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GATE EC · 2022

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GATE EC 2022 Solved Paper

INSIDE

- All 65 questions, in the order of the paper
- The official answer to every question
- A step-by-step solution to every question
- The answer key on one page

Question paper and official answer key: IIT Kharagpur

Online, with the questions linked to their topic notes: **iaeacademy.in/study/papers/ec-2022**

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About this paper

65 questions, 100 marks. Question types: 33 MCQ · 14 MSQ · 18 NAT.

Official answer key: IIT Kharagpur

We placed each question under a subject and topic ourselves; the exam paper does not label them.

This paper online, with the questions linked to their topic notes:
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Answer key

Q	TYPE	MARKS	ANSWER
Q.1	MCQ	1	**C**
Q.2	MCQ	1	**D**
Q.3	MCQ	1	**B**
Q.4	MCQ	1	**B**
Q.5	MCQ	1	**C**
Q.6	MCQ	2	**D**
Q.7	MCQ	2	**C**
Q.8	MCQ	2	**C**
Q.9	MCQ	2	**B**
Q.10	MCQ	2	**B**
Q.11	MCQ	1	**A**
Q.12	MCQ	1	**C**
Q.13	MCQ	1	**B**
Q.14	MCQ	1	**B**
Q.15	MCQ	1	**A**
Q.16	MCQ	1	**A**
Q.17	MCQ	1	**C**
Q.18	MCQ	1	**C**
Q.19	MCQ	1	**C**
Q.20	MCQ	1	**C**
Q.21	MCQ	1	**C**
Q.22	MCQ	1	**A**
Q.23	MCQ	1	**C**
Q.24	MCQ	1	**C**
Q.25	MSQ	1	**A, B**
Q.26	MSQ	1	**B, C**
Q.27	MSQ	1	**B, C**
Q.28	MSQ	1	**C**
Q.29	MSQ	1	**A, B, D**
Q.30	MSQ	1	**A, C**
Q.31	NAT	1	**4.0**
Q.32	NAT	1	**0.50**
Q.33	NAT	1	**0.0**
Q.34	NAT	1	**5.5 to 5.7**
Q.35	NAT	1	**2.99 to 3.01**
Q.36	MCQ	2	**B**
Q.37	MCQ	2	**A**
Q.38	MCQ	2	**A**
Q.39	MCQ	2	**B**
Q.40	MCQ	2	**C**
Q.41	MCQ	2	**C**
Q.42	MCQ	2	**A**
Q.43	MCQ	2	**A**
Q.44	MCQ	2	**A**
Q.45	MSQ	2	**B, D**
Q.46	MSQ	2	**C, D**
Q.47	MSQ	2	**A, C**
Q.48	MSQ	2	**A, C**
Q.49	MSQ	2	**A, C**
Q.50	MSQ	2	**A**
Q.51	MSQ	2	**A**
Q.52	MSQ	2	**Marks to all**
Q.53	NAT	2	**512**
Q.54	NAT	2	**3.9 to 4.1**
Q.55	NAT	2	**7.9 to 8.1**
Q.56	NAT	2	**0.57 to 0.61**
Q.57	NAT	2	**0.70 to 0.80**
Q.58	NAT	2	**-0.55 to -0.45**
Q.59	NAT	2	**-2.1 to -1.9**
Q.60	NAT	2	**-0.11 to -0.09**
Q.61	NAT	2	**1.00**
Q.62	NAT	2	**0.84 to 0.86**
Q.63	NAT	2	**0.22 to 0.25**
Q.64	NAT	2	**1.15 to 1.18**
Q.65	NAT	2	**0.48 to 0.52**

General Aptitude Q.1 to Q.10

Q.1 MCQ · 1 MARK

General Aptitude › Grammar & vocabulary

Mr. X speaks _____ Japanese _____ Chinese.

(A) neither / or

(B) either / nor

(C) neither / nor

(D) also / but

Official answer: **C**

SOLUTION

1. The two blanks call for a correlative conjunction: a pair of words that always go together, one word in each blank.
2. The standard correlative pairs are fixed: *either … or* and *neither … nor*. You cannot swap one half of a pair for the other.
3. (A) *neither / or* puts the negative *neither* with the positive *or*. That pairing is wrong, because *neither* always takes *nor*.
4. (B) *either / nor* has the same fault the other way round: *either* always takes *or*, never *nor*.
5. (D) *also / but* is not a correlative pair. The standard pair is *not only … but also*, and "Mr. X speaks also Japanese but Chinese" is not grammatical.
6. (C) *neither / nor* is a correct pair. "Mr. X speaks neither Japanese nor Chinese" means he speaks neither of the two languages. The sentence is grammatical and makes sense.
7. Only option (C) uses a correct correlative pair, so the answer is (C).

KEY IDEA

Correlative conjunctions come in fixed pairs: *either* goes with *or*, and *neither* goes with *nor*. When the two blanks are split around two nouns joined in this way, check that both halves come from the same pair.

WATCH OUT FOR

Mixing the halves of two pairs, as in *neither … or* (A) or *either … nor* (B). Each one looks nearly right because one word in it is correct. Another mistake is picking (D) because *also* and *but* recall *not only … but also*, a pair that needs *not only* in the first blank.

Q.2 MCQ · 1 MARK

General Aptitude › Numerical

A sum of money is to be distributed among P, Q, R, and S in the proportion 5 : 2 : 4 : 3, respectively.

If R gets Rs. 1000 more than S, what is the share of Q (in Rs.)?

(A) 500

(B) 1000

(C) 1500

(D) 2000

Official answer: **D**

SOLUTION

1. Shares in a given ratio are all multiples of one common unit. Let P, Q, R and S get $5k$, $2k$, $4k$ and $3k$ rupees, for some $k > 0$.
2. The difference between two shares is the difference of their ratio terms times k: $R - S = 4k - 3k = k$.
3. We are told R gets Rs. 1000 more than S, so $R - S = 1000$, which gives $k = 1000$.

4. Q's share is $2k = 2 \times 1000 = 2000$, so Q gets Rs. 2000. That is option (D).
5. Check: the shares are Rs. 5000, 2000, 4000 and 3000 (ratio $5 : 2 : 4 : 3$), and R gets $4000 - 3000 = 1000$ rupees more than S, as the question requires.

KEY IDEA

In a ratio split, every share is (its ratio term) × (one common unit k). A given difference between two shares fixes k straight away: $R - S = (4 - 3)k = k = 1000$. You do not need the total sum at all.

WATCH OUT FOR

Treating the Rs. 1000 difference as if it were the whole sum and dividing it in the ratio (which gives $\frac{2}{14} \times 1000 \approx 143$, not an option). Another slip is to stop at $k = 1000$ and pick option (B), forgetting that Q's share is $2k$, not k.

Q.3 MCQ · 1 MARK

General Aptitude › Geometry

A trapezium has vertices marked as P, Q, R and S (in that order anticlockwise). The side PQ is parallel to side SR.

Further, it is given that, PQ = 11 cm, QR = 4 cm, RS = 6 cm and SP = 3 cm.

What is the shortest distance between PQ and SR (in cm)?

(A) 1.80

(B) 2.40

(C) 4.20

(D) 5.76

Official answer: **B**

SOLUTION

1. The shortest distance between the two parallel sides is the perpendicular height h of the trapezium. Drop perpendiculars from S and R onto PQ, meeting it at M and N, so that SRNM is a rectangle and $MN = SR = 6\ \text{cm}$.
2. The two end pieces of PQ then add up to the difference of the parallel sides: $PM + NQ = PQ - SR = 11 - 6 = 5\ \text{cm}$.
3. Slide the rectangle SRNM out, i.e. move the leg QR a distance of 6 cm along PQ towards P so that it meets the leg SP at S. This leaves a triangle with sides $SP = 3$, $QR = 4$ and base $PM + NQ = 5$ (all in cm). Its height on the base of 5 is the same h.
4. Converse of Pythagoras' theorem: $3^2 + 4^2 = 9 + 16 = 25 = 5^2$, so this triangle has a right angle between the sides 3 and 4.
5. Work out the area of this right triangle in two ways: $\frac{1}{2} \times 3 \times 4 = \frac{1}{2} \times 5 \times h$, so $h = \dfrac{12}{5} = 2.4\ \text{cm}$.
6. Check with lengths: let $PM = a$ and $NQ = b$, with $a + b = 5$. From the two right triangles SMP and RNQ, $h^2 = 9 - a^2 = 16 - b^2$, so $b^2 - a^2 = 7$. Since $b^2 - a^2 = (b - a)(b + a)$, this gives $b - a = \frac{7}{5} = 1.4$, hence $a = 1.8$ and $b = 3.2$. Both are positive, so the feet M and N do lie on PQ as assumed. Then $h^2 = 9 - 3.24 = 5.76$ (and also $16 - 10.24 = 5.76$), so $h = 2.4\ \text{cm}$.
7. The shortest distance between PQ and SR is $2.40\ \text{cm}$, which is option (B).

KEY IDEA

Take the rectangle between the parallel sides out of the trapezium. The two legs (3 and 4) and the difference of the parallel sides (11 - 6 = 5) make a 3-4-5 right triangle, and that triangle has the same height as the trapezium. Working out its area two ways gives $h = \frac{3 \times 4}{5} = 2.4\ \text{cm}$.

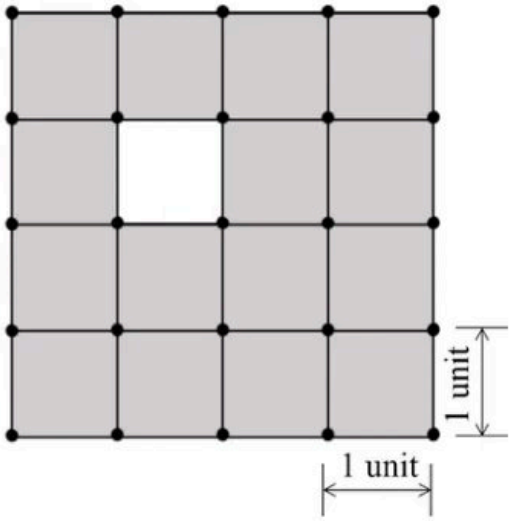
WATCH OUT FOR

The options include two numbers from the working. 5.76 is h^2, so it is what you get if you forget the square root. 1.80 is the end piece PM, not the height. Some students also try the whole length PQ = 11 as the base of the triangle. The base is only the difference 11 - 6 = 5.

Q.4 MCQ · 1 MARK

General Aptitude › Spatial

Q.4 The figure shows a grid formed by a collection of unit squares. The unshaded unit square in the grid represents a hole.



What is the maximum number of squares without a "hole in the interior" that can be formed within the 4×4 grid using the unit squares as building blocks?

(A)	15
(B)	20
(C)	21
(D)	26

Official answer: **B**

SOLUTION

1. Number the rows 1 to 4 from the top and the columns 1 to 4 from the left, so the hole is cell $(2, 2)$. A square built from unit squares as building blocks cannot include the hole anywhere, because the hole is a missing block. So we count only the squares that do not contain cell $(2, 2)$.
2. A 4×4 grid has $(4 - k + 1)^2$ squares of side k: that is $16, 9, 4, 1$ for $k = 1, 2, 3, 4$, or 30 in all if the hole is ignored.
3. 1×1: every unit square except the hole, so $16 - 1 = 15$.
4. 2×2: a square whose top-left cell is (r, c), with $r, c \in \{1, 2, 3\}$, covers rows $r, r + 1$ and columns $c, c + 1$. It contains $(2, 2)$ exactly when $r \in \{1, 2\}$ and $c \in \{1, 2\}$, which gives $2 \times 2 = 4$ squares. So

$9 - 4 = 5$ squares are free of the hole.

5. 3×3: the top-left cell can only be $(1,1), (1,2), (2,1)$ or $(2,2)$. Each of these squares covers rows 2 and 3 and columns 2 and 3, so all 4 contain the hole. That leaves 0.
6. 4×4: the only such square is the whole grid, which contains the hole, so 0.
7. Total:

$$15 + 5 + 0 + 0 = 20$$

The answer is (B).

KEY IDEA

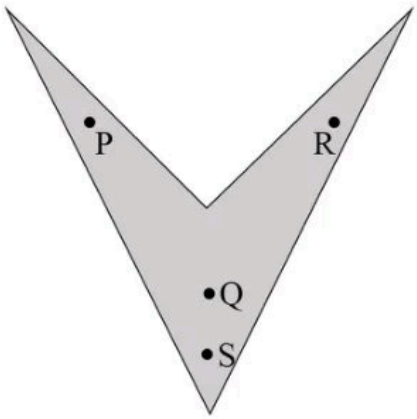
For each size, count the squares by where their top-left cell sits, then subtract the ones that cover the hole. A square of side k with top-left cell (r, c) covers the hole $(2, 2)$ exactly when $r \le 2 \le r + k - 1$ and $c \le 2 \le c + k - 1$. A hole near a corner leaves no room for 3 × 3 squares, because every 3 × 3 square in a 4 × 4 grid covers the two middle rows and the two middle columns.

WATCH OUT FOR

Counting the hole itself as a 1×1 square gives $16 + 5 = 21$, which is option (C). Another mistake is to allow bigger squares that have the hole on their edge rather than at their centre, or to subtract only the 2×2 squares that contain the hole ($30 - 4 = 26$, option (D)). Stopping after the unit squares gives 15, option (A). Because the hole is a missing building block, no square that contains it can be formed at any size.

Q.5 MCQ · 1 MARK

General Aptitude › Spatial

Q.5	An art gallery engages a security guard to ensure that the items displayed are protected. The diagram below represents the plan of the gallery where the boundary walls are opaque. The location the security guard posted is identified such that all the inner space (shaded region in the plan) of the gallery is within the line of sight of the security guard. If the security guard does not move around the posted location and has a 360° view, which one of the following correctly represents the set of ALL possible locations among the locations P, Q, R and S, where the security guard can be posted to watch over the entire inner space of the gallery. 
(A)	P and Q
(B)	Q
(C)	Q and S
(D)	R and S

Official answer: **C**

SOLUTION

1. The guard can see a point X of the gallery only if the straight segment from the guard to X stays inside the shaded region and never crosses a wall. We need every posting from which this holds for all points of the shaded region.
2. Name the corners of the plan: A is the top-left tip, C is the top-right tip, B is the bottom tip, and N is the inward notch. The gallery is the quadrilateral $ANCB$, and N is its only inward-pointing corner. The segment NB runs from the notch straight down the centre line to the bottom tip. It lies inside the

gallery and cuts it into two triangles: ANB (the left arm and the left half of the stem) and NCB (the right arm and the right half of the stem).

3. A triangle is convex: the segment joining any two of its points stays inside it. Q and S both lie on the centre line NB, so each belongs to both triangles. Every point X of the gallery lies in ANB or in NCB. The segment from Q (or from S) to X therefore stays inside that triangle, and so inside the gallery. With a 360° view, Q sees the entire inner space, and so does S.
4. P and R are in the upper parts of the two arms at about the same height, both above the notch N. The straight line from P to R passes above N, through the unshaded V-shaped gap between the arms, so it crosses the inner wall AN and then the inner wall NC. A guard at P therefore cannot see the point R, which is part of the shaded region. By the same line, a guard at R cannot see the point P. Neither P nor R is a valid posting.
5. The set of ALL valid postings is $\{Q, S\}$. Checking the other options: (A) and (D) each include P or R, which fail, and (B) leaves out S, which works. The answer is (C) Q and S.

KEY IDEA

A guard sees the whole room exactly when the segment to every point of the room stays inside it. This room has only one inward corner, the notch. Cut the room along the line from the notch to the bottom tip and you get two convex triangles. Any point on that shared line sees all of both triangles, and so sees the whole room. More generally, every point below both inner walls, extended as full lines, sees the whole room, and Q and S are well inside that region. A point up in one arm is blocked from the other arm by the empty V-shaped gap.

WATCH OUT FOR

Choosing (B) Q alone. Students assume S, near the bottom tip, is too far down or boxed in to see the tips of the arms, but its sight lines to both arms pass below the notch and stay inside the room. The opposite mistake is choosing (A) or (D) because P or R sits inside an arm and seems close to everything. The straight line from one arm to the other passes over the notch through the unshaded gap, so a guard in one arm cannot see the upper part of the other.

Q.6 MCQ · 2 MARKS

General Aptitude › Reading

Mosquitoes pose a threat to human health. Controlling mosquitoes using chemicals may have undesired consequences. In Florida, authorities have used genetically modified mosquitoes to control the overall mosquito population. It remains to be seen if this novel approach has unforeseen consequences.

Which one of the following is the correct logical inference based on the information in the above passage?

(A) Using chemicals to kill mosquitoes is better than using genetically modified mosquitoes because genetic engineering is dangerous

(B) Using genetically modified mosquitoes is better than using chemicals to kill mosquitoes because they do not have any side effects

(C) Both using genetically modified mosquitoes and chemicals have undesired consequences and can be dangerous

(D) Using chemicals to kill mosquitoes may have undesired consequences but it is not clear if using genetically modified mosquitoes has any negative consequence

Official answer: **D**

SOLUTION

1. Pick out what the passage actually claims. (1) Controlling mosquitoes with chemicals **may** have undesired consequences. (2) Florida has used genetically modified (GM) mosquitoes to control the mosquito population. (3) "It remains to be seen" whether the GM approach has unforeseen consequences, so its effects are **not yet known**.

2. The rule for a valid inference: it must follow from the passage alone. It cannot add a comparison or a certainty that the passage does not state.
3. (A) is wrong. The passage never compares the two methods and never says genetic engineering is dangerous. It only says the GM method's consequences are unknown.
4. (B) is wrong. The passage does not claim GM mosquitoes have no side effects. It says their consequences "remain to be seen", which is the opposite of certainty. It also never ranks one method above the other.
5. (C) is wrong. It states that GM mosquitoes **have** undesired consequences, which turns an open question into a stated fact. It also hardens the passage's "may have" for chemicals into a definite "have".
6. (D) is right. It restates both points exactly: chemicals **may** have undesired consequences, and it is **not clear** whether GM mosquitoes have any negative consequence. Both the hedge and the uncertainty are kept, and nothing is added.

KEY IDEA

A logical inference must keep the passage's level of certainty. "May have" stays "may have", and "remains to be seen" means unknown. Any option that ranks the two methods or turns an unknown into a fact goes beyond the passage.

WATCH OUT FOR

Picking (C) because both methods sound risky. The passage never says GM mosquitoes have undesired consequences, only that it is not yet known, and it says chemicals only "may" have them. Students also fall for (A) or (B) because they bring in their own opinion on genetic engineering, which the passage does not give.

Q.7 MCQ · 2 MARKS

General Aptitude › Numerical

Consider the following inequalities.

(i) $2x - 1 > 7$

(ii) $2x - 9 < 1$

Which one of the following expressions below satisfies the above two inequalities?

(A) $x \leq -4$

(B) $-4 < x \leq 4$

(C) $4 < x < 5$

(D) $x \geq 5$

Official answer: **C**

SOLUTION

1. Inequality (i). Adding the same number to both sides keeps the inequality: $2x - 1 > 7 \Rightarrow 2x > 8$. Dividing by a positive number keeps the direction, so $x > 4$.
2. Inequality (ii). In the same way, $2x - 9 < 1 \Rightarrow 2x < 10 \Rightarrow x < 5$.
3. Both must hold, so take the intersection of the two solution sets: $\{x > 4\} \cap \{x < 5\}$ gives $4 < x < 5$. Both ends are strict because both inequalities are strict.
4. Check the options. (A) $x \leq -4$ and (B) $-4 < x \leq 4$ break (i), because every such x is at most 4. (D) $x \geq 5$ breaks (ii). Only (C) $4 < x < 5$ matches.
5. Spot check with $x = 4.5$: $2(4.5) - 1 = 8 > 7$ and $2(4.5) - 9 = 0 < 1$. Both hold.

KEY IDEA

Solve each linear inequality on its own, then intersect the two solution sets. "Satisfies both" means the overlap of the sets, not their union. Keep each end strict or non-strict exactly as the original inequality was.

WATCH OUT FOR

Picking (B) $-4 < x \leq 4$, whether from misreading $2x - 1 > 7$ as $x > -4$ or from treating the endpoint 4 as included. At $x = 4$, $2(4) - 1 = 7$, which is not greater than 7, so 4 is excluded. Another slip is taking the union of the two sets, which is every real number, instead of their intersection.

Q.8 MCQ · 2 MARKS

General Aptitude › Geometry

Four points $P(0, 1)$, $Q(0, -3)$, $R(-2, -1)$, and $S(2, -1)$ represent the vertices of a quadrilateral.

What is the area enclosed by the quadrilateral?

(A) 4

(B) $4\sqrt{2}$

(C) 8

(D) $8\sqrt{2}$

Official answer: **C**

SOLUTION

1. Plot the points. $P(0, 1)$ and $Q(0, -3)$ both lie on the y-axis ($x = 0$). $R(-2, -1)$ and $S(2, -1)$ both lie on the line $y = -1$. R and S are on opposite sides of $x = 0$, and P and Q are on opposite sides of $y = -1$, so segments PQ and RS cross each other. Two crossing segments cannot both be sides of a quadrilateral, so PQ and RS are its diagonals and the boundary runs $P \to S \to Q \to R \to P$. (The letters are not given in boundary order.)
2. Diagonal lengths (distance between two points on the same vertical or horizontal line): $PQ = |1 - (-3)| = 4$ and $RS = |2 - (-2)| = 4$.
3. PQ is vertical and RS is horizontal, so the diagonals are perpendicular. Both have midpoint $(0, -1)$, so they bisect each other.
4. Diagonals that are equal, perpendicular and bisect each other make the quadrilateral a square (a rhombus with equal diagonals).
5. Area of a quadrilateral with perpendicular diagonals:
$$\text{Area} = \frac{1}{2} d_1 d_2 = \frac{1}{2} \times 4 \times 4 = 8$$
6. Check using a side: $PS = \sqrt{(2 - 0)^2 + (-1 - 1)^2} = \sqrt{8} = 2\sqrt{2}$, so the area of the square is $(2\sqrt{2})^2 = 8$. This matches option (C).

KEY IDEA

Plot the points first. Two of them lie on a vertical line and two on a horizontal line, and the two segments cross, so they are the diagonals. When a quadrilateral's diagonals are perpendicular, its area is $\frac{1}{2} d_1 d_2$. Here both diagonals are 4 and bisect each other, so the figure is a square of side $2\sqrt{2}$.

WATCH OUT FOR

Joining the vertices in letter order P, Q, R, S. Segments PQ and RS cross, so that path makes a self-intersecting bow-tie and not the quadrilateral. Taking PQ and RS as sides of length 4 gives 16, which is not an option. Another slip is to use $d_1 d_2 = 16$ without the factor $\frac{1}{2}$. People who get the side $2\sqrt{2}$ right can still pick $4\sqrt{2}$ or $8\sqrt{2}$ by mistake, because those are the square's perimeter $8\sqrt{2}$ or twice the side.

Q.9 MCQ · 2 MARKS

General Aptitude › Reasoning

In a class of five students P, Q, R, S and T, only one student is known to have copied in the exam. The disciplinary committee has investigated the situation and recorded the statements from the students as given below.

Statement of P: R has copied in the exam.

Statement of Q: S has copied in the exam.

Statement of R: P did not copy in the exam.

Statement of S: Only one of us is telling the truth.

Statement of T: R is telling the truth.

The investigating team had authentic information that S never lies.

Based on the information given above, the person who has copied in the exam is

(A) R

(B) P

(C) Q

(D) T

Official answer: **B**

SOLUTION

1. S never lies, so S's statement "Only one of us is telling the truth" is true.
2. S is telling the truth, so S is that one truth-teller. P, Q, R and T are all lying.
3. R is lying, so the opposite of "P did not copy in the exam" holds: **P copied**.
4. Check P: "R has copied in the exam" is false. That fits, because only one student copied and that student is P.
5. Check Q: "S has copied in the exam" is false. That fits too.
6. Check T: "R is telling the truth" is false because R lied, which fits. Every statement is consistent with exactly one truth-teller (S) and exactly one copier (P).
7. Confirm that no other option works. If R, Q or T had copied, P would not have copied, so R's statement "P did not copy in the exam" would be true. R and S would then both be telling the truth, which contradicts S's true statement that only one person is. So the only possible copier is P, option (B).

KEY IDEA

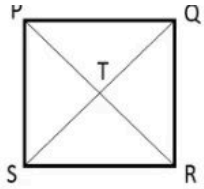
Start with the one statement you know is true: S never lies, so S's claim "Only one of us is telling the truth" holds. S is that one person, which means P, Q, R and T are all lying. Negate R's statement and you get P as the copier. Then check that each of the other lies stays false.

WATCH OUT FOR

Students often trust P's direct accusation ("R has copied") and pick (A). They forget that S's true statement forces everyone except S to be lying. Another slip is to read "Only one of us" as leaving S out. S says "us", so S counts, and S is the one truth-teller.

Q.10 MCQ · 2 MARKS

General Aptitude › Spatial

Q.10	Consider the following square with the four corners and the center marked as P, Q, R, S and T respectively.  Let X, Y and Z represent the following operations: X: rotation of the square by 180 degree with respect to the S-Q axis. Y: rotation of the square by 180 degree with respect to the P-R axis. Z: rotation of the square by 90 degree clockwise with respect to the axis perpendicular, going into the screen and passing through the point T. Consider the following three distinct sequences of operation (which are applied in the left to right order). (1) XYZZ (2) XY (3) ZZZZ Which one of the following statements is correct as per the information provided above?
(A)	The sequence of operations (1) and (2) are equivalent
(B)	The sequence of operations (1) and (3) are equivalent
(C)	The sequence of operations (2) and (3) are equivalent
(D)	The sequence of operations (1), (2) and (3) are equivalent

Official answer: **B**

SOLUTION

1. Fix the starting layout, reading clockwise from the top-left: P (top-left), Q (top-right), R (bottom-right), S (bottom-left), with T at the centre.
2. X is a 180° turn about the diagonal S-Q. A half-turn about a line in the plane of the square is a flip over that line: points on the axis stay put and the other two corners swap. So S and Q stay, and P and R swap. Layout after X (top-left, top-right, bottom-right, bottom-left): R, Q, P, S.

3. Y is a 180° turn about the P-R diagonal. After X, R is at the top-left and P at the bottom-right, so P-R is still the top-left to bottom-right diagonal: the axis is the same line whether you take it as fixed in space or carried along with the labels. The flip keeps the top-left and bottom-right corners and swaps the top-right and bottom-left ones. Layout after XY: R, S, P, Q.
4. Compare with the start (P, Q, R, S). Every corner has moved to the opposite corner: P from top-left to bottom-right, Q from top-right to bottom-left, and so on. So XY is a 180° rotation about T. X alone turns the square face-down and Y turns it face-up again, so XY is an ordinary in-plane half-turn. (Rule: two half-turns about perpendicular lines crossing at T add up to a half-turn about the axis through T perpendicular to the square.)
5. Z is a 90° clockwise turn about the axis through T, so ZZ is a 180° turn and ZZZZ is a 360° turn, which leaves every corner where it started (the identity).
6. Sequence (1): XYZZ is $180^\circ + 180^\circ = 360^\circ$, the identity. Check: starting from R, S, P, Q, one Z (clockwise, so each label moves one corner to the right along the top) gives Q, R, S, P, and a second Z gives P, Q, R, S, the original.
7. Sequence (2): XY is a 180° turn, so the layout ends as R, S, P, Q, not the original.
8. Sequence (3): ZZZZ is $4 \times 90^\circ = 360^\circ$, the identity.
9. So (1) and (3) both return the square to where it started and are equivalent. (2) matches neither of them, which rules out (A), (C) and (D). The answer is option (B).

KEY IDEA

A 180° rotation about a line lying in the square is a flip over that line. Two flips over perpendicular diagonals make a 180° rotation about the centre. So XY equals ZZ, which makes XYZZ equal ZZZZ (a full turn, the identity).

WATCH OUT FOR

Treating X and Y as 180° turns about the centre T instead of flips over the diagonals. Then XY looks like a 360° turn and (D) seems right. Another slip is to think XY undoes itself and leaves the square unchanged, which gives (C). Track the four corner labels step by step: XY sends each corner to the opposite one.

Electronics and Communication Engineering Q.11 to Q.65

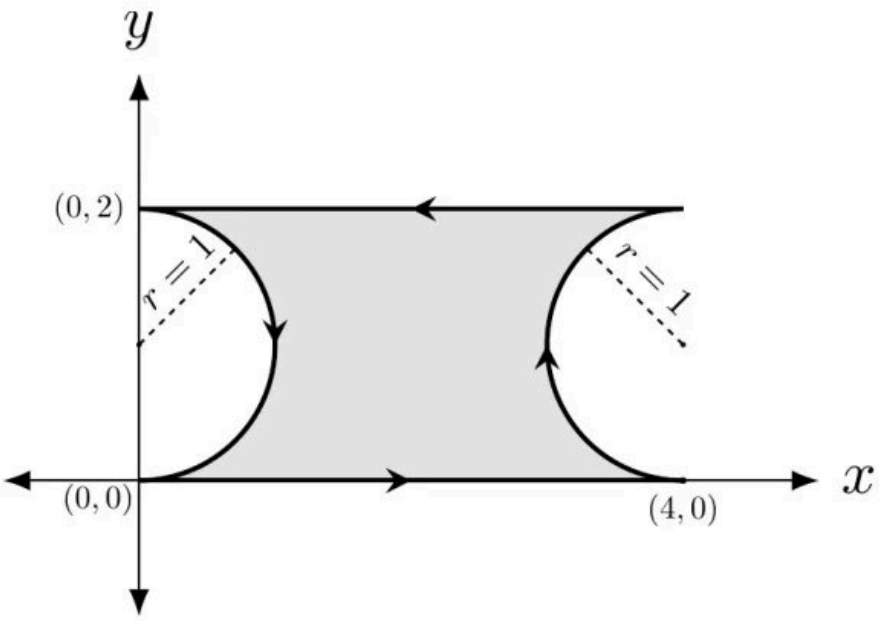
Q.11 MCQ · 1 MARK

Engineering Mathematics › Vector calculus

Q.11 Consider the two-dimensional vector field $\vec{F}(x,y) = x\,\vec{\imath} + y\,\vec{\jmath}$, where $\vec{\imath}$ and $\vec{\jmath}$ denote the unit vectors along the x-axis and the y-axis, respectively. A contour C in the x-y plane, as shown in the figure, is composed of two horizontal lines connected at the two ends by two semicircular arcs of unit radius. The contour is traversed in the counter-clockwise sense. The value of the closed path integral

$$\oint_C \vec{F}(x,y)\cdot(dx\,\vec{\imath} + dy\,\vec{\jmath}\,)$$

is _____.



(A)	0
(B)	1
(C)	$8 + 2\pi$
(D)	-1

Official answer: **A**

SOLUTION

1. Write the field as $\vec{F} = P\,\vec{\imath} + Q\,\vec{\jmath}$ with $P = x$, $Q = y$. With $d\vec{r} = dx\,\vec{\imath} + dy\,\vec{\jmath}$, the integrand is $\vec{F}\cdot d\vec{r} = x\,dx + y\,dy$.

2. Test whether the field is conservative (2-D curl): $\dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y} = 0 - 0 = 0$ everywhere, and P, Q are smooth on the whole plane, which is simply connected. So $\vec{F}$ is a gradient field.
3. Find the potential: $x\,dx + y\,dy = d\phi$ with $\phi(x, y) = \dfrac{x^2 + y^2}{2}$. Check: $\nabla\phi = x\,\vec{i} + y\,\vec{j} = \vec{F}$.
4. Fundamental theorem for line integrals: for a gradient field, $\displaystyle\int \nabla\phi \cdot d\vec{r} = \phi(\text{end}) - \phi(\text{start})$. A closed path ends where it starts, so $\oint_C \nabla\phi \cdot d\vec{r} = 0$.
5. Cross-check with Green's theorem (C is a simple, piecewise-smooth closed curve traversed counter-clockwise):
$$\oint_C (P\,dx + Q\,dy) = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \iint_R 0\,dA = 0.$$
So the shape of the contour, the arc radii and the direction of traversal make no difference.
6. Direct check, piece by piece, using ϕ at the four junctions: $\phi(0,0) = 0$, $\phi(4,0) = 8$, $\phi(4,2) = 10$, $\phi(0,2) = 2$. Bottom line: $8 - 0 = 8$. Right arc: $10 - 8 = 2$. Top line: $2 - 10 = -8$. Left arc: $0 - 2 = -2$. Total: $8 + 2 - 8 - 2 = 0$.
7. Hence $\oint_C \vec{F} \cdot (dx\,\vec{i} + dy\,\vec{j}) = 0$, which is option (A).

KEY IDEA

$\vec{F} = x\,\vec{i} + y\,\vec{j} = \nabla\left(\dfrac{x^2 + y^2}{2}\right)$ is a conservative (gradient) field: its 2-D curl $\dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y}$ is zero everywhere. The line integral of a gradient field depends only on where the path starts and ends, so around any closed contour it is 0. The figure's geometry is a distraction.

WATCH OUT FOR

Picking (C), $8 + 2\pi$. That is the length of C (two lines of length 4 plus two semicircles of length π), which is $\oint_C |d\vec{r}|$, not the work integral $\oint_C \vec{F} \cdot d\vec{r}$. The other trap is working through the four pieces and getting a direction wrong on the top line or an arc, so the pieces fail to cancel. Check the curl first: once it is zero on the whole plane, the closed-path integral is 0 with no parametrisation needed.

Q.12 MCQ · 1 MARK

Engineering Mathematics › Linear algebra

Consider a system of linear equations $Ax = b$, where
$$A = \begin{bmatrix} 1 & -\sqrt{2} & 3 \\ -1 & \sqrt{2} & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

This system of equations admits _____.

(A) a unique solution for x
(B) infinitely many solutions for x
(C) no solutions for x
(D) exactly two solutions for x

Official answer: **C**

SOLUTION

1. Write the augmented matrix:
$$[A \mid b] = \begin{bmatrix} 1 & -\sqrt{2} & 3 & 1 \\ -1 & \sqrt{2} & -3 & 3 \end{bmatrix}$$
2. Row reduce with $R_2 \to R_2 + R_1$:
$$\begin{bmatrix} 1 & -\sqrt{2} & 3 & 1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

3. The second row now reads $0 \cdot x_1 + 0 \cdot x_2 + 0 \cdot x_3 = 4$, i.e. $0 = 4$. No x satisfies it. (You get the same thing by adding the two original equations: the left sides cancel and the right sides add to $1 + 3 = 4$.)
4. Ranks: row 2 of A is -1 times row 1, and row 1 is non-zero, so $\rho(A) = 1$. The reduced augmented matrix has two non-zero rows, one of them $[0\ 0\ 0\ 4]$, so $\rho([A \mid b]) = 2$.
5. Consistency (Rouché–Capelli) theorem: $Ax = b$ has a solution if and only if $\rho(A) = \rho([A \mid b])$. Here $1 \neq 2$, so the system is inconsistent and has no solution. Answer: (C).
6. Option (D) can never be right for a linear system: the number of solutions is always 0, 1 or infinitely many. If x_1 and x_2 were two different solutions, every point $x_1 + t(x_2 - x_1)$ would also be a solution.

KEY IDEA

$Ax = b$ is consistent exactly when $\rho(A) = \rho([A \mid b])$. When the coefficient rows are dependent (here $R_2 = -R_1$), the right-hand sides must follow the same relation ($b_2 = -b_1$). If they do, there are infinitely many solutions. If they don't, as here ($3 \neq -1$), there are none.

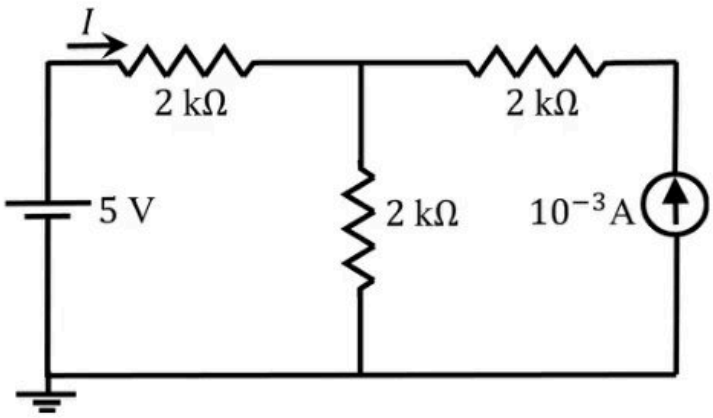
WATCH OUT FOR

Seeing 3 unknowns and only 2 equations and answering (B) 'infinitely many solutions' straight away. Having fewer equations than unknowns rules out a unique solution, but it does not make the system consistent. You still have to compare $\rho(A)$ with $\rho([A \mid b])$. Another mistake is to check only $\rho(A)$ and forget the augmented column.

Q.13 MCQ · 1 MARK

Networks (Electric Circuits) › Laws & node/mesh

Q.13 The current I in the circuit shown is _____.



(A)	1.25×10^{-3}A
(B)	0.75×10^{-3}A
(C)	-0.5×10^{-3}A
(D)	1.16×10^{-3}A

Official answer: **B**

SOLUTION

1. Take the bottom wire as the reference (0 V). The source's longer plate (its positive terminal) is on top, so the top-left node is at 5 V. Let V_A be the voltage of the middle node, where the three 2 kΩ resistors

meet.

2. The right-hand 2 kΩ is in series with an ideal current source, so it carries exactly 10^{-3} A into node A whatever its resistance. Its value does not affect V_A.
3. Write KCL at node A, with current into the node equal to current out of it: $\frac{5 - V_A}{2\ \text{k}\Omega} + 1\ \text{mA} = \frac{V_A}{2\ \text{k}\Omega}$
4. Multiply through by $2\ \text{k}\Omega$: $5 - V_A + 2 = V_A \ \Rightarrow\ V_A = 3.5\ \text{V}$
5. Then $I = \frac{5 - V_A}{2\ \text{k}\Omega} = \frac{1.5\ \text{V}}{2\ \text{k}\Omega} = 0.75 \times 10^{-3}\ \text{A}$
6. Check by superposition. With the 5 V source acting alone (current source open-circuited), the right-hand 2 kΩ carries no current and the source drives two 2 kΩ resistors in series: $I' = \frac{5}{4\ \text{k}\Omega} = 1.25\ \text{mA}$. With the 1 mA source acting alone (5 V source short-circuited), node A sees two equal 2 kΩ paths to ground, so each takes 0.5 mA. The one through the left resistor flows against the arrow for I, so $I'' = -0.5\ \text{mA}$.
7. Adding the two: $I = 1.25 - 0.5 = 0.75\ \text{mA} = 0.75 \times 10^{-3}\ \text{A}$, which confirms option (B).

KEY IDEA

A resistor in series with an ideal current source has no effect on the rest of the circuit, because the source fixes the branch current at 1 mA whatever the resistor's value. That leaves a single unknown node voltage. One KCL equation at node A gives $V_A = 3.5$ V, and $I = (5 - V_A)/2\ \text{k}\Omega$. Superposition gives the same answer: 1.25 mA from the voltage source minus 0.5 mA pushed back by the current source.

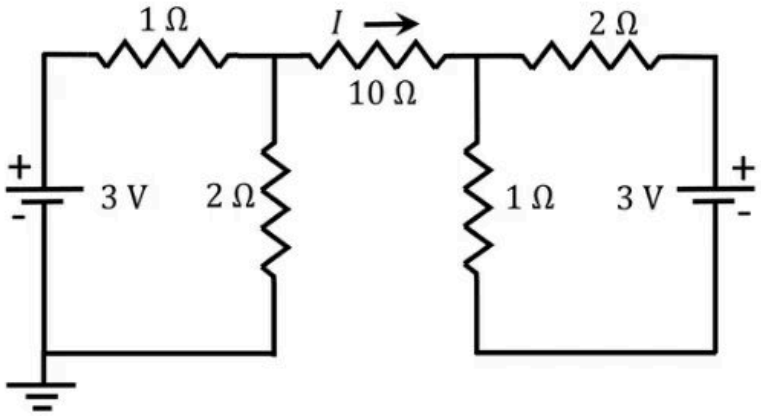
WATCH OUT FOR

Using only the 5 V source's contribution (5 V across 4 kΩ = 1.25 mA, option A), or only the current source's share (−0.5 mA, option C), instead of adding the two. Another mistake is putting the right-hand 2 kΩ in parallel with the middle resistor. It is in series with the current source, so it carries exactly 1 mA and can be ignored when finding V_A.

Q.14 MCQ · 1 MARK

Networks (Electric Circuits) › Laws & node/mesh

Q.14 Consider the circuit shown in the figure. The current I flowing through the $10\ \Omega$ resistor is _____.



(A)	1 A
(B)	0 A
(C)	0.1 A
(D)	-0.1 A

Official answer: **B**

SOLUTION

1. **Look at how the parts are connected.** The right loop (the $2\ \Omega$ resistor, the right-hand $3\ \text{V}$ source and the $1\ \Omega$ resistor) has its own bottom wire. That wire is not joined to the left bottom wire or to ground. The only thing linking the right loop to the rest of the circuit is the $10\ \Omega$ resistor.
2. **KCL on a closed surface drawn around the whole right loop.** Exactly one conductor crosses that surface: the $10\ \Omega$ branch. The net current leaving a closed surface must be zero, so $I = 0$.
3. **Check with the node voltages.** Take the left bottom wire as ground. With $I = 0$, the left loop is a plain series circuit: $i = \dfrac{3}{1+2} = 1\ \text{A}$, so $V_A = 1 \times 2 = 2\ \text{V}$.
4. In the right loop, $i' = \dfrac{3}{2+1} = 1\ \text{A}$, flowing out of the $+$ terminal, through the $2\ \Omega$ resistor into node B and down through the $1\ \Omega$ resistor. So node B sits $1 \times 1 = 1\ \text{V}$ above the right bottom wire. Because $I = 0$, there is no drop across the $10\ \Omega$ resistor, so $V_B = V_A = 2\ \text{V}$ and the floating right bottom wire sits at $1\ \text{V}$. Everything is consistent: the voltage across the $10\ \Omega$ resistor is 0.
5. Therefore $I = 0\ \text{A}$, which is option (B).

KEY IDEA

Draw a closed surface around a subcircuit that is joined to the rest of the network by only one wire. KCL says the net current crossing any closed surface is zero, so the current in that single wire must be zero. A current needs a complete loop, and here there is none. The voltages on each side don't matter.

WATCH OUT FOR

Assuming the two bottom wires are both ground. Students then work out the open-circuit node voltages (V_A = 2 V, V_B = 1 V relative to each bottom wire) and write I = (2 − 1)/10 = 0.1 A, which is option (C), or −0.1 A, option (D), if they flip the direction. This is wrong for two reasons. The right bottom wire is floating, with no common return path. And even if a common return existed, the source resistances would load the circuit and give 1/(10 + 4/3) ≈ 0.088 A, not 0.1 A.

Q.15 MCQ · 1 MARK

Signals and Systems › Fourier transform

The Fourier transform $X(j\omega)$ of the signal

$$x(t) = \frac{t}{(1+t^2)^2}$$

is _____.

(A) $\frac{\pi}{2j}\omega e^{-|\omega|}$

(B) $\frac{\pi}{2}\omega e^{-|\omega|}$

(C) $\frac{\pi}{2j}e^{-|\omega|}$

(D) $\frac{\pi}{2}e^{-|\omega|}$

Official answer: **A**

SOLUTION

1. Standard pair: $e^{-|t|} \longleftrightarrow \frac{2}{1+\omega^2}$, because $\int_{-\infty}^{\infty} e^{-|t|}e^{-j\omega t}\,dt = \frac{1}{1+j\omega} + \frac{1}{1-j\omega} = \frac{2}{1+\omega^2}$.
2. Duality ($g(t) \leftrightarrow G(j\omega) \Rightarrow G(t) \leftrightarrow 2\pi\, g(-\omega)$), with $g(t) = e^{-|t|}$ even: $\frac{2}{1+t^2} \leftrightarrow 2\pi e^{-|\omega|}$, so $\frac{1}{1+t^2} \leftrightarrow \pi e^{-|\omega|}$.
3. Write $x(t)$ as a derivative: $\frac{d}{dt}\left(\frac{1}{1+t^2}\right) = \frac{-2t}{(1+t^2)^2}$, so $x(t) = -\frac{1}{2}\frac{d}{dt}\left(\frac{1}{1+t^2}\right)$.
4. Time-differentiation property ($\frac{d}{dt} \leftrightarrow j\omega$):
$$X(j\omega) = -\frac{1}{2}(j\omega)\,\pi e^{-|\omega|} = -\frac{j\pi}{2}\omega e^{-|\omega|}$$
5. Since $\frac{1}{j} = -j$: $X(j\omega) = \frac{\pi}{2j}\omega e^{-|\omega|}$.
6. Symmetry check: $x(t)$ is real and odd, so $X(j\omega)$ must be purely imaginary and odd in ω. Only option (A) is both: (B) and (D) are real, and (C) is imaginary but even.
7. Answer: (A).

KEY IDEA

Write $x(t)$ as $-\frac{1}{2}\frac{d}{dt}\left(\frac{1}{1+t^2}\right)$. The transform of $\frac{1}{1+t^2}$ is $\pi e^{-|\omega|}$, by duality with $e^{-|t|} \leftrightarrow \frac{2}{1+\omega^2}$. Differentiating in time multiplies the transform by $j\omega$. A real, odd signal must have a purely imaginary, odd transform, and that check alone picks out (A).

WATCH OUT FOR

Getting a sign wrong when converting between j and $\frac{1}{j}$. Since $\frac{1}{j} = -j$, the result $-\frac{j\pi}{2}\omega e^{-|\omega|}$ is exactly $\frac{\pi}{2j}\omega e^{-|\omega|}$. If you drop the minus sign from the derivative ($\frac{d}{dt}\frac{1}{1+t^2} = \frac{-2t}{(1+t^2)^2}$), you get the negative of (A) and may still pick (A) for the wrong reason. Other slips: leaving out the 2π in duality (which changes the constant), or dropping the j and choosing the real option (B).

Q.16 MCQ · 1 MARK

Electronic Devices › Carrier transport

Consider a long rectangular bar of direct bandgap p-type semiconductor. The equilibrium hole density is $10^{17}\ \text{cm}^{-3}$ and the intrinsic carrier concentration is $10^{10}\ \text{cm}^{-3}$. Electron and hole diffusion lengths are $2\ \mu\text{m}$ and $1\ \mu\text{m}$, respectively.

The left side of the bar ($x = 0$) is uniformly illuminated with a laser having photon energy greater than the bandgap of the semiconductor. Excess electron-hole pairs are generated ONLY at $x = 0$ because of the laser. The steady state electron density at $x = 0$ is $10^{14}\ \text{cm}^{-3}$ due to laser illumination. Under these conditions and ignoring electric field, the closest approximation (among the given options) of the steady state electron density at $x = 2\ \mu\text{m}$, is _____.

(A) $0.37 \times 10^{14}\ \text{cm}^{-3}$

(B) $0.63 \times 10^{13}\ \text{cm}^{-3}$

(C) $3.7 \times 10^{14}\ \text{cm}^{-3}$

(D) $10^{3}\ \text{cm}^{-3}$

Official answer: **A**

SOLUTION

1. In the p-type bar, electrons are minority carriers, so their steady-state spatial profile decays with the electron (minority-carrier) diffusion length $L_n = 2\ \mu\text{m}$, not the hole diffusion length, which is irrelevant to the electron density.
2. With no electric field and no generation for $x > 0$, the steady-state minority-carrier diffusion equation reduces to $\dfrac{d^2\Delta n}{dx^2} = \dfrac{\Delta n}{L_n^2}$, whose physically bounded solution in a long bar is $\Delta n(x) = \Delta n(0)\, e^{-x/L_n}$.
3. Check the equilibrium (dark) electron density: $n_0 = \dfrac{n_i^2}{N_A} = \dfrac{(10^{10})^2}{10^{17}} = 10^{3}\ \text{cm}^{-3}$, which is negligible next to $10^{14}\ \text{cm}^{-3}$, so the given $n(0) = 10^{14}\ \text{cm}^{-3}$ is essentially all excess: $n(0) \approx \Delta n(0) = 10^{14}\ \text{cm}^{-3}$.
4. Evaluate at $x = 2\ \mu\text{m} = L_n$, so the exponent is exactly -1:
$$n(2\ \mu\text{m}) = 10^{14} \times e^{-1} = 10^{14} \times 0.3679 \approx 0.37 \times 10^{14}\ \text{cm}^{-3}$$
5. This matches option (A).

KEY IDEA

In a long p-type bar, electrons are minority carriers whose steady-state profile away from the injection point decays exponentially over the electron diffusion length L_n: $\Delta n(x) = \Delta n(0)\, e^{-x/L_n}$.

WATCH OUT FOR

Using the hole diffusion length (1 μm) instead of the electron diffusion length (2 μm) since holes are the majority carrier associated with "p-type," or forgetting to check that the equilibrium electron density $n_0 = n_i^2/N_A$ is negligible compared to the injected value (option D, $10^{3}\ \text{cm}^{-3}$, is exactly that distractor).

Q.17 MCQ · 1 MARK

Electronic Devices › Semiconductors

In a non-degenerate bulk semiconductor with electron density $n = 10^{16}\ \text{cm}^{-3}$, the value of $E_C - E_{Fn} = 200\ \text{meV}$, where E_C and E_{Fn} denote the bottom of the conduction band energy and electron Fermi level energy, respectively. Assume thermal voltage as 26 meV and the intrinsic carrier concentration is $10^{10}\ \text{cm}^{-3}$. For $n = 0.5 \times 10^{16}\ \text{cm}^{-3}$, the closest approximation of the value of $(E_C - E_{Fn})$, among the given options, is ___.

(A) 226 meV

(B) 174 meV

(C) 218 meV

(D) 182 meV

Official answer: **C**

SOLUTION

1. In non-degenerate (Boltzmann) statistics, the electron density is $n = N_C \exp\left(-\dfrac{E_C - E_{Fn}}{kT}\right)$, so $E_C - E_{Fn} = kT \ln(N_C/n)$.
2. For $n_1 = 10^{16}\ \text{cm}^{-3}$: $200\ \text{meV} = 26\ \text{meV} \times \ln(N_C/n_1)$, giving $\ln(N_C/n_1) = 200/26 = 7.6923$.
3. For $n_2 = 0.5 \times 10^{16}\ \text{cm}^{-3} = n_1/2$: $E_C - E_{Fn2} = 26\ \ln(N_C/n_2) = 26\,[\ln(N_C/n_1) + \ln(n_1/n_2)] = 26\,[\ln(N_C/n_1) + \ln 2]$.
4. Substituting: $E_C - E_{Fn2} = 200\ \text{meV} + 26\ \text{meV} \times \ln 2$.
5. $26 \times 0.6931 = 18.02\ \text{meV}$, so $E_C - E_{Fn2} \approx 218.0\ \text{meV}$.
6. This matches option (C), $218\ \text{meV}$. The intrinsic carrier concentration n_i given in the stem is not needed for this shift calculation; it is a distractor here.

KEY IDEA

In non-degenerate (Boltzmann) statistics, $n = N_C \exp\left(-\dfrac{E_C - E_{Fn}}{kT}\right)$, so $E_C - E_{Fn} = kT \ln(N_C/n)$. Since N_C and kT are unchanged between the two cases, going from n_1 to $n_2 = n_1/2$ simply adds $kT \ln 2$ to $E_C - E_{Fn}$. There is no need to solve for N_C at all.

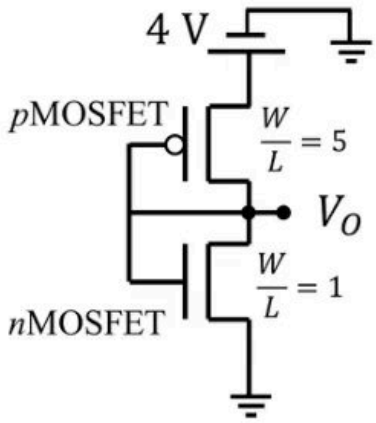
WATCH OUT FOR

Trying to use n_i and the given thermal voltage of 26 meV to compute $E_C - E_{Fn}$ from an intrinsic-level formula (mixing up E_i and E_C relations), or forgetting that halving n only shifts the Fermi level by $kT \ln 2$ relative to the first case; some solvers redo the full N_C calculation and introduce rounding error instead of using the simple shift.

Q.18 MCQ · 1 MARK

Digital Electronics › Logic families

Q.18 Consider the CMOS circuit shown in the figure (substrates are connected to their respective sources). The gate width (W) to gate length (L) ratios $\left(\frac{W}{L}\right)$ of the transistors are as shown. Both the transistors have the same gate oxide capacitance per unit area. For the pMOSFET, the threshold voltage is -1 V and the mobility of holes is $40\ \frac{\text{cm}^2}{\text{V.s}}$. For the nMOSFET, the threshold voltage is 1 V and the mobility of electrons is $300\ \frac{\text{cm}^2}{\text{V.s}}$. The steady state output voltage V_O is _____.



(A) equal to 0 V

(B) more than 2 V

(C) less than 2 V

(D) equal to 2 V

Official answer: **C**

SOLUTION

1. Read the figure. The two gates are tied together and wired to the common drain node, so $V_{GS,n} = V_O$ and $V_{SG,p} = 4 - V_O$. For each device the drain sits at its own gate: $V_{DS,n} = V_{GS,n}$ and $V_{SD,p} = V_{SG,p}$. Whenever a device conducts, $V_{DS} = V_{GS} > V_{GS} - V_T$, so **both transistors are in saturation** (each is diode-connected). Each substrate is tied to its source, so there is no body effect and the thresholds stay at $+1$ V and -1 V.
2. Both devices must be on. If $V_O \le 1$ V the nMOS is off while the pMOS conducts and charges the node upward, so V_O cannot stay there. If $V_O \ge 3$ V the pMOS is off while the nMOS discharges the node. So in steady state $1\text{ V} < V_O < 3\text{ V}$.
3. Saturation (square-law) current, ignoring channel-length modulation: $I_D = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{ov})^2$, where V_{ov} is the overdrive voltage $V_{GS} - V_T$ (for the pMOS, $V_{SG} - |V_{Tp}|$).

4. Strength factors (the common $\frac{1}{2}C_{ox}$ cancels, since both devices have the same oxide capacitance per unit area): $k_n \propto \mu_n(W/L)_n = 300 \times 1 = 300$ and $k_p \propto \mu_p(W/L)_p = 40 \times 5 = 200$.
5. In steady state the same current flows through both devices (nothing is connected to the output):
$$300\,(V_O - 1)^2 = 200\,(4 - V_O - 1)^2 = 200\,(3 - V_O)^2$$
6. Take the positive square root, since both overdrives are positive in the range $1 < V_O < 3$: $\sqrt{1.5}\,(V_O - 1) = 3 - V_O$, with $\sqrt{1.5} \approx 1.2247$.
7. $$V_O = \frac{3 + 1.2247}{1 + 1.2247} = \frac{4.2247}{2.2247} \approx 1.90\ \text{V}$$
8. Check: $V_{GS,n} = 1.90\ \text{V} > 1\ \text{V}$ and $V_{SG,p} = 2.10\ \text{V} > 1\ \text{V}$, so both devices are on and in saturation, as assumed. $V_O \approx 1.90\ \text{V}$, which is less than $2\ \text{V}$.
9. Quick sanity check: the thresholds are symmetric ($\pm 1\ \text{V}$), so V_O would be exactly $V_{DD}/2 = 2\ \text{V}$ only if $k_n = k_p$. Here $k_n > k_p$ (300 against 200), so the stronger nMOS pulls the node below 2 V. Answer (C).

KEY IDEA

With gate tied to drain, a MOSFET that conducts is always in saturation. Here both are, so equate the square-law currents $\mu_n(W/L)_n(V_O - V_{Tn})^2 = \mu_p(W/L)_p(V_{DD} - V_O - |V_{Tp}|)^2$. With symmetric thresholds, the node sits at $V_{DD}/2$ only when $\mu_n(W/L)_n = \mu_p(W/L)_p$. Otherwise the stronger device pulls it towards its own rail. Here $300 > 200$, so the nMOS wins and $V_O < 2\ \text{V}$.

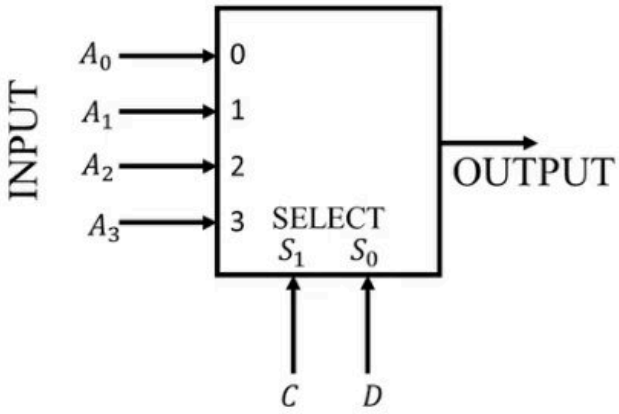
WATCH OUT FOR

Seeing $(W/L)_p = 5$ against $(W/L)_n = 1$ and concluding the pMOS is stronger, which gives (B). Device strength is $\mu \cdot W/L$, not W/L alone. The electron mobility is 7.5 times the hole mobility, so $k_n : k_p = 300 : 200$. Choosing (D) also comes from ignoring the mobilities, or from assuming a CMOS stack always settles at $V_{DD}/2$. Some students treat the circuit as an inverter with an unknown input and pick (A). The figure shorts gate to drain, so there is no free input.

Q.19 MCQ · 1 MARK

Digital Electronics › Combinational

Q.19 Consider the 2-bit multiplexer (MUX) shown in the figure. For OUTPUT to be the XOR of C and D, the values for A_0, A_1, A_2, and A_3 are _____.



(A) $A_0 = 0,\ A_1 = 0,\ A_2 = 1,\ A_3 = 1$

(B) $A_0 = 1,\ A_1 = 0,\ A_2 = 1,\ A_3 = 0$

(C) $A_0 = 0,\ A_1 = 1,\ A_2 = 1,\ A_3 = 0$

(D) $A_0 = 1,\ A_1 = 1,\ A_2 = 0,\ A_3 = 0$

Official answer: **C**

SOLUTION

1. **How a 4-to-1 MUX works:** the output copies the data input whose index is the binary number on the select lines, S_1S_0. Here $S_1 = C$ (the MSB) and $S_0 = D$ (the LSB), so $\text{OUTPUT} = A_{(CD)_2}$.
2. **Write out the output:**
$$\text{OUTPUT} = \overline{C}\,\overline{D}\,A_0 + \overline{C}\,D\,A_1 + C\,\overline{D}\,A_2 + C\,D\,A_3$$
(an overline means complement).
3. **The function we want:** $C \oplus D = \overline{C}\,D + C\,\overline{D}$, which is 1 only for minterms m_1 ($CD = 01$) and m_2 ($CD = 10$).
4. **Match the two expressions minterm by minterm:** each data input A_i appears only in minterm m_i, so it must equal the function's value there: $A_i = f(m_i)$.
5. $CD = 00 \Rightarrow$ input 0 is selected, and $0 \oplus 0 = 0$, so $A_0 = 0$.
6. $CD = 01 \Rightarrow$ input 1 is selected, and $0 \oplus 1 = 1$, so $A_1 = 1$.
7. $CD = 10 \Rightarrow$ input 2 is selected, and $1 \oplus 0 = 1$, so $A_2 = 1$.
8. $CD = 11 \Rightarrow$ input 3 is selected, and $1 \oplus 1 = 0$, so $A_3 = 0$.
9. So $(A_0, A_1, A_2, A_3) = (0, 1, 1, 0)$, which is option (C). Check: substituting into step 2 gives $\text{OUTPUT} = \overline{C}\,D + C\,\overline{D} = C \oplus D$.

KEY IDEA

When the function's variables drive a MUX's select lines, the data inputs are simply the function's truth-table column: $A_i = f(m_i)$, with m_i indexed by S_1S_0. For XOR that column is $0, 1, 1, 0$. XOR is symmetric in C and D, so the answer does not change even if you read S_0 as the MSB, because swapping the select lines only swaps A_1 and A_2, and both are 1.

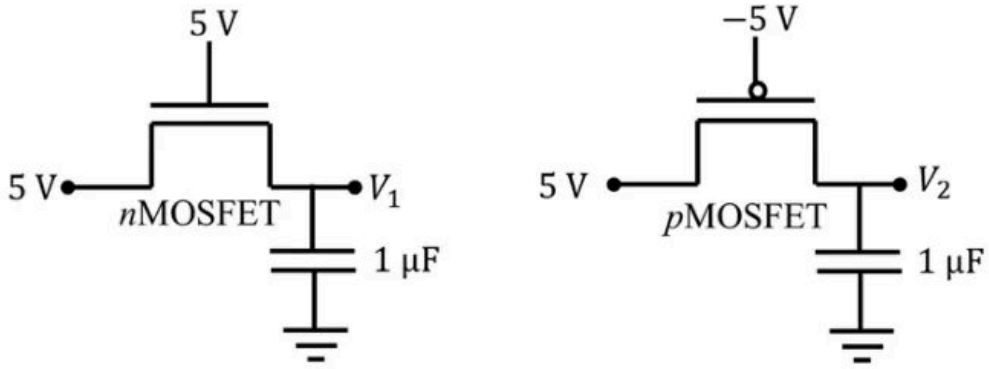
WATCH OUT FOR

Picking (A) (0, 0, 1, 1), which is just C (the MSB select line), or (B) (1, 0, 1, 0), which is $\overline{D}$. This happens when the truth table is written against one variable instead of against the combined index $(S_1S_0)_2$. Picking (D) means confusing XOR with its complement or with $\overline{C}$.

Q.20 MCQ · 1 MARK

Electronic Devices › MOS devices

Q.20 The ideal long channel nMOSFET and pMOSFET devices shown in the circuits have threshold voltages of 1 V and -1 V, respectively. The MOSFET substrates are connected to their respective sources. Ignore leakage currents and assume that the capacitors are initially discharged. For the applied voltages as shown, the steady state voltages are _____.



(A)	$V_1 = 5$ V, $V_2 = 5$ V
(B)	$V_1 = 5$ V, $V_2 = 4$ V
(C)	$V_1 = 4$ V, $V_2 = 5$ V
(D)	$V_1 = 4$ V, $V_2 = -5$ V

Official answer: **C**

SOLUTION

1. NMOS: gate and the left (fixed) terminal are both driven from constant sources; the fixed terminal sits at 5 V while the capacitor node V1 starts at 0 V, so the fixed terminal (higher potential) is the drain and V1 (lower potential) is the source, consistent with substrate tied to source.
2. NMOS conducts, charging the capacitor and raising V1, as long as $V_{GS} = V_G - V_1 = 5 - V_1 > V_{tn} = 1$ V.
3. Steady state for the NMOS is reached when $5 - V_1 = 1$ V $\Rightarrow V_1 = 4$ V; since 4 V < 5 V, the source/drain assignment used is self-consistent throughout the charging process.

4. PMOS: the left (fixed) terminal at 5 V is the highest potential in that circuit (above the gate at -5 V and above V2, which starts at 0 V and can only rise), so it is the source, consistent with substrate tied to source for a pMOS.
5. $V_{SG} = V_S - V_G = 5 - (-5) = 10\ \text{V}$, fixed entirely by the two constant supplies and independent of V2.
6. Since $V_{SG} = 10\ \text{V} \gg |V_{tp}| = 1\ \text{V}$ at all times, the PMOS never turns off due to gate overdrive; it keeps conducting until the drain-source voltage itself vanishes, i.e. until $V_{SD} \to 0$.
7. Steady state for the PMOS: $V_2 = V_S = 5\ \text{V}$.
8. Combined result: $V_1 = 4\ \text{V}$, $V_2 = 5\ \text{V}$, matching option (C).

KEY IDEA

For a MOSFET whose substrate is tied to its own source, the terminal acting as "source" is the one with the lower potential for an nMOS (higher potential for a pMOS), and it is defined by the instantaneous circuit state, not by the schematic labels. Conduction (and hence capacitor charging) continues only as long as $|V_{GS}| > |V_t|$ measured with the source taken this way; steady state is reached when either that inequality just fails (channel pinches off) or the drain-source voltage itself goes to zero (no further current can flow into the capacitor).

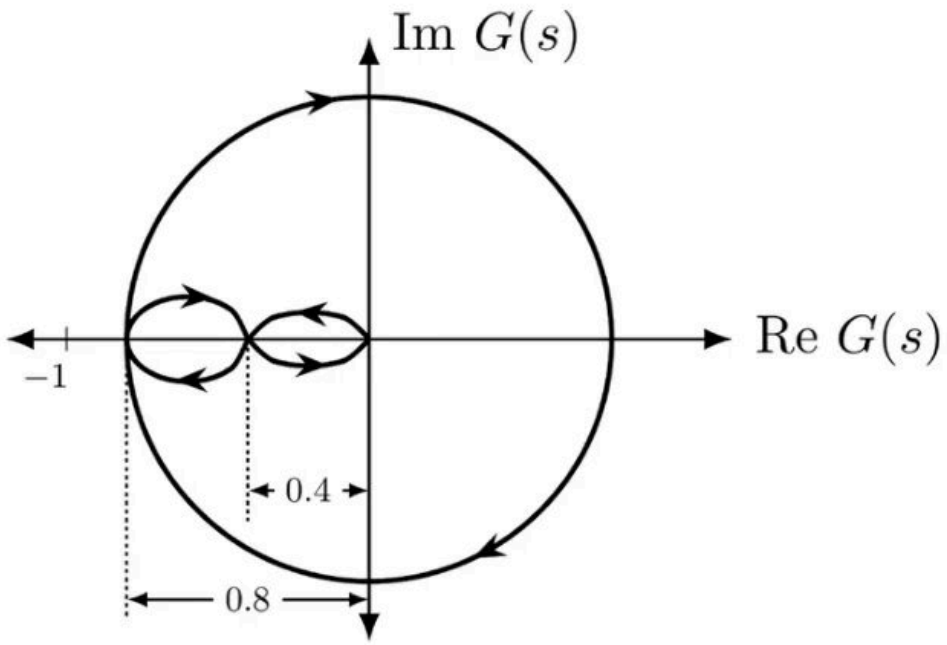
WATCH OUT FOR

Assuming the PMOS turns off once V2 reaches some value related to its own drain voltage (e.g. stopping at 4 V by analogy with the NMOS), instead of recognizing that V_SG = 10 V is fixed by the two constant supplies and always far exceeds |V_tp| = 1 V, so the PMOS keeps conducting until the capacitor is fully charged to the source rail (V2 = 5 V), not until some gate-threshold balance is reached.

Q.21 MCQ · 1 MARK

Control Systems › Nyquist

Q.21 Consider a closed-loop control system with unity negative feedback and $KG(s)$ in the forward path, where the gain $K = 2$. The complete Nyquist plot of the transfer function $G(s)$ is shown in the figure. Note that the Nyquist contour has been chosen to have the clockwise sense. Assume $G(s)$ has no poles on the closed right-half of the complex plane. The number of poles of the closed-loop transfer function in the closed right-half of the complex plane is _____.



(A)	0
(B)	1
(C)	2
(D)	3

Official answer: **C**

SOLUTION

1. **Critical point.** The closed-loop poles are the roots of $1 + KG(s) = 0$, that is $G(s) = -\frac{1}{K} = -\frac{1}{2} = -0.5$. So we count how many times the plot of G encircles -0.5. This is the same as counting encirclements of -1 by the plot of KG. The real-axis crossings of KG are $2(-0.8) = -1.6$ and $2(-0.4) = -0.8$, and these lie on either side of -1.
2. **Nyquist criterion with a clockwise contour.** $Z = P + N$. Here Z is the number of closed-loop poles inside the contour, P is the number of open-loop poles of $KG(s)$ inside it, and N is the net number of **clockwise** encirclements of the critical point.

3. **Open-loop poles.** The question says $G(s)$ has no poles in the closed right-half plane, so $P = 0$ and the contour needs no indentations.
4. **Nothing on the imaginary axis.** The plot meets the real axis only at -0.8, -0.4, 0 and the right-hand crossing of the large curve. It never passes through -0.5, so $1 + KG(j\omega) \neq 0$ for every ω. There are no closed-loop poles on the $j\omega$-axis, and Z is the full count for the closed right-half plane.
5. **Large outer curve.** Its leftmost point is at -0.8 and it goes around the origin, so it encloses -0.5. The arrows show it runs clockwise: the top arc moves right and the lower-right arc moves down and to the left. Contribution: $+1$ clockwise.
6. **Left lobe of the figure-eight.** It runs from -0.8 to -0.4 along the real axis, so -0.5 is inside it. Its upper arc moves right and its lower arc moves left, so it runs clockwise. Contribution: $+1$ clockwise.
7. **Right lobe.** It runs from -0.4 to 0, so it does not contain -0.5. It contributes 0 whichever way it is traversed. (It runs counter-clockwise.)
8. **Check with a ray.** Draw a vertical ray upward from -0.5. It crosses exactly two branches: the upper arc of the left lobe and the upper-left part of the large curve. Both move left to right where they cross, so above the point that is two clockwise crossings. The right lobe never reaches $\mathrm{Re}\, G = -0.5$.
9. **Result.**

$$N = 1 + 1 = 2, \qquad Z = P + N = 0 + 2 = 2$$

The closed-loop system has **2 poles in the closed right-half plane**, so it is unstable. Answer: **(C)**.

KEY IDEA

When the plot shown is of $G(s)$ but the loop gain is $KG(s)$, shift the critical point rather than rescaling the plot. Count clockwise encirclements of $-1/K$ (here -0.5). Then take the direction of every loop from its own arrows and apply $Z = P + N$ for a clockwise contour. Only loops that actually contain the critical point contribute to N.

WATCH OUT FOR

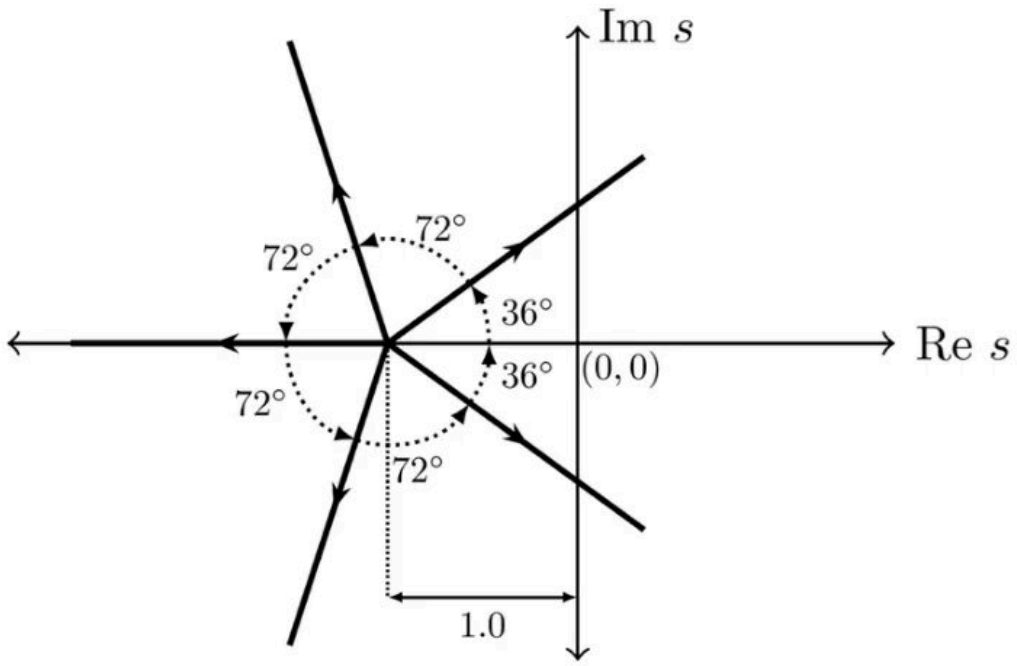
Counting encirclements of -1 instead of $-1/K = -0.5$. Nothing in the figure encloses -1, so this gives $N = 0$ and the wrong answer (A). A second error is treating the figure-eight as one loop, or letting its two lobes cancel. The counter-clockwise right lobe does not contain -0.5, so it cancels nothing, and the clockwise left lobe adds one encirclement to the outer circle's one.

Q.22 MCQ · 1 MARK

Control Systems › Root locus

Q.22 The root-locus plot of a closed-loop system with unity negative feedback and transfer function $KG(s)$ in the forward path is shown in the figure. Note that K is varied from 0 to ∞.

Select the transfer function $G(s)$ that results in the root-locus plot of the closed-loop system as shown in the figure.



(A) $G(s) = \dfrac{1}{(s+1)^5}$

(B) $G(s) = \dfrac{1}{s^5+1}$

(C) $G(s) = \dfrac{s-1}{(s+1)^6}$

(D) $G(s) = \dfrac{s+1}{s^6+1}$

Official answer: **A**

SOLUTION

1. **Read the plot.** Every root-locus branch starts at an open-loop pole (at $K = 0$), and there is one branch for each pole. The figure has five branches, all leaving the single point $s = -1$. So $G(s)$ has $n = 5$ poles, and all five are at $s = -1$: a pole of order 5. No branch ends at a finite point, so there is no finite zero and $m = 0$.

2. **Count the asymptotes.** The five straight branches are $72^\circ = \dfrac{360^\circ}{5}$ apart, at $\pm 36^\circ,\ \pm 108^\circ,\ 180^\circ$. The asymptote-angle rule $\theta_k = \dfrac{(2k+1)180^\circ}{n-m}$ with $n - m = 5$ gives exactly these angles for $k = 0, 1, 2, 3, 4$.
3. **Centroid check.** For option A, $\sigma_A = \dfrac{\sum p - \sum z}{n-m} = \dfrac{5(-1) - 0}{5} = -1$. That is the point where the branches meet.
4. **Real-axis rule.** For $(s+1)^5$, any real point to the left of -1 has 5 poles to its right. That is an odd count, so the point is on the locus. A point between -1 and 0 has no poles or zeros to its right, so it is not. This matches the figure, where the only real-axis branch runs left from -1.
5. **Check the branches exactly.**

   $$1 + \frac{K}{(s+1)^5} = 0 \ \Rightarrow\ (s+1)^5 = -K = K\, e^{j(2k+1)180^\circ} \ \Rightarrow\ s = -1 + K^{1/5}\, e^{j(2k+1)36^\circ}, \quad k = 0, 1, 2, 3, 4$$

   As K goes from 0 to ∞, these are five straight rays from -1 at $36^\circ, 108^\circ, 180^\circ, 252^\circ, 324^\circ$. That is exactly the plot, so A is correct.
6. **Rule out B.** The poles of $s^5 + 1$ are $e^{j(2k+1)36^\circ}$, five points spread around the unit circle centred on the origin. Only one of them is at -1. The five branches would start from five different points, not from one point.
7. **Rule out C and D.** Both have a sixth-degree denominator, so $n = 6$ and there would be six branches, but the figure shows five. There are further mismatches. In C the centroid is $\sigma_A = \dfrac{6(-1) - (+1)}{5} = -1.4 \neq -1$, and the zero at $+1$ would put the segment $(-1,\ 1)$ of the real axis on the locus, with a branch ending at $s = +1$. In D the poles of $s^6 + 1$ are spread around the unit circle, and the zero at -1 means a branch would *end* at -1 rather than leave it. The figure shows none of these.

KEY IDEA

If every open-loop pole is stacked at one point and there are no zeros, $1 + \dfrac{K}{(s+a)^n} = 0$ becomes $(s+a)^n = -K$. The whole locus is then n straight rays from $s = -a$ at angles $\dfrac{(2k+1)180^\circ}{n}$. Count the rays to get n, and read the common point to get a.

WATCH OUT FOR

Picking (B). The poles of $s^5 + 1$ lie at exactly the angles $\pm 36^\circ, \pm 108^\circ, 180^\circ$, the same angles as the rays in the figure, which makes it look right. But those are five separate starting points on the unit circle around the origin, not five directions leaving $s = -1$. The branches of (B) would start at five different places, while every branch in the figure starts at -1. A second slip is to overlook that (C) and (D) have six poles, which means six branches, when the figure shows five.

Q.23 MCQ · 1 MARK

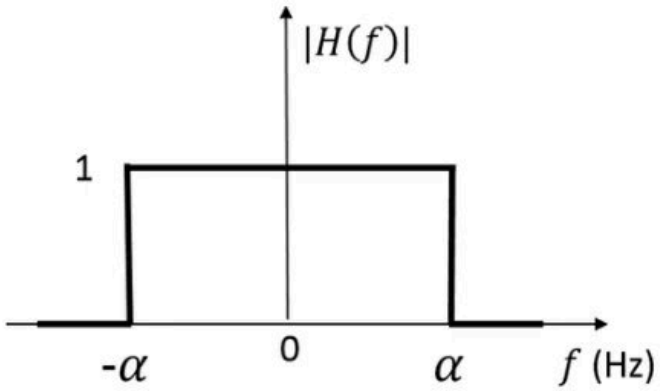
Signals and Systems › Fourier transform

Q.23 The frequency response $H(f)$ of a linear time-invariant system has magnitude as shown in the figure.

Statement I: The system is necessarily a pure delay system for inputs which are bandlimited to $-\alpha \le f \le \alpha$.

Statement II: For any wide-sense stationary input process with power spectral density $S_X(f)$, the output power spectral density $S_Y(f)$ obeys $S_Y(f) = S_X(f)$ for $-\alpha \le f \le \alpha$.

Which one of the following combinations is true?



(A) Statement I is correct, Statement II is correct

(B) Statement I is correct, Statement II is incorrect

(C) Statement I is incorrect, Statement II is correct

(D) Statement I is incorrect, Statement II is incorrect

Official answer: **C**

SOLUTION

1. **What the figure gives.** Only the magnitude is specified: $|H(f)| = 1$ for $|f| \le \alpha$ and 0 outside. The phase $\theta(f) = \angle H(f)$ is not given. So inside the band the response can be $H(f) = e^{j\theta(f)}$ with any phase function $\theta(f)$.
2. **What a pure delay needs.** A pure delay $y(t) = x(t - t_0)$ means $Y(f) = X(f)\, e^{-j2\pi f t_0}$. For every input bandlimited to $|f| \le \alpha$ this requires $H(f) = e^{-j2\pi f t_0}$ across the band: unit magnitude **and** a phase that is linear in f and passes through the origin, $\theta(f) = -2\pi f t_0$. In particular $H(0) = 1$.
3. **Counter-example for Statement I.** Take $H(f) = -1$ for $|f| \le \alpha$ and 0 outside, i.e. $h(t) = -2\alpha\, \mathrm{sinc}(2\alpha t)$, a real impulse response. Its magnitude is exactly the figure. For any input bandlimited to $|f| \le \alpha$ the output is $y(t) = -x(t)$. A pure delay would need $H(0) = e^0 = 1$, but here $H(0) = -1$. Concretely, the constant input $x(t) = 1$ (spectrum $\delta(f)$, inside the band) gives $y(t) = -1$, while $x(t -$

$t_0) = 1$ for every t_0. Any other phase that is not linear through the origin fails in the same way (a Hilbert-type phase $-\frac{\pi}{2}\,\mathrm{sgn}(f)$, for example). The magnitude alone cannot force a pure delay, so **Statement I is incorrect**.

4. **PSD through an LTI system.** For a WSS input, $R_Y(\tau) = h(\tau) * h^*(-\tau) * R_X(\tau)$. Taking Fourier transforms (the PSD is the Fourier transform of the autocorrelation) gives
$$S_Y(f) = H(f)\,H^*(f)\,S_X(f) = |H(f)|^2\,S_X(f).$$
Only $|H(f)|$ appears, so the phase has no effect.
5. **Apply it to the band.** For $-\alpha \le f \le \alpha$ we have $|H(f)|^2 = 1$, so $S_Y(f) = S_X(f)$ for every WSS input, whatever the phase is. (In the counter-example, $y = -x$ gives $R_Y(\tau) = R_X(\tau)$, which agrees.) **Statement II is correct.**
6. Statement I is incorrect and Statement II is correct, so the answer is option **(C)**.

KEY IDEA

For a WSS input, the output power spectrum depends only on $|H(f)|^2$, so a flat unit magnitude over $|f| \le \alpha$ passes $S_X(f)$ through unchanged. Keeping the waveform intact (a pure delay) needs more than that: the phase must also be linear through the origin, $\angle H(f) = -2\pi f t_0$, and a magnitude plot says nothing about phase.

WATCH OUT FOR

Reading a flat unit-magnitude band as "distortionless" and picking (A). This forgets that a pure delay also needs linear phase through the origin: $H(f) = -1$ or a Hilbert-type phase has the same magnitude and is not a delay. A second slip is writing $S_Y = H(f)\,S_X(f)$ instead of $|H(f)|^2 S_X(f)$ and then worrying that the unknown phase makes Statement II false.

Q.24 MCQ · 1 MARK

Electromagnetics › Maxwell's equations

In a circuit, there is a series connection of an ideal resistor and an ideal capacitor. The conduction current (in Amperes) through the resistor is $2\sin(t + \pi/2)$. The displacement current (in Amperes) through the capacitor is _____.

(A) $2\sin(t)$

(B) $2\sin(t + \pi)$

(C) $2\sin(t + \pi/2)$

(D) 0

Official answer: **C**

SOLUTION

1. No charge crosses the gap between the plates of an ideal capacitor, so the current "through" the capacitor is entirely displacement current:
$$i_d = \frac{d}{dt}\int_S \mathbf{D}\cdot d\mathbf{S},$$
where S is a surface lying in the gap.
2. Continuity of total current: take the divergence of the Ampère–Maxwell law $\nabla\times\mathbf{H} = \mathbf{J}_c + \dfrac{\partial \mathbf{D}}{\partial t}$. Since $\nabla\cdot(\nabla\times\mathbf{H}) = 0$, we get
$$\nabla\cdot\left(\mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t}\right) = 0.$$
So the total current (conduction plus displacement) is the same through every cross-section of a single unbranched loop.
3. In the series loop the resistor and the wires carry only conduction current, and the capacitor gap carries only displacement current. Series elements carry the same total current at every instant, so
$$i_d(t) = i_c(t).$$

4. Check at one capacitor plate. If the plate holds charge q, the wire's conduction current is $i_c = \dfrac{dq}{dt}$. By Gauss's law, the flux of $\mathbf{D}$ out of that plate into the gap is $\int_S \mathbf{D}\cdot d\mathbf{S} = q$ (for a parallel-plate capacitor, $D = q/A$). Hence $i_d = \dfrac{d}{dt}\int_S \mathbf{D}\cdot d\mathbf{S} = \dfrac{dq}{dt} = i_c$, and in circuit terms both equal $C\dfrac{dv_C}{dt}$.
5. The equality holds at every instant, so amplitude and phase are both unchanged:

$$i_d = 2\sin(t + \pi/2)\ \text{A}.$$

This is option (C).

KEY IDEA

Maxwell added displacement current so that total current is continuous: $\nabla\cdot(\mathbf{J}_c + \partial\mathbf{D}/\partial t) = 0$. In a series loop the conduction current in the wire and resistor carries on across the capacitor gap as displacement current $d\Psi_D/dt = dq/dt$. The two are equal at every instant, in both amplitude and phase.

WATCH OUT FOR

Picking (D) because "no charge flows through the dielectric". That is true of conduction current, but it is exactly why the gap current is all displacement current, and that current is not zero. Another mistake is picking (A) or (B) by carrying over the 90° voltage–current phase shift of a capacitor. That shift is between the capacitor's voltage and its current. The resistor current and the capacitor current in a series loop are the same current and have no phase difference.

Q.25 MSQ · 1 MARK

Engineering Mathematics › Differential equations

Consider the following partial differential equation (PDE)

$$a\frac{\partial^2 f(x,y)}{\partial x^2} + b\frac{\partial^2 f(x,y)}{\partial y^2} = f(x,y),$$

where a and b are distinct positive real numbers. Select the combination(s) of values of the real parameters ξ and η such that $f(x,y) = e^{(\xi x+\ )}$ is a solution of the given PDE.

[As printed, the second term of the exponent is blank in the paper. Since the options give values for η, the intended function is $f(x,y) = e^{(\xi x+\eta y)}$.]

(A) $\xi = \dfrac{1}{\sqrt{2a}},\ \eta = \dfrac{1}{\sqrt{2b}}$

(B) $\xi = \dfrac{1}{\sqrt{a}},\ \eta = 0$

(C) $\xi = 0,\ \eta = 0$

(D) $\xi = \dfrac{1}{\sqrt{a}},\ \eta = \dfrac{1}{\sqrt{b}}$

Official answer: **A, B**

SOLUTION

1. Try the exponential solution $f = e^{\xi x+\eta y}$. The blank in the printed exponent has to be ηy, because the options give values of η. Each derivative brings down a constant factor: $\dfrac{\partial f}{\partial x} = \xi f$, $\dfrac{\partial^2 f}{\partial x^2} = \xi^2 f$ and $\dfrac{\partial^2 f}{\partial y^2} = \eta^2 f$.
2. Substitute these into the PDE: $a\xi^2 f + b\eta^2 f = f$. Since $e^{\xi x+\eta y} \neq 0$ for every x and y, we can divide by f. This gives the condition

$$a\xi^2 + b\eta^2 = 1.$$

3. (A): $a\cdot\dfrac{1}{2a} + b\cdot\dfrac{1}{2b} = \dfrac{1}{2} + \dfrac{1}{2} = 1$. The condition holds, so (A) is correct.
4. (B): $a\cdot\dfrac{1}{a} + b\cdot 0 = 1$. The condition holds, so (B) is correct.

5. (C): $a \cdot 0 + b \cdot 0 = 0 \neq 1$, so it fails. Here $f = e^0 = 1$, which makes the left side 0 while the right side is 1.
6. (D): $a \cdot \frac{1}{a} + b \cdot \frac{1}{b} = 1 + 1 = 2 \neq 1$, so it fails.
7. The correct options are therefore (A) and (B). In general, any pair (ξ, η) on the ellipse $a\xi^2 + b\eta^2 = 1$ gives a solution. It is an ellipse because $a, b > 0$.

KEY IDEA

With an exponential trial solution $e^{\xi x + \eta y}$, each second partial derivative becomes a multiplication: by ξ^2 for x and by η^2 for y. The PDE then reduces to one algebraic condition, $a\xi^2 + b\eta^2 = 1$, and each option is checked against it.

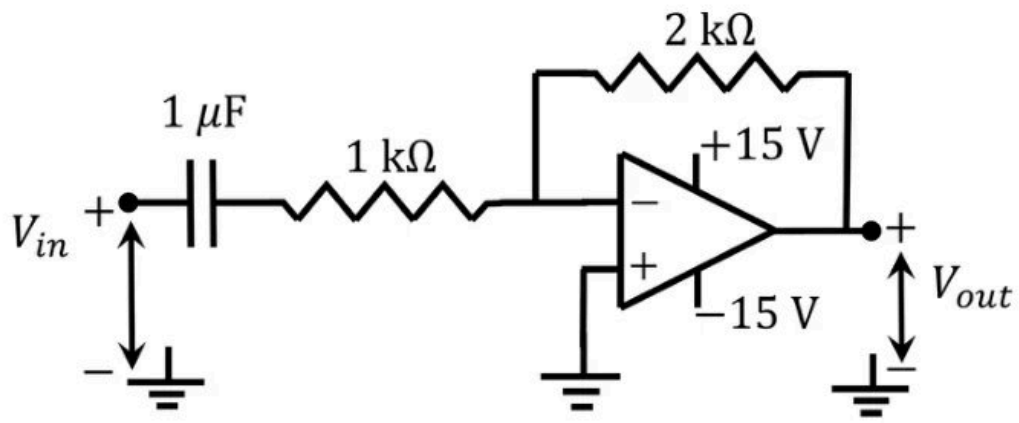
WATCH OUT FOR

One mistake is to pick (D) by requiring each term to match f on its own ($a\xi^2 = 1$ and $b\eta^2 = 1$). Only the sum has to equal 1, and (D) makes it 2. Another is to pick (C) because a zero exponent looks like the "trivial" solution. But $e^0 = 1$, not the zero function, and the constant 1 does not satisfy $0 = 1$.

Q.26 MSQ · 1 MARK

Analog Electronics › Filters & waveforms

Q.26 An ideal OPAMP circuit with a sinusoidal input is shown in the figure. The 3 dB frequency is the frequency at which the magnitude of the voltage gain decreases by 3 dB from the maximum value. Which of the options is/are correct?



(A)	The circuit is a low pass filter.
(B)	The circuit is a high pass filter.
(C)	The 3 dB frequency is 1000 rad/s.
(D)	The 3 dB frequency is $\frac{1000}{3}$ rad/s.

Official answer: **B, C**

SOLUTION

1. Assume an ideal op-amp working in its linear region (the input amplitude is small enough that the output stays inside the ± 15 V supplies). The inputs are then virtually shorted, and because the $+$ input is grounded, the $-$ input sits at 0 V (virtual ground).

2. Input impedance: the capacitor and the $1\ \text{k}\Omega$ resistor are in series, so $Z_1 = R_1 + \dfrac{1}{j\omega C}$ with $R_1 = 1\ \text{k}\Omega$ and $C = 1\ \mu\text{F}$. The feedback impedance is $Z_f = R_f = 2\ \text{k}\Omega$.
3. Inverting-amplifier gain:
$$\frac{V_{out}}{V_{in}} = -\frac{Z_f}{Z_1} = -\frac{R_f}{R_1 + \frac{1}{j\omega C}} = -\frac{R_f}{R_1} \cdot \frac{j\omega R_1 C}{1 + j\omega R_1 C}$$
4. Limiting cases: as $\omega \to 0$ the capacitor acts as an open circuit and $|A| \to 0$. As $\omega \to \infty$ it acts as a short and $|A| \to R_f/R_1 = 2$. The gain rises from zero to a constant maximum, so the circuit is a first-order high-pass filter. (B) is correct and (A) is wrong.
5. Magnitude: $|A(j\omega)| = 2\,\dfrac{\omega R_1 C}{\sqrt{1 + (\omega R_1 C)^2}}$. The maximum is 2. The gain is 3 dB below that maximum when $|A| = 2/\sqrt{2}$, which happens when $\omega R_1 C = 1$.
6. 3 dB frequency:
$$\omega_{3\text{dB}} = \frac{1}{R_1 C} = \frac{1}{(10^3)(10^{-6})} = 1000\ \text{rad/s}$$
So (C) is correct. R_f sets only the passband gain and has no effect on the corner, so the $1000/3\ \text{rad/s}$ in (D) is wrong.

KEY IDEA

Because of the virtual ground at the inverting input, the series R_1–C branch sees V_{in} directly, so the gain is $-Z_f/Z_1$. That gives a single pole at $1/(R_1C)$ and a zero at the origin, which is a high-pass response. The feedback resistor only scales the gain: it sets the passband level R_f/R_1 and does not move the corner.

WATCH OUT FOR

Option (D) comes from putting the total resistance $R_1 + R_f = 3\ \text{k}\Omega$ into the time constant, which gives $1/(3 \times 10^{-3}) = 1000/3\ \text{rad/s}$. The virtual ground isolates R_f from the input branch, so only R_1 and C set the corner. Another mistake is to see a capacitor and call the circuit a low-pass filter. A series input capacitor blocks low frequencies. Low-pass behaviour would need the capacitor in parallel with R_f.

Q.27 MSQ · 1 MARK

Digital Electronics › Boolean logic

Select the Boolean function(s) equivalent to $x + yz$, where x, y, and z are Boolean variables, and $+$ denotes logical OR operation.

(A) $x + z + xy$

(B) $(x + y)(x + z)$

(C) $x + xy + yz$

(D) $x + xz + xy$

Official answer: **B, C**

SOLUTION

1. Target function: $F = x + yz$. It is 1 when $x = 1$, or when $y = z = 1$. (No complements appear anywhere in this question.)
2. (A) Absorption ($x + xy = x$) gives $x + z + xy = x + z$. Test $x = 0, y = 0, z = 1$: $x + z = 1$, but $x + yz = 0 + 0 = 0$. **Not equivalent.**
3. (B) Multiply out (AND distributes over OR) and use idempotence $x \cdot x = x$:
$$(x + y)(x + z) = xx + xz + xy + yz = x + xz + xy + yz = x(1 + z + y) + yz = x + yz$$
since $1 + z + y = 1$. This is exactly the distributive law of OR over AND, $x + yz = (x + y)(x + z)$. **Equivalent.**
4. (C) Absorption: $x + xy = x(1 + y) = x$, so $x + xy + yz = x + yz$. **Equivalent.**

5. (D) Absorption: $x + xz + xy = x(1 + z + y) = x$. Test $x = 0, y = 1, z = 1$: the option gives 0, but $x + yz = 0 + 1 = 1$. **Not equivalent.**
6. Only (B) and (C) reduce to $x + yz$, so the correct options are **B and C**.

KEY IDEA

Reduce each option with absorption ($x + xy = x$) and the OR-over-AND distributive law ($x + yz = (x + y)(x + z)$). To prove an option is NOT equivalent, give one input where it differs from $x + yz$.

WATCH OUT FOR

Rejecting (B) because it looks like a product of sums rather than a sum of products. In Boolean algebra, unlike ordinary algebra, OR distributes over AND, so $(x + y)(x + z) = x + yz$. A second slip: in (C), dropping the yz term along with xy when applying absorption. Only xy is absorbed by x.

Q.28 MSQ · 1 MARK

Digital Electronics › Logic families

Select the correct statement(s) regarding CMOS implementation of NOT gates.

(A) Noise Margin High (NM_H) is always equal to the Noise Margin Low (NM_L), irrespective of the sizing of transistors.

(B) Dynamic power consumption during switching is zero.

(C) For a logical high input under steady state, the nMOSFET is in the linear regime of operation.

(D) Mobility of electrons never influences the switching speed of the NOT gate.

Official answer: **C**

SOLUTION

1. **(A) Noise margins depend on sizing.** By definition $NM_H = V_{OH} - V_{IH}$ and $NM_L = V_{IL} - V_{OL}$. The switching threshold V_M of a CMOS inverter is set by the ratio $\frac{\beta_n}{\beta_p} = \frac{\mu_n (W/L)_n}{\mu_p (W/L)_p}$. Only a symmetric inverter ($\beta_n = \beta_p$, $V_{Tn} = |V_{Tp}|$) has $V_M = V_{DD}/2$ and $NM_H = NM_L$. Change the sizing and V_M shifts, so one margin grows and the other shrinks. The word "always" makes (A) **false**.
2. **(B) Dynamic power is not zero.** Every time the output switches, the load capacitance is charged through the pMOS and discharged through the nMOS:
$$P_{dyn} = \alpha C_L V_{DD}^2 f$$
There is also a short-circuit current while both transistors conduct briefly during the transition. Only the *static* power is ideally zero, so (B) is **false**.
3. **(C) Input HIGH, steady state.** With $V_{in} = V_{DD}$: $V_{SGp} = V_{DD} - V_{DD} = 0 < |V_{Tp}|$, so the pMOS is OFF. For the nMOS, $V_{GSn} = V_{DD} > V_{Tn}$, so it is ON. With the pMOS off there is no path to supply current, so in steady state the output settles at $V_{out} = 0$, giving $V_{DSn} = 0$.
4. Check the region: $V_{DSn} = 0 < V_{GSn} - V_{Tn} = V_{DD} - V_{Tn}$, so the nMOSFET is in the linear (triode) region, acting as a small ON-resistance that holds the output at ground. (C) is **true**.
5. **(D) Electron mobility sets the pull-down speed.** The nMOS drain current is $I_{Dn} \propto \mu_n C_{ox} (W/L)_n$, and the high-to-low propagation delay is $t_{pHL} \approx \frac{C_L \Delta V}{I_{Dn,\text{avg}}}$. A larger μ_n gives a faster discharge, so electron mobility does affect switching speed. (This is also why the pMOS is usually made about 2 to 3 times wider: $\mu_p < \mu_n$.) (D) is **false**.
6. Only statement (C) is correct.

KEY IDEA

A CMOS inverter draws ideally zero *static* current, but it still uses dynamic power $\alpha C_L V_{DD}^2 f$ whenever it switches. The ON transistor passes no current in steady state, so its V_{DS} is zero and it sits in the linear (triode) region, not saturation. Noise margins and switching speed both depend on $\mu_n(W/L)_n$ versus $\mu_p(W/L)_p$. That is why sizing matters, and why electron mobility affects the pull-down delay.

WATCH OUT FOR

Students often mark (B) because they mix up static power (ideally zero) with dynamic switching power (never zero). Another trap is to think the ON nMOS is in saturation because V_{GS} is large. Saturation needs $V_{DS} \geq V_{GS} - V_{Tn}$, and here $V_{DS} = 0$. Some also mark (A) because they remember the symmetric-inverter case, but the word "always" together with "irrespective of sizing" makes it false.

Q.29 MSQ · 1 MARK

Communications › Information theory

Let $H(X)$ denote the entropy of a discrete random variable X taking K possible distinct real values. Which of the following statements is/are necessarily true?

(A) $H(X) \leq \log_2 K$ bits

(B) $H(X) \leq H(2X)$

(C) $H(X) \leq H(X^2)$

(D) $H(X) \leq H(2^X)$

Official answer: **A, B, D**

SOLUTION

1. Entropy depends only on the probability masses $p_1, \ldots, p_K$ of X, not on the values it takes. If g is one-to-one on the values of X, then $g(X)$ has the same set of masses, so $H(g(X)) = H(X)$. If g is many-to-one, some values merge and their masses add, so $H(g(X)) \leq H(X)$.
2. (A) By Jensen's inequality for the concave function $\log_2$: $H(X) = \sum_i p_i \log_2 \dfrac{1}{p_i} \leq \log_2 \left(\sum_i p_i \cdot \dfrac{1}{p_i} \right) = \log_2 K$, with equality for the uniform distribution. So $H(X) \leq \log_2 K$ bits always. True.
3. (B) $Y = 2X$ is one-to-one on the real line, so the masses are unchanged and $H(2X) = H(X)$. Hence $H(X) \leq H(2X)$ holds (with equality). True.
4. (C) $Y = X^2$ is many-to-one when X takes both a and $-a$. Take $X \in \{-1, +1\}$ with probability $\frac{1}{2}$ each: $H(X) = 1$ bit, but $X^2 = 1$ with certainty, so $H(X^2) = 0$. Then $H(X) \leq H(X^2)$ fails, so the statement is not necessarily true.
5. (D) $Y = 2^X$ is strictly increasing, hence one-to-one, so $H(2^X) = H(X)$ and $H(X) \leq H(2^X)$ holds. True.
6. Statements that are necessarily true: A, B and D.

KEY IDEA

Entropy depends only on the probability masses. A one-to-one function of X leaves H unchanged, a many-to-one function can only lower it, and $H(X) \leq \log_2 K$ for K outcomes.

WATCH OUT FOR

Treating X^2 like $2X$ or 2^X. Squaring merges the values a and $-a$, so $H(X^2)$ can be smaller than $H(X)$. The inequality $H(X) \leq H(X^2)$ is therefore not guaranteed.

Q.30 MSQ · 1 MARK

Engineering Mathematics › Differential equations

Consider the following wave equation,

$$\frac{\partial^2 f(x,t)}{\partial t^2} = 10000 \frac{\partial^2 f(x,t)}{\partial x^2}$$

Which of the given options is/are solution(s) to the given wave equation?

(A) $f(x,t) = e^{-(x-100t)^2} + e^{-(x+100t)^2}$

(B) $f(x,t) = e^{-(x-100t)} + 0.5e^{-(x+1000t)}$

(C) $f(x,t) = e^{-(x-100t)} + \sin(x+100t)$

(D) $f(x,t) = e^{j100\pi(-100x+t)} + e^{j100\pi(100x+t)}$

Official answer: **A, C**

SOLUTION

1. Compare with the standard 1-D wave equation $f_{tt} = c^2 f_{xx}$. Here $c^2 = 10000$, so the wave speed is $c = 100$.
2. Define the residual $R[f] = f_{tt} - 10000\, f_{xx}$. f is a solution exactly when $R[f] \equiv 0$, and R is linear: $R[f_1 + f_2] = R[f_1] + R[f_2]$. Now take a travelling-wave term $\phi(x+vt)$, where v may be negative. Then $\phi_{tt} = v^2\phi''$ and $\phi_{xx} = \phi''$, so
$$R[\phi(x+vt)] = (v^2 - 10000)\,\phi''.$$
If ϕ'' is not identically zero, the term is a solution only when $v = \pm 100$, i.e. when it is a function of $x - 100t$ or of $x + 100t$. This is d'Alembert's form $f = g(x-100t) + h(x+100t)$.
3. (A): Both terms use $\phi(u) = e^{-u^2}$, one with $v = -100$ and one with $v = +100$. Each has $R = 0$, so by linearity the sum has $R = 0$. **Solution.**
4. (B): The first term, $e^{-(x-100t)}$, has $v = -100$, so its residual is 0. The second term has $v = 1000$, so its residual is
$$R\left[0.5e^{-(x+1000t)}\right] = (10^6 - 10^4)\cdot 0.5\, e^{-(x+1000t)} = 4.95\times 10^5\, e^{-(x+1000t)}.$$
The total residual is $0 + 4.95\times 10^5\, e^{-(x+1000t)}$, which is never zero. **Not a solution.**
5. (C): $e^{-(x-100t)}$ has $v = -100$ and $\sin(x+100t)$ has $v = +100$. Each residual is 0, so the sum has $R = 0$. **Solution.**
6. (D): Write each term as $e^{j(\omega t + kx)}$ with $\omega = 100\pi$. The first term has $k = -10^4\pi$ and the second has $k = +10^4\pi$. For such a term, $f_{tt} = -\omega^2 f$ and $f_{xx} = -k^2 f$, so
$$R = (10^4 k^2 - \omega^2)\, f = (10^{12}\pi^2 - 10^4\pi^2)\, f = 10^4\pi^2(10^8 - 1)\, f.$$
Both terms have the same factor, so the residual of the whole option is
$$R = 10^4\pi^2(10^8-1)\left[e^{j(100\pi t - 10^4\pi x)} + e^{j(100\pi t + 10^4\pi x)}\right] = 2\cdot 10^4\pi^2(10^8-1)\, e^{j100\pi t}\cos(10^4\pi x).$$
This is not identically zero; at $x = 0$, $t = 0$ it equals $2\cdot 10^4\pi^2(10^8-1)$. The two terms therefore do not cancel each other. **Not a solution.** The reason: $-100x + t = -100\left(x - \dfrac{t}{100}\right)$, so these waves travel at $\dfrac{\omega}{|k|} = \dfrac{1}{100}$, not 100.
7. The correct options are (A) and (C).

KEY IDEA

Every solution of $f_{tt} = c^2 f_{xx}$ has the form $g(x-ct) + h(x+ct)$, with $c = 100$ here. To test a term, write its argument as a multiple of $x + vt$ and check that $v = \pm 100$. If a term fails, work out the residual of the whole sum: the bad terms' residuals could in principle cancel, so check that they do not.

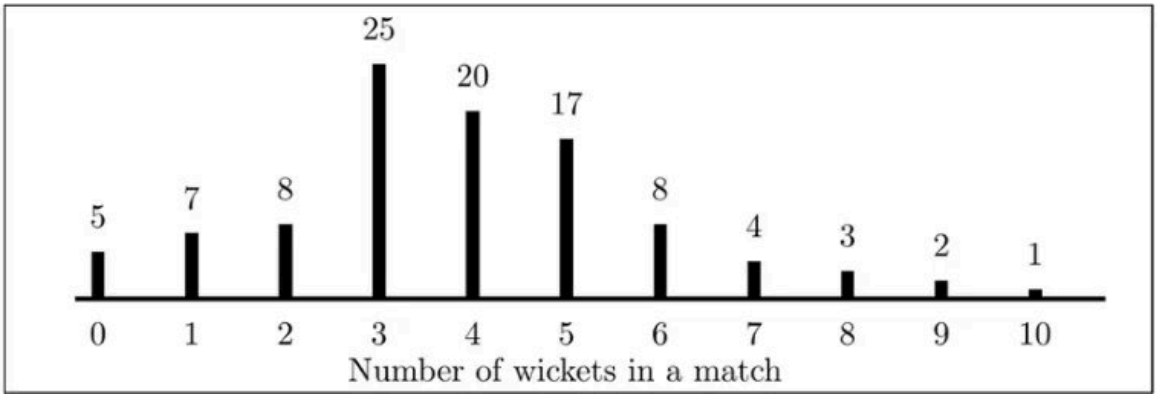
WATCH OUT FOR

In (D), taking the 100 that multiplies x as the wave speed. The argument is $-100x + t = -100\,(x - t/100)$, so the speed is $1/100$. The mistake runs the other way in (B): reading $x + 1000t$ as close enough to $x + 100t$ when its speed is 1000. A third error is to reject a sum only because each of its terms fails. Terms that fail can still cancel, so always compute the residual of the whole expression.

Q.31 NAT · 1 MARK

Engineering Mathematics › Probability & statistics

Q.31 The bar graph shows the frequency of the number of wickets taken in a match by a bowler in her career. For example, in 17 of her matches, the bowler has taken 5 wickets each. The median number of wickets taken by the bowler in a match is _____ (*rounded off to one decimal place*).



Official answer: **4.0**

SOLUTION

1. Read the frequencies off the bars: $f = 5, 7, 8, 25, 20, 17, 8, 4, 3, 2, 1$ for $x = 0, 1, \ldots, 10$ wickets.
2. Total number of matches: $N = 5 + 7 + 8 + 25 + 20 + 17 + 8 + 4 + 3 + 2 + 1 = 100$.
3. The data are discrete (a whole number of wickets per match), so the median is found by position. With N even, it is the mean of the $\left(\frac{N}{2}\right)^{\text{th}}$ and $\left(\frac{N}{2}+1\right)^{\text{th}}$ ordered values, here the 50^{th} and 51^{st} observations.
4. Cumulative frequencies: $x = 0 : 5$, $x = 1 : 12$, $x = 2 : 20$, $x = 3 : 45$, $x = 4 : 65$. So observations 46 to 65 all have $x = 4$.
5. Both the 50^{th} and 51^{st} observations therefore equal 4, and the median is $\dfrac{4+4}{2} = 4.0$ wickets.

KEY IDEA

For a discrete frequency distribution, add up the frequencies and find the value where the running total first reaches the middle position(s), $N/2$ and $N/2+1$. Here $N = 100$, and the running total goes from 45 (up to 3 wickets) to 65 (up to 4 wickets), so the 50th and 51st values are both 4.

WATCH OUT FOR

One mistake is using the grouped-data interpolation formula $L + \frac{N/2 - CF}{f}\,h$ with class 3.5 to 4.5, which gives $3.5 + \frac{50-45}{20} = 3.75 \approx 3.8$. That formula is for continuous class intervals, not for whole-number counts like wickets. Other slips are answering with the mode (3, the tallest bar), the middle axis value (5), or the median of the printed frequencies themselves.

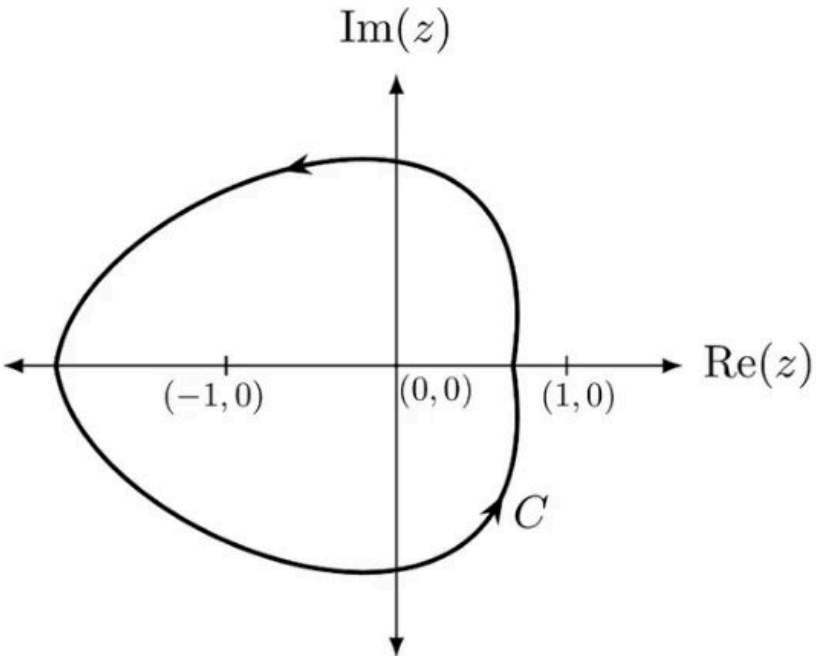
Q.32 NAT · 1 MARK

Engineering Mathematics › Complex analysis

Q.32 A simple closed path C in the complex plane is shown in the figure. If

$$\oint_C \frac{2^z}{z^2-1}\,dz = -i\pi A,$$

where $i = \sqrt{-1}$, then the value of A is _____ (*rounded off to two decimal places*).



Official answer: **0.50**

SOLUTION

1. **Singularities.** $z^2 - 1 = (z-1)(z+1)$, so the integrand has simple poles at $z = 1$ and $z = -1$. The numerator $2^z = e^{z \ln 2}$ is entire and not zero at either point, so there are no other singularities.
2. **Read the figure.** C meets the real axis at about $\mathrm{Re}(z) \approx -1.7$ on the left and $\mathrm{Re}(z) \approx 0.6$ on the right. The right crossing is between the origin and the tick at $(1, 0)$. So $z = -1$ is **inside** C and $z = 1$ is **outside**. The arrows point left on the top arc and up on the lower-right arc, so C is traversed counterclockwise (the positive sense).
3. **Residue at the enclosed pole.** For a simple pole, $\mathrm{Res}_{z=z_0} f = \lim_{z \to z_0} (z - z_0) f(z)$:
$$\mathrm{Res}_{z=-1} \frac{2^z}{(z-1)(z+1)} = \left.\frac{2^z}{z-1}\right|_{z=-1} = \frac{2^{-1}}{-2} = -\frac{1}{4}$$
4. **Cauchy's residue theorem.** The contour is counterclockwise and encloses only $z = -1$:
$$\oint_C \frac{2^z}{z^2-1}\,dz = 2\pi i \left(-\frac{1}{4}\right) = -\frac{i\pi}{2}$$
5. **Match with the given form.** $-i\pi A = -\dfrac{i\pi}{2}$, so $A = \dfrac{1}{2} = 0.50$.

KEY IDEA

Only the poles inside the contour count. The figure puts $z=-1$ inside C and $z=1$ outside, so the integral is $2\pi i$ times the single residue $\text{Res}_{z=-1}=\dfrac{2^{-1}}{-1-1}=-\dfrac{1}{4}$. That gives $-\dfrac{i\pi}{2}$, so $A=\dfrac{1}{2}$.

WATCH OUT FOR

Counting both poles. The residue at $z=1$ is $\dfrac{2^1}{2}=1$, and adding it gives $2\pi i\cdot\dfrac{3}{4}=\dfrac{3\pi i}{2}$, so $A=-1.50$. But the contour crosses the real axis at about 0.6, to the left of $(1,0)$, so $z=1$ is outside. Two other slips give $A=-0.50$: reading the orientation as clockwise, or dropping the sign of $z-1=-2$ at $z=-1$.

Q.33 NAT · 1 MARK

Signals and Systems › LTI & convolution

Let $x_1(t)=e^{-t}u(t)$ and $x_2(t)=u(t)-u(t-2)$, where $u(\cdot)$ denotes the unit step function.

If $y(t)$ denotes the convolution of $x_1(t)$ and $x_2(t)$, then $\lim_{t\to\infty}y(t)=$ _____ (rounded off to one decimal place).

Official answer: **0.0**

SOLUTION

1. $x_2(t)=u(t)-u(t-2)$ is a rectangular pulse of height 1 on $0\le t<2$. Keep the pulse fixed and slide the exponential: $y(t)=\int_{-\infty}^{\infty}x_2(\tau)\,x_1(t-\tau)\,d\tau=\int_0^2 e^{-(t-\tau)}\,u(t-\tau)\,d\tau$.
2. For $t>2$, every τ in $[0,2]$ satisfies $\tau<t$, so $u(t-\tau)=1$ over the whole pulse and
$$y(t)=e^{-t}\int_0^2 e^{\tau}\,d\tau=e^{-t}\left(e^2-1\right),\quad t>2.$$
3. As $t\to\infty$, $e^{-t}(e^2-1)\to 0$, so $\lim_{t\to\infty}y(t)=0$.
4. Check with the final value theorem: $Y(s)=X_1(s)X_2(s)=\dfrac{1}{s+1}\cdot\dfrac{1-e^{-2s}}{s}$.
5. $sY(s)=\dfrac{1-e^{-2s}}{s+1}$ has its only pole at $s=-1$, so it has no poles in the closed right half-plane and the theorem applies: $\lim_{s\to 0}sY(s)=\dfrac{1-1}{1}=0$.
6. Physically, a stable system with impulse response $e^{-t}u(t)$ is driven by an input that switches off at $t=2$, so its output decays to zero. Answer: 0.0.

KEY IDEA

$x_2(t)$ is a finite pulse that ends at $t=2$, not a step. After $t=2$ the output is just the decaying tail $e^{-t}(e^2-1)$ of a stable system, so it tends to zero. The final value theorem confirms this: $sY(s)=\dfrac{1-e^{-2s}}{s+1}$ is 0 at $s=0$.

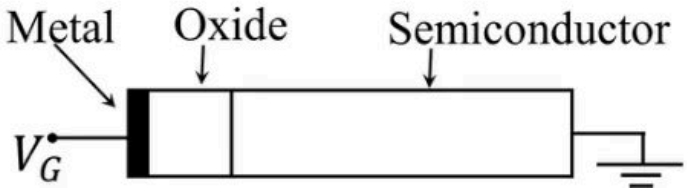
WATCH OUT FOR

Treating $x_2(t)$ as the step $u(t)$, which gives a limit of 1. Other errors: reporting the peak $y(2)=1-e^{-2}\approx 0.9$, or multiplying the areas ($1\times 2=2$). That product is $\int y(t)\,dt$, not the value of $y(t)$ as $t\to\infty$.

Q.34 NAT · 1 MARK

Electronic Devices › MOS devices

Q.34 An ideal MOS capacitor (p-type semiconductor) is shown in the figure. The MOS capacitor is under strong inversion with $V_G = 2$ V. The corresponding inversion charge density (Q_{IN}) is 2.2 μC/cm^2. Assume oxide capacitance per unit area as $C_{OX} = 1.7$ μF/cm^2. For $V_G = 4$ V, the value of Q_{IN} is _____ μC/cm^2 (*rounded off to one decimal place*).



Official answer: **5.5 to 5.7**

SOLUTION

1. State the ideal MOS gate-voltage charge balance: $V_G = V_{FB} + \phi_s + \dfrac{Q_{dep} + Q_{IN}}{C_{OX}}$, with $V_{FB} = 0$ for an ideal capacitor.
2. In strong inversion, ϕ_s (pinned near $2\phi_F$) and Q_{dep} (at its maximum depletion value) saturate and no longer change with further increases in V_G. Hence beyond the onset of strong inversion, $\Delta V_G = \dfrac{\Delta Q_{IN}}{C_{OX}}$.
3. Given: at $V_G = 2$ V, $Q_{IN} = 2.2$ μC/cm^2; $C_{OX} = 1.7$ μF/cm^2.
4. $\Delta V_G = 4\text{ V} - 2\text{ V} = 2\text{ V}$.
5. $\Delta Q_{IN} = C_{OX} \times \Delta V_G = 1.7\ \mu\text{F/cm}^2 \times 2\text{ V} = 3.4\ \mu\text{C/cm}^2$.
6. $Q_{IN}(4\text{V}) = Q_{IN}(2\text{V}) + \Delta Q_{IN} = 2.2 + 3.4 = 5.6\ \mu\text{C/cm}^2$.

KEY IDEA

In an ideal MOS capacitor, gate voltage divides among flat-band voltage (0, ideal case), surface potential, and the voltage dropped across the oxide due to all semiconductor-side charge: $V_G = V_{FB} + \phi_s + \dfrac{Q_{dep} + Q_{IN}}{C_{OX}}$. Once strong inversion is reached, ϕ_s and the depletion charge Q_{dep} saturate (pinned near $2\phi_F$ and its corresponding maximum value) and stop changing with further increases in V_G. So any additional gate voltage beyond the point of strong inversion goes entirely into building up more inversion charge: $\Delta V_G = \dfrac{\Delta Q_{IN}}{C_{OX}}$.

WATCH OUT FOR

Treating the MOS capacitor as a simple linear capacitor and assuming Q_{IN} is proportional to the full gate voltage V_G (i.e., computing $Q_{IN}(4\text{V}) = 2 \times Q_{IN}(2\text{V}) = 4.4\ \mu\text{C/cm}^2$). This ignores that ϕ_s and Q_{dep} have already saturated by the time strong inversion is reached, so only the incremental voltage ΔV_G (not the total V_G) maps to incremental inversion charge.

Q.35 NAT · 1 MARK

Communications › Digital communication

A symbol stream contains alternate QPSK and 16-QAM symbols. If symbols from this stream are transmitted at the rate of 1 mega-symbols per second, the raw (uncoded) data rate is _____ mega-bits per second

(rounded off to one decimal place).

Official answer: **2.99 to 3.01**

SOLUTION

1. Bits per symbol: QPSK has $M = 4$ points, so it carries $\log_2 4 = 2$ bits per symbol. 16-QAM has $M = 16$ points, so it carries $\log_2 16 = 4$ bits per symbol.
2. The stream alternates QPSK, 16-QAM, QPSK, 16-QAM, and so on. Exactly half of the symbols are QPSK and half are 16-QAM.
3. Average bits per symbol: $\bar{b} = \frac{1}{2}(2) + \frac{1}{2}(4) = 3$ bits/symbol.
4. Raw (uncoded) data rate: $R_b = R_s\,\bar{b} = 1 \times 10^6 \text{ symbols/s} \times 3 \text{ bits/symbol} = 3 \times 10^6 \text{ bits/s}$.
5. So the raw rate is 3.0 mega-bits per second. Check over one QPSK and 16-QAM pair: $2 + 4 = 6$ bits in $2\ \mu$s, which is 3 Mbit/s.

KEY IDEA

Bits per symbol of an M-ary constellation is $\log_2 M$. With two alternating constellations, average the bits per symbol over the stream, then multiply by the symbol rate.

WATCH OUT FOR

Using only one constellation, for example 4 bits per symbol for 16-QAM (giving 4 Mbit/s) or 2 bits per symbol for QPSK (giving 2 Mbit/s). Another slip is adding the two rates (2 + 4 = 6) instead of averaging them, because only one symbol is sent in each $1\ \mu$s slot.

Q.36 MCQ · 2 MARKS

Engineering Mathematics › Calculus

The function $f(x) = 8\log_e x - x^2 + 3$ attains its minimum over the interval $[1, e]$ at $x =$ _____.

(Here $\log_e x$ is the natural logarithm of x.)

(A) 2

(B) 1

(C) e

(D) $\dfrac{1+e}{2}$

Official answer: **B**

SOLUTION

1. **Closed-interval extremum rule:** f is continuous on $[1, e]$, so its absolute minimum there is reached either at a critical point inside the interval or at an endpoint. We find the critical points, then compare values.
2. **Critical point:** $f'(x) = \dfrac{8}{x} - 2x = 0 \Rightarrow x^2 = 4 \Rightarrow x = 2$ (we drop $x = -2$, which lies outside the domain $x > 0$). Since $e \approx 2.718$, the point $x = 2$ lies inside $[1, e]$.
3. **Second-derivative test:** $f''(x) = -\dfrac{8}{x^2} - 2 < 0$ for every $x > 0$, so $x = 2$ is a local **maximum**, not a minimum. Also, f is concave on the whole interval, so its minimum on $[1, e]$ has to be at an endpoint. (Equivalently: $f' > 0$ on $[1, 2)$ and $f' < 0$ on $(2, e]$, so f rises and then falls.)
4. **Evaluate the endpoints:** $f(1) = 8(0) - 1 + 3 = 2$ and $f(e) = 8(1) - e^2 + 3 = 11 - e^2 \approx 11 - 7.389 = 3.611$.
5. **Compare:** $f(1) = 2 < f(e) \approx 3.611$, and the interior value is $f(2) = 8\ln 2 - 1 \approx 4.545$. So the absolute minimum on $[1, e]$ is at $x = 1$, which is option (B).

KEY IDEA

On a closed interval, the absolute minimum is at a critical point or at an endpoint. Here the only critical point, $x = 2$, is a maximum, because $f'' < 0$ everywhere. A concave function takes its minimum at an endpoint, so you only have to compare $f(1) = 2$ with $f(e) = 11 - e^2 \approx 3.61$.

WATCH OUT FOR

Setting $f'(x) = 0$, getting $x = 2$ and picking option (A) without checking which kind of extremum it is. $f''(2) = -4 < 0$, so $x = 2$ is where f is largest on the interval. The other trap is picking $x = e$ because it is the right-hand endpoint, without actually comparing $f(1)$ and $f(e)$.

Q.37 MCQ · 2 MARKS

Engineering Mathematics › Linear algebra

Let α, β be two non-zero real numbers and v_1, v_2 be two non-zero real vectors of size 3×1. Suppose that v_1 and v_2 satisfy $v_1^T v_2 = 0$, $v_1^T v_1 = 1$, and $v_2^T v_2 = 1$. Let A be the 3×3 matrix given by:

$$A = \alpha v_1 v_1^T + \beta v_2 v_2^T$$

The eigenvalues of A are _____.

(A) $0,\ \alpha,\ \beta$

(B) $0,\ \alpha + \beta,\ \alpha - \beta$

(C) $0,\ \dfrac{\alpha + \beta}{2},\ \sqrt{\alpha\beta}$

(D) $0,\ 0,\ \sqrt{\alpha^2 + \beta^2}$

Official answer: **A**

SOLUTION

1. The conditions $v_1^T v_1 = 1$, $v_2^T v_2 = 1$ and $v_1^T v_2 = 0$ say that v_1, v_2 are **orthonormal** vectors in $\mathbb{R}^3$.
2. Apply A to v_1. Matrix multiplication is associative, and $v_1^T v_1$, $v_2^T v_1$ are scalars:
   $$Av_1 = \alpha v_1 (v_1^T v_1) + \beta v_2 (v_2^T v_1) = \alpha v_1 (1) + \beta v_2 (0) = \alpha v_1$$
   So v_1 is an eigenvector with eigenvalue α.
3. In the same way,
   $$Av_2 = \alpha v_1 (v_1^T v_2) + \beta v_2 (v_2^T v_2) = 0 + \beta v_2 = \beta v_2$$
   So v_2 is an eigenvector with eigenvalue β.
4. In $\mathbb{R}^3$, take a non-zero v_3 orthogonal to both v_1 and v_2 (for example $v_3 = v_1 \times v_2$, which is non-zero because v_1, v_2 are orthonormal). Then $v_1^T v_3 = v_2^T v_3 = 0$, so
   $$Av_3 = \alpha v_1 (v_1^T v_3) + \beta v_2 (v_2^T v_3) = 0 = 0 \cdot v_3$$
   So v_3 is an eigenvector with eigenvalue 0.
5. The vectors v_1, v_2, v_3 are non-zero and mutually orthogonal, so they are linearly independent and form a basis of $\mathbb{R}^3$. A 3×3 matrix with three independent eigenvectors has exactly those three eigenvalues (counted with multiplicity), so the eigenvalues of A are α, β and 0.
6. Check with the trace: $\mathrm{tr}(A) = \alpha\ \mathrm{tr}(v_1 v_1^T) + \beta\ \mathrm{tr}(v_2 v_2^T) = \alpha(v_1^T v_1) + \beta(v_2^T v_2) = \alpha + \beta$, which equals $0 + \alpha + \beta$. Also A is a sum of two rank-1 matrices, so $\mathrm{rank}(A) \le 2$. That makes $\det A = 0$, so one eigenvalue must be 0.
7. The eigenvalues of A are $0, \alpha, \beta$: option (A).

KEY IDEA

v_1 and v_2 are orthonormal, so $A = \alpha v_1 v_1^T + \beta v_2 v_2^T$ is a spectral decomposition. The matrix $v_i v_i^T$ projects onto v_i, so $Av_1 = \alpha v_1$ and $Av_2 = \beta v_2$. Any vector orthogonal to both is sent to 0, which gives the third eigenvalue 0. Checks: trace $= \alpha + \beta$, and rank ≤ 2.

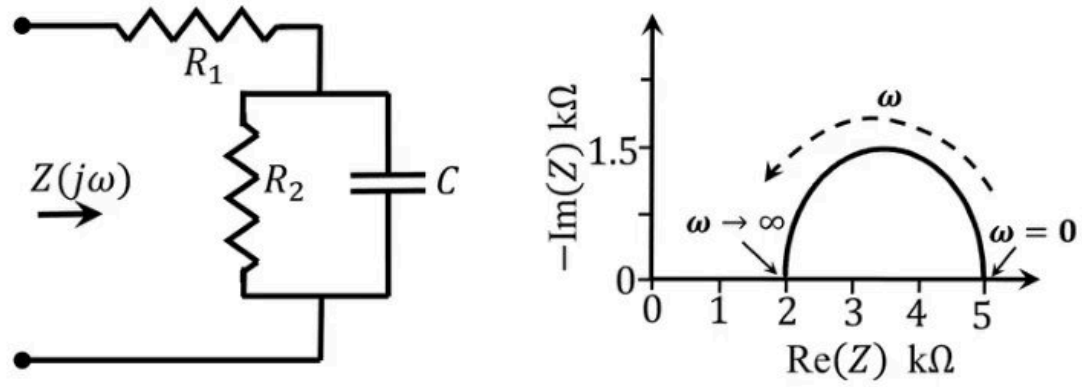
WATCH OUT FOR

Adding or combining α and β (options B, C, D) as if the two rank-1 terms mix. Because $v_1 \perp v_2$, each term acts only on its own direction, so the eigenvalues stay separate as α and β. Some students also forget the third eigenvalue. A 3×3 matrix of rank at most 2 always has 0 as an eigenvalue.

Q.38 MCQ · 2 MARKS

Networks (Electric Circuits) › AC, power & resonance

Q.38 For the circuit shown, the locus of the impedance $Z(j\omega)$ is plotted as ω increases from zero to infinity. The values of R_1 and R_2 are:



(A)	$R_1 = 2\ \text{k}\Omega,\ R_2 = 3\ \text{k}\Omega$
(B)	$R_1 = 5\ \text{k}\Omega,\ R_2 = 2\ \text{k}\Omega$
(C)	$R_1 = 5\ \text{k}\Omega,\ R_2 = 2.5\ \text{k}\Omega$
(D)	$R_1 = 2\ \text{k}\Omega,\ R_2 = 5\ \text{k}\Omega$

Official answer: **A**

SOLUTION

1. Write the impedance of R_1 in series with $R_2 \parallel C$:

$$Z(j\omega) = R_1 + \frac{R_2}{1 + j\omega C R_2}$$

2. Low-frequency end ($\omega = 0$, the capacitor is an open circuit): $Z(0) = R_1 + R_2$. The plot marks $\omega = 0$ at $\mathrm{Re}(Z) = 5\ \text{k}\Omega$, so $R_1 + R_2 = 5\ \text{k}\Omega$.

3. High-frequency end ($\omega \to \infty$, the capacitor is a short circuit across R_2): $Z(\infty) = R_1$. The plot marks $\omega \to \infty$ at $\mathrm{Re}(Z) = 2\ \mathrm{k\Omega}$, so $R_1 = 2\ \mathrm{k\Omega}$.
4. Therefore $R_2 = 5 - 2 = 3\ \mathrm{k\Omega}$.
5. Check against the height of the arc. Since
$$Z - \left(R_1 + \frac{R_2}{2}\right) = \frac{R_2}{2} \cdot \frac{1 - j\omega C R_2}{1 + j\omega C R_2},$$
and the last fraction always has magnitude 1, the locus is a semicircle of radius $R_2/2$ centred at $R_1 + R_2/2$. Its imaginary part is negative, so it lies above the axis on a $-\mathrm{Im}(Z)$ plot. Its highest point, reached at $\omega = 1/(R_2 C)$, is $-\mathrm{Im}(Z) = R_2/2 = 1.5\ \mathrm{k\Omega}$, which matches the plot.
6. Hence $R_1 = 2\ \mathrm{k\Omega}$ and $R_2 = 3\ \mathrm{k\Omega}$: option (A).

KEY IDEA

Read the two ends of the locus. At $\omega = 0$ the capacitor is an open circuit, so $Z = R_1 + R_2$. As $\omega \to \infty$ it shorts out R_2, so $Z = R_1$. The locus of $R_2 \parallel C$ is a semicircle of diameter R_2, so the arc's height, $R_2/2$, gives an independent check.

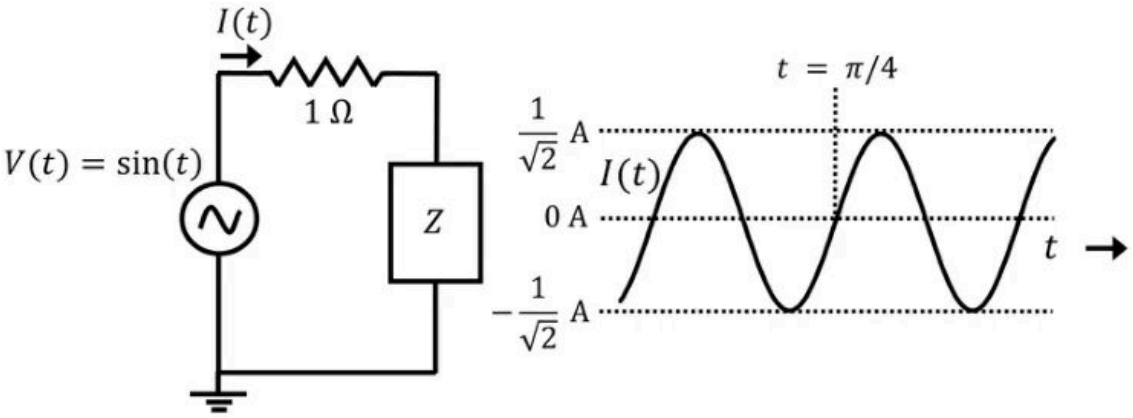
WATCH OUT FOR

Reading the ends of the locus the wrong way round, so that $R_1 = 5\ \mathrm{k\Omega}$ and $R_2 = 2\ \mathrm{k\Omega}$ (option B). Another mistake is taking the arc's height of $1.5\ \mathrm{k\Omega}$ as the radius of the whole locus measured from the origin. A third is treating the right-hand end (5) as R_2 on its own rather than $R_1 + R_2$, which gives option D. The capacitor removes R_2 at high frequency, so the high-frequency end (2) is R_1 alone.

Q.39 MCQ · 2 MARKS

Networks (Electric Circuits) › AC, power & resonance

Q.39 Consider the circuit shown in the figure with input $V(t)$ in volts. The sinusoidal steady state current $I(t)$ flowing through the circuit is shown graphically (where t is in seconds). The circuit element Z can be _____.



(A) a capacitor of 1 F

(B) an inductor of 1 H

(C) a capacitor of $\sqrt{3}$ F

(D) an inductor of $\sqrt{3}$ H

Official answer: **B**

SOLUTION

1. **Source phasor.** $V(t) = \sin t$ gives $\omega = 1$ rad/s and a peak of $V_m = 1$ V. Taking sin as the phase reference, the rms phasor is $\mathbf{V} = \frac{1}{\sqrt{2}}\angle 0^\circ$ V (rms).
2. **Amplitude from the plot.** The current just touches the dotted lines at $\pm\frac{1}{\sqrt{2}}$ A, so its peak is $I_m = \frac{1}{\sqrt{2}}$ A and its rms value is $\frac{I_m}{\sqrt{2}} = \frac{1}{2}$ A. Then $|Z_{total}| = \frac{V_m}{I_m} = \frac{1}{1/\sqrt{2}} = \sqrt{2}\,\Omega$. The ratio of the rms values gives the same answer.
3. **Phase from the plot.** Write $I(t) = I_m \sin(t + \phi)$. This crosses zero going upward when $t + \phi$ is a multiple of 2π. The plot puts a rising zero crossing at $t = \pi/4$, so $\phi = -\pi/4$ (up to a multiple of 2π). That gives $I(t) = \frac{1}{\sqrt{2}} \sin\left(t - \frac{\pi}{4}\right)$ A. The current lags the voltage by 45°: $\mathbf{I} = \frac{1}{2}\angle -45^\circ$ A (rms).
4. **Total impedance from Ohm's law in phasor form.**

$$Z_{total} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{\frac{1}{\sqrt{2}}\angle 0^\circ}{\frac{1}{2}\angle -45^\circ} = \sqrt{2}\angle 45^\circ = \sqrt{2}\left(\cos 45^\circ + j \sin 45^\circ\right) = 1 + j1\ \Omega$$

5. **Series combination.** The $1\ \Omega$ resistor and Z are in series, so $Z_{total} = 1 + Z$. That gives $Z = j1\ \Omega$. This is a purely reactive impedance with positive reactance, so Z is an inductor.
6. **Element value.** $j\omega L = j1$ with $\omega = 1\ \text{rad/s}$ gives $L = 1\ \text{H}$. Check: $|1 + j1| = \sqrt{2}\ \Omega$ and the lag is $\tan^{-1}(\omega L/R) = \tan^{-1}(1) = 45^\circ$. Both match the plot. The other options fail. A $1\ \text{F}$ capacitor gives $Z_{total} = 1 - j1\ \Omega$, so the current would lead by 45° and its rising zero crossing would be at $t = -\pi/4$. Either $\sqrt{3}$ element gives $|Z_{total}| = \sqrt{1+3} = 2\ \Omega$, so the peak current would be $\frac{1}{2}\ \text{A}$, not $\frac{1}{\sqrt{2}}\ \text{A}$.

Answer: option (B), an inductor of 1 H.

KEY IDEA

In a series circuit driven by a sinusoid, the source phasor divided by the current phasor is the total impedance. The two readings come straight off the plot. The ratio of peak voltage to peak current gives |Z_total|. The labelled time of the current's rising zero crossing, multiplied by ω, gives the angle. A rising zero crossing at a positive time means the current lags, so the circuit is inductive. Subtract the known resistor, and what is left is jωL (inductor) or –j/(ωC) (capacitor).

WATCH OUT FOR

Reading the rising zero crossing at t = +π/4 as a lead. The current reaches that point of its cycle π/4 s later than the source does, so it lags. Calling it a lead gives the 1 F capacitor (A). A second trap is mixing peak and rms values. 1/√2 A on the plot is the current's PEAK. Dividing the source's rms value (1/√2 V) by that peak gives |Z_total| = 1 Ω, which leaves nothing for Z and matches no option. A third slip is forgetting that the reactance is ωL. Here ω = 1 rad/s, so X = 1 Ω means L = 1 H, but that only works because ω happens to be 1.

Q.40 MCQ · 2 MARKS

Electronic Devices › MOS devices

Consider an ideal long channel nMOSFET (enhancement-mode) with gate length 10 µm and width 100 µm. The product of electron mobility (μ_n) and oxide capacitance per unit area (C_OX) is $\mu_n C_{OX}$ = 1 mA/V². The threshold voltage of the transistor is 1 V. For a gate-to-source voltage V_GS = [2 – sin(2t)] V and drain-to-source voltage V_DS = 1 V (substrate connected to the source), the maximum value of the drain-to-source current is _____.

(A) 40 mA

(B) 20 mA

(C) 15 mA

(D) 5 mA

Official answer: **C**

SOLUTION

1. **Aspect ratio.** $\frac{W}{L} = \frac{100\ \mu\text{m}}{10\ \mu\text{m}} = 10.$
2. **Overdrive voltage.** $V_{ov}(t) = V_{GS} - V_{tn} = (2 - \sin(2t)) - 1 = 1 - \sin(2t)$. Since $\sin(2t) \in [-1, 1]$, the overdrive swings over $V_{ov} \in [0, 2]\ \text{V}$.
3. **Region check.** $V_{DS} = 1\ \text{V}$ is fixed, so the transistor is in saturation only while $V_{DS} \geq V_{ov}$, i.e. while $V_{ov} \leq 1\ \text{V}$. When $V_{ov} > 1\ \text{V}$, the device operates in the triode (linear) region instead, because $V_{DS} < V_{ov}$ there.
4. **Saturation branch ($V_{ov} \leq 1\ \text{V}$).**
$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)V_{ov}^2 = \frac{1}{2}(1)(10)V_{ov}^2 = 5V_{ov}^2\ \text{mA}$$
which is increasing on this range, reaching $5(1)^2 = 5\ \text{mA}$ at $V_{ov} = 1\ \text{V}$.
5. **Triode branch ($V_{ov} > 1\ \text{V}$).**

$$I_D = \mu_n C_{ox}\left(\frac{W}{L}\right)\left[V_{ov}V_{DS} - \frac{V_{DS}^2}{2}\right] = 10\,(V_{ov} - 0.5)\ \text{mA}$$

using $V_{DS} = 1\ \text{V}$.

6. **Continuity check at the boundary** $V_{ov} = 1\ \text{V}$**.** Saturation gives $5(1)^2 = 5\ \text{mA}$; triode gives $10(1 - 0.5) = 5\ \text{mA}$. The two branches agree, confirming the region split and formulas are applied correctly.
7. **Locating the true maximum.** On the triode branch, $I_D = 10(V_{ov} - 0.5)$ increases monotonically with V_{ov}, so the overall maximum current occurs at the largest achievable overdrive, $V_{ov} = 2\ \text{V}$ (reached when $\sin(2t) = -1$), which lies in the triode region since $2 > V_{DS} = 1$.
8. **Final value.**

$$I_{D,\max} = 10(2 - 0.5) = 10 \times 1.5 = 15\ \text{mA}$$

which matches option (C).

KEY IDEA

A time-varying gate voltage makes the overdrive $V_{ov}(t)$ swing over a range; with V_{DS} fixed, the device moves between saturation and triode as V_{ov} crosses V_{DS}, and since I_D keeps rising with V_{ov} in triode too, the true maximum must be checked at the extreme of V_{ov}, not assumed to sit in saturation.

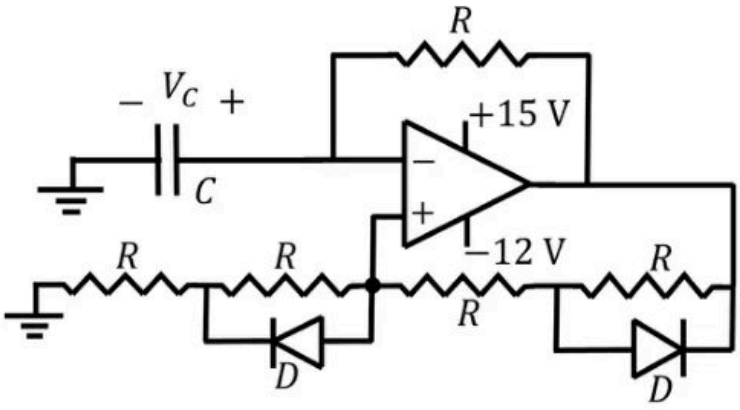
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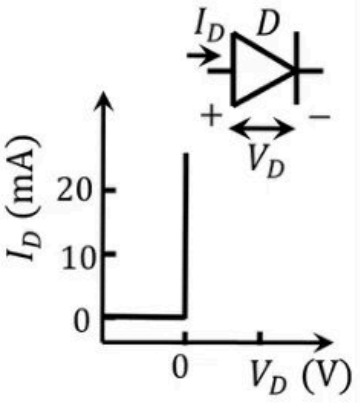
Assuming the device stays in saturation for the whole cycle and blindly maximizing $I_D = \frac{1}{2}\mu_n C_{ox}(W/L)V_{ov}^2$ at $V_{ov,max} = 2\ \text{V}$, which gives $20\ \text{mA}$ (option B). This ignores that for $V_{ov} > V_{DS} = 1\ \text{V}$ the transistor has already left saturation and entered the triode region, where the saturation formula no longer applies.

Q.41 MCQ · 2 MARKS

Analog Electronics › Filters & waveforms

Q.41 For the following circuit with an ideal OPAMP, the difference between the maximum and the minimum values of the capacitor voltage (V_c) is _____.





(A)	15 V
(B)	27 V
(C)	13 V
(D)	14 V

Official answer: **C**

SOLUTION

1. **Model.** The op-amp is ideal: its inputs draw no current, and with positive feedback to its $+$ input it acts as a Schmitt trigger. Its output therefore sits at a rail, $V_o = +15$ V or $V_o = -12$ V. The capacitor charges through the feedback R towards V_o. Its $+$ terminal is the inverting input and its $-$ terminal is grounded, so $V_C = v_-$. The I_D–V_D plot says each diode is an ideal switch: $I_D = 0$ for $V_D < 0$ (an open circuit), and it conducts at $V_D = 0$ (a short).
2. **Label the ladder.** From ground: R to node A, R to node P (which is v_+), R to node B, then R to V_o. D_1 sits across A–P with its anode at P and its cathode at A. D_2 sits across B–V_o with its anode at B and its cathode at V_o. No current enters the $+$ input, so the ladder is a plain voltage divider.
3. **Output at $+15$ V.** Current flows down the ladder from V_o to ground. D_2: current flows from V_o to B through its resistor, so $V_B < V_o$ and $V_{D2} = V_B - V_o < 0$. D_2 is OFF. D_1: current reaches P and must continue towards A, so $V_P > V_A$ and D_1 is forward biased. It turns ON and shorts the A–P resistor, carrying current from P to A (forward, which is consistent). The path is $V_o \to R \to B \to R \to P \to D_1 \to A \to R \to$ ground: a total of $3R$, with R below P.
4. **Upper threshold, by the voltage divider rule:**

$$V_{UT} = (+15) \cdot \frac{R}{3R} = +5 \text{ V}$$

5. **Output at -12 V.** Current now flows up the ladder from ground to V_o. D_1: current flows from A to P through its resistor, so $V_A > V_P$ and $V_{D1} = V_P - V_A < 0$. D_1 is OFF. D_2: current reaching B must continue towards V_o, so $V_B > V_o$ and D_2 is forward biased. It turns ON and shorts the B–V_o resistor, carrying current from B to V_o (forward, which is consistent). The path is ground $\to R \to A \to R \to P \to R \to B \to D_2 \to V_o$: a total of $3R$, with $2R$ below P.
6. **Lower threshold, by the voltage divider rule:**

$$V_{LT} = (-12) \cdot \frac{2R}{3R} = -8 \text{ V}$$

7. **Capacitor swing.** With $V_o = +15$ V, the capacitor charges towards $+15$ V. When $v_- = V_C$ passes $v_+ = +5$ V, the output flips to -12 V. The capacitor then discharges towards -12 V. When V_C falls below $v_+ = -8$ V, the output flips back to $+15$ V. Both thresholds lie strictly between the rails, so each is always reached and the circuit keeps oscillating. In steady state $V_{C,\max} = +5$ V and $V_{C,\min} = -8$ V.
8. **Result:**

$$V_{C,\max} - V_{C,\min} = 5 - (-8) = 13 \text{ V}$$

This is option (C).

KEY IDEA

This is an RC relaxation oscillator: the capacitor on the inverting input charges towards whichever rail the output is at, and the output flips when V_C crosses v_+. So V_C swings between the two thresholds, not between the rails. The diodes make the thresholds asymmetric. At $+15$ V output, D_1 shorts the A–P resistor, so v_+ is $\frac{1}{3}$ of the output. At -12 V output, D_2 shorts the B–V_o resistor, so v_+ is $\frac{2}{3}$ of the output. The thresholds are therefore $+5$ V and -8 V.

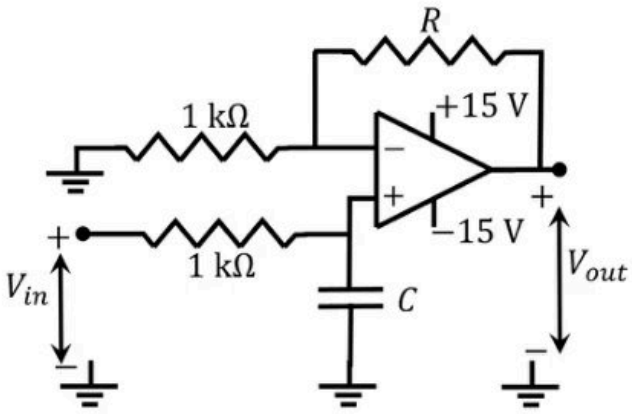
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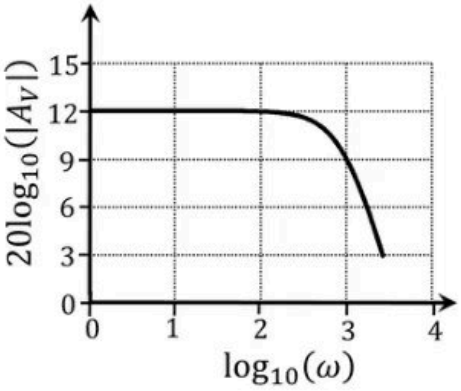
(1) Taking the rails as the capacitor's extremes gives $15 - (-12) = 27$ V, option (B). The capacitor stops at the thresholds, because the output flips before V_C gets anywhere near a rail. (2) Swapping the diode states, for example by misreading which end is the anode, puts D_2 ON at $+15$ V (giving $+10$ V) and D_1 ON at -12 V (giving -4 V). That yields 14 V, option (D). Always check each diode's state against the direction the ladder current actually flows. (3) Ignoring the diodes gives a $4R$ ladder with $2R$ below P: thresholds of $+7.5$ V and -6 V, and a swing of 13.5 V, which is not an option.

Q.42 MCQ · 2 MARKS

Analog Electronics › Filters & waveforms

Q.42 A circuit with an ideal OPAMP is shown. The Bode plot for the magnitude (in dB) of the gain transfer function ($A_V(j\omega) = V_{out}(j\omega)/V_{in}(j\omega)$) of the circuit is also provided (here, ω is the angular frequency in rad/s). The values of R and C are _____.





(A) $R = 3\ \text{k}\Omega,\ C = 1\ \mu\text{F}$

(B) $R = 1\ \text{k}\Omega,\ C = 3\ \mu\text{F}$

(C) $R = 4\ \text{k}\Omega,\ C = 1\ \mu\text{F}$

(D) $R = 3\ \text{k}\Omega,\ C = 2\ \mu\text{F}$

Official answer: **A**

SOLUTION

1. **Assumptions.** The op-amp is ideal: infinite input impedance, zero input current, infinite open-loop gain. It works in its linear region, so the ±15 V rails play no part.
2. **Input RC network (voltage divider).** The op-amp draws no input current, so the series 1 kΩ resistor and C form an unloaded low-pass divider:
$$V_+ = V_{in}\,\frac{1/(j\omega C)}{1\ \text{k}\Omega + 1/(j\omega C)} = \frac{V_{in}}{1 + j\omega(1\ \text{k}\Omega)C}$$
3. **Non-inverting amplifier.** Negative feedback holds a virtual short at the inputs, so $V_- = V_+$. R and the 1 kΩ resistor to ground divide the output: $V_- = V_{out}\,\frac{1\ \text{k}\Omega}{1\ \text{k}\Omega + R}$. Therefore $V_{out} = \left(1 + \frac{R}{1\ \text{k}\Omega}\right) V_+$.
4. **Overall transfer function.**
$$A_V(j\omega) = \left(1 + \frac{R}{1\ \text{k}\Omega}\right)\frac{1}{1 + j\omega/\omega_c}, \qquad \omega_c = \frac{1}{(1\ \text{k}\Omega)\,C}$$

This is a first-order low-pass response. It is flat at the DC gain, 3 dB below that level at ω_c, and falls toward a 20 dB/decade asymptote beyond. R sets only the DC gain. C, with the series 1 kΩ, sets only the corner.

5. **DC gain from the plot.** The flat level is 12 dB, so $|A_V(0)| = 10^{12/20} \approx 3.98 \approx 4$. Then $1 + R/1\ \text{k}\Omega = 4$, which gives $R = 3\ \text{k}\Omega$. This rules out (B), whose gain of 2 would sit at about 6 dB, and (C), whose gain of 5 would sit at about 14 dB.
6. **Corner frequency from the plot.** The curve reaches 3 dB below the flat level ($12 - 3 = 9$ dB) at $\log_{10}\omega = 3$, so $\omega_c = 10^3\ \text{rad/s}$.
7. **Solve for C.** $C = \dfrac{1}{(1\ \text{k}\Omega)\,\omega_c} = \dfrac{1}{10^3 \times 10^3} = 10^{-6}\ \text{F} = 1\ \mu\text{F}$.
8. **Check (D).** With $C = 2\ \mu\text{F}$ the corner would be $\omega_c = 500\ \text{rad/s}$ ($\log_{10}\omega \approx 2.7$). At $\omega = 10^3\ \text{rad/s}$ the magnitude would be $12 - 10\log_{10}(1 + 2^2) \approx 5$ dB, not the 9 dB on the plot. So the answer is $R = 3\ \text{k}\Omega,\ C = 1\ \mu\text{F}$, option (A).

KEY IDEA

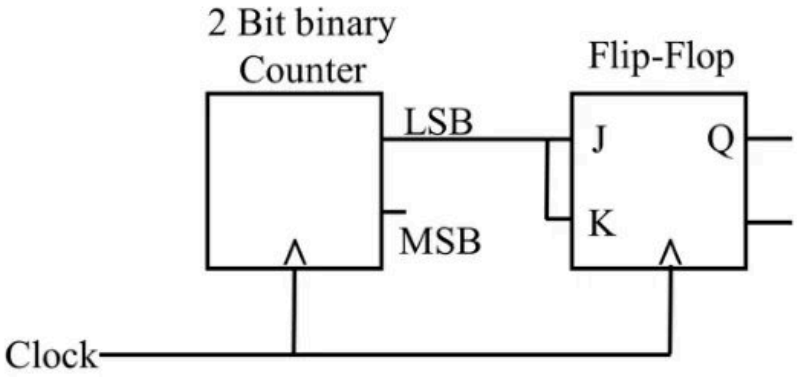
The ideal op-amp draws no current, so the input RC is an unloaded first-order low-pass with $\omega_c = 1/((1\ \text{k}\Omega)C)$. The non-inverting stage multiplies it by $1 + R/1\ \text{k}\Omega$ and does not move the pole. Read the flat level for R (12 dB is a gain of 4, so $R = 3\ \text{k}\Omega$). Read the point 3 dB below that level for C (9 dB at $\omega = 10^3\ \text{rad/s}$, so $C = 1\ \mu\text{F}$).

WATCH OUT FOR

One slip is using the inverting-gain formula $R/1\ \text{k}\Omega = 4$, which gives $R = 4\ \text{k}\Omega$ and lands on option (C). Another is putting the feedback R into the time constant; only the series 1 kΩ and C set ω_c. A third is taking the corner from the wrong point: where the curve first visibly bends (about $\log_{10}\omega = 2.3$), or where it reaches 3 dB absolute (about $\log_{10}\omega = 3.4$). The corner is the point 3 dB below the flat level, which is 9 dB here.

Q.43 MCQ · 2 MARKS

Digital Electronics › Sequential

Q.43	For the circuit shown, the clock frequency is f_0 and the duty cycle is 25%. For the signal at the Q output of the Flip-Flop, _____.
	
(A)	frequency is $f_0/4$ and duty cycle is 50%
(B)	frequency is $f_0/4$ and duty cycle is 25%
(C)	frequency is $f_0/2$ and duty cycle is 50%
(D)	frequency is f_0 and duty cycle is 25%

Official answer: **A**

SOLUTION

1. **Reduce the flip-flop.** J and K are tied together, and both get the counter LSB Q_0. A JK flip-flop with $J = K$ acts as a T flip-flop whose toggle input is Q_0:
$$Q^+ = Q_0 \oplus Q$$
It toggles on a clock edge when $Q_0 = 1$ and holds when $Q_0 = 0$.
2. **How the counter LSB behaves.** In a binary counter the LSB toggles on every clock edge: $Q_0 = 0, 1, 0, 1, \ldots$ Write the clock period as $T_0 = 1/f_0$. Q_0 is then a square wave with period $2T_0$, i.e. frequency $f_0/2$ and duty cycle 50%. The MSB is not connected and plays no part.
3. **Same-edge sampling.** The counter and the flip-flop trigger on the same rising edge. At each edge the flip-flop acts on the value Q_0 had just before the edge, not the value the counter moves to at that edge.
4. **Tabulate the rising edges.** Start from $Q_0 = 0$, $Q = 0$.
- Edge 1: $J = K = 0$, so Q holds at 0 (and Q_0 goes to 1).
- Edge 2: $J = K = 1$, so Q toggles to 1 (and Q_0 goes to 0).
- Edge 3: $J = K = 0$, so Q holds at 1 (and Q_0 goes to 1).
- Edge 4: $J = K = 1$, so Q toggles to 0 (and Q_0 goes to 0).

After that the pattern repeats. A different starting state only shifts the waveform in time and does not change its shape.

5. **Period and duty cycle of Q.** Q toggles on every second rising edge. It stays high for $2T_0$, then low for $2T_0$, so its period is $4T_0$:
$$f_Q = \frac{1}{4T_0} = \frac{f_0}{4}, \qquad D = \frac{2T_0}{4T_0} = 50\%$$
6. **Why the clock's 25% duty cycle does not matter.** Every change in Q_0 and in Q happens on a rising edge, and rising edges are exactly one period T_0 apart whatever the clock's duty cycle is. The 25% therefore has no effect on the output. Q has frequency $f_0/4$ and a 50% duty cycle, which is option (A).

KEY IDEA

With $J = K$ tied to the counter LSB, the JK flip-flop works as a T flip-flop fed by an $f_0/2$ square wave. It toggles on every second rising edge, which divides the clock frequency by 4. Every transition lands on a rising edge, and rising edges are always one period apart, so the output duty cycle is 50% whatever the input's duty cycle is.

WATCH OUT FOR

Three traps. (1) Assuming the clock's 25% duty cycle carries through to the output, which gives option (B). Edge-triggered outputs change only on rising edges, so the clock's high time drops out. (2) Assuming the flip-flop toggles on every clock edge, which gives $f_0/2$, option (C). It toggles only on edges where the LSB is 1. (3) Using the LSB's value just after an edge instead of just before it. On a shared clock the flip-flop sees the pre-edge value.

Q.44 MCQ · 2 MARKS

Control Systems › Routh–Hurwitz

Consider an even polynomial $p(s)$ given by
$$p(s) = s^4 + 5s^2 + 4 + K,$$
where K is an unknown real parameter. The complete range of K for which $p(s)$ has all its roots on the imaginary axis is _____.

(A) $-4 \le K \le \frac{9}{4}$

(B) $-3 \le K \le \frac{9}{2}$

(C) $-6 \le K \le \frac{5}{4}$

(D) $-5 \le K \le 0$

Official answer: **A**

SOLUTION

1. $p(s)$ is even, so its roots are symmetric about the origin: every root s_0 comes with $-s_0$. If a root had a non-zero real part, either it or its mirror $-s_0$ would lie in the RHP. So all roots are on the $j\omega$ axis exactly when $p(s)$ has no RHP roots.
2. Routh array: $p(s)$ has no s^3 or s^1 terms, so the s^3 row is entirely zero. $p(s)$ is then its own auxiliary polynomial, $A(s) = s^4 + 5s^2 + (4 + K)$. Replace the zero row with the coefficients of $\frac{dA}{ds} = 4s^3 + 10s$.
3. Complete the array:
$$\begin{array}{ll} s^4: & 1 \quad 5 \quad 4+K \\ s^3: & 4 \quad 10 \\ s^2: & \frac{5}{2} \quad 4+K \\ s^1: & \frac{9-4K}{5/2} \\ s^0: & 4+K \end{array}$$

Here s^2: $\dfrac{4\cdot 5 - 1\cdot 10}{4} = \frac{5}{2}$, and s^1: $\dfrac{\frac{5}{2}\cdot 10 - 4(4+K)}{5/2} = \dfrac{9-4K}{5/2}$.

4. The number of sign changes in the first column from the s^3 row down equals the number of RHP roots. There are none when $9 - 4K > 0$ and $4 + K > 0$, that is, for $-4 < K < \dfrac{9}{4}$. By the mirror symmetry there are then no LHP roots either. For $K > \frac{9}{4}$ the s^1 entry is negative, giving two sign changes and so two RHP roots. For $K < -4$ the s^0 entry is negative, giving one sign change and one RHP root. The two endpoints make a first-column entry zero, so they are checked directly in step 6.
5. Check with $x = s^2$: $x^2 + 5x + (4+K) = 0$. For $s = \pm j\sqrt{-x}$ to be imaginary (or zero), both x-roots must be real and ≤ 0. Real roots: $25 - 4(4+K) \ge 0 \Rightarrow K \le \frac{9}{4}$. Both non-positive: the sum is $-5 < 0$ and the product $4 + K \ge 0 \Rightarrow K \ge -4$.
6. Endpoints are included. At $K = -4$, $p(s) = s^2(s^2+5)$ and the roots are $s = 0, 0, \pm j\sqrt{5}$. The root $s = 0$ lies on the imaginary axis. At $K = \frac{9}{4}$, $p(s) = \left(s^2 + \frac{5}{2}\right)^2$ and the roots are $s = \pm j\sqrt{5/2}$, each repeated. In both cases every root is on the axis.
7. So the complete range is $-4 \le K \le \dfrac{9}{4}$, which is option (A).

KEY IDEA

The roots of an even polynomial are symmetric about the origin, so 'all roots on the $j\omega$ axis' means the same as 'no RHP roots'. In the Routh array the s^3 row vanishes and is rebuilt from dA/ds. Substituting $x = s^2$ turns the question into one about a quadratic: it needs real roots that are both ≤ 0. That gives $K \le 9/4$ from the discriminant and $K \ge -4$ from the product.

WATCH OUT FOR

Checking only that the coefficients are positive ($4 + K \ge 0$) and missing the discriminant condition $9 - 4K \ge 0$. Without it the s^2-roots become complex and the roots of $p(s)$ leave the axis in mirror pairs. A second common error is to drop the endpoints. At $K = -4$ there is a double root at $s = 0$, which is still on the imaginary axis. At $K = 9/4$ the roots are the repeated pair $\pm j\sqrt{5/2}$. A third is to give up when the s^3 row comes out all zero, instead of replacing it with the derivative of the auxiliary polynomial.

Q.45 MSQ · 2 MARKS

Engineering Mathematics › Calculus

Consider the following series:

$$\sum_{n=1}^{\infty} \frac{n^d}{c^n}$$

For which of the following combinations of c, d values does this series converge?

(A) $c = 1,\ d = -1$

(B) $c = 2,\ d = 1$

(C) $c = 0.5,\ d = -10$

(D) $c = 1,\ d = -2$

Official answer: **B, D**

SOLUTION

1. Let $a_n = \dfrac{n^d}{c^n}$. Every option has $c > 0$, so every term is positive. Apply the **ratio test**:

$$\frac{a_{n+1}}{a_n} = \left(\frac{n+1}{n}\right)^d \cdot \frac{1}{c} \longrightarrow \frac{1}{c} \quad (n \to \infty),$$

for any value of d, because $\left(\frac{n+1}{n}\right)^d \to 1$.

2. So the series **converges if** $c > 1$ (the limit is less than 1) and **diverges if** $0 < c < 1$ (the limit is greater than 1), for every d. A power n^d can never outgrow an exponential c^n, and can never hold one back.
3. When $c = 1$ the ratio test gives a limit of exactly 1, so it tells us nothing. The series becomes $\sum n^d = \sum \frac{1}{n^{-d}}$, a **p-series** with $p = -d$. It converges if and only if $p > 1$, that is, $d < -1$.
4. (A) $c = 1,\ d = -1$: $\sum \frac{1}{n}$ is the harmonic series ($p = 1$), which **diverges**.
5. (B) $c = 2,\ d = 1$: $\sum \frac{n}{2^n}$ has ratio limit $\frac{1}{2} < 1$, so it **converges**. (Its sum is 2.)
6. (C) $c = 0.5,\ d = -10$: $a_n = \frac{n^{-10}}{(1/2)^n} = \frac{2^n}{n^{10}} \to \infty$, and the ratio limit is $2 > 1$. The terms do not even go to zero (the n**-th term test**), so the series **diverges**.
7. (D) $c = 1,\ d = -2$: $\sum \frac{1}{n^2}$ is a p-series with $p = 2 > 1$, so it **converges**.
8. The series converges for **(B)** and **(D)**.

KEY IDEA

For $c > 0$, the ratio test gives $\lim a_{n+1}/a_n = 1/c$ whatever d is. The exponential decides: the series converges for $c > 1$ and diverges for $0 < c < 1$. Only at $c = 1$ does d matter, where the series is the p-series $\sum 1/n^{-d}$, which converges only when $d < -1$.

WATCH OUT FOR

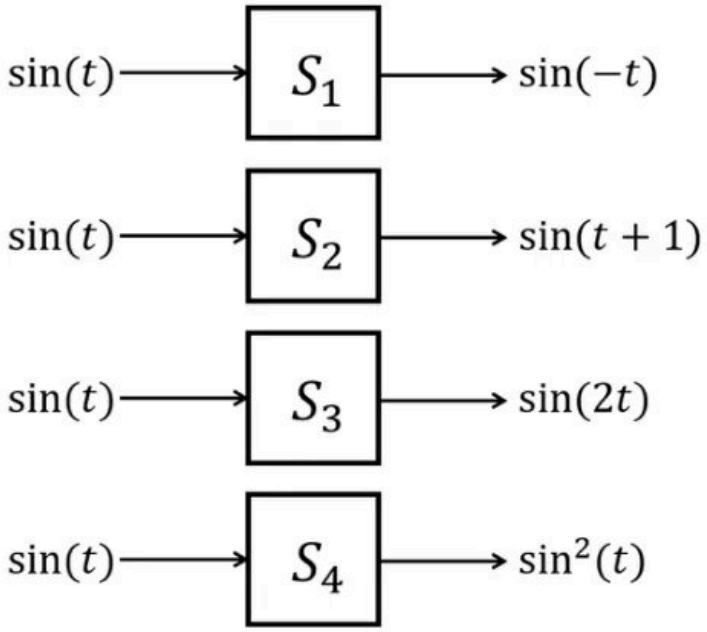
Picking (A) because $1/n \to 0$. Terms going to zero is necessary but not sufficient, and the harmonic series ($p = 1$) diverges. The other trap is picking (C) because the power n^{-10} looks strongly decaying. With $c = 0.5$ the factor $1/c^n = 2^n$ grows exponentially and swamps any power of n, so the terms blow up.

Q.46 MSQ · 2 MARKS

Signals and Systems › LTI & convolution

Q.46 The outputs of four systems (S_1, S_2, S_3, and S_4) corresponding to the input signal $\sin(t)$, for all time t, are shown in the figure.

Based on the given information, which of the four systems is/are definitely NOT LTI (linear and time-invariant)?



(A) S_1

(B) S_2

(C) S_3

(D) S_4

Official answer: **C, D**

SOLUTION

1. **Eigenfunction property of LTI systems:** write $\sin(t) = \dfrac{e^{jt} - e^{-jt}}{2j}$. An LTI system maps $e^{j\omega t}$ to $H(j\omega)e^{j\omega t}$, so its response to $\sin(t)$ is $\dfrac{H(j1)e^{jt} - H(-j1)e^{-jt}}{2j}$. For a real system this is $|H(j1)|\sin\left(t + \angle H(j1)\right)$. Either way the output contains only the **same frequency** $\omega = 1\ \text{rad/s}$, scaled and phase-shifted. An LTI system cannot create a new frequency or a DC term.

2. S_1: $\sin(-t) = -\sin(t) = \sin(t+\pi)$. This is the same frequency with gain $|H(j1)| = 1$ and phase π. The LTI system $y(t) = -x(t)$ produces exactly this output. (So could the time reversal $y(t) = x(-t)$, which is not time-invariant, but one input cannot tell the two apart.) So S_1 is **not definitely** non-LTI.
3. S_2: $\sin(t+1)$ is the input phase-shifted by 1 rad at the same frequency. The LTI system $y(t) = x(t+1)$ (a time advance with $H(j\omega) = e^{j\omega}$) produces it. That system is non-causal, but causality is not part of being LTI. So S_2 is **not definitely** non-LTI.
4. S_3: $\sin(2t)$ has frequency 2 rad/s, and that frequency is not in the input. No LTI system can do this, so S_3 is **definitely NOT LTI**.
5. S_4:
$$\sin^2(t) = \frac{1}{2} - \frac{1}{2}\cos(2t)$$
This has a DC term and a 2 rad/s component, and neither is in the input. So S_4 is **definitely NOT LTI**.
6. **Cross-check without frequency response.** Since $\sin(t-\pi) = -\sin(t)$, the input delayed by π is the same signal as the input negated. If a system is LTI and $\sin(t) \to y(t)$, time-invariance says the response to $\sin(t-\pi)$ is $y(t-\pi)$, and linearity says the response to $-\sin(t)$ is $-y(t)$. So we need $y(t-\pi) = -y(t)$ for all t. S_1: $\sin(\pi - t) = \sin(t) = -\sin(-t)$, which holds. S_2: $\sin(t+1-\pi) = -\sin(t+1)$, which holds. S_3: $\sin(2t - 2\pi) = \sin(2t) \neq -\sin(2t)$, which fails. S_4: $\sin^2(t-\pi) = \sin^2(t) \neq -\sin^2(t)$, which fails. This agrees with steps 2 to 5.
7. One input-output pair can prove that a system is **not** LTI, but it can never prove that a system **is** LTI. That is why the question says "definitely NOT": S_1 and S_2 are consistent with an LTI system, while S_3 and S_4 rule one out. Answer: **(C) and (D)**.

KEY IDEA
A sinusoid is an eigenfunction of every LTI system, so an LTI output for $\sin(t)$ can only be $\sin(t)$ scaled and phase-shifted: the same frequency, no new harmonics and no DC. A single input-output pair can prove a system is not LTI, but it can never prove that a system is LTI.

WATCH OUT FOR
Marking S_1 because $\sin(-t)$ looks like time reversal. In fact $\sin(-t) = -\sin(t)$, which the LTI system $y = -x$ produces. Marking S_2 because $x(t+1)$ is non-causal is the same kind of mistake: causality is not part of being LTI. Another error is reading the question as asking which systems ARE LTI, when one input can only show whether an LTI system is possible.

Q.47 MSQ · 2 MARKS

Electronic Devices › Carrier transport

Select the CORRECT statement(s) regarding semiconductor devices.

(A) Electrons and holes are of equal density in an intrinsic semiconductor at equilibrium.

(B) Collector region is generally more heavily doped than Base region in a BJT.

(C) Total current is spatially constant in a two terminal electronic device in dark under steady state condition.

(D) Mobility of electrons always increases with temperature in Silicon beyond 300 K.

Official answer: **A, C**

SOLUTION

1. Statement (A): By definition of an intrinsic semiconductor at thermal equilibrium, $n = p = n_i$: electron and hole densities are equal by symmetry (no dopants). TRUE.
2. Statement (B): The standard BJT doping hierarchy is $N_{D,E} \gg N_{A,B} > N_{D,C}$: the emitter is doped heaviest for high injection efficiency, while the collector is kept lightly doped (relative to the base) so it can support the reverse-biased collector–base junction with a high breakdown voltage and good Early voltage. So the collector is NOT more heavily doped than the base. FALSE.

3. Statement (C): In steady state the continuity equation gives $\frac{\partial \rho}{\partial t} = 0$ everywhere, and a two-terminal device has only one current path, so by Kirchhoff's current law the total current (drift + diffusion, electrons + holes summed) must be identical at every cross-section of the device, spatially constant. TRUE.
4. Statement (D): Above the impurity-scattering-dominated low-temperature regime, electron mobility in Silicon is limited by lattice (phonon) scattering, whose rate increases with temperature; consequently μ_n decreases with T (roughly $\mu_n \propto T^{-3/2}$) for T well above 300 K. Mobility does NOT always increase with temperature. FALSE.
5. Hence the correct statements are (A) and (C).

KEY IDEA

Apply the four separate physical facts: intrinsic carrier symmetry, standard BJT doping hierarchy (emitter heaviest, collector lightest), KCL/continuity in steady state, and phonon-scattering-limited mobility above ~300 K in Si.

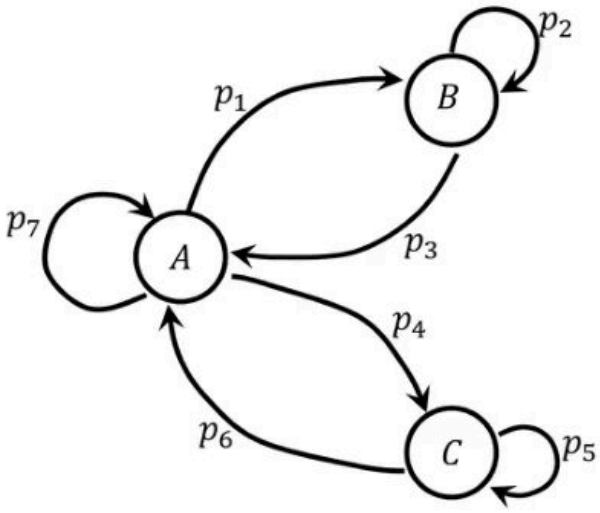
WATCH OUT FOR

Assuming the collector must be more heavily doped than the base because it is physically larger, or assuming mobility keeps rising with temperature by analogy with intrinsic carrier concentration (which does rise with T). Mobility and carrier concentration have opposite temperature dependences above ~300 K in Si.

Q.48 MSQ · 2 MARKS

Engineering Mathematics › Probability & statistics

Q.48 A state transition diagram with states $A, B,$ and $C,$ and transition probabilities $p_1, p_2, \ldots, p_7$ is shown in the figure (*e.g.*, p_1 denotes the probability of transition from state A to B). For this state diagram, select the statement(s) which is/are universally true.



(A) $p_2 + p_3 = p_5 + p_6$

(B) $p_1 + p_3 = p_4 + p_6$

(C) $p_1 + p_4 + p_7 = 1$

/(D) $p_2 + p_5 + p_7 = 1$

Official answer: **A, C**

SOLUTION

1. **The rule.** From any state the chain must go somewhere at the next step, so the probabilities on the arrows **leaving** a state add up to 1. (Each row of the transition matrix sums to 1.) The figure shows every allowed transition, so a move with no arrow has probability 0.
2. **Read the arrows leaving each state from the figure.** Leaving A: p_1 (to B), p_4 (to C), p_7 (back to A). Leaving B: p_2 (back to B), p_3 (to A). Leaving C: p_5 (back to C), p_6 (to A).
3. **The three equations that always hold:**

$$p_1 + p_4 + p_7 = 1 \quad (\text{state } A)$$
$$p_2 + p_3 = 1 \quad (\text{state } B)$$
$$p_5 + p_6 = 1 \quad (\text{state } C)$$

4. **(A)** $p_2 + p_3 = 1 = p_5 + p_6$, so (A) is always true.
5. **(C)** $p_1 + p_4 + p_7 = 1$ is exactly the equation for state A, so (C) is always true.
6. **(B) is false in general.** Take $p_1 = 0.5,\ p_4 = 0.2,\ p_7 = 0.3$ and $p_2 = p_3 = p_5 = p_6 = 0.5$. All three equations hold, but $p_1 + p_3 = 1.0$ while $p_4 + p_6 = 0.7$. The two sides are not equal.
7. **(D) is false in general.** The three self-loops belong to three different states, so no equation ties them together. Take $p_2 = p_5 = p_7 = 0.5$, with $p_3 = p_6 = 0.5$ and $p_1 = p_4 = 0.25$. Every equation holds ($0.25 + 0.25 + 0.5 = 1$ for A), but $p_2 + p_5 + p_7 = 1.5 \neq 1$.
8. So the statements that are always true are **(A)** and **(C)**.

KEY IDEA

In a Markov chain's state diagram, the probabilities on the arrows leaving a state (its self-loop included) add up to 1. Only statements that follow from these per-state equations are universally true. To rule out any other statement, one valid set of numbers that breaks it is enough.

WATCH OUT FOR

Summing the arrows coming INTO a state instead of the ones leaving it. That leads to equations like $p_1 + p_2 = 1$ for B, and it makes (B) or (D) look plausible. A second trap is marking (D) because it collects "all the self-loops". The self-loops belong to different states, so nothing forces them to add up to 1.

Q.49 MSQ · 2 MARKS

Digital Electronics › Boolean logic

Consider a Boolean gate (D) where the output Y is related to the inputs A and B as, $Y = A + \overline{B}$, where $+$ denotes logical OR operation. The Boolean inputs '0' and '1' are also available separately. Using instances of only D gates and inputs '0' and '1', _____ (select the correct option(s)).

(A) NAND logic can be implemented

(B) OR logic cannot be implemented

(C) NOR logic can be implemented

(D) AND logic cannot be implemented

Official answer: **A, C**

SOLUTION

1. Write the gate as a function. Here an overline means complement: $D(A, B) = A + \overline{B}$.
2. Tie input A to the constant '0': $D(0, B) = 0 + \overline{B} = \overline{B}$. One D gate therefore works as an **inverter** (NOT gate).
3. Feed the inverter's output into input B of a second D gate: $D\big(A, D(0, B)\big) = A + \overline{\overline{B}} = A + B$. So **OR can be implemented**, and (B) is false.
4. OR and NOT together form a functionally complete set, so any Boolean function can be built from D gates and the constants. The next three steps give the explicit circuits.
5. NAND, using De Morgan's law $\overline{AB} = \overline{A} + \overline{B}$: $D\big(D(0, A), B\big) = \overline{A} + \overline{B} = \overline{AB}$. So (A) is true.
6. NOR: $D\big(0, A + B\big) = 0 + \overline{A + B} = \overline{A + B}$, where $A + B$ is the OR circuit from step 3. So (C) is true.
7. AND: $D\big(0, \overline{AB}\big) = \overline{\overline{AB}} = AB$, where $\overline{AB}$ is the NAND circuit from step 5. So AND can be implemented, and (D) is false.
8. Correct options: (A) and (C).

KEY IDEA

$D(A,B) = A + \overline{B}$ is the implication gate $B \to A$. On its own it cannot make NOT, because it outputs 1 whenever both inputs are 1. The constant '0' breaks that: $D(0,B) = \overline{B}$. Once you have NOT, $D(A,\overline{B}) = A + B$ gives OR, and {OR, NOT} is functionally complete, so every gate can be built.

WATCH OUT FOR

Stopping at "D is not a universal gate". Implication with no constants is indeed not complete, and that leads students to tick (B) or (D). The question explicitly supplies '0', which turns D into an inverter and makes the set complete. A second trap is reading (B) and (D) as true statements: both say "cannot", and both are false because OR and AND can be built.

Q.50 MSQ · 2 MARKS

Control Systems › Time response

Two linear time-invariant systems with transfer functions

$$G_1(s) = \frac{10}{s^2 + s + 1} \quad \text{and} \quad G_2(s) = \frac{10}{s^2 + s\sqrt{10} + 10}$$

have unit step responses $y_1(t)$ and $y_2(t)$, respectively. Which of the following statements is/are true?

(A) $y_1(t)$ and $y_2(t)$ have the same percentage peak overshoot.

(B) $y_1(t)$ and $y_2(t)$ have the same steady-state value.

(C) $y_1(t)$ and $y_2(t)$ have the same damped frequency of oscillation.

(D) $y_1(t)$ and $y_2(t)$ have the same 2% settling time.

Official answer: **A**

SOLUTION

1. Write each denominator in the standard form $s^2 + 2\zeta\omega_n s + \omega_n^2$. For $G_1(s) = \dfrac{10}{s^2 + s + 1}$: $\omega_n^2 = 1 \Rightarrow \omega_n = 1\ \text{rad/s}$, and $2\zeta\omega_n = 1 \Rightarrow \zeta = 0.5$.
2. For $G_2(s) = \dfrac{10}{s^2 + \sqrt{10}\,s + 10}$: $\omega_n^2 = 10 \Rightarrow \omega_n = \sqrt{10}\ \text{rad/s}$, and $2\zeta\sqrt{10} = \sqrt{10} \Rightarrow \zeta = 0.5$. Both systems are underdamped with the same damping ratio.
3. Percentage peak overshoot is measured relative to the steady-state value, so it does not depend on the gain, and it depends only on ζ:
$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} = e^{-\pi/\sqrt{3}} \approx 0.163$$
That is about 16.3% for both. So (A) is true.
4. Steady-state value of a unit-step response (final value theorem) equals the DC gain $G(0)$: $G_1(0) = \dfrac{10}{1} = 10$, while $G_2(0) = \dfrac{10}{10} = 1$. They differ, so (B) is false.
5. Damped frequency $\omega_d = \omega_n\sqrt{1-\zeta^2}$: $\omega_{d1} = 1 \cdot \dfrac{\sqrt{3}}{2} \approx 0.866\ \text{rad/s}$, while $\omega_{d2} = \sqrt{10} \cdot \dfrac{\sqrt{3}}{2} \approx 2.74\ \text{rad/s}$. They differ, so (C) is false.
6. 2% settling time $t_s \approx \dfrac{4}{\zeta\omega_n}$: $t_{s1} = \dfrac{4}{0.5 \times 1} = 8\ \text{s}$, while $t_{s2} = \dfrac{4}{0.5\sqrt{10}} \approx 2.53\ \text{s}$. The conclusion does not depend on the approximation: with the same ζ, the second response is the first one time-compressed by the factor $\sqrt{10}$, so every time measure (exact settling time included) is $\sqrt{10}$ times smaller. They differ, so (D) is false.
7. Only (A) holds. The two systems share ζ but have different ω_n and different DC gains.

KEY IDEA

For a standard second-order system, percentage overshoot depends only on ζ. The time scale (ω_d, t_s, t_p) is set by ω_n, and the steady-state value is set by the DC gain $G(0)$. Two systems with the same ζ but different ω_n and gain therefore share only the percentage overshoot.

WATCH OUT FOR

Seeing the same numerator (10) in both transfer functions and assuming the steady-state values match, so marking (B). The steady-state value is $G(0)$, numerator divided by the constant term of the denominator: $10/1 = 10$ against $10/10 = 1$. A second slip is thinking a different gain changes the percentage overshoot. It does not, because overshoot is measured relative to the final value.

Q.51 MSQ · 2 MARKS

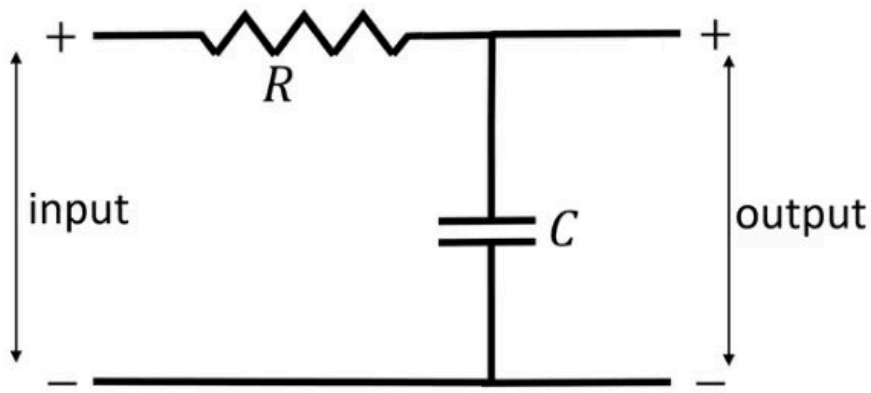
Communications › Analog modulation

Q.51 Consider an FM broadcast that employs the pre-emphasis filter with frequency response

$$H_{pe}(\omega) = 1 + \frac{j\omega}{\omega_0},$$

where $\omega_0 = 10^4$ rad/sec.

For the network shown in the figure to act as a corresponding de-emphasis filter, the appropriate pair(s) of (R, C) values is/are _____.



(A) $R = 1\ k\Omega,\ C = 0.1\ \mu F$

(B) $R = 2\ k\Omega,\ C = 1\ \mu F$

(C) $R = 1\ k\Omega,\ C = 2\ \mu F$

(D) $R = 2\ k\Omega,\ C = 0.5\ \mu F$

Official answer: **A**

SOLUTION

1. A de-emphasis filter must undo the pre-emphasis, so $H_{de}(\omega) = \dfrac{1}{H_{pe}(\omega)} = \dfrac{1}{1 + j\omega/\omega_0}$.
2. The network is a series R with the output taken across the shunt C, so by the voltage divider $H(\omega) = \dfrac{1/(j\omega C)}{R + 1/(j\omega C)} = \dfrac{1}{1 + j\omega RC}$.
3. Comparing the two forms, $RC = \dfrac{1}{\omega_0} = \dfrac{1}{10^4} = 10^{-4}$ s.

4. (A) $1\,\text{k}\Omega \times 0.1\,\mu\text{F} = 10^{3} \times 10^{-7} = 10^{-4}$ s, which matches.
5. (B) $2 \times 10^{3} \times 10^{-6} = 2 \times 10^{-3}$ s, which does not match.
6. (C) $10^{3} \times 2 \times 10^{-6} = 2 \times 10^{-3}$ s, which does not match.
7. (D) $2 \times 10^{3} \times 0.5 \times 10^{-6} = 10^{-3}$ s, which does not match.
8. Only (A) gives $RC = 10^{-4}$ s, so (A) is the only correct pair.

KEY IDEA
De-emphasis is the inverse of pre-emphasis. A series-R, shunt-C low-pass has $H = 1/(1 + j\omega RC)$, so the time constant must equal $1/\omega_0$.

WATCH OUT FOR
Using $RC = \omega_0$ instead of $1/\omega_0$, or reading ω_0 as a frequency in Hz and so multiplying by 2π. Another slip is mixing up μF and nF when multiplying the values.

Q.52 MSQ · 2 MARKS

Electromagnetics › Waveguides & fibres

A waveguide consists of two infinite parallel plates (perfect conductors) at a separation of 10^{-4} cm, with air as the dielectric. Assume the speed of light in air to be 3×10^{8} m/s. The frequency/frequencies of TM waves which can propagate in this waveguide is/are _____.

(A) 6×10^{15} Hz
(B) 0.5×10^{12} Hz
(C) 8×10^{14} Hz
(D) 1×10^{13} Hz

Official answer: **Marks to all**

SOLUTION

1. Plate separation: $d = 10^{-4}\,\text{cm} = 10^{-6}\,\text{m}$. The dielectric is air, so $c = 3 \times 10^{8}\,\text{m/s}$.
2. Put the plates at $y = 0$ and $y = d$. In a TM wave, E_z is tangential to both perfectly conducting plates, so it must be zero at $y = 0$ and $y = d$. That gives $E_z \propto \sin\left(\frac{n\pi y}{d}\right)$ with $n = 0, 1, 2, \ldots$ The phase constant of the TM_n mode is
$$\beta_n = \sqrt{\left(\frac{2\pi f}{c}\right)^2 - \left(\frac{n\pi}{d}\right)^2}$$
It is real, so the mode propagates, only when $f > f_{c,n} = \frac{nc}{2d}$. Below that frequency β_n is imaginary and the mode is evanescent.
3. For $n = 0$ the longitudinal field is $E_z = 0$, so that mode is the TEM mode, not a TM wave. A true TM wave needs $E_z \neq 0$, so the lowest TM mode is TM_1.
4. Cut-off frequency of TM_1:
$$f_{c,1} = \frac{c}{2d} = \frac{3 \times 10^{8}}{2 \times 10^{-6}} = 1.5 \times 10^{14}\,\text{Hz}$$
Every higher TM mode has a higher cut-off, $f_{c,n} = n f_{c,1}$. So a frequency carries some TM wave exactly when $f > 1.5 \times 10^{14}\,\text{Hz}$.
5. (A) $6 \times 10^{15}\,\text{Hz} = 40 \times (1.5 \times 10^{14}\,\text{Hz})$, which is above $f_{c,1}$, so it propagates. TM_1 through TM_{39} all propagate. (C) $8 \times 10^{14}\,\text{Hz} \approx 5.33 \times (1.5 \times 10^{14}\,\text{Hz})$, which is above $f_{c,1}$, so it propagates. TM_1 through TM_5 propagate.
6. (B) $0.5 \times 10^{12}\,\text{Hz}$ and (D) $1 \times 10^{13}\,\text{Hz}$ are both below $1.5 \times 10^{14}\,\text{Hz}$. No TM mode propagates at these frequencies: all of them are evanescent.
7. So the frequencies at which TM waves propagate are (A) and (C).
8. About the official key: the organising institute awarded marks to all candidates for this question and published no reason. One reading changes the answer. Some texts label the TEM mode as "TM_0", with

$f_c = 0$. If TM_0 is counted as a TM wave, every option propagates. Under the usual definition of a TM wave ($E_z \neq 0$), only (A) and (C) propagate.

KEY IDEA

Between parallel plates, the TM_n cut-off is $f_{c,n} = nc/(2d)$. The $n = 0$ mode has $E_z = 0$, which makes it TEM, so the lowest true TM mode is TM_1 with $f_c = c/(2d)$. A TM wave propagates only at frequencies above that cut-off.

WATCH OUT FOR

Leaving the separation in the wrong unit. Taking $d = 10^{-4}\,\text{m}$ instead of $10^{-6}\,\text{m}$ gives $f_c = 1.5 \times 10^{12}\,\text{Hz}$, which wrongly adds (D). Two other errors: using $f_c = c/d$ instead of $c/(2d)$, or counting the TEM ($n = 0$) mode as a TM wave and ticking all four options.

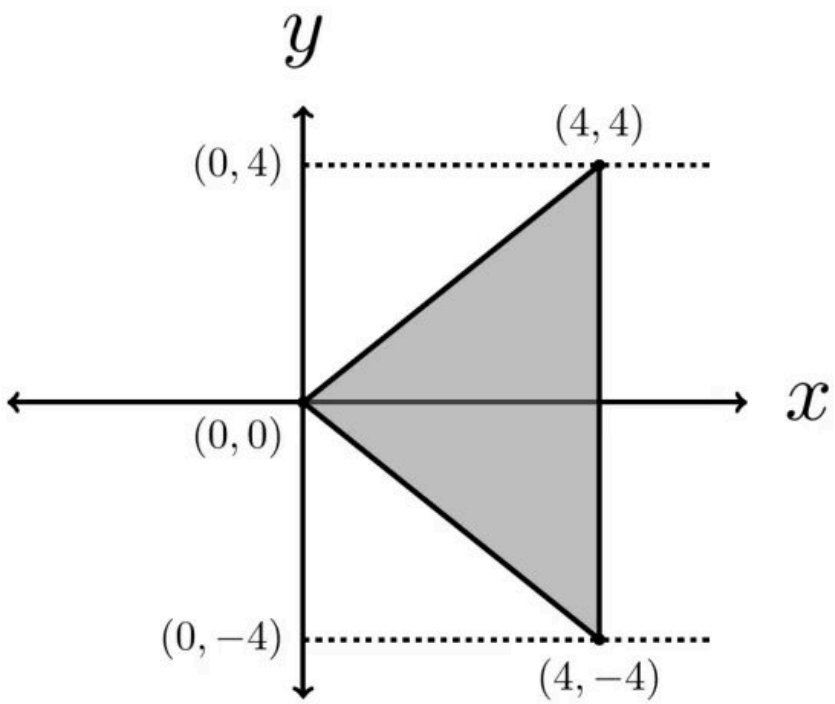
Q.53 NAT · 2 MARKS

Engineering Mathematics › Calculus

Q.53 The value of the integral

$$\iint_D 3(x^2+y^2)dxdy,$$

where D is the shaded triangular region shown in the diagram, is _____ (*rounded off to the nearest integer*).



Official answer: **512**

SOLUTION

1. **Describe the region.** The triangle has vertices $(0,0)$, $(4,4)$ and $(4,-4)$. The line $y=x$ bounds it above, the line $y=-x$ bounds it below, and the line $x=4$ closes it on the right. Cut it into vertical strips: for each x with $0 \le x \le 4$, y runs from $-x$ to x.
2. **Set up the iterated integral.** The integrand is continuous on this closed, bounded region, so by Fubini's theorem we can integrate over y first:
$$I = \int_0^4 \int_{-x}^{x} 3(x^2+y^2)\,dy\,dx$$
3. **Inner integral in y.** The integrand x^2+y^2 is even in y and the limits are symmetric, so
$$\int_{-x}^{x}(x^2+y^2)\,dy = 2\int_0^x (x^2+y^2)\,dy = 2\left(x^2\cdot x + \frac{x^3}{3}\right) = \frac{8x^3}{3}$$

4. **Multiply by the constant 3.** $3 \cdot \dfrac{8x^3}{3} = 8x^3$.

5. **Outer integral in x.**

$$I = \int_0^4 8x^3\,dx = \Big[2x^4\Big]_0^4 = 2 \times 256 = 512$$

6. **Answer.** The value is exactly 512, so rounding changes nothing: $I = 512$.

KEY IDEA

Read the region off the figure and slice it the easy way. With vertical strips, the triangle bounded by $y = x$, $y = -x$ and $x = 4$ becomes $0 \le x \le 4$, $-x \le y \le x$, and one iterated integral covers it. Horizontal strips would split it at $y = 0$ into two pieces. The integrand is even in y and the region is symmetric about the x-axis, so the inner integral is twice the integral from 0 to x.

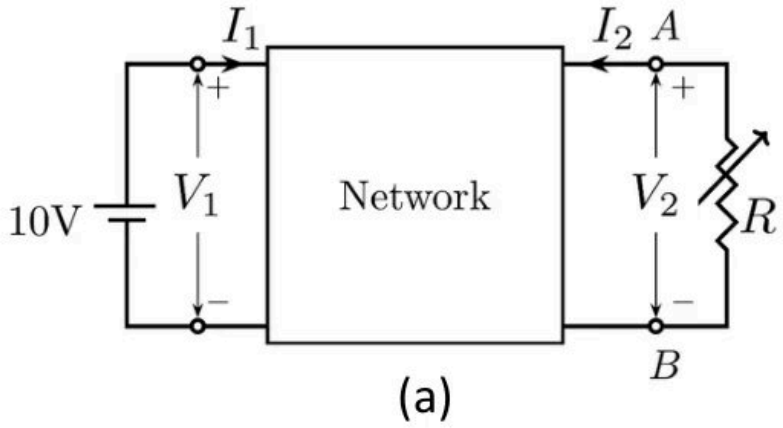
WATCH OUT FOR

Integrating over only the upper half, $0 \le y \le x$, gives 256. So does integrating $x^2 + y^2$ over the whole triangle and forgetting the factor 3 in front (that also gives 256, and 256/3 if only one of the two errors is made). Another slip is using $\int_0^x y^2\,dy = x^2$ in place of $x^3/3$. Some students also try polar coordinates, where $x = 4$ becomes $r = 4\sec\theta$ for $-\pi/4 \le \theta \le \pi/4$. That leads to a harder $\sec^4\theta$ integral with more chances to go wrong.

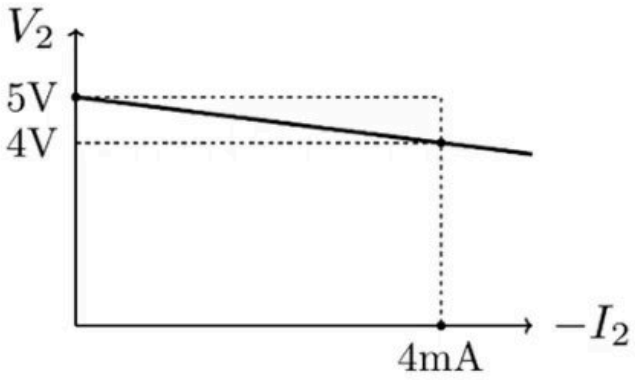
Q.54 NAT · 2 MARKS

Q.54 A linear **2**-port network is shown in Fig. (a). An ideal DC voltage source of **10** V is connected across Port **1**. A variable resistance R is connected across Port **2**. As R is varied, the measured voltage and current at Port **2** is shown in Fig. (b) as a V_2 versus $-I_2$ plot. Note that for $V_2 = 5$ V, $I_2 = 0$ mA, and for $V_2 = 4$ V, $I_2 = -4$ mA.

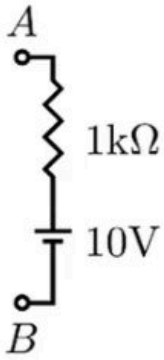
When the variable resistance R at Port **2** is replaced by the load shown in Fig. (c), the current I_2 is _____ mA (*rounded off to one decimal place*).



(a)



(b)



(c)

Official answer: **3.9 to 4.1**

SOLUTION

1. **Port 2 is a Thevenin source.** The network is linear and port 1 is held at a fixed 10 V by an ideal source, so port 2 behaves as a linear one-port. Its V_2–I_2 characteristic is a straight line, as Fig. (b) shows. At $I_2 = 0$ (open circuit), $V_{th} = V_2 = 5$ V, with + at A.
2. **Thevenin resistance from the second point.** I_2 is defined as flowing *into* the network at A, so $I_2 = -4$ mA means 4 mA flows *out* of A into R. That current drops $R_{th}(-I_2)$ across the Thevenin resistance, so $V_2 = V_{th} - R_{th}(-I_2)$ and

$$R_{th} = \frac{5-4}{4\times10^{-3}} = 250\ \Omega$$

This is the magnitude of the slope of the line in Fig. (b).

3. **Port-2 relation.** With I_2 in amperes, taken into the network at A:

$$V_2 = 5 + 250\,I_2$$

Check: $I_2 = -4\times10^{-3}$ gives $V_2 = 5 - 1 = 4$ V, which matches Fig. (b).

4. **Load relation from Fig. (c).** The battery's positive (long) plate faces the resistor side and its negative plate is at B. The current that enters the network at A is the current that leaves the load at A, so it flows up through the load: through the battery from $-$ to $+$ (a 10 V rise), then through the $1\ \text{k}\Omega$ resistor towards A (a drop of $1000\,I_2$). Hence

$$V_2 = V_{AB} = 10 - 1000\,I_2$$

5. **Solve the two equations (one KVL loop).**

$$5 + 250\,I_2 = 10 - 1000\,I_2$$

$$I_2 = \frac{10-5}{1000+250} = \frac{5}{1250} = 4\times10^{-3}\ \text{A} = 4.0\ \text{mA}$$

6. **Check.** $V_2 = 5 + 250(0.004) = 6$ V and $10 - 1000(0.004) = 6$ V, so the two sides agree. I_2 comes out positive because the 10 V load battery is stronger than the 5 V Thevenin source, so it pushes current into the network at A. The answer is $I_2 = 4.0\ \text{mA}$.

KEY IDEA

With port 1 held at a fixed voltage, a linear 2-port looks like a plain Thevenin source at port 2. The V_2 versus $-I_2$ line gives both parameters: its intercept is $V_{th} = 5$ V and the magnitude of its slope is $R_{th} = 250\ \Omega$. Once the Thevenin source is joined to the new load (1 kΩ in series with 10 V), the circuit is a single KVL loop: $I_2 = (10-5)/(1000+250)$.

WATCH OUT FOR

The usual mistake is getting the direction of I_2 wrong. I_2 is drawn *into* the network at A, so the given point $I_2 = -4$ mA means current flows out of the network into R. If you take the port relation as $V_2 = 5 - 250\,I_2$, or add the load battery the wrong way round (treating the sources as series-aiding, for example $(10+5)/1250 = 12$ mA), you get the wrong sign or the wrong magnitude. A second mistake is to write $R_{th} = 4\ \text{V}/4\ \text{mA} = 1\ \text{k}\Omega$ from the second point alone. That is the load resistance at that point, not the Thevenin resistance. R_{th} is the slope, $(5-4)/4$ mA $= 250\ \Omega$.

Q.55 NAT · 2 MARKS

Signals and Systems › DTFT & DFT

For a vector $\bar{x} = [x[0], x[1], \ldots, x[7]]$, the 8-point discrete Fourier transform (DFT) is denoted by $\bar{X} = DFT(\bar{x}) = [X[0], X[1], \ldots, X[7]]$, where

$$X[k] = \sum_{n=0}^{7} x[n] \exp\left(-j\frac{2\pi}{8}nk\right).$$

Here, $j = \sqrt{-1}$. If $\bar{x} = [1, 0, 0, 0, 2, 0, 0, 0]$ and $\bar{y} = DFT(DFT(\bar{x}))$, then the value of $y[0]$ is _____ (rounded off to one decimal place).

Official answer: **7.9 to 8.1**

SOLUTION

1. **Double-DFT property.** Applying the N-point DFT twice gives the input scaled by N and circularly time-reversed:

$$DFT(DFT(x))[n] = N\,x[(-n)_N]$$

This holds because

$$\sum_{k=0}^{N-1} X[k]\, e^{-j\frac{2\pi}{N}nk} = \sum_{m=0}^{N-1} x[m] \sum_{k=0}^{N-1} e^{-j\frac{2\pi}{N}(m+n)k}$$

and the inner sum is N when $m + n \equiv 0 \pmod{N}$ and 0 otherwise.

2. At $n = 0$ the reversal has no effect, because $(-0)_8 = 0$. So
$$y[0] = 8\, x[0] = 8 \times 1 = 8$$
3. **Direct check.** The only non-zero samples are $x[0] = 1$ and $x[4] = 2$, and $e^{-j\frac{2\pi}{8}(4)k} = e^{-j\pi k} = (-1)^k$. So
$$X[k] = 1 + 2(-1)^k$$
4. That gives $\bar{X} = [3, -1, 3, -1, 3, -1, 3, -1]$.
5. **Second DFT at index 0.** At index 0 every exponential equals 1, so the DFT there is just the sum of the samples:
$$y[0] = \sum_{k=0}^{7} X[k] = 4(3) + 4(-1) = 8$$
6. Both methods give the same result: $y[0] = 8.0$.

KEY IDEA

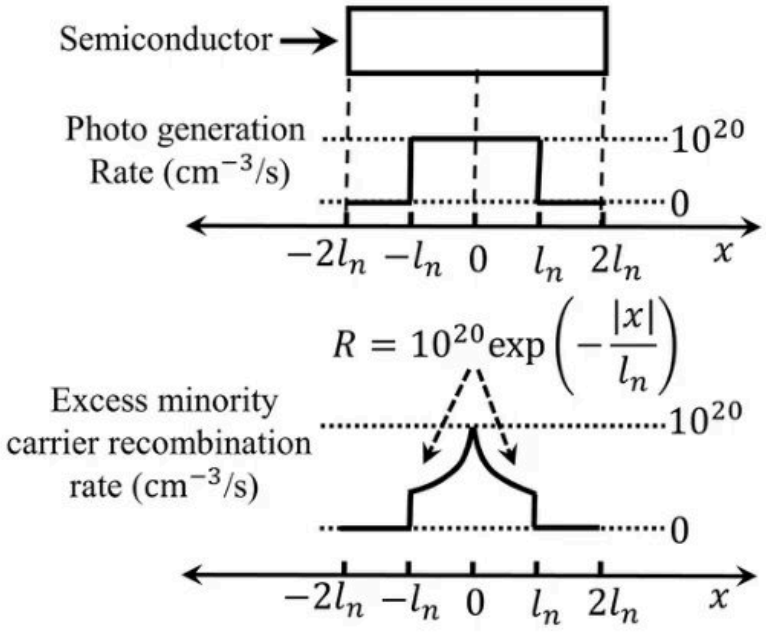
Taking the N-point DFT twice gives $N\, x[(-n)_N]$, the input scaled by N and circularly reversed. Index 0 does not move under that reversal, so $y[0] = N\, x[0]$. Also, the DFT at index 0 is simply the sum of the input samples.

WATCH OUT FOR

Two errors are common. The first is to assume that $DFT(DFT(x))$ gives back x, which leads to the answer 1. That confuses the forward DFT with the inverse DFT, which has the opposite sign in the exponent and a $1/N$ factor. The second is to forget the factor N and use the time-reversal alone. Some students also add $x[4]$ in as well, getting 3 or 24, when $x[4]$ actually lands at $y[4] = 16$ and not at $y[0]$.

Q.56 NAT · 2 MARKS

Q.56 A p-type semiconductor with zero electric field is under illumination (low level injection) in steady state condition. Excess minority carrier density is zero at $x = \pm 2l_n$, where $l_n = 10^{-4}$ cm is the diffusion length of electrons. Assume electronic charge, $q = -1.6 \times 10^{-19}$ C. The profiles of photo-generation rate of carriers and the recombination rate of excess minority carriers (R) are shown. Under these conditions, the magnitude of the current density due to the photo-generated electrons at $x = +2l_n$ is _____ mA/cm^2 (*rounded off to two decimal places*).



Official answer: **0.57 to 0.61**

SOLUTION

1. Read the two profiles from the figure. The generation rate is $G(x) = 10^{20}\ \text{cm}^{-3}\text{s}^{-1}$ for $|x| < l_n$ and 0 for $l_n < |x| < 2l_n$. The recombination rate is $R(x) = 10^{20}e^{-|x|/l_n}\ \text{cm}^{-3}\text{s}^{-1}$ for $|x| < l_n$ and 0 for $l_n < |x| < 2l_n$. It drops to zero as a step at $x = \pm l_n$ and does not decay all the way to $\pm 2l_n$.
2. Write the steady-state continuity equation for electrons, with zero field and $q = 1.6 \times 10^{-19}$ C as a magnitude:
$$0 = \frac{1}{q}\frac{dJ_n}{dx} + G(x) - R(x) \quad \Rightarrow \quad \frac{dJ_n}{dx} = q\,[R(x) - G(x)]$$
3. Use symmetry. G and R are even in x, and the boundary conditions $n'(\pm 2l_n) = 0$ are symmetric too, so the excess electron profile $n'(x)$ is even. The diffusion current $J_n = qD_n\,\dfrac{dn'}{dx}$ is then odd in x, which gives $J_n(0) = 0$.
4. Integrate from 0 to $2l_n$:
$$J_n(2l_n) = J_n(0) + q\int_0^{2l_n} [R(x) - G(x)]\,dx = q\int_0^{2l_n} [R(x) - G(x)]\,dx$$
5. Total recombination per unit area on the right half. R is nonzero only on $0 < x < l_n$:

$$\int_0^{2l_n} R\,dx = \int_0^{l_n} 10^{20} e^{-x/l_n}\,dx = 10^{20} l_n\,(1 - e^{-1})$$

6. Total generation per unit area on the right half:
$$\int_0^{2l_n} G\,dx = \int_0^{l_n} 10^{20}\,dx = 10^{20} l_n$$
7. Take the difference:
$$\int_0^{2l_n} (R - G)\,dx = 10^{20} l_n \left[(1 - e^{-1}) - 1\right] = -10^{20} l_n\, e^{-1}$$
Physically, the electrons generated in $0 < x < 2l_n$ but not recombined there, $10^{20} l_n e^{-1}$ per unit area per second, must leave through $x = 2l_n$. Between l_n and $2l_n$ we have $G = R = 0$, so the current is constant in that region.
8. So $|J_n(2l_n)| = q \times 10^{20} \times l_n \times e^{-1}$, with $q = 1.6 \times 10^{-19}$ C and $l_n = 10^{-4}$ cm:
$$|J_n| = (1.6 \times 10^{-19})(10^{20})(10^{-4})(0.3679)\ \text{A/cm}^2$$
9. Compute: $(1.6 \times 10^{-19})(10^{20}) = 16$, and $16 \times 10^{-4} = 1.6 \times 10^{-3}$. Then $1.6 \times 10^{-3} \times 0.3679 = 5.886 \times 10^{-4}\ \text{A/cm}^2$.
10. Convert: $5.886 \times 10^{-4}\ \text{A/cm}^2 = 0.5886\ \text{mA/cm}^2 \approx 0.59\ \text{mA/cm}^2$.

KEY IDEA

In steady state with no field, the continuity equation says $dJ_n/dx = q(R - G)$. Because the profiles are symmetric, $J_n(0) = 0$. So the current leaving through $x = 2l_n$ is q times the carriers generated but not recombined between 0 and $2l_n$, per unit area per second. You get it by integrating $G - R$; you never need to solve the diffusion equation for $n'(x)$.

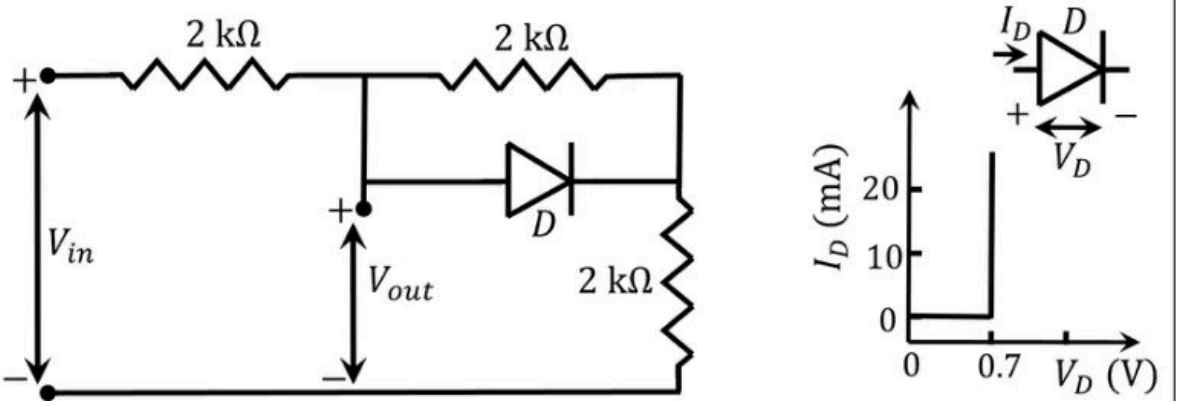
WATCH OUT FOR

The usual error is assuming $R = 10^{20} e^{-|x|/l_n}$ continues to $x = \pm 2l_n$. The figure shows R stepping down to zero at $x = \pm l_n$. Integrating R out to $2l_n$ gives e^{-2} in place of e^{-1} and the wrong answer $0.22\ \text{mA/cm}^2$. A second trap is using $q = -1.6 \times 10^{-19}$ C with its sign and reporting a negative value: the question asks for the magnitude.

Q.57 NAT · 2 MARKS

Analog Electronics › Diode circuits

Q.57 A circuit and the characteristics of the diode (D) in it are shown. The ratio of the minimum to the maximum small signal voltage gain $\frac{\partial V_{out}}{\partial V_{in}}$ is _____ *(rounded off to two decimal places).*



Official answer: **0.70 to 0.80**

SOLUTION

1. **Diode model from the characteristic.** For $V_D < 0.7\,\text{V}$, $I_D = 0$, so the diode is an open circuit and its small-signal resistance is $r_d = \infty$. At $V_D = 0.7\,\text{V}$ the curve is vertical: the voltage stays at 0.7 V

whatever the current, so $r_d = \dfrac{\partial V_D}{\partial I_D} = 0$. For small signals it is a short.

2. **Circuit structure.** V_{in} drives a series $2\,\text{k}\Omega$ resistor to node A, where V_{out} is taken. Between A and B the second $2\,\text{k}\Omega$ resistor sits in parallel with D (anode at A). A third $2\,\text{k}\Omega$ resistor runs from B to ground. So V_{out} is the voltage across ($2\,\text{k}\Omega \parallel$ D) in series with the bottom $2\,\text{k}\Omega$.
3. **Diode OFF.** With $I_D = 0$ the same current $I = \dfrac{V_{in}}{6\,\text{k}\Omega}$ flows through all three resistors. By voltage division
$$\frac{\partial V_{out}}{\partial V_{in}} = \frac{2+2}{2+2+2} = \frac{2}{3} \approx 0.667$$
This holds while $V_{AB} = \dfrac{V_{in}}{3} < 0.7\,\text{V}$, i.e. for $V_{in} < 2.1\,\text{V}$. That range includes every negative V_{in}, where D is reverse biased.
4. **Diode ON ($V_{in} > 2.1\,\text{V}$).** Now D holds $V_A - V_B = 0.7\,\text{V}$. All the current entering A through the series resistor reaches B, through the upper resistor or through D, and then flows through the bottom resistor. So
$$\frac{V_{in} - V_A}{2\,\text{k}\Omega} = \frac{V_B}{2\,\text{k}\Omega} = \frac{V_A - 0.7}{2\,\text{k}\Omega} \;\Rightarrow\; V_A = \frac{V_{in} + 0.7}{2}$$
Consistency check: $I_D = \dfrac{V_B - 0.7}{2\,\text{k}\Omega} = \dfrac{V_{in} - 2.1}{4}\,\text{mA} > 0$ for $V_{in} > 2.1\,\text{V}$, so the diode is indeed conducting.
5. **Small-signal gain with the diode ON.** From $V_{out} = V_A = \dfrac{V_{in} + 0.7}{2}$,
$$\frac{\partial V_{out}}{\partial V_{in}} = \frac{1}{2}$$
The same result follows directly from the small-signal model: $r_d = 0$ shorts the parallel pair, leaving the divider $\dfrac{2}{2+2} = \dfrac{1}{2}$. The constant 0.7 V only shifts the operating point and adds nothing to the incremental gain.
6. **Ratio of minimum to maximum gain.** The gain is $\dfrac{2}{3}$ in one region and $\dfrac{1}{2}$ in the other, so
$$\frac{G_{min}}{G_{max}} = \frac{1/2}{2/3} = \frac{3}{4} = 0.75$$

KEY IDEA

With a piecewise-linear diode, the small-signal gain is constant within each region. Below 0.7 V the diode is an open ($r_d = \infty$). On the vertical part of the curve it is a short ($r_d = 0$). Each case reduces to a plain resistive divider. The fixed 0.7 V drop only sets where the diode switches (at $V_{in} = 2.1\,\text{V}$) and has no effect on either gain.

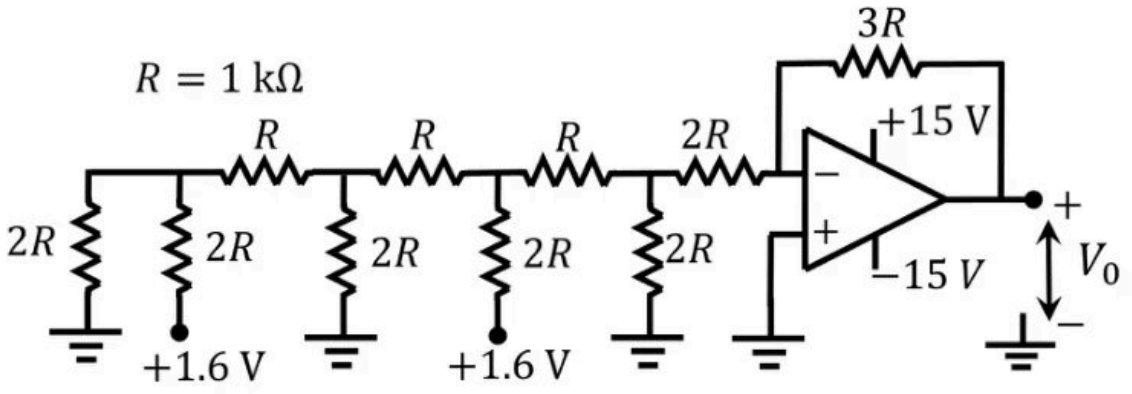
WATCH OUT FOR

Taking V_{out} at node B (across the bottom resistor only) gives gains $\frac{1}{3}$ and $\frac{1}{2}$ and a ratio of 0.67, which is outside the key. The printed V_{out} terminal is at node A, the diode's anode. Other slips are inverting the ratio (max/min = 1.33), or letting the 0.7 V drop enter the incremental gain when it only shifts the operating point.

Q.58 NAT · 2 MARKS

Digital Electronics › ADC & DAC

Q.58 Consider the circuit shown with an ideal OPAMP. The output voltage V_0 is _____ V (*rounded off to two decimal places*).



Official answer: **-0.55 to -0.45**

SOLUTION

1. The op-amp is ideal and its (+) input is grounded, so the (−) input is a virtual ground at 0 V. The ladder's final series $2R$ therefore ends at 0 V. Reduce the ladder from the left, one node at a time, using Thévenin equivalents.
2. Node A has $2R$ to 1.6 V in parallel with $2R$ to ground. Thévenin: $V_{th} = \dfrac{1.6}{2} = 0.8$ V, $R_{th} = 2R \parallel 2R = R$. Adding the series R gives 0.8 V behind $2R$.
3. Node B adds $2R$ to ground: $V_{th} = \dfrac{0.8 + 0}{2} = 0.4$ V, $R_{th} = R$. Adding the series R gives 0.4 V behind $2R$.
4. Node C adds $2R$ to 1.6 V. Two equal resistances average their source voltages (Millman's theorem): $V_{th} = \dfrac{0.4 + 1.6}{2} = 1.0$ V, $R_{th} = R$. Adding the series R gives 1.0 V behind $2R$.
5. Node D adds $2R$ to ground: $V_{th} = \dfrac{1.0 + 0}{2} = 0.5$ V, $R_{th} = R$.
6. This equivalent source drives the final series $2R$ into the virtual ground, so the input current is
$$I = \frac{0.5}{R + 2R} = \frac{0.5\ \text{V}}{3\ \text{k}\Omega} = \frac{1}{6}\ \text{mA}$$
7. No current enters the ideal op-amp, so all of I flows through the feedback $3R$:
$$V_0 = -I \cdot 3R = -\frac{1}{6}\ \text{mA} \times 3\ \text{k}\Omega = -0.5\ \text{V}$$
This is well inside the ± 15 V rails, so the op-amp is not saturated.
8. Check: the voltage at node D is $V_D = 0.5 - IR = 0.5 - \dfrac{1}{6} = \dfrac{1}{3}$ V, and the current in the series $2R$ is $\dfrac{V_D}{2R} = \dfrac{1/3\ \text{V}}{2\ \text{k}\Omega} = \dfrac{1}{6}$ mA, the same current. Full nodal analysis gives the same node voltages: $V_A = \dfrac{2}{3}$, $V_B = \dfrac{8}{15}$, $V_C = \dfrac{2}{3}$, $V_D = \dfrac{1}{3}$ V. So $V_0 = -0.50$ V.

KEY IDEA

This is an R-2R ladder DAC feeding an inverting amplifier. Because the (−) input is a virtual ground, reduce the ladder from its far end. At each node, two equal $2R$ branches combine into a Thévenin source equal to the average of their two voltages, with resistance R, and the next series R turns that back into a source behind $2R$. Each stage halves the weight of every bit to its left. The final current, $V_{th}/(R+2R)$, flows entirely through $3R$, so $V_0 = -3R \cdot I$.

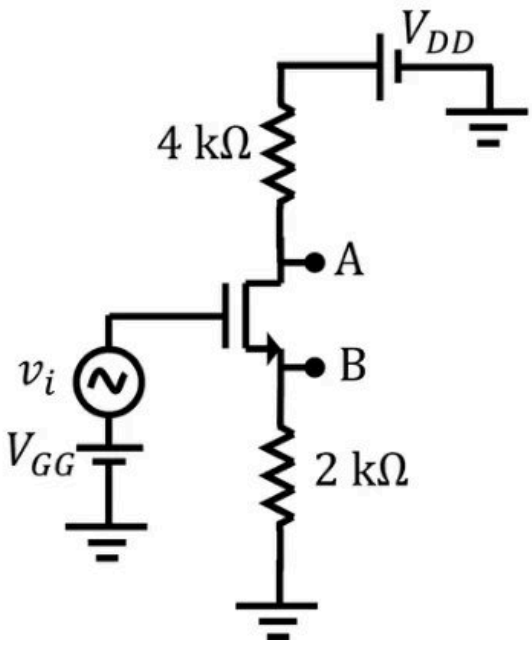
WATCH OUT FOR

The usual mistake is to treat the circuit as a standard DAC in which the last $2R$ goes to ground and the output is taken at node D, then use $V_0 = -\frac{R_f}{R}V_D$ with the wrong resistance. In this circuit the last $2R$ is in series and goes to the virtual ground, so the current is $0.5/(3\ \text{k}\Omega)$, not $0.5/(2\ \text{k}\Omega)$ or $0.5/(1\ \text{k}\Omega)$. A second mistake is to forget the extra $2R$ to ground at node A. Node A has two shunt branches, so its Thévenin voltage is 0.8 V, not 1.6 V. Dropping the minus sign of the inverting amplifier gives $+0.5$, which is outside the key.

Q.59 NAT · 2 MARKS

Analog Electronics › Amplifiers

Q.59 Consider the circuit shown with an ideal long channel *n*MOSFET (enhancement-mode, substrate is connected to the source). The transistor is appropriately biased in the saturation region with V_{GG} and V_{DD} such that it acts as a linear amplifier. v_i is the small-signal ac input voltage. v_A and v_B represent the small-signal voltages at the nodes A and B, respectively. The value of $\frac{v_A}{v_B}$ is _____ (*rounded off to one decimal place*).



Official answer: **-2.1 to -1.9**

SOLUTION

1. Assumptions: the MOSFET is in saturation. It is an ideal long-channel device, so $r_o \to \infty$ (no channel-length modulation). The body is tied to the source, so $v_{bs} = 0$ and there is no body effect ($g_{mb}v_{bs} = 0$). The small-signal model is then a single dependent current source $i_d = g_m v_{gs}$ from drain to source, with the gate open. V_{DD} and V_{GG} are dc sources, so both are ac ground.
2. The gate draws no current and there is no r_o path, so the same small-signal current i_d flows down through the $4\,\text{k}\Omega$ drain resistor, through the channel, and down through the $2\,\text{k}\Omega$ source resistor.

3. Node A (drain): i_d flows from the ac-grounded V_{DD} rail into A through $4\,\text{k}\Omega$. By Ohm's law, $v_A = 0 - i_d(4\,\text{k}\Omega) = -i_d(4\,\text{k}\Omega)$.
4. Node B (source): the same i_d flows from B to ground through $2\,\text{k}\Omega$, so $v_B = +i_d(2\,\text{k}\Omega)$.
5. Take the ratio. i_d cancels, and with it g_m and v_i:
$$\frac{v_A}{v_B} = \frac{-i_d\,(4\,\text{k}\Omega)}{i_d\,(2\,\text{k}\Omega)} = -\frac{R_D}{R_S} = -2.0$$
6. Check with the individual gains. With $v_{gs} = v_i - v_B$ and $v_B = g_m v_{gs} R_S$, we get $v_{gs} = \dfrac{v_i}{1 + g_m R_S}$. So $\dfrac{v_A}{v_i} = -\dfrac{g_m R_D}{1 + g_m R_S}$ and $\dfrac{v_B}{v_i} = \dfrac{g_m R_S}{1 + g_m R_S}$. Their ratio is $-R_D/R_S = -2$ at any bias point, as long as the device stays in saturation.

KEY IDEA

With $r_o \to \infty$ and no body effect, one small-signal current i_d flows through both R_D and R_S. The drain voltage is $-i_d R_D$ (current flows into the node from ac ground) and the source voltage is $+i_d R_S$ (current flows out of the node to ground). So the drain-to-source voltage ratio is simply $-R_D/R_S$. You don't need g_m, the bias point or v_i.

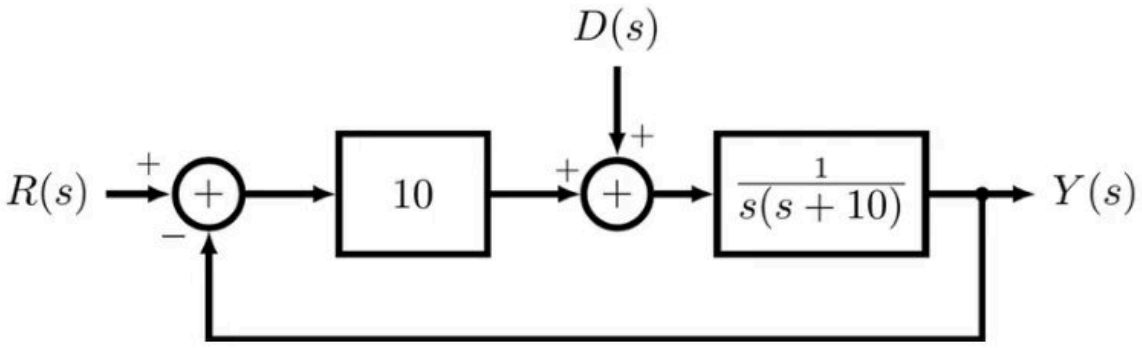
WATCH OUT FOR

Answering $+2$ by dropping the sign: the drain (common-source output) is inverted and the source (source-follower output) is not, so the ratio is negative. Another mistake is working out v_A/v_i or v_B/v_i separately and getting stuck because g_m is not given. It cancels in the ratio. Adding an r_o or a body-effect term also breaks the single-current argument, but the question rules both out ('ideal long channel', 'substrate connected to the source').

Q.60 NAT · 2 MARKS

Control Systems › Time response

Q.60 The block diagram of a closed-loop control system is shown in the figure. $R(s)$, $Y(s)$, and $D(s)$ are the Laplace transforms of the time-domain signals $r(t)$, $y(t)$, and $d(t)$, respectively. Let the error signal be defined as $e(t) = r(t) - y(t)$. Assuming the reference input $r(t) = 0$ for all t, the steady-state error $e(\infty)$, due to a unit step disturbance $d(t)$, is _____ (*rounded off to two decimal places*).



Official answer: **-0.11 to -0.09**

SOLUTION

1. With $r(t) = 0$, the error is $E(s) = R(s) - Y(s) = -Y(s)$. Let the controller gain be $G_c = 10$ and the plant be $G_p(s) = \dfrac{1}{s(s+10)}$.

2. The plant input is the sum of the controller output and the disturbance:
$$Y(s) = G_p(s)\,[D(s) + 10E(s)] = G_p(s)\,[D(s) - 10Y(s)]$$
3. Collecting $Y(s)$ gives the disturbance-to-output transfer function:
$$\frac{Y(s)}{D(s)} = \frac{G_p}{1 + 10G_p} = \frac{1}{s(s+10) + 10} = \frac{1}{s^2 + 10s + 10}$$
4. Stability check before using the final value theorem: $s^2 + 10s + 10 = 0$ gives $s = -5 \pm \sqrt{15} \approx -1.13,\ -8.87$. Both closed-loop poles are in the left half-plane (for a second-order polynomial, all coefficients positive is sufficient), so the final value theorem applies.
5. Final value theorem with $D(s) = \dfrac{1}{s}$:
$$y(\infty) = \lim_{s \to 0} s \cdot \frac{1}{s} \cdot \frac{1}{s^2 + 10s + 10} = \frac{1}{10} = 0.1$$
6. Hence $e(\infty) = r(\infty) - y(\infty) = 0 - 0.1 = -0.10$.

KEY IDEA

Treat the disturbance as the input and find $Y(s)/D(s) = \dfrac{G_p}{1 + G_c G_p}$. Whether a step disturbance is rejected depends on integrators placed BEFORE the point where the disturbance enters, not on the system type as a whole. Here the only integrator is in the plant, after the disturbance, so the step is not rejected: the output settles at $1/G_c = 1/10$, and because $r = 0$ the error is the negative of that, $e(\infty) = -y(\infty) = -0.1$.

WATCH OUT FOR

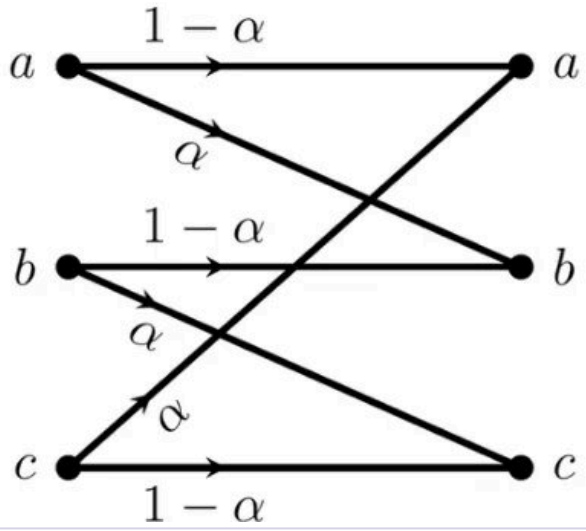
Two errors are common. The first is reporting $+0.10$, which forgets that $e = r - y = -y$ when $r = 0$. The second is answering 0 on the grounds that the loop is type 1. The integrator sits after the disturbance entry point, so it gives zero error for a step REFERENCE but cannot reject a step DISTURBANCE. Only an integrator in the controller, before the disturbance enters, would drive this error to zero.

Q.61 NAT · 2 MARKS

Communications › Information theory

Q.61 The transition diagram of a discrete memoryless channel with three input symbols and three output symbols is shown in the figure. The transition probabilities are as marked.

The parameter α lies in the interval $[0.25, 1]$. The value of α for which the capacity of this channel is maximized, is _____ (*rounded off to two decimal places*).



Official answer: **1.00**

SOLUTION

1. Read the transition probabilities from the figure. From input a: to a with $1-\alpha$, to b with α. From input b: to b with $1-\alpha$, to c with α. From input c: to c with $1-\alpha$, to a with α. With rows as inputs and columns as outputs a, b, c:
$$P = \begin{bmatrix} 1-\alpha & \alpha & 0 \\ 0 & 1-\alpha & \alpha \\ \alpha & 0 & 1-\alpha \end{bmatrix}$$
2. Every row is a permutation of $(1-\alpha,\ \alpha,\ 0)$, and so is every column. The channel is therefore symmetric, and the uniform input distribution $P(x) = \frac{1}{3}$ achieves capacity.
3. With a uniform input, each output has $P(y) = \frac{1}{3}\left[(1-\alpha) + \alpha + 0\right] = \frac{1}{3}$. The output is uniform on three symbols, so $H(Y) = \log_2 3$ bits, the largest value possible.
4. For each input, the output entropy is the entropy of the row $(1-\alpha, \alpha, 0)$, and the zero entry adds nothing:
$$H(Y|X=x) = -(1-\alpha)\log_2(1-\alpha) - \alpha\log_2\alpha = h(\alpha)$$
This is the same for every x, so $H(Y|X) = h(\alpha)$.
5. Capacity:
$$C(\alpha) = \log_2 3 - h(\alpha) \text{ bits per channel use}$$
6. $C(\alpha)$ is largest when $h(\alpha)$ is smallest. The binary entropy $h(\alpha)$ rises from 0 at $\alpha = 0$ to its maximum 1 at $\alpha = 0.5$, then falls back to 0 at $\alpha = 1$.
7. On the allowed interval $[0.25, 1]$: $h(0.25) \approx 0.811$, $h(0.5) = 1$, $h(1) = 0$. Since $h(\alpha) \geq 0$ everywhere and equals 0 only at $\alpha = 1$ in this interval, the minimum is at $\alpha = 1$.
8. At $\alpha = 1$ the channel is the noiseless cyclic map $a \to b,\ b \to c,\ c \to a$, with $C = \log_2 3 \approx 1.585$ bits. Hence capacity is maximized at $\alpha = 1.00$.

KEY IDEA
The channel is symmetric, so the uniform input is optimal and $C = \log_2 3 - h(\alpha)$. Capacity is largest where the binary entropy $h(\alpha)$ is smallest, which on $[0.25, 1]$ is $\alpha = 1$.

WATCH OUT FOR
Setting $\alpha = 0.5$ or looking for a stationary point of $C(\alpha)$. At $\alpha = 0.5$ the binary entropy is largest, so capacity is smallest there. The interval $[0.25, 1]$ excludes $\alpha = 0$, which would be the other noiseless case, so the answer sits at the endpoint $\alpha = 1$.

Q.62 NAT · 2 MARKS

Communications › Error control

Consider communication over a memoryless binary symmetric channel using a (7, 4) Hamming code. Each transmitted bit is received correctly with probability $(1 - \epsilon)$, and flipped with probability ϵ. For each codeword transmission, the receiver performs minimum Hamming distance decoding, and correctly decodes the message bits if and only if the channel introduces at most one bit error. For $\epsilon = 0.1$, the probability that a transmitted codeword is decoded correctly is _____ (rounded off to two decimal places).

Official answer: **0.84 to 0.86**

SOLUTION

1. A $(7, 4)$ Hamming code has $d_{\min} = 3$, so minimum-distance decoding corrects every pattern of at most one bit error. The question states that the message is decoded correctly if and only if the channel introduces at most one error in the 7 transmitted bits.
2. The channel is memoryless, so the 7 bit flips are independent, each with probability ϵ. The number of errors k in a codeword is binomial: $P(k) = \binom{7}{k}\epsilon^k(1-\epsilon)^{7-k}$.
3. $$P_{\text{correct}} = P(0) + P(1) = (1-\epsilon)^7 + 7\epsilon(1-\epsilon)^6$$
4. For $\epsilon = 0.1$: $P(0) = 0.9^7 = 0.478297$.
5. $P(1) = 7(0.1)(0.9)^6 = 0.7 \times 0.531441 = 0.372009$.
6. $$P_{\text{correct}} = 0.478297 + 0.372009 = 0.850306 \approx 0.85$$

KEY IDEA
Decoding is correct exactly when the block has 0 or 1 bit errors, so add the first two terms of a Binomial(7, 0.1) distribution.

WATCH OUT FOR
Keeping only the no-error term $(0.9)^7 \approx 0.48$ and forgetting that the Hamming code also corrects every single-bit error. Another slip is using $\binom{7}{1}$ without the factor $(1-\epsilon)^6$ for the six correct bits.

Q.63 NAT · 2 MARKS

Communications › Digital communication

Consider a channel over which either symbol x_A or symbol x_B is transmitted. Let the output of the channel Y be the input to a maximum likelihood (ML) detector at the receiver. The conditional probability density functions for Y given x_A and x_B are: $f_{Y|x_A}(y) = e^{-(y+1)}u(y+1)$, $f_{Y|x_B}(y) = e^{(y-1)}\,(1 - u(y-1))$, where $u(\cdot)$ is the standard unit step function. The probability of symbol error for this system is _____ (rounded off to two decimal places).

Official answer: **0.22 to 0.25**

SOLUTION

1. Equally likely symbols are assumed (the question gives no priors, and ML detection is the same as MAP for equal priors). The supports are $f_A(y) = e^{-(y+1)}$ for $y > -1$ (zero otherwise) and $f_B(y) = e^{y-1}$ for $y < 1$ (zero otherwise).
2. ML rule: decide x_A if $f_A(y) > f_B(y)$, otherwise decide x_B.
3. For $y < -1$ only f_B is nonzero, so decide x_B. For $y > 1$ only f_A is nonzero, so decide x_A.

4. In the overlap $-1 < y < 1$: $f_A > f_B \iff -(y+1) > y-1 \iff y < 0$. So decide x_A for $-1 < y < 0$ and x_B for $0 < y < 1$.
5. Decision regions: x_A on $(-1, 0) \cup (1, \infty)$ and x_B on $(-\infty, -1) \cup (0, 1)$.
6. Error given x_A: Y lands in the x_B region. Only $(0, 1)$ has nonzero probability under f_A, so
$$P(e|x_A) = \int_0^1 e^{-(y+1)}\, dy = e^{-1}\left(1 - e^{-1}\right) \approx 0.2325.$$
7. Error given x_B: Y lands in the x_A region. Only $(-1, 0)$ has nonzero probability under f_B, so
$$P(e|x_B) = \int_{-1}^0 e^{y-1}\, dy = e^{-1}\left(1 - e^{-1}\right) \approx 0.2325.$$
8. Average over the equal priors:
$$P_e = \tfrac{1}{2}(0.2325) + \tfrac{1}{2}(0.2325) = e^{-1}\left(1 - e^{-1}\right) \approx 0.23.$$

KEY IDEA

The ML detector picks the symbol with the larger likelihood at each y. Compare the two densities on their overlap, then integrate each density over the other symbol's decision region.

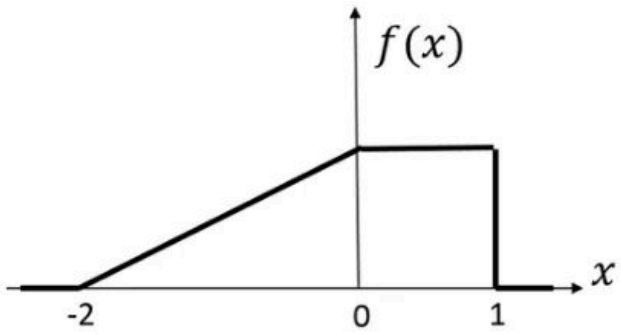
WATCH OUT FOR

Forgetting that each density is zero outside its support. Students then integrate over the wrong region, for example $(-\infty, -1)$ for x_A, or $(1, \infty)$ for x_B, or they pick the wrong threshold. Another slip is to treat the decision boundary as $y = \pm 1$ instead of solving $f_A = f_B$, which gives $y = 0$.

Q.64 NAT · 2 MARKS

Communications › Digital communication

Q.64 Consider a real valued source whose samples are independent and identically distributed random variables with the probability density function, $f(x)$, as shown in the figure.



Consider a 1 bit quantizer that maps positive samples to value α and others to value β. If α^* and β^* are the respective choices for α and β that minimize the mean square quantization error, then $(\alpha^* - \beta^*) =$ _____ (*rounded off to two decimal places*).

Official answer: **1.15 to 1.18**

SOLUTION

1. The total area under the pdf must be 1. The triangle from $x = -2$ to 0 has area $\frac{1}{2} \cdot 2 \cdot h = h$, and the rectangle from 0 to 1 has area $1 \cdot h = h$. So $2h = 1$ and $h = \frac{1}{2}$.
2. The pdf is $f(x) = \dfrac{x+2}{4}$ for $-2 \le x \le 0$ (it rises linearly from 0 to $\frac{1}{2}$), and $f(x) = \frac{1}{2}$ for $0 < x \le 1$. It is zero elsewhere.
3. The quantizer has two cells: $X > 0$ (output α) and $X \le 0$ (output β). For a fixed cell, the mean square error is smallest when the output level is the centroid (conditional mean) of that cell. So $\alpha^* = E[X \mid X > 0]$ and $\beta^* = E[X \mid X \le 0]$.
4. Positive cell: $P(X > 0) = \frac{1}{2} \cdot 1 = \frac{1}{2}$. Here the pdf is constant, so X is uniform on $(0, 1]$ and $\alpha^* = \frac{1}{2}$.
5. Non-positive cell: $P(X \le 0) = \frac{1}{2}$ (the triangle's area). The first moment over this cell is
$$\int_{-2}^{0} x \, \frac{x+2}{4} \, dx = \frac{1}{4}\left[\frac{x^3}{3} + x^2\right]_{-2}^{0} = \frac{1}{4}\left(0 - \left(-\frac{8}{3} + 4\right)\right) = -\frac{1}{3}.$$
6. So $\beta^* = \dfrac{-1/3}{1/2} = -\dfrac{2}{3}$.
7. $\alpha^* - \beta^* = \dfrac{1}{2} + \dfrac{2}{3} = \dfrac{7}{6} \approx 1.17$.

KEY IDEA
For a 1-bit minimum-MSE quantizer, each output level is the centroid (conditional mean) of its cell. The unit area fixes the unlabelled pdf height, then each centroid is a first moment divided by a cell probability.

WATCH OUT FOR
Forgetting to divide the first moment by the cell probability, or taking the midpoint of the triangular part (-1) instead of its centroid at $-\frac{2}{3}$. Another slip is skipping the normalisation step and using a wrong height for the pdf.

Q.65 NAT · 2 MARKS

Electromagnetics › Electrostatics

In an electrostatic field, the electric displacement density vector, $\vec{D}$, is given by

$$\vec{D}(x, y, z) = (x^3\,\vec{i} + y^3\,\vec{j} + xy^2\,\vec{k})\ \mathrm{C/m^2},$$

where $\vec{i}$, $\vec{j}$, $\vec{k}$ are the unit vectors along x-axis, y-axis, and z-axis, respectively. Consider a cubical region R centered at the origin with each side of length 1 m, and vertices at $(\pm 0.5\ \mathrm{m}, \pm 0.5\ \mathrm{m}, \pm 0.5\ \mathrm{m})$. The electric charge enclosed within R is _____ C (rounded off to two decimal places).

Official answer: **0.48 to 0.52**

SOLUTION

1. **Gauss's law, point form:** $\nabla \cdot \mathbf{D} = \rho_v$, the volume charge density in $\mathrm{C/m^3}$. So the enclosed charge is $Q = \int_R (\nabla \cdot \mathbf{D})\, dV$, which by the divergence theorem equals the net outward flux $\oint_S \mathbf{D} \cdot d\mathbf{S}$.
2. **Divergence:**
$$\nabla \cdot \mathbf{D} = \frac{\partial(x^3)}{\partial x} + \frac{\partial(y^3)}{\partial y} + \frac{\partial(xy^2)}{\partial z} = 3x^2 + 3y^2 + 0$$
The z-component xy^2 does not depend on z, so it contributes nothing.
3. **Volume integral over the cube** $-0.5 \le x, y, z \le 0.5$:
$$\int_{-0.5}^{0.5} x^2\, dx = \frac{(0.5)^3 - (-0.5)^3}{3} = \frac{0.25}{3} = \frac{1}{12}$$
and the same value holds for $\int_{-0.5}^{0.5} y^2\, dy$.
4. **Collect the terms:** each side has length $1\ \mathrm{m}$, so the two remaining integrations each contribute a factor of 1:
$$Q = 3 \cdot \frac{1}{12} \cdot 1 \cdot 1 + 3 \cdot \frac{1}{12} \cdot 1 \cdot 1 = \frac{1}{4} + \frac{1}{4} = 0.5\ \mathrm{C}$$
5. **Check by surface flux:** on the faces $x = \pm 0.5$, $D_x = x^3 = \pm 0.125$ and the outward normal is $\pm \hat{a}_x$, so $\mathbf{D} \cdot \hat{n} = +0.125$ on both. Each face has area $1\ \mathrm{m^2}$, so the pair gives $0.25\ \mathrm{C}$. The faces $y = \pm 0.5$ give another $0.25\ \mathrm{C}$ in the same way. On the faces $z = \pm 0.5$, $D_z = xy^2$ is the same on both faces but the outward normals point in opposite directions, so those two fluxes cancel. Total $= 0.50\ \mathrm{C}$, which agrees with the volume integral.

KEY IDEA
Enclosed charge is the volume integral of $\nabla \cdot \mathbf{D}$ (Gauss's law in point form plus the divergence theorem). Here $\nabla \cdot \mathbf{D} = 3x^2 + 3y^2$. Over a cube centred at the origin, $\int_{-0.5}^{0.5} x^2\, dx = 1/12$, so each term gives $1/4$ and $Q = 0.5\ \mathrm{C}$.

WATCH OUT FOR
Integrating from 0 to 1 instead of from -0.5 to 0.5. That gives $\int_0^1 x^2\, dx = 1/3$ and $Q = 2\ \mathrm{C}$, because the cube is centred at the origin, not placed at a corner. Other slips: differentiating xy^2 with respect to x or y instead of z, which adds a spurious term, or using $\int_{-0.5}^{0.5} x^2\, dx = 2(0.5)^3 = 0.25$ and forgetting to divide by 3.

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