IAE ACADEMY INSTITUTE FOR ENGINEERS

GATE CS

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Theory of Computation Formula Sheet

INSIDE

- 28 formulas in 5 topics
- When to use each one
- Taken from the free topic notes on iaeacademy.in

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Regular expressions and finite automata

COMPLEMENT OF A DFA LANGUAGE

$$L(M') = L(M)^c \text{ where } M' = (Q, \Sigma, \delta, q_0, Q \setminus F)$$

Valid only for a complete DFA. Swapping the accepting states of an NFA does not in general complement its language.

PRODUCT CONSTRUCTION (UNION, INTERSECTION, DIFFERENCE)

$$|Q| \le m \cdot n \text{ states for } L(M_1) \cap L(M_2),\ L(M_1) \cup L(M_2),\ L(M_1) \setminus L(M_2)$$

Run complete DFAs with m and n states together; states are pairs. Accept when both components accept (intersection), at least one accepts (union), the first accepts and the second does not (difference), or exactly one accepts (symmetric difference).

NUMBER OF DFAS ON N STATES

$$\#\text{DFAs} = n^{nk} \cdot 2^n$$

For a fixed set of n labelled states, a fixed start state and an alphabet of k symbols: each of the nk transitions has n choices, and F is any of the 2^n subsets of Q. This counts machines, not languages; many machines accept the same language. Example: $n = 2$, $k = 2$ gives $2^4 \cdot 2^2 = 64$.

REMOVING EPSILON-MOVES (EPSILON-NFA TO NFA)

$$\delta'(q, a) = \text{ECLOSE}\Big(\bigcup_{p \in \text{ECLOSE}(q)} \delta(p, a)\Big), \qquad q \in F' \iff \text{ECLOSE}(q) \cap F \neq \emptyset$$

Same states and start state. Each new edge stands for 'ε-moves, then a, then ε-moves'; a state becomes accepting if an accepting state can be reached from it by ε-moves alone.

SIZE BOUND OF THE SUBSET CONSTRUCTION

$$\text{DFA states} \le 2^n \text{ for an NFA with } n \text{ states}$$

The bound is tight: the language 'the n-th symbol from the right is 1' has an $(n+1)$-state NFA whose minimal DFA has 2^n states. Often far fewer subsets are reachable.

MINIMAL DFA SIZE (MYHILL–NERODE)

$$\#\text{states of the minimal complete DFA} = \#\text{equivalence classes of } x \sim_L y \iff \forall z:\ (xz \in L \Leftrightarrow yz \in L)$$

Each class of prefixes is one state. L is regular exactly when there are finitely many classes.

IDENTITIES FOR $\EMPTYSET$ AND $\VAREPSILON$

$$r + \emptyset = r, \quad r\varepsilon = \varepsilon r = r, \quad r\emptyset = \emptyset r = \emptyset, \quad \emptyset^* = \varepsilon^* = \varepsilon, \quad r + r = r$$

Union is commutative and associative; concatenation is associative but not commutative.

IDENTITIES FOR STAR

$$(r^*)^* = r^*, \quad rr^* = r^*r = r^+, \quad \varepsilon + rr^* = r^*, \quad (r + \varepsilon)^* = r^*$$

r^+ means one or more repetitions.

STAR OF A UNION

$$(r + s)^* = (r^*s^*)^* = (r^* + s^*)^* = (r^*s)^*r^* = r^*(sr^*)^*$$

All five denote the strings formed by any mixture of r-strings and s-strings. $(r + s)^*$ is NOT equal to $r^* + s^*$.

SHIFTING IDENTITY

$$r(sr)^* = (rs)^*r$$

Both sides are r followed by alternating s, r blocks. $(rs)^* \neq (sr)^*$ in general.

DISTRIBUTIVITY

$$r(s+t) = rs + rt, \quad (s+t)r = sr + tr$$

Concatenation distributes over union on both sides; union does not distribute over concatenation ($r + st \neq (r + s)(r + t)$ in general).

ARDEN'S RULE

$$R = Q + RP \Rightarrow R = QP^*, \qquad R = Q + PR \Rightarrow R = P^*Q$$

When $\varepsilon \notin L(P)$ the stated solution is the unique one. When $\varepsilon \in L(P)$ it is still a solution (the smallest) but not the only one. The first form fits Arden's method below, where each equation lists the strings that reach a state.

SIZE OF THOMPSON'S MACHINE

$$\#\text{states} \leq 2\,m \text{ for a regular expression with } m \text{ symbols and operators}$$

Linear in the length of the expression. Each symbol (a, ε or $\emptyset$) and each union or star adds two states; concatenation adds none.

OUTPUT LENGTH

$$|\text{out}_{\text{Moore}}(w)| = |w| + 1, \qquad |\text{out}_{\text{Mealy}}(w)| = |w|$$

Holds for every input w because both machines are deterministic and complete.

Regular languages: closure properties and the pumping lemma

COMPLEMENT OF A REGULAR LANGUAGE

$$\overline{L} = \Sigma^* \setminus L: \qquad Q' = Q, \quad F' = Q \setminus F$$

Applies to a total DFA only. If some transitions are missing, first add a dead (trap) state and send every missing transition to it. The complement DFA has the same states, so the minimal total DFAs of L and L^c have the same number of states. Swapping accepting states in an NFA does not complement it: convert the NFA to a DFA first (the subset construction can need up to 2^n states for an n-state NFA).

PRODUCT CONSTRUCTION

$$\delta\big((p,q),a\big) = \big(\delta_1(p,a),\, \delta_2(q,a)\big), \qquad \text{start } (s_1, s_2)$$

For total DFAs with m and n states, the product has mn states. A pair is accepting when both components accept for intersection, when either accepts for union, and when the first accepts and the second does not for difference.

INVERSE HOMOMORPHISM

$$\delta'(q,a) = \hat{\delta}\big(q,\, h(a)\big)$$

Here $\hat{\delta}$ is the extended transition function on strings, with $\hat{\delta}(q,\varepsilon) = q$. The new DFA has the same states, start state and accepting states as the DFA for L, and it accepts $h^{-1}(L) = \{w : h(w) \in L\}$.

PUMPING LEMMA FOR REGULAR LANGUAGES

$$L \text{ regular } \Rightarrow\ \exists\, p \geq 1\ \forall w \in L \text{ with } |w| \geq p\ \ \exists\, x, y, z:\ w = xyz,\ |xy| \leq p,\ |y| \geq 1, \text{ and } xy^iz \in L\ \forall i \geq 0$$

The implication runs one way only. A language can satisfy the conditions and still be non-regular, so the lemma can prove non-regularity but never regularity. For example, $\{a^ib^jc^k : i = 0 \text{ or } j = k\}$ satisfies the lemma with $p = 1$ (pump the first symbol), yet it is not regular: its intersection with ab^*c^* is $\{ab^nc^n : n \geq 0\}$. The pumping length p can be taken as the number of states of any DFA for L.

MYHILL-NERODE THEOREM

$$L \text{ regular} \iff \sim_L \text{ has finitely many equivalence classes}$$

Unlike the pumping lemma, this is an iff. The number of classes equals the number of states of the minimal total DFA for L, counting the dead state if the minimal total DFA has one.

Context-free grammars and push-down automata

DERIVATION LENGTH IN CHOMSKY NORMAL FORM

$$\text{steps to derive } w \text{ in CNF} = 2|w| - 1$$

For $|w| = n \geq 1$ and any derivation of w from S in a CNF grammar: $n-1$ steps use $A \to BC$ and n steps use $A \to a$. In the parse tree there are $n-1$ variable nodes with two variable children and n variable nodes whose only child is a terminal leaf, so $2n-1$ variable nodes, one per step.

DERIVATION LENGTH IN GREIBACH NORMAL FORM

$$\text{steps to derive } w \text{ in GNF} = |w|$$

For $|w| = n \geq 1$; each step adds exactly one terminal.

EQUIVALENCE OF CFGS AND PDAS

$$\text{CFLs} = \text{languages accepted by NPDAs}$$

Every CFG has an equivalent NPDA and every NPDA has an equivalent CFG, with either acceptance mode.

Context-free languages: closure properties and the pumping lemma

INTERSECTION WITH A REGULAR LANGUAGE

$$L \text{ CFL},\ R \text{ regular} \Rightarrow L \cap R \text{ is a CFL}$$

Proved by running a PDA and a DFA together; the same holds for DCFLs. Hence $L - R = L \cap \overline{R}$ is a CFL, but $R - L$ need not be.

PUMPING LEMMA FOR CONTEXT-FREE LANGUAGES

$$L \text{ CFL} \Rightarrow \exists p \geq 1\ \forall z \in L \text{ with } |z| \geq p\ \exists u,v,w,x,y:\ z = uvwxy,\ |vwx| \leq p,\ |vx| \geq 1,\ uv^iwx^iy \in L \text{ for all } i \geq 0$$

A necessary condition only: it can prove a language is not context-free, never that it is. If L is generated by a Chomsky normal form grammar with k variables, $p = 2^k$ works.

Turing machines, decidability and undecidability

LANGUAGE OF A TURING MACHINE

$$L(M) = \{\, w \in \Sigma^* \mid M \text{ accepts } w \,\}$$

Only accepted strings count; strings on which M loops or rejects are outside L(M).

RECURSIVE = RE AND CO-RE

$$L \text{ recursive} \iff L \text{ is RE and } \overline{L} \text{ is RE}$$

Here the complement is taken with respect to Σ^*. To decide, run both recognisers in parallel (alternating steps); exactly one of them accepts: the one for L if $w \in L$, the one for $\overline{L}$ otherwise.

REDUCTION PRINCIPLE (MANY-ONE)

$$A \leq_m B \text{ and } B \text{ decidable} \Rightarrow A \text{ decidable}$$

Use the contrapositive: if $A \leq_m B$ and A is undecidable, B is undecidable. Also: $A \leq_m B$ and B RE implies A RE; $A \leq_m B$ and A not RE implies B not RE. Since $A \leq_m B$ if and only if $\overline{A} \leq_m \overline{B}$, reducing A_{TM} to $\overline{B}$ proves B is not RE. Direction matters.

RICE'S THEOREM

$$P \text{ non-trivial and semantic} \Rightarrow \{\langle M \rangle \mid L(M) \in P\} \text{ undecidable}$$

P is a property of the language L(M). Non-trivial: some TM languages have it and some do not. Semantic: it depends only on L(M), not on the machine's structure.

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