis is warped by e^t, not simply shifted, so a delay at the input does not appear as the same delay at the output. The system is **time varying**.

6. Non-causal and time varying, which is option (B).

KEY IDEA

When the output is the input evaluated at a warped time $y(t) = x(f(t))$: it is causal only if $f(t) \le t$ for every t, and time invariant only if $f(t) = t - c$ (a pure shift). Here $f(t) = e^t$ is greater than t everywhere and is not a pure shift, so the system is non-causal and time varying.

WATCH OUT FOR

Delaying the output wrongly by replacing t with $t - t_0$ only in the input and writing $y(t - t_0) = x(e^t - t_0)$, which makes the system look time invariant. Another slip is checking causality only at $t = 0$, where $e^0 = 1 > 0$ is easy to overlook, or assuming a very negative t makes e^t lie in the past. In fact $e^t > t$ for every t.

Q.27 MCQ · 1 MARK

Signals and Systems › Fourier transform

Let $m(t)$ be a strictly band-limited signal with bandwidth B and energy E. Assuming $\omega_0 = 10B$, the energy in the signal $m(t)\cos\omega_0 t$ is

(A) $\dfrac{E}{4}$

(B) $\dfrac{E}{2}$

(C) E

(D) $2E$

Official answer: **B**

SOLUTION

1. Modulation property of the Fourier transform: with $m(t) \leftrightarrow M(\omega)$,
$$x(t) = m(t)\cos\omega_0 t \;\leftrightarrow\; X(\omega) = \tfrac{1}{2}\left[M(\omega - \omega_0) + M(\omega + \omega_0)\right]$$
2. $M(\omega)$ is zero outside $|\omega| \le W$, where W is the bandwidth in rad/s ($W = B$ if B is in rad/s, $W = 2\pi B$ if B is in Hz). The two shifted copies occupy $\omega_0 - W \le \omega \le \omega_0 + W$ and $-\omega_0 - W \le \omega \le -\omega_0 + W$.
3. The copies are disjoint when the lower edge of the right copy lies above the upper edge of the left copy: $\omega_0 - W > -\omega_0 + W$, i.e. $\omega_0 > W$. With $\omega_0 = 10B$ this holds whichever unit B is in, since $10B > B$ and $10B > 2\pi B \approx 6.28B$. So the two copies do not overlap.
4. Parseval's theorem:
$$E_x = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega = \frac{1}{4}\cdot\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[|M(\omega-\omega_0)|^2 + |M(\omega+\omega_0)|^2 + 2\,\mathrm{Re}\{M(\omega-\omega_0)M^*(\omega+\omega_0)\}\right] d\omega$$
The cross term is zero at every ω, because wherever one copy is non-zero the other is zero.
5. A frequency shift does not change the area under $|M|^2$, so each shifted copy carries energy E:
$$E_x = \frac{1}{4}(E + E) = \frac{E}{2}$$
6. Time-domain check: $\cos^2\omega_0 t = \tfrac{1}{2}(1 + \cos 2\omega_0 t)$, so
$$E_x = \frac{1}{2}\int_{-\infty}^{\infty} m^2(t)\, dt + \frac{1}{2}\int_{-\infty}^{\infty} m^2(t)\cos 2\omega_0 t\, dt$$
The second integral is the real part of the Fourier transform of $m^2(t)$ evaluated at $2\omega_0$. $m^2(t)$ is band-limited to $|\omega| \le 2W$ and $2\omega_0 > 2W$, so that integral is zero and $E_x = E/2$. Answer: (B).

KEY IDEA

Multiplying by $\cos\omega_0 t$ splits the spectrum into two half-amplitude copies at $\pm\omega_0$. When ω_0 exceeds the signal bandwidth the copies do not overlap. Each copy then carries $\left(\tfrac{1}{2}\right)^2 E = E/4$, so the total is $E/2$. It is the same as saying the average of $\cos^2$ is $\tfrac{1}{2}$.

WATCH OUT FOR

Picking E because $|\cos\omega_0 t| \le 1$ and the cosine "only shifts the spectrum", which forgets the factor $\tfrac{1}{2}$ on each copy. Or picking $E/4$ by squaring that $\tfrac{1}{2}$ and counting only one of the two copies. The given $\omega_0 = 10B$ is there to guarantee there is no spectral overlap. Without it the cross term would not vanish and the energy would not be exactly $E/2$.

Q.28 MCQ · 1 MARK

Signals and Systems › Fourier transform

The Fourier transform $X(\omega)$ of $x(t) = e^{-t^2}$ is

Note: $\int_{-\infty}^{\infty} e^{-y^2}\,dy = \sqrt{\pi}$

(A) $\sqrt{\pi}\, e^{\frac{\omega^2}{2}}$

(B) $\dfrac{e^{-\frac{\omega^2}{4}}}{2\sqrt{\pi}}$

(C) $\sqrt{\pi}\, e^{-\frac{\omega^2}{4}}$

(D) $\sqrt{\pi}\, e^{-\frac{\omega^2}{2}}$

Official answer: **C**

SOLUTION

1. Start from the definition of the Fourier transform:
$$X(\omega) = \int_{-\infty}^{\infty} e^{-t^2} e^{-j\omega t}\,dt = \int_{-\infty}^{\infty} e^{-(t^2 + j\omega t)}\,dt$$
2. Complete the square in the exponent:
$$t^2 + j\omega t = \left(t + \frac{j\omega}{2}\right)^2 + \frac{\omega^2}{4}$$
because $\left(\frac{j\omega}{2}\right)^2 = -\frac{\omega^2}{4}$, so the square is corrected by adding back $\frac{\omega^2}{4}$.
3. Take the constant factor out of the integral:
$$X(\omega) = e^{-\frac{\omega^2}{4}} \int_{-\infty}^{\infty} e^{-\left(t + \frac{j\omega}{2}\right)^2}\,dt$$
4. Substitute $y = t + \frac{j\omega}{2}$, so $dy = dt$. This moves the path of integration to a line parallel to the real axis. Because e^{-z^2} is analytic everywhere and $\left|e^{-(\pm R + jv)^2}\right| = e^{-R^2 + v^2} \to 0$ as $R \to \infty$ for any fixed v, the short vertical sides of the shifted contour contribute nothing, and the integral equals the given Gaussian integral:
$$\int_{-\infty}^{\infty} e^{-\left(t + \frac{j\omega}{2}\right)^2}\,dt = \int_{-\infty}^{\infty} e^{-y^2}\,dy = \sqrt{\pi}$$
5. Therefore
$$X(\omega) = \sqrt{\pi}\, e^{-\frac{\omega^2}{4}}$$
6. Quick checks on the options. First, $X(0) = \int_{-\infty}^{\infty} x(t)\,dt = \sqrt{\pi}$. Option (B) gives $X(0) = \frac{1}{2\sqrt{\pi}}$, so it is out. Second, $x(t)$ is absolutely integrable, so $X(\omega) \to 0$ as $|\omega| \to \infty$. Option (A) grows without bound, so it is out. Options (C) and (D) both pass these checks, and the derivation settles it: the exponent is $-\frac{\omega^2}{4}$, not $-\frac{\omega^2}{2}$. Answer: (C).

KEY IDEA

A Gaussian transforms to a Gaussian. Complete the square in $t^2 + j\omega t$ and the leftover term gives the factor $e^{-\omega^2/4}$. The remaining integral is the given $\sqrt{\pi}$. In general $e^{-at^2} \leftrightarrow \sqrt{\frac{\pi}{a}}\, e^{-\frac{\omega^2}{4a}}$ for $a > 0$; this question is the case $a = 1$.

WATCH OUT FOR

Getting the sign of the completed-square term wrong. Since $\left(\frac{j\omega}{2}\right)^2 = -\frac{\omega^2}{4}$, the leftover is $+\frac{\omega^2}{4}$ inside $e^{-(\cdot)}$, which gives a decaying $e^{-\omega^2/4}$. Dropping the j produces the growing $e^{+\omega^2/4}$ (the trap in (A)). Another slip is to use the Gaussian-density form $e^{-t^2/2} \leftrightarrow \sqrt{2\pi}\, e^{-\omega^2/2}$, which leads to the $-\frac{\omega^2}{2}$ exponent in (D). Setting $\omega = 0$ is a fast check: it must give $\int x(t)\,dt = \sqrt{\pi}$.

Q.29 MCQ · 1 MARK

Signals and Systems › Fourier transform

In the table shown below, match the signal type with its spectral characteristics.

SIGNAL TYPE	SPECTRAL CHARACTERISTICS
(i) Continuous, aperiodic	(a) Continuous, aperiodic
(ii) Continuous, periodic	(b) Continuous, periodic
(iii) Discrete, aperiodic	(c) Discrete, aperiodic
(iv) Discrete, periodic	(d) Discrete, periodic

(A) $(i) \rightarrow (a), (ii) \rightarrow (b), (iii) \rightarrow (c), (iv) \rightarrow (d)$
(B) $(i) \rightarrow (a), (ii) \rightarrow (c), (iii) \rightarrow (b), (iv) \rightarrow (d)$
(C) $(i) \rightarrow (d), (ii) \rightarrow (b), (iii) \rightarrow (c), (iv) \rightarrow (a)$
(D) $(i) \rightarrow (a), (ii) \rightarrow (c), (iii) \rightarrow (d), (iv) \rightarrow (b)$

Official answer: **B**

SOLUTION

1. Duality rule between the two domains: **periodic** in one domain ⇔ **discrete** in the other, and **discrete** (sampled) in one domain ⇔ **periodic** in the other. The complementary pairs also hold: **aperiodic** in one domain ⇔ **continuous** in the other, and **continuous** in one domain ⇔ **aperiodic** in the other.
2. (i) Continuous-time, aperiodic $x(t)$: the transform is the CTFT $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$. Aperiodic in time gives a continuous spectrum, and continuous time gives an aperiodic spectrum, so (i) $\rightarrow$ (a).
3. (ii) Continuous-time, periodic $x(t)$ with period T_0: the tool is the continuous-time Fourier series, whose spectrum has lines only at $k\omega_0$, where $\omega_0 = \frac{2\pi}{T_0}$. Periodic in time gives a discrete spectrum, and continuous time gives an aperiodic spectrum, so (ii) $\rightarrow$ (c).
4. (iii) Discrete-time, aperiodic $x[n]$: the transform is the DTFT $X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$, which satisfies $X(e^{j(\Omega+2\pi)}) = X(e^{j\Omega})$ because $e^{-j2\pi n} = 1$ for every integer n. Discrete time gives a periodic spectrum, and aperiodic time gives a continuous spectrum, so (iii) $\rightarrow$ (b).
5. (iv) Discrete-time, periodic $x[n]$ with period N: the tool is the discrete-time Fourier series (the DFT form). Its coefficients c_k exist only at integer k and repeat, $c_{k+N} = c_k$, so the spectrum is discrete and periodic: (iv) $\rightarrow$ (d).
6. The matching is (i) $\rightarrow$ (a), (ii) $\rightarrow$ (c), (iii) $\rightarrow$ (b), (iv) $\rightarrow$ (d), which is option (B).

KEY IDEA

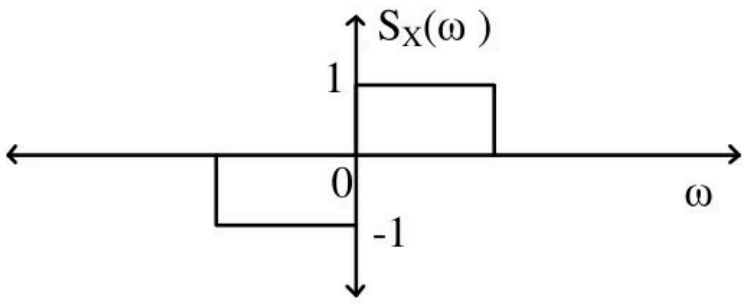
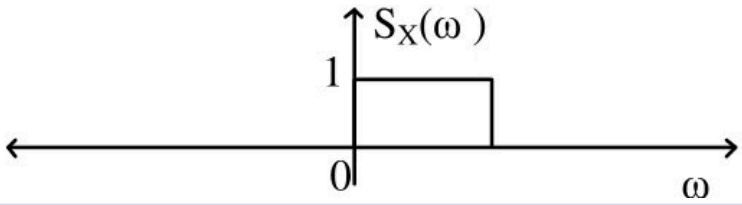
Periodicity and discreteness trade places between time and frequency. A signal that is periodic in one domain is discrete in the other, and one that is discrete in one domain is periodic in the other. So CTFT gives continuous and aperiodic, CTFS gives discrete and aperiodic, DTFT gives continuous and periodic, and DTFS/DFT gives discrete and periodic.

WATCH OUT FOR

Picking option (A) by assuming the spectrum keeps the time signal's own properties, e.g. that a continuous periodic signal has a continuous periodic spectrum. The properties swap roles: periodicity in time makes the spectrum discrete (ii → c), and sampling in time makes the spectrum periodic (iii → b). Option (D) wrongly gives the DTFT of an aperiodic sequence a discrete spectrum. The DTFT is a continuous function of Ω; only a periodic sequence has a discrete spectrum.

Q.30 MSQ · 1 MARK

Communications › Random processes

Q.30	For a real signal, which of the following is/are valid power spectral density/densities?
(A)	$S_X(\omega) = \dfrac{2}{9+\omega^2}$
(B)	$S_X(\omega) = e^{-\omega^2}\cos^2\omega$
(C)	
(D)	

Official answer: **A, B**

SOLUTION

1. A valid power spectral density of a real signal must satisfy three conditions: (i) $S_X(\omega) \geq 0$ for all ω; (ii) $S_X(-\omega) = S_X(\omega)$, because $R_X(\tau)$ is real and even for a real signal; (iii) $\int_{-\infty}^{\infty} S_X(\omega)\, d\omega < \infty$, because the average power is finite. The spectrum here is the two-sided PSD in rad/s.
2. (A) $S_X(\omega) = \dfrac{2}{9+\omega^2}$ is positive and even in ω. Also
$$\int_{-\infty}^{\infty} \frac{2\, d\omega}{9+\omega^2} = \frac{2}{3}\left[\tan^{-1}\frac{\omega}{3}\right]_{-\infty}^{\infty} = \frac{2\pi}{3} < \infty.$$
It is the transform of $\frac{1}{3}e^{-3|\tau|}$, since $e^{-a|\tau|} \leftrightarrow \dfrac{2a}{a^2+\omega^2}$ with $a = 3$. Valid.
3. (B) $S_X(\omega) = e^{-\omega^2}\cos^2\omega$ is a product of non-negative terms, so it is ≥ 0. It is even, because $\cos^2(-\omega) = \cos^2\omega$. It is integrable, since $0 \leq S_X(\omega) \leq e^{-\omega^2}$ and $\int e^{-\omega^2} d\omega = \sqrt{\pi}$. Valid.
4. (C) The plot is $+1$ over a band of positive ω and -1 over a band of negative ω. It takes negative values, which violates $S_X(\omega) \geq 0$, and it is odd rather than even. Not valid.
5. (D) The plot is non-zero only for $\omega > 0$, so $S_X(-\omega) \neq S_X(\omega)$. A one-sided spectrum corresponds to a complex autocorrelation, which cannot occur for a real signal. Not valid.
6. Hence the valid power spectral densities are (A) and (B).

KEY IDEA
A PSD of a real signal must be non-negative, even in omega, and have finite area. Check each option against these three conditions.

WATCH OUT FOR
Accepting (D) because it is non-negative and has finite area, and forgetting that a real signal needs a symmetric (even) PSD. Accepting (C) because it looks symmetric in shape, when it is odd and goes negative.

Q.31 NAT · 1 MARK

Digital Electronics › ADC & DAC

The signal-to-noise ratio (SNR) of an ADC with a full-scale sinusoidal input is given to be 61.96 dB. The resolution of the ADC is _____ bits (*rounded off to the nearest integer*).

Official answer: **10**

SOLUTION

1. Take an ideal n-bit ADC whose full-scale range is $2V_m$ (a sine of peak V_m just fills it). The step size is $\Delta = \dfrac{2V_m}{2^n}$. The quantisation error is uniformly distributed over $\left(-\frac{\Delta}{2}, \frac{\Delta}{2}\right)$, so the quantisation noise power is $\dfrac{\Delta^2}{12}$.
2. A full-scale sinusoid has power $\dfrac{V_m^2}{2}$, so
$$\text{SNR} = \frac{V_m^2/2}{\Delta^2/12} = \frac{V_m^2}{2} \cdot \frac{12 \cdot 2^{2n}}{4V_m^2} = \frac{3}{2}\,2^{2n}$$
3. In decibels:
$$\text{SNR}_{\text{dB}} = 10\log_{10}\left(\tfrac{3}{2}\right) + 20n\log_{10} 2 = 1.76 + 6.02\,n \text{ dB}$$
4. Set this equal to the given value: $6.02\,n + 1.76 = 61.96$, so $6.02\,n = 60.20$.
5. $n = \dfrac{60.20}{6.02} = 10$. (With the unrounded constants 1.7609 and 6.0206, $n = \dfrac{60.1991}{6.0206} \approx 9.999$, which also rounds to 10.) The resolution of the ADC is 10 bits.

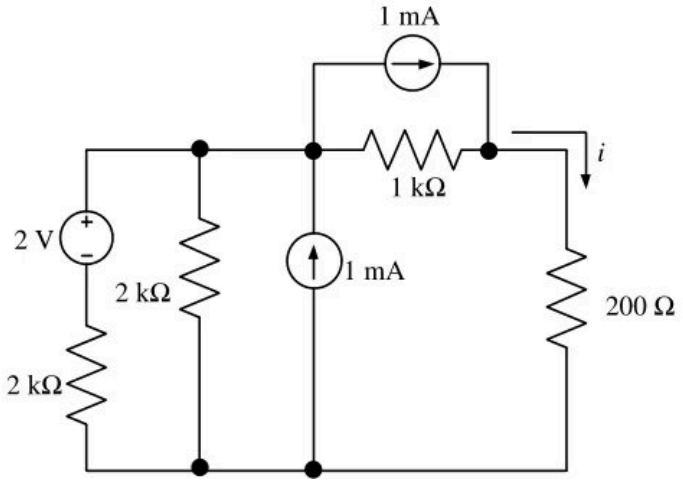
KEY IDEA
For an ideal n-bit ADC driven by a full-scale sine wave, the signal power is $V_m^2/2$ and the quantisation noise power is $\Delta^2/12$ with $\Delta = 2V_m/2^n$. That gives $\text{SNR}_{\text{dB}} = 6.02\,n + 1.76$. Solve it for n.

WATCH OUT FOR
Using the rough '6 dB per bit' rule and leaving out the 1.76 dB term: $61.96/6.02 \approx 10.3$ happens to round to the right answer here, but the method is wrong and fails on other values. Taking the signal power as V_m^2 instead of $V_m^2/2$, or the full-scale range as V_m instead of $2V_m$, moves the constant away from 1.76 dB.

Q.32 NAT · 1 MARK

Networks (Electric Circuits) › Laws & node/mesh

Q.32 In the circuit shown below, the current i flowing through 200 Ω resistor is _____ mA (*rounded off to two decimal places*).



Official answer: **1.30 to 1.40**

SOLUTION

1. Take the bottom rail as ground. Call the top-left node A (voltage V_A) and the node at the right end of the $1\ \text{k}\Omega$ resistor B (voltage V_B). Work in volts, $\text{k}\Omega$ and mA.
2. Source transformation on the left: the $2\ \text{V}$ source (+ towards A) in series with $2\ \text{k}\Omega$ becomes a $\frac{2}{2} = 1\ \text{mA}$ source into A in parallel with $2\ \text{k}\Omega$.
3. Combine everything between A and ground: the currents add, $1 + 1 = 2\ \text{mA}$ into A (the transformed source plus the upward $1\ \text{mA}$ source), and the resistances combine, $2\ \text{k}\Omega \parallel 2\ \text{k}\Omega = 1\ \text{k}\Omega$. Back to Thevenin form, that is $2 \times 1 = 2\ \text{V}$ behind $1\ \text{k}\Omega$.
4. Source transformation on the $1\ \text{mA}$ source in parallel with the $1\ \text{k}\Omega$ resistor between A and B: it becomes a $1 \times 1 = 1\ \text{V}$ source in series with $1\ \text{k}\Omega$. The current source pushes current towards B, so the + terminal is on the B side: a rise of $1\ \text{V}$ going from A to B, which adds to the $2\ \text{V}$.
5. That leaves one loop: $2 + 1 = 3\ \text{V}$ behind $1 + 1 = 2\ \text{k}\Omega$, driving the $200\ \Omega = 0.2\ \text{k}\Omega$ load.
$$i = \frac{2+1}{1+1+0.2}\ \text{mA} = \frac{3}{2.2}\ \text{mA} = \frac{15}{11}\ \text{mA} \approx 1.364\ \text{mA}$$
6. Check by nodal analysis (currents leaving each node). KCL at A: $\frac{V_A - 2}{2} + \frac{V_A}{2} - 1 + (V_A - V_B) + 1 = 0 \Rightarrow 2V_A - V_B = 1$. KCL at B: $(V_A - V_B) + 1 = \frac{V_B}{0.2} \Rightarrow V_A = 6V_B - 1$. Substituting, $2(6V_B - 1) - V_B = 1$, so $11V_B = 3$, $V_B = \frac{3}{11}\ \text{V}$ and $i = \frac{V_B}{0.2} = \frac{15}{11}\ \text{mA}$.
7. Rounded to two decimal places, $i = 1.36\ \text{mA}$.

KEY IDEA

Use repeated source transformation. Everything between node A and ground reduces to $2\ \text{V}$ behind $1\ \text{k}\Omega$. The $1\ \text{mA}$ source across the $1\ \text{k}\Omega$ becomes a series $1\ \text{V}$ source whose + terminal faces B. That leaves a single loop: $3\ \text{V}$ across $2.2\ \text{k}\Omega$. Two-node nodal analysis gives the same result.

WATCH OUT FOR

Getting the polarity of the transformed $1\ \text{mA} \parallel 1\ \text{k}\Omega$ source wrong. A current source pointing from A to B becomes a voltage source with its + terminal at B, so it adds to the $2\ \text{V}$. Subtracting it gives $\frac{1}{2.2} \approx 0.45\ \text{mA}$. A second trap is dropping the upward $1\ \text{mA}$ source when combining the branches at A, which gives $\frac{2}{2.2} \approx 0.91\ \text{mA}$.

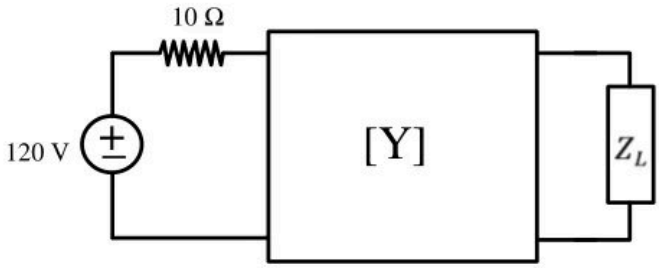
Q.33 NAT · 1 MARK

Networks (Electric Circuits) › Two-port

Q.33 For the two port network shown below, the [Y]-parameters is given as

$$[Y] = \frac{1}{100}\begin{bmatrix} 2 & -1 \\ -1 & 4/3 \end{bmatrix} S$$

The value of load impedance Z_L, in Ω, for maximum power transfer will be _____ (*rounded off to the nearest integer*).



Official answer: **80**

SOLUTION

1. **Setup.** Maximum power transfer needs $Z_L = Z_{th}^*$, where Z_{th} is the impedance seen looking into port 2 with the 120 V source set to zero. The source is DC and every y-parameter is real, so Z_{th} is a pure resistance and $Z_L = Z_{th}$. From the matrix: $y_{11} = 0.02$ S, $y_{12} = y_{21} = -0.01$ S, $y_{22} = \frac{4}{300}$ S.
2. **Kill the source.** With the 120 V source replaced by a short, port 1 is closed through the 10 Ω resistor alone. I_1 enters the upper input terminal after flowing through the resistor from the (now zero-volt) source node, so $0 - V_1 = 10\,I_1$, i.e. $V_1 = -10\,I_1$ and $I_1 = -0.1\,V_1$.
3. **Port-1 Y equation:** $I_1 = y_{11}V_1 + y_{12}V_2$ gives $-0.1V_1 = 0.02V_1 - 0.01V_2$, so $0.12V_1 = 0.01V_2$ and $V_1 = \frac{V_2}{12}$.
4. **Port-2 Y equation:**

$$I_2 = y_{21}V_1 + y_{22}V_2 = -0.01 \cdot \frac{V_2}{12} + \frac{4}{300}V_2 = \left(-\frac{1}{1200} + \frac{16}{1200}\right)V_2 = \frac{V_2}{80}$$

5. **Thevenin impedance:** $Z_{th} = \frac{V_2}{I_2} = 80\ \Omega$. The same result comes from the terminated-two-port formula with source admittance $Y_s = \frac{1}{10}$ S:

$$Y_{out} = y_{22} - \frac{y_{12}y_{21}}{y_{11}+Y_s} = \frac{4}{300} - \frac{10^{-4}}{0.12} = 0.01333 - 0.00083 = 0.0125\ \text{S}$$

so $Z_{th} = \dfrac{1}{0.0125} = 80\ \Omega$.

6. **Result:** $Z_L = Z_{th}^* = 80\ \Omega$. As a check, the open-circuit port-2 voltage: with $I_2 = 0$, port 2 gives $0 = -0.01V_1 + \dfrac{4}{300}V_2$, so $V_1 = \dfrac{4}{3}V_2$; port 1 gives $\dfrac{120 - V_1}{10} = 0.02V_1 - 0.01V_2$, i.e. $12 = 0.12V_1 - 0.01V_2 = 0.16V_2 - 0.01V_2 = 0.15V_2$, so $V_{th} = 80$ V and $P_{max} = \dfrac{80^2}{4 \times 80} = 20$ W.

KEY IDEA

For maximum power transfer the load must equal the conjugate of the Thevenin impedance seen at port 2. To find it, set the source to zero, which leaves port 1 closed through the 10 Ω source resistance, so $I_1 = -V_1/10$. Put that into the Y equations and read off V_2/I_2. In one line: $Y_{out} = y_{22} - \dfrac{y_{12}y_{21}}{y_{11}+Y_s}$.

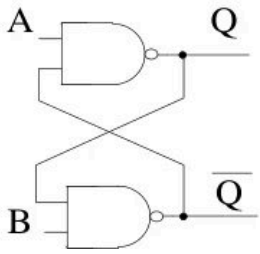
WATCH OUT FOR

Using $1/y_{22} = 75\ \Omega$, which ignores the 10 Ω source resistance that loads port 1 (that is the output impedance with port 1 shorted). A second slip is the sign in $V_1 = -10I_1$: I_1 is defined as entering the network, so it flows through the resistor toward the port. Writing $V_1 = +10I_1$ gives $Y_{out} = \dfrac{4}{300} - \dfrac{10^{-4}}{0.02 - 0.1}$ and $Z_{th} \approx 69\ \Omega$, which is wrong.

Q.34 NAT · 1 MARK

Digital Electronics › Sequential

Q.34 For the circuit shown below, the propagation delay of each NAND gate is 1 ns. The critical path delay, in ns, is _____ (*rounded off to the nearest integer*).



Official answer: **02**

SOLUTION

1. **Identify the circuit.** The two cross-coupled NAND gates form an $\overline{S}\,\overline{R}$ latch with active-low inputs. $A = 0$ sets it ($Q = 1$) and $B = 0$ resets it ($Q = 0$). An overline marks the complement, e.g. $\overline{Q}$.
2. **Critical path means the longest input-to-output path before both outputs settle.** A change on an input passes through its own gate to change that gate's output. The new output then travels along the feedback wire into the other gate, which changes the second output.
3. **Trace a set operation.** Start from the stored state $Q = 0$, $\overline{Q} = 1$ with $A = B = 1$. This state is consistent: the top NAND sees $(1, 1)$ and gives 0, and the bottom NAND sees $(0, 1)$ and gives 1. Now let A go $1 \to 0$. The top NAND has a 0 on an input, so $Q = 1$ after 1 ns.
4. **The second gate responds.** The bottom NAND now sees $Q = 1$ and $B = 1$, so $\overline{Q} = 0$ after another 1 ns.
5. **The feedback closes the loop with no further change.** $\overline{Q} = 0$ goes back into the top NAND, which already has $A = 0$, so Q stays at 1. Nothing else switches and the latch is stable. Resetting through B is the same by symmetry.
6. **Critical path delay.**

$$t_{cp} = t_{pd,\text{NAND1}} + t_{pd,\text{NAND2}} = 1 + 1 = 2\ \text{ns}$$

KEY IDEA

In a cross-coupled NAND (SR) latch, the worst case is an input change that switches its own gate and then, through the feedback wire, switches the other gate. That makes two gate delays. Once the second output reaches the first gate, it only confirms the value already there, so the loop does not keep going round.

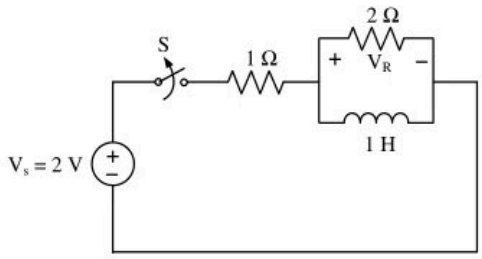
WATCH OUT FOR

Answering 1 ns, which counts only the gate the input drives and ignores that $\overline{Q}$ settles one gate delay later. The opposite mistake is thinking the feedback loop adds delays without end (or a third delay). The feedback signal arrives at the first gate without changing its output, so the circuit is settled after 2 ns.

Q.35 NAT · 1 MARK

Networks (Electric Circuits) › Transients

Q.35 In the circuit shown below, switch S was closed for a long time. If the switch is opened at $t = 0$, the maximum magnitude of the voltage V_R, in volts, is _____ (*rounded off to the nearest integer*).



Official answer: **04**

SOLUTION

1. Before the switch opens ($t < 0$, DC steady state): the inductor behaves as a short circuit. It is in parallel with the $2\,\Omega$ resistor, so the resistor has zero voltage across it and carries no current: $V_R(0^-) = 0$.
2. Inductor current in steady state (the short bypasses the $2\,\Omega$, leaving only the $1\,\Omega$ in the loop): $i_L(0^-) = \dfrac{V_s}{1\,\Omega} = \dfrac{2}{1} = 2\ \text{A}$, flowing through the inductor from the left node to the right node.
3. An inductor current cannot change instantaneously: $i_L(0^+) = i_L(0^-) = 2\ \text{A}$, still flowing left to right through the inductor.
4. For $t > 0$ the open switch disconnects the source and the $1\,\Omega$ resistor. The inductor and the $2\,\Omega$ resistor now form a closed loop on their own, so the inductor current must return through the resistor from the right node to the left node.
5. Voltage across the resistor at $t = 0^+$ ($+$ at the left node): current flows through the resistor from right to left, so the right node is at the higher potential and $V_R(0^+) = -\,i_L(0^+) \times 2\,\Omega = -2 \times 2 = -4\ \text{V}$.
6. Source-free RL decay with $\tau = \dfrac{L}{R} = \dfrac{1}{2}\ \text{s}$:
$$V_R(t) = -4\,e^{-2t}\ \text{V}, \quad t > 0$$
The magnitude falls monotonically, so it is largest at $t = 0^+$ (and $V_R = 0$ for all $t < 0$).
7. Maximum magnitude: $|V_R|_{\max} = 4\ \text{V}$.

KEY IDEA

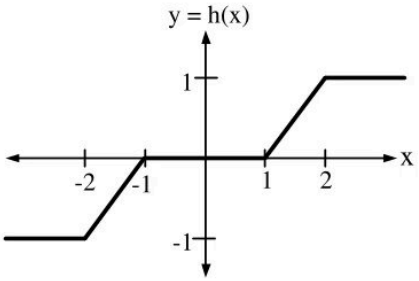
The inductor current stays at its pre-switching value (2 A, set only by the 1 Ω resistor because the inductor shorts the 2 Ω). Once the switch opens, that current has nowhere to go except back through the 2 Ω resistor, so the resistor voltage jumps from 0 to 2 A × 2 Ω = 4 V in magnitude and then decays with $\tau = L/R = 0.5$ s.

WATCH OUT FOR

Answering 0 V because the 2 Ω resistor carried no current before switching. Resistor voltage can jump; only the inductor current is continuous. A second mistake is using the full 3 Ω series path to find $i_L(0^-)$ (giving 2/3 A and $|V_R| \approx 1.33$ V). In DC steady state the inductor shorts out the 2 Ω, so $i_L(0^-) = 2$ A. The sign of $V_R(0^+)$ is negative, but the question asks for the magnitude.

Q.36 MCQ · 2 MARKS

Engineering Mathematics › Probability & statistics

Q.36	A random variable X, distributed normally as $N(0,1)$, undergoes the transformation Y = h(X), given in the figure. The form of the probability density function of Y is (In the options given below, a, b, c are non-zero constants and $g(y)$ is piece-wise continuous function)
	
(A)	$a\delta(y-1)+b\delta(y+1)+g(y)$
(B)	$a\delta(y+1)+b\delta(y)+c\delta(y-1)+g(y)$
(C)	$a\delta(y+2)+b\delta(y)+c\delta(y-2)+g(y)$
(D)	$a\delta(y+2)+b\delta(y-2)+g(y)$

Official answer: **B**

SOLUTION

1. Read the figure as a piecewise function: $h(x) = -1$ for $x \le -2$, $h(x) = x + 1$ for $-2 \le x \le -1$, $h(x) = 0$ for $-1 \le x \le 1$, $h(x) = x - 1$ for $1 \le x \le 2$, and $h(x) = 1$ for $x \ge 2$. So Y only takes values in $[-1, 1]$.
2. Rule: wherever h is flat over an interval of x with non-zero probability, all of that probability lands on one value y_0. In the PDF of Y this appears as an impulse $P(\text{interval})\,\delta(y - y_0)$.

3. The flat piece $x \le -2$ maps to $y = -1$: $P(Y = -1) = P(X \le -2) = \Phi(-2) \approx 0.0228 \neq 0$. This gives the term $a\,\delta(y+1)$.
4. The flat piece $-1 \le x \le 1$ (the part of the graph lying on the x-axis) maps to $y = 0$: $P(Y = 0) = P(-1 \le X \le 1) = 2\Phi(1) - 1 \approx 0.6827 \neq 0$. This gives the term $b\,\delta(y)$.
5. The flat piece $x \ge 2$ maps to $y = 1$: $P(Y = 1) = P(X \ge 2) = 1 - \Phi(2) \approx 0.0228 \neq 0$. This gives the term $c\,\delta(y-1)$.
6. The two sloped pieces are one-to-one with slope 1, so they give an ordinary (continuous) density. For $-1 < y < 0$: $x = y - 1$ and $\left|\frac{dx}{dy}\right| = 1$, so $f_Y(y) = f_X(y-1)$. For $0 < y < 1$: $x = y + 1$, so $f_Y(y) = f_X(y+1)$. Together these make the piecewise-continuous part $g(y)$, which is zero outside $(-1, 1)$.
7. Collecting the terms:
$$f_Y(y) = a\,\delta(y+1) + b\,\delta(y) + c\,\delta(y-1) + g(y),$$
with $a = \Phi(-2)$, $b = 2\Phi(1) - 1$, $c = 1 - \Phi(2)$, all non-zero. Check: $a + b + c + \int g(y)\,dy \approx 0.0228 + 0.6827 + 0.0228 + 2(0.1359) = 1$. This is option (B).

KEY IDEA

When a transformation $Y = h(X)$ is constant at y_0 over an interval of x that has non-zero probability, Y has a point mass at y_0, and its PDF picks up an impulse $P(\text{interval})\,\delta(y - y_0)$. Where h is strictly monotonic, the density is continuous: $f_Y(y) = f_X(x)/|h'(x)|$. So you get one delta for each flat level of the graph, and they sit at the output values (y), not at the x breakpoints.

WATCH OUT FOR

There are two traps. First, placing the impulses at the x-axis breakpoints ± 2 instead of at the output levels $y = \pm 1$, which leads to (C) or (D). Second, overlooking that the segment lying along the x-axis for $-1 \le x \le 1$ is also a flat piece. It carries about 68% of the probability, so it produces the impulse $b\,\delta(y)$ at $y = 0$. Missing it leads to (A).

Q.37 MCQ · 2 MARKS

Engineering Mathematics › Vector calculus

The value of the line integral $\int_P^Q (z^2\,dx + 3y^2\,dy + 2xz\,dz)$ along the straight line joining the points $P\,(1, 1, 2)$ and $Q\,(2, 3, 1)$ is

(A) 20

(B) 24

(C) 29

(D) -5

Official answer: **B**

SOLUTION

1. Write the integrand as $\vec{F} \cdot d\vec{r}$ with $\vec{F} = (z^2,\ 3y^2,\ 2xz)$ and test whether the field is conservative. The curl components are $\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z} = 0 - 0 = 0$, $\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x} = 2z - 2z = 0$ and $\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = 0 - 0 = 0$. So $\nabla \times \vec{F} = \vec{0}$ everywhere in $\mathbb{R}^3$: the field is conservative and the integral does not depend on the path.
2. Find the potential. $z^2\,dx + 2xz\,dz = d(xz^2)$ and $3y^2\,dy = d(y^3)$, so $\phi(x, y, z) = xz^2 + y^3$. Check: $\nabla\phi = (z^2,\ 3y^2,\ 2xz) = \vec{F}$.
3. By the fundamental theorem for line integrals, $\int_P^Q \nabla\phi \cdot d\vec{r} = \phi(Q) - \phi(P)$. Here $\phi(Q) = \phi(2, 3, 1) = 2 \cdot 1^2 + 3^3 = 29$ and $\phi(P) = \phi(1, 1, 2) = 1 \cdot 2^2 + 1^3 = 5$.
4. So the integral is $29 - 5 = 24$.

5. Check along the given straight line: $x = 1 + t,\ y = 1 + 2t,\ z = 2 - t$ for $0 \le t \le 1$, so $dx = dt$, $dy = 2\,dt$, $dz = -dt$. Then
$$\int_0^1 \left[(2-t)^2 + 6(1+2t)^2 - 2(1+t)(2-t)\right] dt = \frac{7}{3} + 26 - \frac{13}{3} = 24$$
which agrees.
6. The value is 24, which is option (B).

KEY IDEA

$\nabla \times \vec{F} = \vec{0}$, so $\vec{F} = (z^2, 3y^2, 2xz)$ is the gradient of $\phi = xz^2 + y^3$. The integral therefore depends only on its endpoints: $\phi(Q) - \phi(P) = 29 - 5 = 24$. Saying the path is a straight line only reassures you. It does not change the answer.

WATCH OUT FOR

Stopping at $\phi(Q) = 29$ and forgetting to subtract $\phi(P) = 5$ picks the distractor (C). A sign error with $dz = -dt$ when parametrising also goes wrong: z falls from 2 to 1 along the line, so the $2xz\,dz$ term has to be negative. Another mistake is adding xz^2 twice when building the potential, once from $z^2\,dx$ and once from $2xz\,dz$. It is a single term.

Q.38 MCQ · 2 MARKS

Engineering Mathematics › Linear algebra

Let $\boldsymbol{x}$ be an $n \times 1$ real column vector with length $l = \sqrt{\boldsymbol{x}^T\boldsymbol{x}}$. The trace of the matrix $P = \boldsymbol{x}\boldsymbol{x}^T$ is

(A) l^2

(B) $\dfrac{l^2}{4}$

(C) l

(D) $\dfrac{l^2}{2}$

Official answer: **A**

SOLUTION

1. Write $\boldsymbol{x} = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}^T$ with real entries. The outer product $P = \boldsymbol{x}\boldsymbol{x}^T$ is an $n \times n$ matrix whose entries are $P_{ij} = x_i x_j$.
2. The trace is the sum of the diagonal entries: $\mathrm{tr}(P) = \sum_{i=1}^{n} P_{ii} = \sum_{i=1}^{n} x_i^2$.
3. The inner product is also $\boldsymbol{x}^T\boldsymbol{x} = \sum_{i=1}^{n} x_i^2$, so $\mathrm{tr}(\boldsymbol{x}\boldsymbol{x}^T) = \boldsymbol{x}^T\boldsymbol{x}$. The cyclic property $\mathrm{tr}(AB) = \mathrm{tr}(BA)$ gives this directly: $\mathrm{tr}(\boldsymbol{x}\boldsymbol{x}^T) = \mathrm{tr}(\boldsymbol{x}^T\boldsymbol{x})$, and $\boldsymbol{x}^T\boldsymbol{x}$ is a 1×1 matrix, so its trace is the number itself.
4. The length is defined as $l = \sqrt{\boldsymbol{x}^T\boldsymbol{x}}$, so $\boldsymbol{x}^T\boldsymbol{x} = l^2$.
5. Therefore
$$\mathrm{tr}(P) = l^2,$$
which is option (A).
6. Check: take $\boldsymbol{x} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, so $l = \sqrt{9 + 16} = 5$. Then $P = \begin{bmatrix} 9 & 12 \\ 12 & 16 \end{bmatrix}$ and $\mathrm{tr}(P) = 9 + 16 = 25 = l^2$.

KEY IDEA

The diagonal entries of the outer product $\boldsymbol{x}\boldsymbol{x}^T$ are x_i^2, so its trace is $\sum x_i^2 = \boldsymbol{x}^T\boldsymbol{x}$, the squared length. You can also get this from $\mathrm{tr}(AB) = \mathrm{tr}(BA)$: it turns the $n \times n$ outer product into the 1×1 inner product.

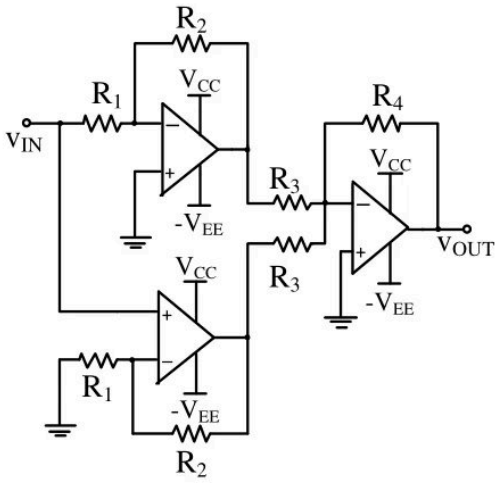
WATCH OUT FOR

Treating the trace as the length itself and choosing l (option C). That forgets that l is defined as the square root of $\boldsymbol{x}^T\boldsymbol{x}$. The other trap is mixing up the outer product $\boldsymbol{x}\boldsymbol{x}^T$ (an $n \times n$ matrix) with the inner product $\boldsymbol{x}^T\boldsymbol{x}$ (a scalar) and then guessing a scaling factor such as $\frac{1}{2}$ or $\frac{1}{4}$ that is not there.

Q.39 MCQ · 2 MARKS

Analog Electronics › Op-amps

Q.39 The $\frac{V_{OUT}}{V_{IN}}$ of the circuit shown below is



(A) $-\frac{R_4}{R_3}$

(B) $\frac{R_4}{R_3}$

(C) $1+\frac{R_4}{R_3}$

(D) $1-\frac{R_4}{R_3}$

Official answer: **A**

SOLUTION

1. Assume ideal op-amps (infinite input resistance, zero output resistance, infinite open-loop gain) that stay in their linear region, well inside the V_{CC} and $-V_{EE}$ rails. Each stage has negative feedback from its output to its inverting input, so each has a virtual short across its inputs and draws no input current.
2. Top op-amp (inverting amplifier): its + input is grounded, so its − input is at $0\ \mathrm{V}$. KCL at the − node (current in through R_1 equals current out through R_2): $\frac{V_{IN}-0}{R_1}=\frac{0-V_1}{R_2}$, which gives

$$V_1 = -\frac{R_2}{R_1} V_{IN}$$

3. Bottom op-amp (non-inverting amplifier): V_{IN} is wired straight to its + input, so its − input is also at V_{IN}. KCL at the − node: $\frac{V_{IN} - 0}{R_1} = \frac{V_2 - V_{IN}}{R_2}$, which gives
$$V_2 = \left(1 + \frac{R_2}{R_1}\right) V_{IN}$$
4. Right op-amp (inverting summer): both R_3 resistors meet at its − input. That input is a virtual ground because the + input is grounded. KCL at the − node: $\frac{V_1}{R_3} + \frac{V_2}{R_3} = -\frac{V_{OUT}}{R_4}$, so
$$V_{OUT} = -\frac{R_4}{R_3}(V_1 + V_2)$$
5. Add the two first-stage outputs. The R_2/R_1 terms cancel:
$$V_1 + V_2 = \left(-\frac{R_2}{R_1} + 1 + \frac{R_2}{R_1}\right) V_{IN} = V_{IN}$$
6. So
$$\frac{V_{OUT}}{V_{IN}} = -\frac{R_4}{R_3}$$
which is option (A). The result does not depend on R_1 or R_2.

KEY IDEA

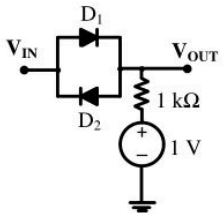
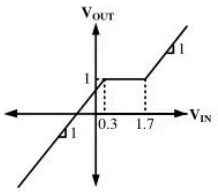
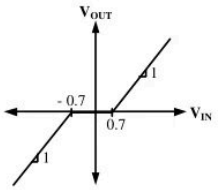
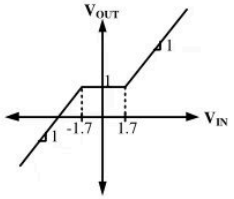
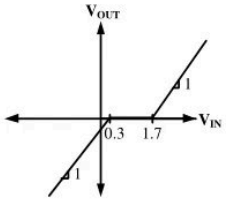
With the same R_1 and R_2, the inverting gain $-R_2/R_1$ and the non-inverting gain $1 + R_2/R_1$ always add to exactly 1. An equal-weight summer that adds the two outputs therefore sees just V_{IN}, and the whole circuit reduces to the summer's inverting gain $-R_4/R_3$.

WATCH OUT FOR

One mistake is to read the bottom op-amp as a second inverting stage because it also has an R_1 and an R_2. In fact V_{IN} drives its + input, so it is non-inverting, and R_1 goes to ground. Misread this way, the two outputs add to $-2(R_2/R_1)V_{IN}$ and R_2/R_1 does not cancel. Another mistake is to forget that the output stage is an inverting summer (its + input is grounded) and choose (B) $+R_4/R_3$. Treating it as a non-inverting stage gives (C).

Q.40 MCQ · 2 MARKS

Analog Electronics › Diode circuits

Q.40	In the circuit shown below, D_1 and D_2 are silicon diodes with cut-in voltage of 0.7 V. V_{IN} and V_{OUT} are input and output voltages in volts. The transfer characteristic is
	
(A)	
(B)	
(C)	
(D)	

Official answer: **A**

SOLUTION

1. Model and assumptions: V_{IN} is driven by an ideal voltage source. Each diode uses the constant-voltage-drop model: it is off until its forward voltage reaches the cut-in value of $0.7\,\mathrm{V}$, and after that it holds a constant $0.7\,\mathrm{V}$ drop with no series resistance. The output is open-circuited, so any current through the diodes must also flow in the $1\,\mathrm{k\Omega}$ resistor.
2. Both diodes off: no current flows in the $1\,\mathrm{k\Omega}$ resistor, so there is no drop across it and $V_{OUT} = 1\,\mathrm{V}$ (the source voltage).

3. D_1 (anode at V_{IN}, cathode at V_{OUT}) turns on when $V_{IN} - V_{OUT} \geq 0.7$. With $V_{OUT} = 1$ this means $V_{IN} \geq 1.7\,\text{V}$. While it conducts, KVL across D_1 gives $V_{OUT} = V_{IN} - 0.7$, a line of slope 1. Consistency check: the current from V_{OUT} down through the resistor is $\dfrac{V_{OUT} - 1}{1\,\text{k}\Omega} = \dfrac{V_{IN} - 1.7}{1\,\text{k}\Omega} \geq 0$, which is forward current in D_1, as required.
4. D_2 (anode at V_{OUT}, cathode at V_{IN}) turns on when $V_{OUT} - V_{IN} \geq 0.7$. With $V_{OUT} = 1$ this means $V_{IN} \leq 0.3\,\text{V}$. While it conducts, $V_{OUT} = V_{IN} + 0.7$, again a line of slope 1. Consistency check: the current from the source up through the resistor into D_2 is $\dfrac{1 - V_{OUT}}{1\,\text{k}\Omega} = \dfrac{0.3 - V_{IN}}{1\,\text{k}\Omega} \geq 0$, which is forward current in D_2.
5. The two diodes are antiparallel, so they never conduct at the same time. The complete characteristic is

$$V_{OUT} = \begin{cases} V_{IN} + 0.7, & V_{IN} \leq 0.3 \\ 1, & 0.3 \leq V_{IN} \leq 1.7 \\ V_{IN} - 0.7, & V_{IN} \geq 1.7 \end{cases}$$

6. Check: the pieces meet at the breakpoints ($0.3 + 0.7 = 1$ and $1.7 - 0.7 = 1$), so the curve is continuous. At $V_{IN} = 0$, $V_{OUT} = 0.7$, so the lower line crosses the V_{IN} axis at $V_{IN} = -0.7$, to the left of the origin. This is option (A): flat at $V_{OUT} = 1$ between $V_{IN} = 0.3$ and $V_{IN} = 1.7$, with slope 1 on both sides.

KEY IDEA

With both diodes off, no current flows in the resistor, so the output sits at the $1\,\text{V}$ source voltage, not at $0\,\text{V}$. Each diode switches on once V_{IN} is $0.7\,\text{V}$ away from that $1\,\text{V}$ level: D_1 at $V_{IN} = 1.7\,\text{V}$ and D_2 at $V_{IN} = 0.3\,\text{V}$. The result is a flat segment centred on $1\,\text{V}$, with the output following the input at unit slope on either side.

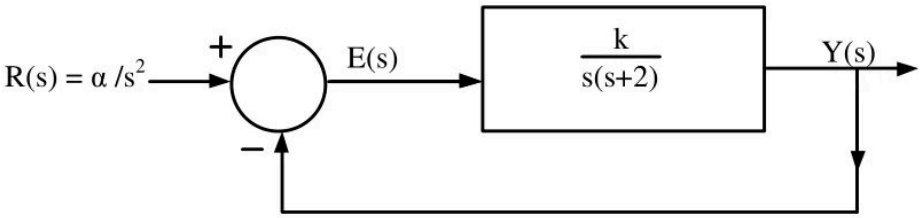
WATCH OUT FOR

Measuring the diode thresholds from $0\,\text{V}$ instead of from the $1\,\text{V}$ bias. That gives a dead zone of $\pm 0.7\,\text{V}$ around the origin (option B) or a flat region at $\pm 1.7\,\text{V}$ (option C). The other trap is taking $V_{OUT} = 0$ while both diodes are off (option D). With no current in the $1\,\text{k}\Omega$ resistor, the output equals the source voltage, $1\,\text{V}$.

Q.41 MCQ · 2 MARKS

Control Systems › Time response

Q.41 A closed loop system is shown in the figure where $k > 0$ and $\alpha > 0$. The steady state error due to a ramp input $(R(s) = \alpha/s^2)$ is given by



(A) $\frac{2\alpha}{k}$

(B) $\frac{\alpha}{k}$

(C) $\frac{\alpha}{2k}$

(D) $\frac{\alpha}{4k}$

Official answer: **A**

SOLUTION

1. The loop has unity feedback and $G(s) = \dfrac{k}{s(s+2)}$. $G(s)$ has one pole at the origin, so the system is type 1. A type-1 system follows a ramp with a finite, nonzero steady-state error.
2. Check stability first, because the final value theorem only applies to a stable loop. The characteristic equation is $1 + G(s) = 0 \Rightarrow s^2 + 2s + k = 0$. For a second-order polynomial, all coefficients positive is enough for stability, and that holds for every $k > 0$.

3. Velocity error constant:

$$K_v = \lim_{s\to 0} s\,G(s) = \lim_{s\to 0} \frac{k}{s+2} = \frac{k}{2}$$

4. Apply the final value theorem to $E(s) = \dfrac{R(s)}{1+G(s)}$:

$$e_{ss} = \lim_{s\to 0} s \cdot \frac{\alpha/s^2}{1 + \dfrac{k}{s(s+2)}} = \lim_{s\to 0} \frac{\alpha(s+2)}{s(s+2)+k} = \frac{2\alpha}{k}$$

5. Cross-check with the shortcut for a ramp of slope α: $e_{ss} = \dfrac{\alpha}{K_v} = \dfrac{\alpha}{k/2} = \dfrac{2\alpha}{k}$. The answer is option (A).

KEY IDEA

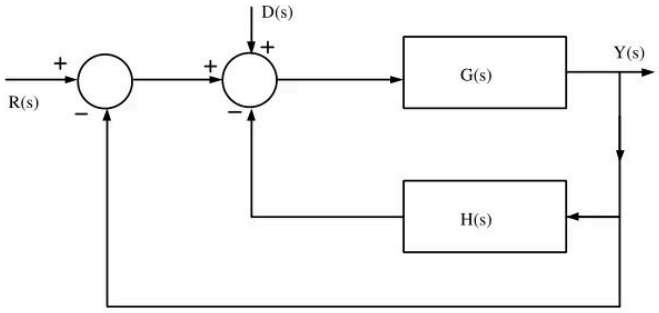
In a stable type-1 unity-feedback loop, a ramp $R(s) = \alpha/s^2$ leaves a steady-state error $e_{ss} = \alpha/K_v$, where $K_v = \lim_{s\to 0} sG(s)$. Here the $(s+2)$ factor makes $K_v = k/2$, not k.

WATCH OUT FOR

Taking $K_v = k$ by ignoring the $(s+2)$ factor, which gives α/k (option B). Another is using $e_{ss} = \alpha K_v$ instead of α/K_v, which gives $\alpha/(2k)$ (option C). And the final value theorem should not be used before checking that the closed loop is stable.

Q.42 MCQ · 2 MARKS

Control Systems › Block diagrams & SFG

Q.42	In the following block diagram, R(s) and D(s) are two inputs. The output Y(s) is expressed as $Y(s) = G_1(s)R(s) + G_2(s)D(s)$. $G_1(s)$ and $G_2(s)$ are given by
	
(A)	$G_1(s) = \dfrac{G(s)}{1 + G(s) + G(s)H(s)}$ and $G_2(s) = \dfrac{G(s)}{1 + G(s) + G(s)H(s)}$
(B)	$G_1(s) = \dfrac{G(s)}{1 + G(s) + H(s)}$ and $G_2(s) = \dfrac{G(s)}{1 + G(s) + H(s)}$
(C)	$G_1(s) = \dfrac{G(s)}{1 + G(s) + H(s)}$ and $G_2(s) = \dfrac{G(s)}{1 + G(s) + G(s)H(s)}$
(D)	$G_1(s) = \dfrac{G(s)}{1 + G(s) + G(s)H(s)}$ and $G_2(s) = \dfrac{G(s)}{1 + G(s) + H(s)}$

Official answer: **A**

SOLUTION

1. Write the signal leaving each summing junction. Junction 1 subtracts the unity feedback from the output: $E_1 = R - Y$.
2. Junction 2 adds the disturbance and subtracts the H feedback: $E_2 = E_1 + D - HY = R + D - Y - HY$.
3. The forward block gives $Y = G\,E_2 = G\,(R + D - Y - HY)$.

4. Collect the Y terms on the left:
$$Y\,(1 + G + GH) = G\,R + G\,D$$
5. So
$$Y(s) = \frac{G}{1 + G + GH}\,R(s) + \frac{G}{1 + G + GH}\,D(s)$$
6. Check with Mason's gain formula. There are two loops, with gains $-G$ (through the unity outer feedback) and $-GH$ (through H). Both pass through G, so they touch, and $\Delta = 1 + G + GH$. From R and from D the only forward path is through G alone, and it touches both loops, so $\Delta_1 = 1$ for each input. Both transfer functions are therefore $G/(1 + G + GH)$.
7. Therefore $G_1(s) = G_2(s) = \dfrac{G(s)}{1 + G(s) + G(s)H(s)}$, which is option (A).

KEY IDEA

Both inputs enter the loop ahead of the single forward block G, and nothing but a summing junction separates them from it, so they share one forward path (gain G) and one characteristic denominator. Each feedback loop's gain is the product around the loop: $-G \cdot 1$ for the outer loop and $-G \cdot H$ for the inner one. That gives $\Delta = 1 + G + GH$, and so $G_1 = G_2 = G/(1 + G + GH)$.

WATCH OUT FOR

Writing the inner loop's contribution as H in place of the loop gain GH. The feedback signal HY goes back through G before it reaches Y, so the term is GH. Dropping the G gives the $1 + G + H$ denominators in options (B), (C) and (D). A second slip is to assume the disturbance sees a different denominator because D enters inside the outer loop. Junction 2 is fed straight from junction 1, so D and the outer feedback act at effectively the same point and both loops still act on it.

Q.43 MCQ · 2 MARKS

Control Systems › State space

The state equation of a second order system is
$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t), \quad \boldsymbol{x}(0) \text{ is the initial condition.}$$
Suppose λ_1 and λ_2 are two distinct eigenvalues of A and $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are the corresponding eigenvectors. For constants α_1 and α_2, the solution, $\boldsymbol{x}(t)$, of the state equation is

(A) $\displaystyle\sum_{i=1}^{2} \alpha_i e^{\lambda_i t}\boldsymbol{v}_i$

(B) $\displaystyle\sum_{i=1}^{2} \alpha_i e^{2\lambda_i t}\boldsymbol{v}_i$

(C) $\displaystyle\sum_{i=1}^{2} \alpha_i e^{3\lambda_i t}\boldsymbol{v}_i$

(D) $\displaystyle\sum_{i=1}^{2} \alpha_i e^{4\lambda_i t}\boldsymbol{v}_i$

Official answer: **A**

SOLUTION

1. With no input, the solution of $\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t)$ is
$$\boldsymbol{x}(t) = e^{At}\boldsymbol{x}(0), \qquad e^{At} = \sum_{k=0}^{\infty} \frac{A^k t^k}{k!}$$
2. The eigenvectors are linearly independent because $\lambda_1 \neq \lambda_2$. Suppose $c_1\boldsymbol{v}_1 + c_2\boldsymbol{v}_2 = \boldsymbol{0}$. Multiply on the left by $(A - \lambda_2 I)$ to get $c_1(\lambda_1 - \lambda_2)\boldsymbol{v}_1 = \boldsymbol{0}$, so $c_1 = 0$, and then $c_2 = 0$. Two independent vectors in a two-dimensional state space form a basis. (If A is real, its eigenvalues can be a complex-conjugate pair. The eigenvectors and the α_i are then complex, so the basis is of $\mathbb{C}^2$, not only $\mathbb{R}^2$.) So the initial condition has a unique expansion

$$\boldsymbol{x}(0) = \alpha_1 \boldsymbol{v}_1 + \alpha_2 \boldsymbol{v}_2$$

3. Use the eigenvector property. From $A\boldsymbol{v}_i = \lambda_i \boldsymbol{v}_i$ we get $A^k \boldsymbol{v}_i = \lambda_i^k \boldsymbol{v}_i$ for every k. Put this into the series:

$$e^{At}\boldsymbol{v}_i = \sum_{k=0}^{\infty} \frac{\lambda_i^k t^k}{k!}\, \boldsymbol{v}_i = e^{\lambda_i t}\boldsymbol{v}_i$$

4. e^{At} is linear, so

$$\boldsymbol{x}(t) = e^{At}\left(\alpha_1 \boldsymbol{v}_1 + \alpha_2 \boldsymbol{v}_2\right) = \alpha_1 e^{\lambda_1 t}\boldsymbol{v}_1 + \alpha_2 e^{\lambda_2 t}\boldsymbol{v}_2 = \sum_{i=1}^{2} \alpha_i e^{\lambda_i t}\boldsymbol{v}_i$$

   This is option (A).

5. Check by substitution:

$$\dot{\boldsymbol{x}} = \sum_{i=1}^{2} \alpha_i \lambda_i e^{\lambda_i t}\boldsymbol{v}_i = \sum_{i=1}^{2} \alpha_i e^{\lambda_i t} A\boldsymbol{v}_i = A\boldsymbol{x}$$

   At $t = 0$ it gives $\boldsymbol{x}(0) = \alpha_1 \boldsymbol{v}_1 + \alpha_2 \boldsymbol{v}_2$. So both the equation and the initial condition are satisfied.

6. Rule out (B), (C) and (D). Take $\boldsymbol{y}(t) = \sum_{i=1}^{2} \alpha_i e^{k\lambda_i t}\boldsymbol{v}_i$ with $k = 2, 3, 4$. Then

$$\dot{\boldsymbol{y}} - A\boldsymbol{y} = \sum_{i=1}^{2} (k-1)\, \alpha_i \lambda_i\, e^{k\lambda_i t}\boldsymbol{v}_i$$

   Here $\boldsymbol{v}_1, \boldsymbol{v}_2$ are independent, $e^{k\lambda_i t} \neq 0$ and $k - 1 \neq 0$. So this is zero only if $\alpha_i \lambda_i = 0$ for both i. That is a special case (for example, $\boldsymbol{x}(0)$ along an eigenvector whose eigenvalue is zero), not the solution for a general initial condition. In fact $e^{k\lambda_i t}\boldsymbol{v}_i$ are the modes of $\dot{\boldsymbol{x}} = kA\boldsymbol{x}$, a different system. Options (B), (C) and (D) are wrong.

7. Answer: (A) $\sum_{i=1}^{2} \alpha_i e^{\lambda_i t}\boldsymbol{v}_i$

KEY IDEA

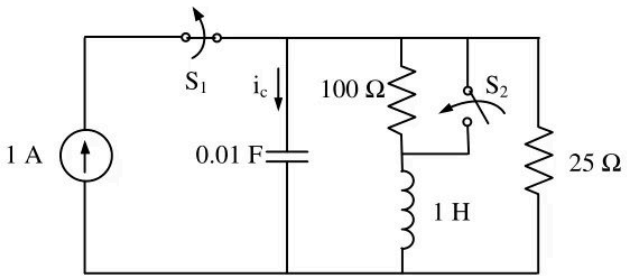
This is modal decomposition. Along an eigenvector, the state transition matrix acts as a scalar: $e^{At}\boldsymbol{v}_i = e^{\lambda_i t}\boldsymbol{v}_i$. Distinct eigenvalues give independent eigenvectors, so any $\boldsymbol{x}(0)$ splits as $\sum \alpha_i \boldsymbol{v}_i$. Each part then evolves on its own as the mode $e^{\lambda_i t}\boldsymbol{v}_i$, and the free response is the weighted sum of these modes.

WATCH OUT FOR

The usual trap is thinking "second order" puts a 2 into the exponent and choosing (B). The order of the system is the number of modes (two terms in the sum), not a factor on λ_i. The term $e^{2\lambda_i t}\boldsymbol{v}_i$ is a mode of $\dot{\boldsymbol{x}} = 2A\boldsymbol{x}$, not of $\dot{\boldsymbol{x}} = A\boldsymbol{x}$. Substituting it back leaves an extra factor of 2. A second slip is to take the α_i as the components of $\boldsymbol{x}(0)$. They are its coordinates in the eigenvector basis.

Q.44 MCQ · 2 MARKS

Networks (Electric Circuits) › Transients

Q.44	The switch S_1 was closed and S_2 was open for a long time. At $t = 0$, switch S_1 is opened and S_2 is closed, simultaneously. The value of $i_c(0^+)$, in amperes, is
	
(A)	1
(B)	-1
(C)	0.2
(D)	0.8

Official answer: **B**

SOLUTION

1. **Steady state for** $t < 0$ (S_1 closed, S_2 open): the capacitor acts as an open circuit and the inductor as a short circuit. The 1 A source then feeds two parallel paths: the $100\ \Omega$ resistor (in series with the shorted inductor) and the $25\ \Omega$ resistor.
2. Top-rail voltage, which is also the capacitor voltage:

$$v_C(0^-) = 1 \times \frac{100 \times 25}{100 + 25} = 20\ \text{V}$$

3. Inductor current, flowing downward through the $100\ \Omega$ branch: $i_L(0^-) = \dfrac{20}{100} = 0.2\ \text{A}$. Check: the $25\ \Omega$ resistor carries $\dfrac{20}{25} = 0.8\ \text{A}$, and $0.2 + 0.8 = 1\ \text{A}$, which is the source current.
4. **Continuity rules:** $v_C(0^+) = v_C(0^-) = 20\ \text{V}$ and $i_L(0^+) = i_L(0^-) = 0.2\ \text{A}$. At $t = 0^+$ the capacitor therefore behaves as a $20\ \text{V}$ source and the inductor as a $0.2\ \text{A}$ current source.
5. **Circuit at $t = 0^+$:** S_1 is in series with the current source, so opening it disconnects the source from the network. Closing S_2 shorts the $100\ \Omega$ resistor, so that resistor carries no current and the inductor is connected directly between the rails. The capacitor, the inductor and the $25\ \Omega$ resistor are now in parallel, all at $20\ \text{V}$. The inductor voltage jumps from 0 to $20\ \text{V}$, which is allowed: only the inductor current has to be continuous.
6. Current in the $25\ \Omega$ resistor: $i_{25}(0^+) = \dfrac{v_C(0^+)}{25} = \dfrac{20}{25} = 0.8\ \text{A}$, flowing downward.
7. **KCL at the top node**, taking every branch current downward (the direction of the marked i_c). No source current enters, because S_1 is open:

$$i_c(0^+) + i_L(0^+) + i_{25}(0^+) = 0$$

8. $$i_c(0^+) = -(0.2 + 0.8) = -1\ \text{A}$$

   The minus sign means the capacitor is discharging, so current flows upward through it, against the marked arrow. This is option (B). The values $0.01\ \text{F}$ and $1\ \text{H}$ are not needed to find the $t = 0^+$ value.

KEY IDEA

Work out $v_C(0^-)$ and $i_L(0^-)$ from the DC steady state, with the capacitor open and the inductor shorted. At $t = 0^+$, replace the capacitor with a voltage source of $v_C(0^-)$ and the inductor with a current source of $i_L(0^-)$, redraw the circuit for the new switch positions, and apply KCL. The sign of i_c is set by the marked arrow direction.

WATCH OUT FOR

Answering $+1$ by dropping the sign: the capacitor current actually flows upward, against the marked i_c. Other traps: choosing 0.8 because you forgot the inductor keeps drawing its $0.2\ \text{A}$ after the switch; choosing 0.2 because you ignored the $25\ \Omega$ resistor; still treating the capacitor as open at $t = 0^+$ and getting $i_c = 0$; and thinking the open S_1 leaves the $1\ \text{A}$ source in the circuit.

Q.45 MCQ · 2 MARKS

Communications › Analog modulation

Let a frequency modulated (FM) signal $x(t) = A\cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(\lambda)\, d\lambda\right)$, where $m(t)$ is a message signal of bandwidth W. It is passed through a non-linear system with output $y(t) = 2x(t) + 5(x(t))^2$. Let B_T denote the FM bandwidth. The minimum value of ω_c required to recover $x(t)$ from $y(t)$ is

(A) $B_T + W$

(B) $\frac{3}{2}B_T$

(C) $2B_T + W$

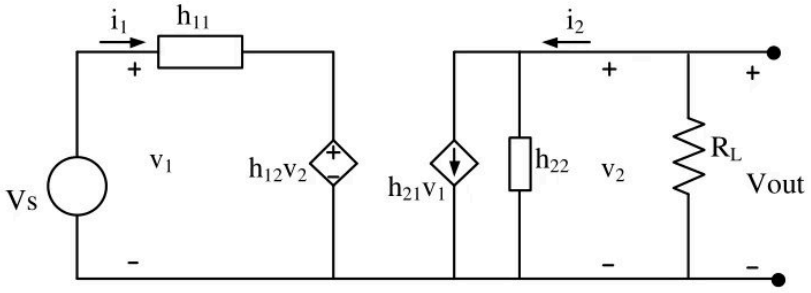
(D) $\frac{5}{2}B_T$

Official answer: **B**

Q.46 MCQ · 2 MARKS

Networks (Electric Circuits) › Two-port

Q.46 The h-parameters of a two port network are shown below. The condition for the maximum small signal voltage gain $\frac{V_{out}}{V_s}$ is



(A) $h_{11} = 0, h_{12} = 0, h_{21} =$ very high and $h_{22} = 0$

(B) $h_{11} =$ very high, $h_{12} = 0, h_{21} =$ very high and $h_{22} = 0$

(C) $h_{11} = 0, h_{12} =$ very high, $h_{21} =$ very high and $h_{22} = 0$

(D) $h_{11} = 0, h_{12} = 0, h_{21} =$ very high and $h_{22} =$ very high

Official answer: **Marks to all**

SOLUTION

1. **What the stem fixes.** The stem says the network is drawn with its h-parameters. By definition, its port equations are then

$$v_1 = h_{11} i_1 + h_{12} v_2, \qquad i_2 = h_{21} i_1 + h_{22} v_2$$

Here h_{21} is the ratio i_2/i_1 with the output shorted ($v_2 = 0$), which is a pure number. The input half of the figure is exactly the first equation: h_{11} in series with the dependent source $h_{12} v_2$.

2. **As printed, the figure has no single answer.** The output source is printed as $h_{21}v_1$, not $h_{21}i_1$. Take that label literally. Then $v_1 = V_s$, because its + and – marks sit across the source terminals. KCL at the output top node, with $v_2 = -i_2 R_L$, gives $v_2 = -R_L(h_{21}V_s + h_{22}v_2)$, so
$$\frac{V_{out}}{V_s} = \frac{-h_{21}R_L}{1 + h_{22}R_L}$$
Neither h_{11} nor h_{12} appears. Options (A), (B) and (C) all have h_{21} very high and $h_{22} = 0$, so they give the same gain, $-h_{21}R_L$ (reading $h_{11} = 0$ as the limit $h_{11} \to 0$). Only (D) gives less. With the printed label, three options are equally correct, and the official key awards marks to all candidates. The label is also inconsistent in itself: a source $h_{21}v_1$ would make h_{21} a conductance in siemens, which an h-parameter is not. The rest of this solution uses the h-parameter model the stem names, with the output source $h_{21}i_1$.
3. **Source and load.** The ideal source drives port 1 directly, so $v_1 = V_s$. At port 2, i_2 is drawn flowing from R_L into the network. The current down through R_L is therefore $-i_2$, and
$$v_2 = V_{out} = -i_2 R_L$$
4. **Output side.** Substitute $i_2 = -v_2/R_L$ into $i_2 = h_{21}i_1 + h_{22}v_2$:
$$-\frac{v_2}{R_L} = h_{21}i_1 + h_{22}v_2 \quad \Rightarrow \quad i_1 = -\frac{(1 + h_{22}R_L)\,v_2}{h_{21}R_L}$$
5. **Input side.** Substitute this i_1 into $V_s = h_{11}i_1 + h_{12}v_2$:
$$V_s = v_2\left(h_{12} - \frac{h_{11}(1 + h_{22}R_L)}{h_{21}R_L}\right)$$
The voltage gain is then
$$A_v = \frac{V_{out}}{V_s} = \frac{-h_{21}R_L}{h_{11}(1 + h_{22}R_L) - h_{12}h_{21}R_L}$$
6. **How to compare the options.** All four options have h_{21} very high. Each of (B), (C) and (D) differs from (A) in exactly one parameter, so compare them with (A) one parameter at a time.
7. h_{22}**: (A) against (D).** With $h_{12} = 0$,
$$|A_v| = \frac{h_{21}R_L}{h_{11}(1 + h_{22}R_L)}$$
A very high h_{22} divides the gain by $1 + h_{22}R_L$, because the shunt admittance carries away current that would otherwise flow through R_L. So $h_{22} = 0$, and (D) is out.
8. h_{12}**: (A) against (C).** With $h_{22} = 0$,
$$A_v = \frac{-h_{21}R_L}{h_{11} - h_{12}h_{21}R_L}$$
As $h_{11} \to 0$ this tends to $\dfrac{1}{h_{12}}$, which is tiny when h_{12} is very high. The feedback source $h_{12}v_2$ caps the gain at about $1/h_{12}$, and only $h_{12} = 0$ removes the cap. So (C) is out.
9. h_{11}**: (A) against (B).** With $h_{12} = 0$ and $h_{22} = 0$,
$$A_v = -\frac{h_{21}R_L}{h_{11}}$$
The input current is $i_1 = V_s/h_{11}$, and the output source feeds h_{21} times that current into R_L. For the same h_{21}, a very high h_{11} chokes i_1 and lowers the gain, while $h_{11} \to 0$ raises it without bound. So (B) is out.
10. **Answer (A):** $h_{11} = 0$, $h_{12} = 0$, h_{21} very high and $h_{22} = 0$, which gives $A_v = -h_{21}R_L/h_{11}$. Read $h_{11} = 0$ as the ideal limit $h_{11} \to 0$. Exactly zero, together with $h_{12} = 0$, would put a short circuit across the ideal source V_s.

KEY IDEA

Write the gain in h-parameters, $A_v = \dfrac{-h_{21}R_L}{h_{11}(1+h_{22}R_L) - h_{12}h_{21}R_L}$. Each wrong option changes exactly one parameter from (A), so compare one parameter at a time. h_{22} is a shunt that steals load current. h_{12} is feedback that caps the gain near $1/h_{12}$. h_{11} sets $i_1 = V_s/h_{11}$, the current that the output source multiplies by h_{21}.

WATCH OUT FOR

Choosing (B) from the rule that a voltage amplifier needs a high input resistance. That rule is about not loading a source that has its own internal resistance. Here V_s is ideal, so there is no loading to avoid: h_{11} only sets $i_1 = V_s/h_{11}$, and a larger h_{11} means less gain. A second trap is solving the figure literally with $h_{21}v_1$, which makes h_{11} and h_{12} irrelevant, and then quietly switching to $h_{21}i_1$ to rule out an option.

Q.47 MCQ · 2 MARKS

Signals and Systems › Fourier series

Consider a discrete-time periodic signal with period $N = 5$. Let the discrete-time Fourier series (DTFS) representation be $x[n] = \sum_{k=0}^{4} a_k e^{\frac{jk2\pi n}{5}}$, where $a_0 = 1$, $a_1 = 3j$, $a_2 = 2j$, $a_3 = -2j$ and $a_4 = -3j$. The value of the sum $\sum_{n=0}^{4} x[n] \sin\frac{4\pi n}{5}$ is

(A) -10

(B) 10

(C) -2

(D) 2

Official answer: **A**

SOLUTION

1. DTFS analysis equation: $a_k = \dfrac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-jk\frac{2\pi}{N}n}$. With $N = 5$ this gives $\sum_{n=0}^{4} x[n]\, e^{-jk\frac{2\pi}{5}n} = 5a_k$.
2. Euler's formula: $\sin\dfrac{4\pi n}{5} = \dfrac{1}{2j}\left(e^{j\frac{4\pi n}{5}} - e^{-j\frac{4\pi n}{5}}\right)$.
3. Identify each exponential with an analysis kernel. Since $e^{-j2\pi n} = 1$ for integer n: $e^{j\frac{4\pi n}{5}} = e^{j\frac{4\pi n}{5}} e^{-j2\pi n} = e^{-j\frac{2\pi(3)n}{5}}$, which is the $k = 3$ kernel. The term $e^{-j\frac{4\pi n}{5}} = e^{-j\frac{2\pi(2)n}{5}}$ is the $k = 2$ kernel.
4. So
$$\sum_{n=0}^{4} x[n]\sin\frac{4\pi n}{5} = \frac{1}{2j}(5a_3 - 5a_2) = \frac{5}{2j}(-2j - 2j) = \frac{5(-4j)}{2j} = -10.$$
5. Cross-check by writing $x[n]$ out. Because $a_4 = a_1^*$ and $a_3 = a_2^*$, $x[n]$ is real, and $e^{j\frac{2\pi(4)n}{5}} = e^{-j\frac{2\pi n}{5}}$, $e^{j\frac{2\pi(3)n}{5}} = e^{-j\frac{4\pi n}{5}}$. So $x[n] = 1 + 2\,\mathrm{Re}\left(3j\, e^{j\frac{2\pi n}{5}}\right) + 2\,\mathrm{Re}\left(2j\, e^{j\frac{4\pi n}{5}}\right) = 1 - 6\sin\frac{2\pi n}{5} - 4\sin\frac{4\pi n}{5}$.
6. Orthogonality over one period: the constant term and the $-6\sin\frac{2\pi n}{5}$ term both sum to zero against $\sin\frac{4\pi n}{5}$. Only the $-4\sin\frac{4\pi n}{5}$ term is left, and $\sum_{n=0}^{4}\sin^2\frac{4\pi n}{5} = \sum_{n=0}^{4}\frac{1-\cos\frac{8\pi n}{5}}{2} = \frac{5}{2}$. That gives $-4 \times \frac{5}{2} = -10$, the same result. Answer: option (A).

KEY IDEA

A sum of the form $\sum_{n=0}^{N-1} x[n]\, e^{-jk\frac{2\pi}{N}n}$ is just Na_k, so you never need to compute $x[n]$. Split the sine into two complex exponentials with Euler's formula. Then use N -periodicity in k (here $a_{-2} = a_3$) to match each exponential to a coefficient you were given.

WATCH OUT FOR

The usual slip is sending $e^{+j\frac{4\pi n}{5}}$ to a_2 instead of a_3. The analysis kernel carries a minus sign, so $e^{+j\frac{4\pi n}{5}}$ belongs to $k = -2 \equiv 3$. Swapping a_2 and a_3 gives $+10$, option (B). Two other mistakes are leaving out the factor $N = 5$ from the analysis equation, which gives ± 2 (options C and D), and forgetting the $\frac{1}{2j}$ from Euler's formula.

Q.48 MCQ · 2 MARKS

Signals and Systems › DTFT & DFT

Let an input $x[n]$ having discrete time Fourier transform $X(e^{j\Omega}) = 1 - e^{-j\Omega} + 2e^{-3j\Omega}$ be passed through an LTI system. The frequency response of the LTI system is $H(e^{j\Omega}) = 1 - \frac{1}{2}e^{-j2\Omega}$. The output $y[n]$ of the system is

(A) $\delta[n] + \delta[n-1] - \frac{1}{2}\delta[n-2] - \frac{5}{2}\delta[n-3] + \delta[n-5]$

(B) $\delta[n] - \delta[n-1] - \frac{1}{2}\delta[n-2] - \frac{5}{2}\delta[n-3] + \delta[n-5]$

(C) $\delta[n] - \delta[n-1] - \frac{1}{2}\delta[n-2] + \frac{5}{2}\delta[n-3] - \delta[n-5]$

(D) $\delta[n] + \delta[n-1] + \frac{1}{2}\delta[n-2] + \frac{5}{2}\delta[n-3] + \delta[n-5]$

Official answer: **C**

SOLUTION

1. Convolution property of the DTFT: for an LTI system, $Y(e^{j\Omega}) = X(e^{j\Omega})\,H(e^{j\Omega})$.
2. Write $u = e^{-j\Omega}$, so $X = 1 - u + 2u^3$ and $H = 1 - \frac{1}{2}u^2$. By the time-shift property, a term $a\,u^k = a\,e^{-jk\Omega}$ is the DTFT of $a\,\delta[n-k]$.
3. Multiply the two polynomials term by term:
$$\begin{aligned} Y &= (1 - u + 2u^3)\left(1 - \tfrac{1}{2}u^2\right) \\ &= 1 - u + 2u^3 - \tfrac{1}{2}u^2 + \tfrac{1}{2}u^3 - u^5 \end{aligned}$$
4. Collect like powers. The only overlap is at u^3, where $2 + \frac{1}{2} = \frac{5}{2}$:
$$Y(e^{j\Omega}) = 1 - e^{-j\Omega} - \tfrac{1}{2}e^{-j2\Omega} + \tfrac{5}{2}e^{-j3\Omega} - e^{-j5\Omega}$$
5. Take the inverse DTFT term by term:
$$y[n] = \delta[n] - \delta[n-1] - \tfrac{1}{2}\delta[n-2] + \tfrac{5}{2}\delta[n-3] - \delta[n-5]$$
6. Check the signs against the options. Only option (C) has $+\frac{5}{2}$ on $\delta[n-3]$ and -1 on $\delta[n-5]$ (A, B and D all have $+\delta[n-5]$).

KEY IDEA

When both $X(e^{j\Omega})$ and $H(e^{j\Omega})$ are finite sums of powers of $e^{-j\Omega}$, the output transform is just their polynomial product, and each $e^{-jk\Omega}$ turns back into $\delta[n-k]$. You can check it in the time domain too: $h[n] = \delta[n] - \frac{1}{2}\delta[n-2]$, so $y[n] = x[n] - \frac{1}{2}x[n-2]$ with $x[n] = \delta[n] - \delta[n-1] + 2\delta[n-3]$. That gives the same five terms.

WATCH OUT FOR

Sign slips in the cross terms. The product $(-e^{-j\Omega})(-\frac{1}{2}e^{-j2\Omega}) = +\frac{1}{2}e^{-j3\Omega}$ adds to $2e^{-j3\Omega}$ to give $+\frac{5}{2}$, not $-\frac{5}{2}$. Also, $(2e^{-j3\Omega})(-\frac{1}{2}e^{-j2\Omega}) = -e^{-j5\Omega}$ is negative. Getting either sign wrong leads to option (A) or (B).

Q.49 MCQ · 2 MARKS

Signals and Systems › Fourier transform

Let $x(t) = 10\cos(10.5Wt)$ be passed through an LTI system having impulse response $h(t) = \pi\left(\dfrac{\sin Wt}{\pi t}\right)^2 \cos 10Wt$. The output of the system is

(A) $\left(\dfrac{15W}{4}\right)\cos(10.5Wt)$

(B) $\left(\dfrac{15W}{2}\right)\cos(10.5Wt)$

(C) $\left(\dfrac{15W}{8}\right)\cos(10.5Wt)$

(D) $(15W)\cos(10.5Wt)$

Official answer: **A**

SOLUTION

1. **Standard pair.** $\dfrac{\sin Wt}{\pi t} \longleftrightarrow$ a rectangle of height 1 on $|\omega| < W$, zero elsewhere.
2. **Multiplication property.** A product in time becomes $\dfrac{1}{2\pi}\times$ a convolution in frequency. The rectangle convolved with itself is a triangle with peak $2W$ (the overlap length) at $\omega = 0$, falling to zero at $|\omega| = 2W$. So
$$\left(\frac{\sin Wt}{\pi t}\right)^2 \longleftrightarrow \frac{1}{2\pi}(2W)\left(1-\frac{|\omega|}{2W}\right) = \frac{W}{\pi}\left(1-\frac{|\omega|}{2W}\right), \quad |\omega| < 2W$$
3. **Scale by π.** $g(t) = \pi\left(\dfrac{\sin Wt}{\pi t}\right)^2 \longleftrightarrow G(\omega) = W\left(1-\dfrac{|\omega|}{2W}\right)$ for $|\omega| < 2W$, and 0 otherwise.
4. **Modulation property.** $h(t) = g(t)\cos 10Wt \longleftrightarrow H(\omega) = \frac{1}{2}\left[G(\omega-10W) + G(\omega+10W)\right]$. These are two triangles centred at $\pm 10W$, each extending $2W$ on either side of its centre. Since $10W > 2W$ they do not overlap.
5. **Evaluate at the input frequency** $\omega_0 = 10.5W$. The second term needs $G(20.5W) = 0$, so
$$H(j10.5W) = \tfrac{1}{2}\,G(0.5W) = \tfrac{1}{2}\,W\left(1-\frac{0.5W}{2W}\right) = \tfrac{1}{2}\cdot\frac{3W}{4} = \frac{3W}{8}$$
6. **Phase.** $h(t)$ is real and even, so $H(\omega)$ is real; here it is also non-negative, so the phase shift is zero.
7. **Sinusoidal steady state of an LTI system.**
$$y(t) = 10\,|H(j\omega_0)|\cos(\omega_0 t) = 10\cdot\frac{3W}{8}\cos(10.5Wt) = \frac{15W}{4}\cos(10.5Wt)$$
This is option (A).

KEY IDEA

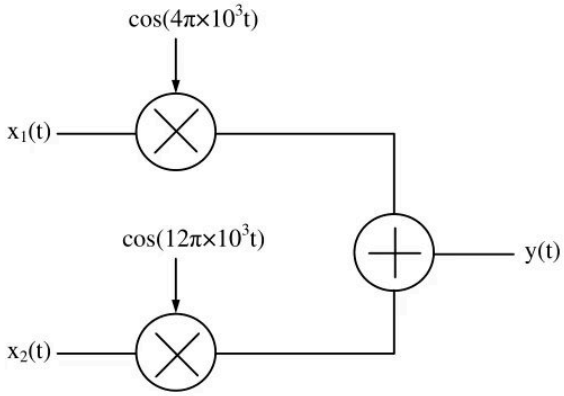
A sinusoid through an LTI system comes out as the same sinusoid, scaled by $|H(j\omega_0)|$ and shifted by $\angle H(j\omega_0)$. Here $\pi\left(\frac{\sin Wt}{\pi t}\right)^2$ has a triangular spectrum of height W reaching zero at $\pm 2W$. Multiplying by $\cos 10Wt$ moves half of that triangle to $\pm 10W$. The input at $10.5W$ lies $0.5W$ from the centre of one copy, so $H = \frac{1}{2}\cdot W\left(1-\frac{0.5W}{2W}\right) = \frac{3W}{8}$.

WATCH OUT FOR

Two mistakes are common. (1) Leaving out the $\frac{1}{2\pi}$ in the multiplication property, or taking the triangle's peak as W rather than the overlap length $2W$. Either error shifts $G(0)$ away from W and leads to option (B) or (D). (2) Forgetting the factor $\frac{1}{2}$ from the modulation property. That gives $|H| = \frac{3W}{4}$ and an output of $\frac{15W}{2}\cos(10.5Wt)$, which is option (B).

Q.50 MCQ · 2 MARKS

Signals and Systems › Sampling

Q.50	Let $x_1(t)$ and $x_2(t)$ be two band-limited signals having bandwidth $B = 4\pi \times 10^3$ rad/s each. In the figure below, the Nyquist sampling frequency, in rad/s, required to sample y(t), is
	
(A)	$20\pi \times 10^3$
(B)	$40\pi \times 10^3$
(C)	$8\pi \times 10^3$
(D)	$32\pi \times 10^3$

Official answer: **D**

SOLUTION

1. From the figure, $y(t) = x_1(t)\cos(4\pi \times 10^3 t) + x_2(t)\cos(12\pi \times 10^3 t)$. Each $x_i(t)$ is low-pass with bandwidth B, so $X_i(\omega) = 0$ for $|\omega| > 4\pi \times 10^3$ rad/s.
2. Modulation property: $x(t)\cos(\omega_c t) \leftrightarrow \frac{1}{2}\left[X(\omega - \omega_c) + X(\omega + \omega_c)\right]$. Multiplying by the cosine moves a copy of the spectrum to $\pm\omega_c$, and each copy still extends B either side of its centre.
3. Top branch, $\omega_c = 4\pi \times 10^3$ rad/s: the copies occupy $0 \le |\omega| \le 4\pi \times 10^3 + 4\pi \times 10^3 = 8\pi \times 10^3$ rad/s.

4. Bottom branch, $\omega_c = 12\pi \times 10^3$ rad/s: the copies occupy $12\pi \times 10^3 - 4\pi \times 10^3 \le |\omega| \le 12\pi \times 10^3 + 4\pi \times 10^3$, that is $8\pi \times 10^3 \le |\omega| \le 16\pi \times 10^3$ rad/s.
5. The adder adds the two spectra (the Fourier transform is linear). Together they fill $0 \le |\omega| \le 16\pi \times 10^3$ rad/s with no gap, so $y(t)$ is a low-pass signal whose highest frequency is $\omega_{\max} = 16\pi \times 10^3$ rad/s.
6. Nyquist sampling frequency for a low-pass signal:

$$\omega_s = 2\,\omega_{\max} = 2 \times 16\pi \times 10^3 = 32\pi \times 10^3 \text{ rad/s}$$

7. This is option (D).

KEY IDEA

Multiplying by $\cos(\omega_c t)$ shifts a baseband spectrum of bandwidth B so it sits at $\omega_c \pm B$. The highest frequency in $y(t)$ is therefore the largest carrier plus B: $12\pi \times 10^3 + 4\pi \times 10^3 = 16\pi \times 10^3$ rad/s. The top branch fills the band from 0 up to $8\pi \times 10^3$, so there is no empty band that band-pass sampling could use, and the Nyquist rate is simply $2\omega_{\max}$.

WATCH OUT FOR

Answering $2B = 8\pi \times 10^3$ (option C), which ignores the fact that modulation moves the spectrum up to the carrier frequency. Another mistake is taking the DSB width $2B$ as the one-sided extent above the carrier. That gives a highest frequency of $12\pi \times 10^3 + 8\pi \times 10^3 = 20\pi \times 10^3$, and doubling it gives $40\pi \times 10^3$ (option B). A third is to apply band-pass sampling to the bottom branch alone. The sum $y(t)$ also contains the top branch, which occupies 0 to $8\pi \times 10^3$ rad/s.

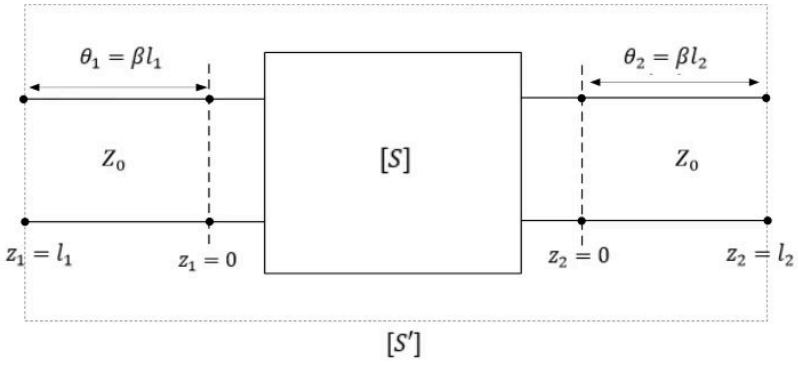
Q.51 MCQ · 2 MARKS

Electromagnetics › Transmission lines

Q.51 The S-parameters of a two port network is given as

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

with reference to Z_0. Two lossless transmission line sections of electrical lengths $\theta_1 = \beta l_1$ and $\theta_2 = \beta l_2$ are added to the input and output ports for measurement purposes, respectively. The S-parameters $[S']$ of the resultant two port network is



(A) $\begin{bmatrix} S_{11}e^{-j2\theta_1} & S_{12}e^{-j(\theta_1+\theta_2)} \\ S_{21}e^{-j(\theta_1+\theta_2)} & S_{22}e^{-j2\theta_2} \end{bmatrix}$

(B) $\begin{bmatrix} S_{11}e^{j2\theta_1} & S_{12}e^{-j(\theta_1+\theta_2)} \\ S_{21}e^{-j(\theta_1+\theta_2)} & S_{22}e^{j2\theta_2} \end{bmatrix}$

(C) $\begin{bmatrix} S_{11}e^{j2\theta_1} & S_{12}e^{j(\theta_1+\theta_2)} \\ S_{21}e^{j(\theta_1+\theta_2)} & S_{22}e^{j2\theta_2} \end{bmatrix}$

(D) $\begin{bmatrix} S_{11}e^{-j2\theta_1} & S_{12}e^{j(\theta_1+\theta_2)} \\ S_{21}e^{j(\theta_1+\theta_2)} & S_{22}e^{-j2\theta_2} \end{bmatrix}$

Official answer: **A**

SOLUTION

1. Let a_1', b_1' and a_2', b_2' be the incident and reflected wave amplitudes at the new reference planes $z_1 = l_1$ and $z_2 = l_2$, and a_1, b_1, a_2, b_2 the ones at the original planes $z_1 = z_2 = 0$. The added lines have the same characteristic impedance Z_0 as the reference impedance, so they add no reflection, only phase.
2. Phase delay on a matched lossless line: a wave that travels a length l picks up the factor $e^{-j\beta l}$. The incident wave travels from the new plane in to the old one, and the reflected wave travels from the old plane out to the new one:

$$a_1 = a_1' e^{-j\theta_1}, \quad b_1' = b_1 e^{-j\theta_1}, \quad a_2 = a_2' e^{-j\theta_2}, \quad b_2' = b_2 e^{-j\theta_2}$$

Equivalently, $a_1' = a_1 e^{j\theta_1}$ and $a_2' = a_2 e^{j\theta_2}$.

3. Input reflection (set $a_2' = 0$, which gives $a_2 = a_2' e^{-j\theta_2} = 0$): the original network gives $b_1 = S_{11} a_1$. Then

$$S_{11}' = \frac{b_1'}{a_1'} = \frac{S_{11}\, a_1 e^{-j\theta_1}}{a_1 e^{j\theta_1}} = S_{11} e^{-j2\theta_1}$$

The reflected signal crosses line 1 twice: in and back out.

4. Forward transmission (again $a_2' = 0$, so $a_2 = 0$): $b_2 = S_{21} a_1$. Then

$$S_{21}' = \frac{b_2'}{a_1'} = \frac{S_{21}\, a_1 e^{-j\theta_2}}{a_1 e^{j\theta_1}} = S_{21} e^{-j(\theta_1+\theta_2)}$$

The signal crosses line 1 once and line 2 once.

5. By the same argument with $a_1' = 0$ (so $a_1 = 0$), driving from port 2: $S_{22}' = S_{22} e^{-j2\theta_2}$ and $S_{12}' = S_{12} e^{-j(\theta_1+\theta_2)}$.
6. In matrix form:

$$[S'] = \begin{bmatrix} S_{11} e^{-j2\theta_1} & S_{12} e^{-j(\theta_1+\theta_2)} \\ S_{21} e^{-j(\theta_1+\theta_2)} & S_{22} e^{-j2\theta_2} \end{bmatrix}$$

The magnitudes are unchanged; only phase lag is added. This is option (A).

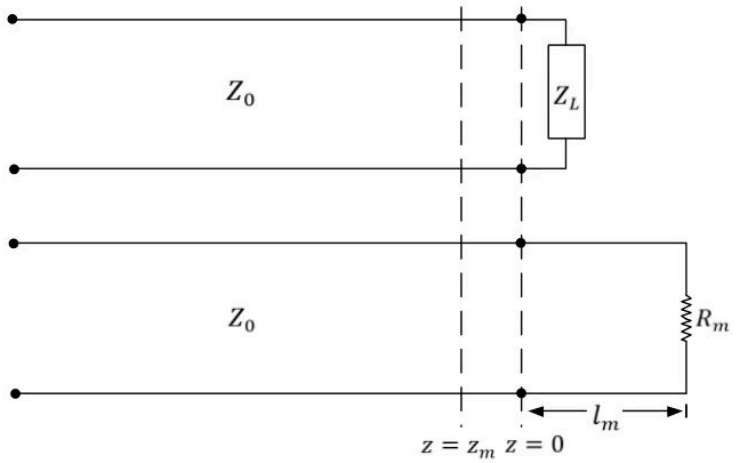
KEY IDEA

Moving the reference planes outward along matched lossless lines of electrical lengths θ_1 and θ_2 keeps every $|S_{ij}|$ the same and multiplies each entry by the phase lag of the path its signal travels: $S_{ij}' = S_{ij}\, e^{-j(\theta_i+\theta_j)}$. A reflection crosses its own line twice, giving $e^{-j2\theta_i}$. A transmission crosses each line once, giving $e^{-j(\theta_1+\theta_2)}$.

WATCH OUT FOR

Using $e^{+j2\theta}$ on the reflection terms (options B and C). A positive exponent is de-embedding: it moves the reference plane in towards the network, which removes line length. Adding line only ever adds phase lag. A second mistake is to give the transmission terms a different sign from the reflection terms (option D). Every entry picks up delay, because every signal path gets longer.

Q.52 MSQ · 2 MARKS

Q.52	The standing wave ratio on a 50 Ω lossless transmission line terminated in an unknown load impedance is found to be 2.0. The distance between successive voltage minima is 30 cm and the first minimum is located at 10 cm from the load. Z_L can be replaced by an equivalent length l_m and terminating resistance R_m of the same line. The value of R_m and l_m, respectively, are
	
(A)	R_m=100 Ω, l_m= 20 cm
(B)	R_m= 25 Ω, l_m= 20 cm
(C)	R_m=100 Ω, l_m= 5 cm
(D)	R_m= 25 Ω, l_m= 5 cm

Official answer: **B, C**

SOLUTION

1. Successive voltage minima are $\lambda/2$ apart, so $\lambda/2 = 30\ \text{cm}$, which gives $\lambda = 60\ \text{cm}$ and $\beta = \dfrac{2\pi}{60}\ \text{rad/cm}$. A voltage maximum sits midway between two minima, $\lambda/4 = 15\ \text{cm}$ from each.
2. On a lossless line the impedance is purely resistive at the extremes of the standing wave. At a voltage minimum the voltage is smallest and the current largest, so $Z = \dfrac{V_{min}}{I_{max}} = Z_0\dfrac{1-|\Gamma|}{1+|\Gamma|} = \dfrac{Z_0}{S} = \dfrac{50}{2} = 25\ \Omega$.

At a voltage maximum, $Z = Z_0 S = 50 \times 2 = 100\ \Omega$.

3. On a lossless line, $|\Gamma(d)| = \dfrac{S-1}{S+1}$ stays the same everywhere, and the phase of $\Gamma(d) = \Gamma_L e^{-j2\beta d}$ changes steadily with distance. So the impedance at every point is fixed by S and the position of one minimum. Carry the same pattern on past the load, as in the lower figure. Minima repeat every $30\ \text{cm}$, so the real minimum $10\ \text{cm}$ on the generator side of the load has a copy $30 - 10 = 20\ \text{cm}$ beyond the load. The maximum between these two minima lies $20 - 15 = 5\ \text{cm}$ beyond the load.
4. Now end the extended line at either point with the resistance the line shows there. A $25\ \Omega$ resistor gives $\Gamma = \dfrac{25-50}{25+50} = -\dfrac{1}{3}$: $S = 2$ with a minimum at the resistor. Placed $20\ \text{cm}$ beyond the load, it puts minima $20\ \text{cm}$ beyond the load and $10\ \text{cm}$ before it, which is exactly the measured pattern. A $100\ \Omega$ resistor gives $\Gamma = +\dfrac{1}{3}$: $S = 2$ with a maximum at the resistor. Placed $5\ \text{cm}$ beyond the load, it puts the next minimum $15\ \text{cm}$ nearer the generator, at $10\ \text{cm}$ before the load, which is again the measured pattern. The same pattern means the same impedance at $z = 0$, namely Z_L. That gives two equivalents: $R_m = 25\ \Omega$ with $l_m = 20\ \text{cm}$, and $R_m = 100\ \Omega$ with $l_m = 5\ \text{cm}$.
5. Check by finding Z_L directly. $|\Gamma_L| = \dfrac{S-1}{S+1} = \dfrac{1}{3}$. At a voltage minimum $\theta_\Gamma - 2\beta d_{min} = -\pi$, where $2\beta d_{min} = 2 \cdot \dfrac{2\pi}{60} \cdot 10 = \dfrac{2\pi}{3}$, so $\theta_\Gamma = -\dfrac{\pi}{3}$ and $\Gamma_L = \dfrac{1}{6} - j\dfrac{\sqrt{3}}{6}$. Then
$$Z_L = 50\,\frac{1+\Gamma_L}{1-\Gamma_L} = 50\,\frac{7 - j\sqrt{3}}{5 + j\sqrt{3}} = \frac{50\,(8 - j3\sqrt{3})}{7} \approx (57.14 - j37.12)\ \Omega.$$
6. The input impedance of a length l of line ending in R is $Z_{in} = Z_0\,\dfrac{R + jZ_0 \tan\beta l}{Z_0 + jR\tan\beta l}$. For $R_m = 25\ \Omega$ and $l_m = 20\ \text{cm}$: $\beta l_m = 120^\circ$ and $\tan\beta l_m = -\sqrt{3}$, so
$$Z_{in} = 50\,\frac{25 - j50\sqrt{3}}{50 - j25\sqrt{3}} = 50\,\frac{1 - j2\sqrt{3}}{2 - j\sqrt{3}} = \frac{50\,(8 - j3\sqrt{3})}{7}.$$
For $R_m = 100\ \Omega$ and $l_m = 5\ \text{cm}$: $\beta l_m = 30^\circ$ and $\tan\beta l_m = \dfrac{1}{\sqrt{3}}$, so
$$Z_{in} = 50\,\frac{100 + j\frac{50}{\sqrt{3}}}{50 + j\frac{100}{\sqrt{3}}} = 50\,\frac{2\sqrt{3} + j}{\sqrt{3} + j2} = \frac{50\,(8 - j3\sqrt{3})}{7}.$$
Both equal Z_L exactly.
7. Options A and D pair each resistance with the wrong length. $100\ \Omega$ at $20\ \text{cm}$ and $25\ \Omega$ at $5\ \text{cm}$ both give $Z_{in} = \dfrac{50\,(8 + j3\sqrt{3})}{13} \approx (30.77 + j19.99)\ \Omega \neq Z_L$. Each puts a maximum where the measured pattern has a minimum, or the other way round. The correct options are B and C.

KEY IDEA

On a lossless line the impedance at any point depends only on the SWR and where the minima are. It is purely resistive at the extremes of the standing wave: Z_0/S at a voltage minimum and $Z_0 S$ at a voltage maximum. So any load can be replaced by extending the line to a minimum or a maximum of its own standing-wave pattern and ending it there in that resistance. Because minima repeat every $\lambda/2$, the minimum 10 cm towards the generator has a copy 20 cm beyond the load, and there is a maximum 5 cm beyond the load.

WATCH OUT FOR

Using the 10 cm distance itself as l_m, or measuring the extension towards the generator instead of past the load. A second trap is pairing $Z_0/S = 25\ \Omega$ with the position of a maximum or $Z_0 S = 100\ \Omega$ with a minimum (options A and D). Both give $(30.77 + j19.99)\ \Omega$, not Z_L. Finally, this is an MSQ: marking only one of B and C loses the marks, because both equivalents are valid.

Q.53 MSQ · 2 MARKS

Electromagnetics › Plane waves

The electric field of a plane electromagnetic wave is

$$\mathbf{E} = \hat{a}_x C_{1x}\cos(\omega t - \beta z) + \hat{a}_y C_{1y}\cos(\omega t - \beta z + \theta) \quad \mathrm{V/m}.$$

Which of the following combination(s) will give rise to a left handed elliptically polarized (LHEP) wave?

(A) $C_{1x} = 1,\ C_{1y} = 1,\ \theta = \pi/4$

(B) $C_{1x} = 2,\ C_{1y} = 1,\ \theta = \pi/2$

(C) $C_{1x} = 1,\ C_{1y} = 2,\ \theta = 3\pi/2$

(D) $C_{1x} = 2,\ C_{1y} = 1,\ \theta = 3\pi/4$

Official answer: **A, B, D**

SOLUTION

1. The phase term is $\omega t - \beta z$, so the wave travels in the $+z$ direction. Look at the plane $z = 0$: $E_x = C_{1x}\cos\omega t$ and $E_y = C_{1y}\cos(\omega t + \theta)$, so E_y leads E_x by θ.
2. **Sense of rotation.** The vector $\mathbf{E} \times \dfrac{d\mathbf{E}}{dt}$ points along the axis about which $\mathbf{E}$ turns, by the right-hand rule. Its z-component is
$$\begin{aligned} E_x\frac{dE_y}{dt} - E_y\frac{dE_x}{dt} &= -\omega C_{1x}C_{1y}\left[\cos\omega t\,\sin(\omega t + \theta) - \cos(\omega t + \theta)\sin\omega t\right] \\ &= -\omega C_{1x}C_{1y}\sin\theta \end{aligned}$$
This does not depend on t, so the field never reverses its direction of rotation. If $\sin\theta > 0$ the z-component is negative, so $\mathbf{E}$ turns in the negative sense about $+z$ (the positive sense about $-z$).
3. **Handedness (IEEE convention).** Point the thumb along the direction of travel ($+z$). The wave is left-handed if the fingers of the left hand curl the way $\mathbf{E}$ turns, which is the negative sense about $+z$: $\left(\mathbf{E} \times \dfrac{d\mathbf{E}}{dt}\right) \cdot \hat{a}_z < 0$. So we need $\sin\theta > 0$, i.e. $0 < \theta < \pi$ (E_y leads E_x). For $\pi < \theta < 2\pi$, $\sin\theta < 0$ and the wave is right-handed.
4. **Elliptical, not linear or circular.** The polarization is linear if $\theta = 0$ or π. It is circular only if $C_{1x} = C_{1y}$ **and** $\theta = \pi/2$ or $3\pi/2$. Every other combination with both amplitudes non-zero is elliptical.
5. (A) $C_{1x} = C_{1y} = 1$, $\theta = \pi/4$. The amplitudes are equal but $\theta \neq \pi/2$, so the wave is elliptical (an ellipse tilted at $45°$). $\sin(\pi/4) = \frac{1}{\sqrt{2}} > 0$, so it is left-handed: **LHEP**.
6. (B) $C_{1x} = 2$, $C_{1y} = 1$, $\theta = \pi/2$. The amplitudes differ, so the wave is elliptical (an upright ellipse, major axis along x). $\sin(\pi/2) = 1 > 0$, so it is left-handed: **LHEP**.
7. (C) $C_{1x} = 1$, $C_{1y} = 2$, $\theta = 3\pi/2$. The wave is elliptical, but $\sin(3\pi/2) = -1 < 0$: E_y lags E_x by $\pi/2$ and the wave is right-handed elliptical (RHEP). Not correct.
8. (D) $C_{1x} = 2$, $C_{1y} = 1$, $\theta = 3\pi/4$. The wave is elliptical and $\sin(3\pi/4) = \frac{1}{\sqrt{2}} > 0$, so it is left-handed: **LHEP**.
9. The left-handed elliptical cases are (A), (B) and (D).

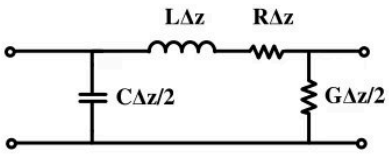
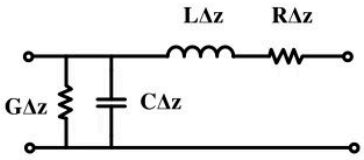
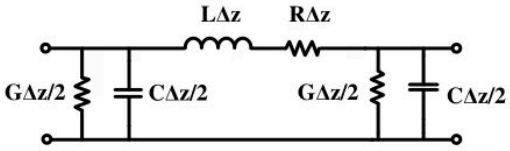
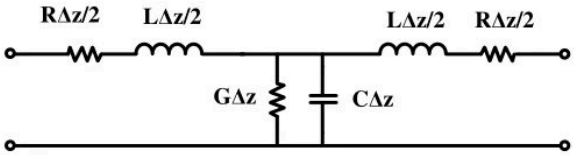
KEY IDEA

For $\mathbf{E} = \hat{a}_x C_{1x}\cos(\omega t - \beta z) + \hat{a}_y C_{1y}\cos(\omega t - \beta z + \theta)$, travelling along $+z$, $(\mathbf{E} \times d\mathbf{E}/dt) \cdot \hat{a}_z = -\omega C_{1x}C_{1y}\sin\theta$ sets the handedness. With $\sin\theta > 0$ (E_y leads E_x) the wave is left-handed in the IEEE convention, and with $\sin\theta < 0$ it is right-handed. The polarization is elliptical unless $\theta = 0$ or π (linear), or the amplitudes are equal and $\theta = \pi/2$ or $3\pi/2$ (circular).

WATCH OUT FOR

One mistake is using the optics convention, where the wave is viewed coming towards the observer. That reverses the handedness, so it would pick (C) alone. Another is rejecting (A) because its amplitudes are equal: equal amplitudes give a circle only when $\theta = \pi/2$ or $3\pi/2$, and with $\theta = \pi/4$ the path is a tilted ellipse. It is also a mistake to call (B) circular just because $\theta = \pi/2$, since its amplitudes are unequal.

Q.54 MSQ · 2 MARKS

Q.54	The following circuit(s) representing a lumped element equivalent of an infinitesimal section of a transmission line is/are
(A)	
(B)	
(C)	
(D)	

Official answer: **B, C, D**

SOLUTION

1. A section of length Δz must carry the full per-unit-length parameters: a total series impedance of $(R + j\omega L)\Delta z$ and a total shunt admittance of $(G + j\omega C)\Delta z$.
2. Apply KVL and KCL to any such section, divide by Δz and let $\Delta z \to 0$. This gives the telegrapher's equations

$$-\frac{\partial V}{\partial z} = (R + j\omega L)\, I, \qquad -\frac{\partial I}{\partial z} = (G + j\omega C)\, V$$

How the elements are split or placed (L-, T- or π-section) changes ΔV and ΔI only by terms of order $(\Delta z)^2$. After dividing by Δz those terms are of order Δz, so they vanish in the limit. The only thing that matters is that the totals are right.

3. (A): the series arm is correct ($L\Delta z$, $R\Delta z$), but the only shunt elements are one $C\Delta z/2$ and one $G\Delta z/2$. The total shunt admittance is $\left(\frac{G}{2} + j\omega\frac{C}{2}\right)\Delta z$, half the required value. It would give $-\dfrac{\partial I}{\partial z} = \dfrac{1}{2}(G + j\omega C)\,V$, which is wrong. Not valid.
4. (B): shunt $G\Delta z \parallel C\Delta z$, then series $L\Delta z$ and $R\Delta z$. The totals are $(R + j\omega L)\Delta z$ and $(G + j\omega C)\Delta z$. This is the standard L-section. Valid.
5. (C): series $L\Delta z$ and $R\Delta z$, with $G\Delta z/2 \parallel C\Delta z/2$ at each end. The total shunt admittance is $2 \times \dfrac{(G + j\omega C)\Delta z}{2} = (G + j\omega C)\Delta z$. This is a symmetric π-section. Valid.
6. (D): series $\dfrac{R\Delta z}{2} + j\omega\dfrac{L\Delta z}{2}$ on each side, giving a total of $(R + j\omega L)\Delta z$, with $G\Delta z \parallel C\Delta z$ in the middle. This is a symmetric T-section. Valid.
7. The correct options are (B), (C) and (D).

KEY IDEA

A lumped model of a length Δz of line is valid if its series elements add up to $(R + j\omega L)\Delta z$ and its shunt elements add up to $(G + j\omega C)\Delta z$. The layout (L, T or π, split or unsplit) only changes terms of order $(\Delta z)^2$, which disappear when you derive the telegrapher's equations.

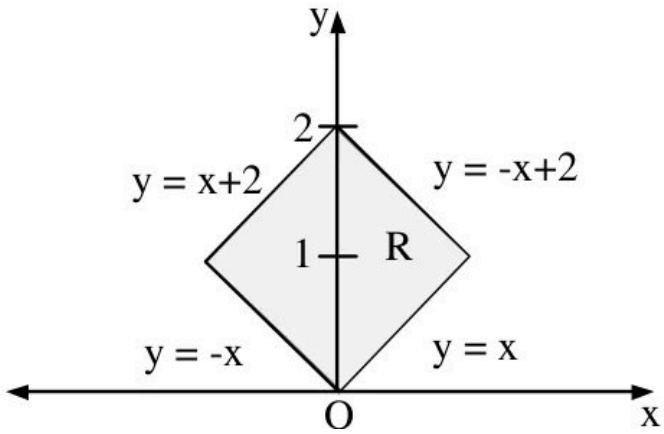
WATCH OUT FOR

Accepting (A) because it contains all four symbols L, R, C, G and looks like a π-section. It has only one $C\Delta z/2$ and one $G\Delta z/2$, so the shunt admittance is halved. The opposite mistake is rejecting (B) because it is not symmetric. An unsymmetric L-section still gives the correct telegrapher's equations in the limit $\Delta z \to 0$.

Q.55 NAT · 2 MARKS

Engineering Mathematics › Calculus

Q.55 The value of the integral $\iint_R xy\,dx\,dy$ over the region R, given in the figure, is _____ (*rounded off to the nearest integer*).



Official answer: **0**

SOLUTION

1. From the figure, R is the square with vertices $(0, 0)$, $(1, 1)$, $(0, 2)$ and $(-1, 1)$. The lines $y = x$ and $y = -x + 2$ meet at $(1, 1)$, and $y = -x$ and $y = x + 2$ meet at $(-1, 1)$. The y-axis splits R into two mirror-image halves.

2. **Symmetry check:** replacing x by $-x$ turns $y = x$ into $y = -x$ and $y = -x + 2$ into $y = x + 2$, so R is symmetric about the y-axis. The integrand $f(x, y) = xy$ is odd in x, since $f(-x, y) = -f(x, y)$. The two halves therefore cancel and the integral is 0. Both halves are worked out below as a check.
3. **Right half** ($0 \le x \le 1$, $x \le y \le 2 - x$):
$$\int_0^1 x \left[\frac{y^2}{2}\right]_x^{2-x} dx = \int_0^1 x \, \frac{(2-x)^2 - x^2}{2} \, dx = \int_0^1 x \, \frac{4 - 4x}{2} \, dx = \int_0^1 (2x - 2x^2) \, dx = 1 - \frac{2}{3} = \frac{1}{3}$$
4. **Left half** ($-1 \le x \le 0$, $-x \le y \le x + 2$):
$$\int_{-1}^0 x \, \frac{(x+2)^2 - x^2}{2} \, dx = \int_{-1}^0 x \, \frac{4x + 4}{2} \, dx = \int_{-1}^0 (2x^2 + 2x) \, dx = \frac{2}{3} + (-1) = -\frac{1}{3}$$
5. Adding the halves: $\frac{1}{3} + \left(-\frac{1}{3}\right) = 0$, so $\iint_R xy \, dx \, dy = 0$.

KEY IDEA
If a region is symmetric about the y-axis and the integrand is odd in x ($f(-x, y) = -f(x, y)$), the double integral is zero. The contribution from each point (x, y) cancels the one from $(-x, y)$. Always check for symmetry before integrating.

WATCH OUT FOR
A common error is integrating over the right half only (where $x \ge 0$ and the integrand is positive), which gives $1/3$ and rounds to 0 by luck, without seeing why the full answer is exactly zero. Another is taking y from x to $x + 2$ across the whole square, which mixes up the edges that bound each half. The upper and lower boundaries change at $x = 0$, so the integral has to be split there.

Q.56 NAT · 2 MARKS

Electronic Devices › Carrier transport

In an extrinsic semiconductor, the hole concentration is given to be $1.5n_i$ where n_i is the intrinsic carrier concentration of $1 \times 10^{10}\ \text{cm}^{-3}$. The ratio of electron to hole mobility for equal hole and electron drift current is given as _____ (rounded off to two decimal places).

Official answer: **2.20 to 2.30**

SOLUTION

1. Given: hole concentration $p = 1.5\, n_i$, with $n_i = 1 \times 10^{10}\ \text{cm}^{-3}$.
2. Mass-action law (valid for any non-degenerate extrinsic semiconductor in equilibrium): $np = n_i^2 \Rightarrow n = \frac{n_i^2}{p} = \frac{n_i^2}{1.5\, n_i} = \frac{n_i}{1.5}$.
3. Drift current densities under the same applied field E: $J_n = q\, n\, \mu_n E$ and $J_p = q\, p\, \mu_p E$.
4. Equal drift currents: $J_n = J_p \Rightarrow n\mu_n = p\mu_p \Rightarrow \frac{\mu_n}{\mu_p} = \frac{p}{n}$.
5. Substitute: $\frac{p}{n} = \frac{1.5\, n_i}{n_i / 1.5} = 1.5 \times 1.5 = 2.25$.
6. Hence $\frac{\mu_n}{\mu_p} = 2.25$.

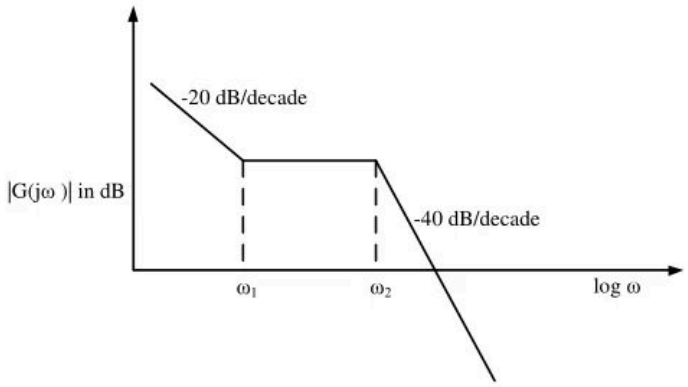
KEY IDEA
In an extrinsic semiconductor, electron and hole concentrations obey the mass-action law $np = n_i^2$, so knowing one concentration (here $p = 1.5n_i$) fixes the other. Equal drift current densities for electrons and holes (same applied field E) then give a direct relation between n, p and the mobility ratio.

WATCH OUT FOR
Inverting the ratio (computing μ_p/μ_n instead of μ_n/μ_p) or forgetting to apply the mass-action law and instead assuming $n = p$, which would (incorrectly) give a ratio of 1.

Q.57 NAT · 2 MARKS

Control Systems › Bode & margins

Q.57 The asymptotic magnitude Bode plot of a minimum phase system is shown in the figure. The transfer function of the system is $(s) = \frac{k(s+z)^a}{s^b(s+p)^c}$, where k, z, p, a, b and c are positive constants. The value of $(a + b + c)$ is _____(*rounded off to the nearest integer*).



Official answer: **4**

SOLUTION

1. **Read the form of the transfer function.**

$$G(s) = \frac{k(s+z)^a}{s^b(s+p)^c}$$

Each factor bends the asymptotic magnitude plot in its own way. s^b in the denominator sets the starting slope at $-20b\,\mathrm{dB/decade}$. $(s+z)^a$ adds $+20a\,\mathrm{dB/decade}$ above $\omega = z$. $(s+p)^c$ adds $-20c\,\mathrm{dB/decade}$ above $\omega = p$.

2. **The starting slope gives the number of poles at the origin.** The plot starts at $-20\,\mathrm{dB/decade}$, so $-20b = -20$ and $b = 1$.

3. **Corner at ω_1: the slope goes from -20 to $0\,\mathrm{dB/decade}$,** a change of $+20\,\mathrm{dB/decade}$. A slope that rises comes from the zero factor. So $z = \omega_1$ and $20a = 20$, which gives $a = 1$.

4. **Corner at ω_2: the slope goes from 0 to $-40\,\mathrm{dB/decade}$,** a change of $-40\,\mathrm{dB/decade}$. A slope that falls comes from the pole factor. So $p = \omega_2$ and $-20c = -40$, which gives $c = 2$. The given form has only one non-origin pole factor, so the $-40\,\mathrm{dB/decade}$ drop has to be one repeated pole of order 2.

5. **Why the magnitude plot is enough.** A magnitude plot cannot tell a left-half-plane zero from a right-half-plane zero. The system is stated to be minimum phase, though, and $z, p > 0$ puts every zero and pole in the left half plane. That rules out the ambiguity, and the transfer function is

$$G(s) = \frac{k(s+\omega_1)}{s(s+\omega_2)^2}$$

6. **Sum:** $a + b + c = 1 + 1 + 2 = 4$.

KEY IDEA

In an asymptotic Bode magnitude plot, the starting slope is $-20\,\mathrm{dB/decade}$ for each pole at the origin. After that, the slope rises by $+20\,\mathrm{dB/decade}$ for each zero at its corner and falls by $-20\,\mathrm{dB/decade}$ for each pole at its corner. So the power of each factor is the change in slope at its corner divided by 20. Minimum phase is what lets the magnitude plot fix the transfer function uniquely.

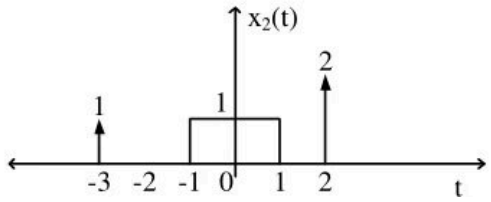
WATCH OUT FOR

Reading the final slope of $-40\,\mathrm{dB/decade}$ as the change at ω_2 and adding the origin pole again, which gives $c = 3$ or $b = 2$. The change at ω_2 is measured from the flat $0\,\mathrm{dB/decade}$ segment, so it is -40 and $c = 2$. A second mistake is taking the flat segment to mean there is no zero. Going from -20 to 0 is a $+20\,\mathrm{dB/decade}$ change, so there is one zero ($a = 1$).

Q.58 NAT · 2 MARKS

Signals and Systems › LTI & convolution

Q.58 Let $x_1(t) = u(t + 1.5) - u(t - 1.5)$ and $x_2(t)$ is shown in the figure below. For $y(t) = x_1(t) * x_2(t)$, the $\int_{-\infty}^{\infty} y(t)dt$ is _____ *(rounded off to the nearest integer)*.



Official answer: **15**

SOLUTION

1. Area property of convolution: for $y(t) = x_1(t) * x_2(t)$,
$$\int_{-\infty}^{\infty} y(t)\,dt = \left(\int_{-\infty}^{\infty} x_1(t)\,dt\right)\left(\int_{-\infty}^{\infty} x_2(t)\,dt\right)$$
This is $Y(0) = X_1(0)\,X_2(0)$ in the Fourier domain, since $X(0) = \int_{-\infty}^{\infty} x(t)\,dt$.
2. $x_1(t) = u(t + 1.5) - u(t - 1.5)$ is a unit-height pulse on $-1.5 < t < 1.5$, so its area is $1 \times 3 = 3$.
3. Read $x_2(t)$ from the figure: $x_2(t) = \delta(t + 3) + [u(t + 1) - u(t - 1)] + 2\delta(t - 2)$.
4. Area of x_2: the impulse at $t = -3$ contributes its strength 1, the pulse contributes $1 \times 2 = 2$, and the impulse at $t = 2$ contributes its strength 2. Total $= 1 + 2 + 2 = 5$.
5. So
$$\int_{-\infty}^{\infty} y(t)\,dt = 3 \times 5 = 15$$

KEY IDEA

The area under a convolution equals the product of the areas of the two signals, because $Y(0) = X_1(0)X_2(0)$. You never need to work out $y(t)$ itself. An impulse adds its strength (the number printed on the arrow) to the area.

WATCH OUT FOR

Doing the convolution by hand, and slipping on the overlapping pieces as the pulse slides past the impulses. Also: counting the impulses as zero area because they have zero width, reading the impulse at $t = 2$ as strength 1 instead of 2, or taking the width of x_1 as 1.5 instead of 3.

Q.59 NAT · 2 MARKS

Communications › Random processes

Let $X(t)$ be a white Gaussian noise with power spectral density $\frac{1}{2}$ W/Hz. If $X(t)$ is input to an LTI system with impulse response $e^{-t}u(t)$. The average power of the system output is _____ W (rounded off to two

decimal places).

Official answer: **0.24 to 0.26**

SOLUTION

1. The input is white noise with two-sided power spectral density $S_X(f) = \dfrac{N_0}{2} = \dfrac{1}{2}$ W/Hz, flat for all f.
2. The output PSD is $S_Y(f) = S_X(f)\,|H(f)|^2$.
3. The impulse response $h(t) = e^{-t}u(t)$ has frequency response $H(f) = \dfrac{1}{1 + j2\pi f}$, so $|H(f)|^2 = \dfrac{1}{1 + 4\pi^2 f^2}$.
4. The output power is
$$P_Y = \int_{-\infty}^{\infty} S_Y(f)\,df = \frac{1}{2}\int_{-\infty}^{\infty} \frac{df}{1 + 4\pi^2 f^2}.$$
5. By Parseval's theorem,
$$\int_{-\infty}^{\infty} |H(f)|^2\,df = \int_{-\infty}^{\infty} h^2(t)\,dt = \int_{0}^{\infty} e^{-2t}\,dt = \frac{1}{2}.$$
6. Therefore $P_Y = \dfrac{1}{2} \times \dfrac{1}{2} = 0.25$ W.
7. Check with the autocorrelation: $R_Y(\tau) = \dfrac{N_0}{2}\big(h(\tau) * h(-\tau)\big)$, so $R_Y(0) = \dfrac{N_0}{2}\displaystyle\int h^2(t)\,dt = 0.25$ W.
   Direct integration agrees: $\displaystyle\int_{-\infty}^{\infty} \frac{df}{1 + 4\pi^2 f^2} = \frac{1}{2\pi} \cdot \pi = \frac{1}{2}$.

KEY IDEA
For white noise of two-sided PSD $N_0/2$ through an LTI system, output power is $\frac{N_0}{2}\int h^2(t)\,dt$ (Parseval). You need no frequency-domain integral.

WATCH OUT FOR
Treating $1/2$ W/Hz as one-sided, or using N_0 in place of $N_0/2$. That doubles the answer to 0.5. Another slip is integrating over $f \geq 0$ only.

Q.60 NAT · 2 MARKS

Electromagnetics › Plane waves

A transparent dielectric coating is applied to glass ($\varepsilon_r = 4, \mu_r = 1$) to eliminate the reflection of red light ($\lambda_0 = 0.75\ \mu$m). The minimum thickness of the dielectric coating, in μm, that can be used is _____ (rounded off to two decimal places).

Official answer: **0.12 to 0.14**

SOLUTION

1. **Refractive indices.** For a lossless medium, $n = \sqrt{\mu_r \varepsilon_r}$. Air: $n_1 = 1$. Glass: $n_3 = \sqrt{1 \times 4} = 2$.
2. **Condition for zero reflection (quarter-wave matching).** At normal incidence from air onto glass through a single coating layer, reflection is zero only when the coating acts as a quarter-wave transformer between the two media. In impedance form this is $\eta_2 = \sqrt{\eta_1 \eta_3}$. For non-magnetic media $\eta = \eta_0/n$ (with $\eta_0 \approx 377\ \Omega$), so
$$n_2 = \sqrt{n_1 n_3} = \sqrt{1 \times 2} = \sqrt{2} \approx 1.414 \quad (\varepsilon_{r2} = 2)$$
3. **Thickness condition.** The coating thickness must be an odd multiple of a quarter of the wavelength *inside the coating*:
$$d = (2m + 1)\frac{\lambda_2}{4}, \qquad \lambda_2 = \frac{\lambda_0}{n_2}, \quad m = 0, 1, 2, \ldots$$
4. **Minimum thickness** (take $m = 0$):
$$d_{\min} = \frac{\lambda_0}{4 n_2} = \frac{0.75\ \mu\text{m}}{4\sqrt{2}} = \frac{0.75}{5.657}\ \mu\text{m} \approx 0.1326\ \mu\text{m}$$

5. **Check.** The wave reflected at the coating–glass face travels an extra distance $2d = \lambda_2/2$, which adds a phase of $2\beta_2 d = \pi$. Both interfaces go from lower to higher index, so both reflections pick up the same sign. The two reflected waves therefore arrive out of phase. Because $n_2 = \sqrt{n_1 n_3}$, the two interface reflection coefficients are equal: $\dfrac{1-\sqrt{2}}{1+\sqrt{2}}$ at air–coating and $\dfrac{\sqrt{2}-2}{\sqrt{2}+2} = \dfrac{1-\sqrt{2}}{1+\sqrt{2}}$ at coating–glass. The exact transmission-line result agrees: with $d = \lambda_2/4$, the impedance seen from the air side is $\eta_{in} = \eta_2^2/\eta_3 = \eta_1$, so the net reflection is zero.
6. Rounded to two decimal places, $d_{\min} \approx 0.13\ \mu\text{m}$.

KEY IDEA

An anti-reflection coating is a quarter-wave transformer. Its index must be the geometric mean of the two media it sits between, $n_2 = \sqrt{n_1 n_3}$, and its thickness must be a quarter of the wavelength inside the coating, $\lambda_0/(4n_2)$, not a quarter of the free-space wavelength.

WATCH OUT FOR

Using the free-space wavelength gives $\lambda_0/4 = 0.1875\ \mu\text{m}$, which rounds to 0.19. Using the glass index instead of the coating index gives $\lambda_0/(4\times 2) \approx 0.094\ \mu\text{m}$. Both answers fall outside the key. Another slip is taking $n = \varepsilon_r$ (4) rather than $n = \sqrt{\mu_r \varepsilon_r}$ (2).

Q.61 NAT · 2 MARKS

Electronic Devices › Semiconductors

In a semiconductor device, the Fermi-energy level is 0.35 eV above the valence band energy. The effective density of states in the valence band at $T = 300\ \text{K}$ is $1 \times 10^{19}\ \text{cm}^{-3}$. The thermal equilibrium hole concentration in silicon at 400 K is _____ $\times 10^{13}\ \text{cm}^{-3}$ (rounded off to two decimal places). Given kT at 300 K is 0.026 eV.

Official answer: **60.00 to 70.00**

SOLUTION

1. Given: Fermi level above valence band edge, $E_F - E_V = 0.35\ \text{eV}$ (taken as unchanged with temperature, since no doping concentration is given to compute its shift). Effective density of states in the valence band, $N_V(300\ \text{K}) = 1 \times 10^{19}\ \text{cm}^{-3}$. Thermal voltage, $kT|_{300\ \text{K}} = 0.026\ \text{eV}$.
2. Hole concentration: $p = N_V\, e^{-(E_F - E_V)/kT}$.
3. N_V scales with temperature as $T^{3/2}$:
$$N_V(400\ \text{K}) = N_V(300\ \text{K}) \left(\frac{400}{300}\right)^{3/2} = 1 \times 10^{19} \times 1.5396 = 1.5396 \times 10^{19}\ \text{cm}^{-3}$$
4. kT scales linearly with temperature:
$$kT|_{400\ \text{K}} = 0.026 \times \frac{400}{300} = 0.034667\ \text{eV}$$
5. Compute the exponent:
$$\frac{E_F - E_V}{kT|_{400\ \text{K}}} = \frac{0.35}{0.034667} = 10.096$$
6. Compute p:
$$p = 1.5396 \times 10^{19} \times e^{-10.096} = 1.5396 \times 10^{19} \times 4.124 \times 10^{-5}$$
7. $$p \approx 6.349 \times 10^{14}\ \text{cm}^{-3} = 63.49 \times 10^{13}\ \text{cm}^{-3}$$

KEY IDEA

Hole concentration is $p = N_V e^{-(E_F - E_V)/kT}$. Both N_V (through the $T^{3/2}$ density-of-states scaling) and kT change with temperature, while the doping-fixed energy gap $(E_F - E_V)$ is taken as unchanged between 300 K and 400 K.

WATCH OUT FOR

Scaling only N_V with temperature but forgetting to also scale kT (i.e. plugging $400\,\mathrm{K}$ carrier density with the $300\,\mathrm{K}$ value of $kT = 0.026\,\mathrm{eV}$ in the exponential), which changes the exponent from about 10.10 to about 13.46 and gives an answer far outside the accepted range.

Q.62 NAT · 2 MARKS

Digital Electronics › ADC & DAC

A sample and hold circuit is implemented using a resistive switch and a capacitor with a time constant of 1 µs. The time for the sampling switch to stay closed to charge a capacitor adequately to a full scale voltage of 1 V with 12-bit accuracy is _____ µs (rounded off to two decimal places).

Official answer: **8.30 to 8.34**

SOLUTION

1. While the switch is closed, the capacitor charges through the switch resistance as a first-order RC circuit: $v_C(t) = V_{FS}\left(1 - e^{-t/\tau}\right)$, with $\tau = 1\ \mu\mathrm{s}$ and $V_{FS} = 1\ \mathrm{V}$.
2. The error still left after time t is $V_{FS} - v_C(t) = V_{FS}\, e^{-t/\tau}$.
3. 12-bit accuracy means this error must be no bigger than one LSB: $\mathrm{LSB} = \dfrac{V_{FS}}{2^{12}} = \dfrac{1}{4096}\ \mathrm{V} \approx 0.244\ \mathrm{mV}$.
4. Set the condition: $V_{FS}\, e^{-t/\tau} \le \dfrac{V_{FS}}{2^{12}} \ \Rightarrow\ e^{t/\tau} \ge 2^{12}$.
5. Take the natural log of both sides:
$$t \ge \tau \ln\left(2^{12}\right) = 12\,\tau \ln 2 = 12 \times 0.6931 \times 1\ \mu\mathrm{s}$$
6. $t = 8.318\ \mu\mathrm{s} \approx 8.32\ \mu\mathrm{s}$. The 1 V full-scale value cancels, so the answer depends only on τ and the number of bits N: $t = N\tau \ln 2$.

KEY IDEA

An RC sample-and-hold settles exponentially. The capacitor has settled to N-bit accuracy once the leftover error $V_{FS}e^{-t/\tau}$ has fallen to one LSB, $V_{FS}/2^N$. That gives $t = \tau \ln(2^N) = N\tau \ln 2$, which does not depend on the full-scale voltage.

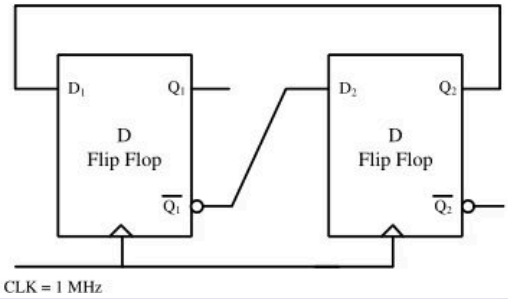
WATCH OUT FOR

Using a half-LSB error limit, $V_{FS}/2^{13}$, gives $13 \ln 2 \approx 9.01\ \mu\mathrm{s}$, which falls outside the key. The key here uses a one-LSB limit. Other slips: taking $\log_{10}$ where the natural log is needed ($12 \log_{10} 2 \approx 3.61\ \mu\mathrm{s}$), and setting $e^{-t/\tau}$ equal to $1/12$ when it should be $1/2^{12}$.

Q.63 NAT · 2 MARKS

Digital Electronics › Sequential

Q.63 In a given sequential circuit, initial states are $Q_1 = 1$ and $Q_2 = 0$. For a clock frequency of 1 MHz, the frequency of signal Q_2 in kHz, is _____ (*rounded off to the nearest integer*).



Official answer: **250, or 500**

SOLUTION

1. Read the connections from the figure: $D_1 = Q_2$ (the feedback wire over the top) and $D_2 = \overline{Q_1}$ (the diagonal wire from the bubbled output; the bar marks the complement). The Q_1 and $\overline{Q_2}$ outputs are unused stubs. Both flip-flops take the same clock, so they change state together on each clock edge.
2. D flip-flop characteristic equation: $Q^+ = D$. So $Q_1^+ = Q_2$ and $Q_2^+ = \overline{Q_1}$.
3. Step the state (Q_1Q_2) forward from the given start 10. Clock 1: $Q_1^+ = Q_2 = 0$, $Q_2^+ = \overline{1} = 0$, giving 00.
4. Clock 2: $Q_1^+ = 0$, $Q_2^+ = \overline{0} = 1$, giving 01. Clock 3: $Q_1^+ = 1$, $Q_2^+ = \overline{0} = 1$, giving 11. Clock 4: $Q_1^+ = 1$, $Q_2^+ = \overline{1} = 0$, giving 10, which is the starting state again.
5. The sequence $10 \to 00 \to 01 \to 11 \to 10$ repeats every 4 clock pulses. The loop through the two flip-flops has exactly one inversion, so this is a 2-bit twisted-ring (Johnson) counter, which has $2n = 4$ states for $n = 2$.
6. Over one cycle Q_2 reads $0, 0, 1, 1$. That is a square wave (50% duty cycle) whose period is 4 clock periods.
7. $$f_{Q_2} = \frac{f_{clk}}{4} = \frac{1\ \text{MHz}}{4} = 250\ \text{kHz}$$

KEY IDEA

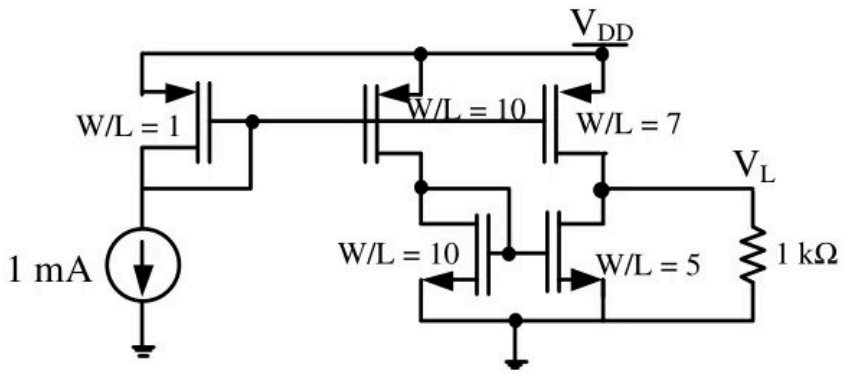
Write $Q^+ = D$ for each flip-flop and step the state table from the given initial state until it repeats. The number of states in the cycle is the division ratio: an output that repeats every N clocks has frequency f_{clk}/N. A shift loop with exactly one inversion (a twisted-ring, or Johnson, counter) of n flip-flops has $2n$ states, so here $N = 4$.

WATCH OUT FOR

Reading the diagonal wire as coming from Q_1 instead of $\overline{Q_1}$. With no inversion the loop is a plain ring counter, $10 \to 01 \to 10$, which gives a 2-state cycle and 500 kHz. (The official key accepts both 250 and 500, but the figure as printed gives 250.) A second slip is to count state changes of Q_2 rather than full periods: Q_2 changes every 2 clocks, but one full period is 4 clocks.

Q.64 NAT · 2 MARKS

Q.64 In the circuit below, the voltage V_L is _____ V (*rounded off to two decimal places*).



Official answer: **2.00**

SOLUTION

1. **Assumptions.** Every transistor is in saturation. Channel-length modulation is neglected ($\lambda = 0$). All PMOS devices share one $\mu_p C_{ox}$ and V_{tp}, and all NMOS devices share one $\mu_n C_{ox}$ and V_{tn}. Every source sits on a supply rail, so there is no body effect. With these assumptions, transistors that share a gate-source voltage carry drain currents in proportion to their W/L: $I_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{GS} - V_t)^2 \propto \frac{W}{L}$. The labels M_1 to M_5 and node X are not printed on the figure. They are used here only to name the devices.
2. **Reference branch.** The diode-connected PMOS M_1 ($W/L = 1$) carries all of the source current, $I_{REF} = 1\ \text{mA}$. That current fixes the V_{SG} shared by M_1, M_2 and M_3. M_1 is always saturated because $V_{SD} = V_{SG} > V_{SG} - |V_{tp}|$.
3. **PMOS mirror (equal V_{SG}, so $I_D \propto W/L$).** $I_{D2} = \dfrac{10}{1}(1\ \text{mA}) = 10\ \text{mA}$ and $I_{D3} = \dfrac{7}{1}(1\ \text{mA}) = 7\ \text{mA}$.
4. **Second reference.** Node X has only one other path, so all 10 mA from M_2 flows into the diode-connected NMOS M_4 ($W/L = 10$). This sets the V_{GS} that M_4 and M_5 share.

5. **NMOS mirror.** $I_{D5} = \frac{5}{10}(10\text{ mA}) = 5\text{ mA}$. M_5 draws this current out of the V_L node and sends it to ground.
6. **KCL at the output node.** M_3 pushes 7 mA into the node and M_5 pulls 5 mA out. The only other path is the load, so the difference goes through it to ground: $I_L = 7 - 5 = 2\text{ mA}$.
7. **Ohm's law.**

$$V_L = I_L R_L = (2\text{ mA})(1\text{ k}\Omega) = 2.00\text{ V}$$

This answer holds only if M_3 and M_5 stay saturated with their drains at 2 V. V_{DD} and the thresholds are not given, so the question intends ideal mirror operation, and that is assumed here.

KEY IDEA

This is a cascade of current mirrors. When transistors share a gate-source voltage and are in saturation with $\lambda = 0$, their drain currents scale with W/L. The PMOS mirror multiplies the 1 mA reference to 10 mA and 7 mA. The NMOS mirror then scales the 10 mA to 5 mA. KCL at the output node leaves $7 - 5 = 2\text{ mA}$ for the 1 kΩ load.

WATCH OUT FOR

There are three common slips. The first is taking the NMOS ratio against the 1 mA source instead of the 10 mA branch, which gives $I_{D5} = 5/1 \times 1 = 5$ mA by luck or $5/10 \times 1 = 0.5$ mA by mistake. The second is scaling the current through M_5 from M_3's 7 mA, when M_5 is mirrored from M_4's branch. The third is adding the two currents at the output node (12 mA, so 12 V) when M_3 sources current into the node and M_5 sinks current out of it.

Q.65 NAT · 2 MARKS

Communications › Information theory

Q.65 The frequency of occurrence of 8 symbols (a-h) is shown in the table below. A symbol is chosen and it is determined by asking a series of "yes/no" questions which are assumed to be truthfully answered. The average number of questions when asked in the most efficient sequence, to determine the chosen symbol, is _____ (*rounded off to two decimal places*).

Symbols	a	b	c	d	e	f	g	h
Frequency of occurrence	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{128}$

Official answer: **1.97 to 1.99**

SOLUTION

1. Each yes/no question gives at most 1 bit, so the most efficient questioning scheme is an optimal binary prefix code (Huffman code). The average number of questions equals the average codeword length $\bar{L}$ of that code.
2. The probabilities $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \frac{1}{128}$ sum to 1, and each one is a negative integer power of 2 (dyadic).
3. Huffman merging: the two least likely symbols g, h merge to $\frac{1}{128} + \frac{1}{128} = \frac{1}{64}$. This joins f ($\frac{1}{64}$) to give $\frac{1}{32}$, then e gives $\frac{1}{16}$, and so on up to 1. The tree is a single chain, so the codeword lengths are

$1, 2, 3, 4, 5, 6, 7, 7$ for a, b, c, d, e, f, g, h.

4. Check: for a dyadic source the length is exactly $-\log_2 p_i$ (for g and h, $-\log_2 \frac{1}{128} = 7$), so $\bar{L}$ equals the entropy H and the code meets the lower bound.
5. $$\bar{L} = \frac{1}{2}(1) + \frac{1}{4}(2) + \frac{1}{8}(3) + \frac{1}{16}(4) + \frac{1}{32}(5) + \frac{1}{64}(6) + 2 \cdot \frac{1}{128}(7)$$
6. $$\bar{L} = 0.5 + 0.5 + 0.375 + 0.25 + 0.15625 + 0.09375 + 0.109375 = 1.984375$$
7. Rounded to two decimal places, the average number of questions is ≈ 1.98 (inside the official range 1.97 to 1.99).

KEY IDEA

The most efficient yes/no questioning scheme is the Huffman code. Its average length is the expected number of questions, and for a dyadic source it equals the entropy $H = \sum p_i \log_2 \frac{1}{p_i}$.

WATCH OUT FOR

Giving the entropy of a source that is not dyadic, or assuming a fixed-length code with 3 questions for 8 symbols. Another slip is giving both of the two rarest symbols length 6 instead of 7.

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