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Sensors and Industrial Instrumentation Formula Sheet

INSIDE

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- When to use each one
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Sensing principles and signal conditioning

NON-LINEARITY

$$\text{Non-linearity } (\%\,\text{FS}) = \frac{\Delta y_{\max}}{y_{FS}} \times 100$$

Delta y_max is the largest deviation of the actual output from the reference straight line.

LOADED POTENTIOMETER

$$\frac{V_o}{V_s} = \frac{x}{1 + \dfrac{R_p}{R_L}\,x(1-x)}$$

R_p is the total track resistance. Reduces to x for R_L infinite. The error is zero at both ends. For R_L much larger than R_p: as a fraction of the ideal reading the error is about (R_p/R_L)x(1-x), largest at x = 0.5; as a fraction of V_s (full scale) it is about (R_p/R_L)x^2(1-x), largest at x = 2/3, where it is about 0.148 R_p/R_L.

GAUGE FACTOR

$$G = \frac{\Delta R/R}{\varepsilon} = 1 + 2\nu + \frac{\Delta\rho/\rho}{\varepsilon}$$

nu is Poisson's ratio of the gauge material. 1 + 2nu is the dimensional effect; the last term is the piezoresistive effect. For metal foil gauges the resistivity term is small and G is about 2. Semiconductor gauges have G of order 100 or more, mainly from the piezoresistive term, but are more temperature-sensitive and less linear.

EQUAL-ARM BRIDGE, SMALL CHANGES

$$V_o \approx \frac{V_s}{4}\left(\frac{\Delta R_1}{R} - \frac{\Delta R_2}{R} + \frac{\Delta R_3}{R} - \frac{\Delta R_4}{R}\right)$$

Valid for all arms initially R and ΔR much less than R; sign of V_o depends on which terminals are taken as output.

QUARTER-BRIDGE NON-LINEARITY

$$\frac{V_{o,\text{exact}}}{V_{o,\text{linear}}} = \frac{1}{1 + \frac{1}{2}\frac{\Delta R}{R}}$$

Equal-arm bridge with one active arm; the fractional error is about (ΔR/R)/2. Half and full bridges with equal and opposite changes, each pair in series across the supply, have no such error; axial-plus-transverse arrangements keep a small one.

THREE-OP-AMP INSTRUMENTATION AMPLIFIER

$$V_o = \left(1 + \frac{2R_1}{R_g}\right)\frac{R_3}{R_2}\,(V_2 - V_1)$$

Input-stage resistors R_1 (two equal), gain resistor R_g, difference stage with matched R_2 at the inputs and R_3 as feedback and ground resistors. Gain is set by R_g alone when R_3/R_2 is fixed.

PARALLEL-PLATE CAPACITANCE

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

Fringing neglected. Sensing by varying plate gap d (output C is non-linear in d), overlap area A (linear in displacement), or dielectric (varying the effective ε_r).

DIFFERENTIAL VARIABLE-GAP CAPACITOR

$$\frac{C_1 - C_2}{C_1 + C_2} = \frac{x}{d_0}$$

Exactly linear in displacement x of the centre plate; needs a ratio-measuring circuit such as a bridge or ratiometric amplifier.

DIFFERENTIAL CAPACITOR IN A BRIDGE

$$V_o = \frac{V_s}{2}\,\frac{C_1 - C_2}{C_1 + C_2} = \frac{V_s}{2}\,\frac{x}{d_0}$$

C_1 and C_2 in series across the AC excitation V_s, the other half of the bridge two equal impedances, V_o measured across the detector diagonal by a high-impedance detector. The phase of V_o (0 or 180 degrees) shows the direction of x.

VARIABLE-RELUCTANCE INDUCTANCE

$$L = \frac{N^2}{\mathcal{R}}, \qquad \mathcal{R} \approx \frac{l_g}{\mu_0 A_g}$$

N turns, l_g the total air-gap length, A_g the gap area; valid when the core's reluctance is negligible beside the gap's. L is then proportional to 1/l_g, so a single-gap sensor is non-linear; a differential (push-pull) pair in a bridge improves linearity, as for the capacitor.

PIEZOELECTRIC VOLTAGE SENSITIVITY

$$g = \frac{d}{\varepsilon_0 \varepsilon_r}, \qquad V = g\,t\,p$$

t is the crystal thickness, p = F/A the applied stress; g in V·m/N. Follows from q = dF and C_p = ε_0 ε_r A/t, for an open-circuit crystal with no cable or amplifier load.

VOLTAGE-AMPLIFIER CONNECTION

$$V = \frac{q}{C_p + C_c + C_a} = \frac{dF}{C_p + C_c + C_a}$$

Mid-band value, leakage neglected. Output changes if the cable is changed; this is the reason a charge amplifier is preferred.

IDEAL CHARGE AMPLIFIER

$$V_o = -\frac{q}{C_f}$$

Op-amp with feedback capacitor C_f; independent of cable capacitance when the op-amp gain is high. Sign is inverting.

CHARGE AMPLIFIER WITH FINITE OP-AMP GAIN

$$V_o = -\frac{A_{ol}\,q}{C_t + (1 + A_{ol})\,C_f}, \qquad C_t = C_p + C_c + C_a$$

A_ol is the op-amp's open-loop gain. When (1 + A_ol)C_f is much larger than C_t, V_o tends to −q/C_f, which is why the cable capacitance hardly matters.

CHARGE AMPLIFIER WITH FEEDBACK RESISTOR

$$\frac{V_o}{q} = -\frac{j\omega R_f}{1 + j\omega R_f C_f}, \qquad f_c = \frac{1}{2\pi R_f C_f}$$

R_f in parallel with C_f gives a high-pass response; at f_c the gain magnitude is 3 dB below its mid-band value 1/C_f. A steady (DC) force gives a charge that leaks away, so true static force cannot be measured.

INDUCED EMF

$$e = -N\frac{d\Phi}{dt}, \qquad e = Blv$$

Faraday's law for a coil of N turns linking flux Φ; Blv for a conductor of length l moving at speed v, with B, l and v mutually perpendicular. Electromagnetic velocity pickups and flowmeters use this principle.

TOOTHED-WHEEL (VARIABLE-RELUCTANCE) TACHOMETER

$$f = \frac{n\,T}{60}$$

A ferromagnetic wheel with T teeth turning at n rev/min past a magnetic pickup gives pulses at frequency f in Hz. The pulse amplitude also rises with speed, so the speed is read from the frequency.

HALL VOLTAGE

$$V_H = R_H\,\frac{IB}{t} = \frac{IB}{n\,q\,t}$$

B perpendicular to the plate and to the current; n is carrier density, q the carrier charge magnitude, so this gives the magnitude of V_H. The sign of R_H follows the carrier sign (negative for electrons), which identifies the carrier type. V_H reverses sign when B (or I) is reversed. Semiconductors give a larger V_H than metals because n is much smaller.

Displacement, velocity, acceleration, force, torque and vibration

LOADED POTENTIOMETER

$$\frac{V_o}{V_s} = \frac{k}{1 + k(1-k)\,R_p/R_L}, \qquad k = \frac{x}{L}$$

R_p is the total track resistance and R_L the load across the wiper. For $R_L \to \infty$ the output is $V_s k$, which is linear. A finite load makes it nonlinear. For $R_L \gg R_p$ the error (unloaded minus loaded output, as a fraction of V_s) is approximately $k^2(1-k)\,R_p/R_L$, largest near $k = 2/3$, where it is about $0.148\,R_p/R_L$.

CAPACITIVE SENSITIVITY AND DIFFERENTIAL CAPACITOR

$$\frac{dC}{dd} = -\frac{\varepsilon_0 \varepsilon_r A}{d^2} = -\frac{C}{d}, \qquad \frac{C_1 - C_2}{C_1 + C_2} = \frac{x}{d_0}$$

The first result is for a change of gap: sensitivity rises as the gap shrinks. The second is for a centre plate moved by x from the middle of two fixed plates, each at gap d_0 when centred, so $C_1 = \varepsilon_0\varepsilon_r A/(d_0 - x)$ and $C_2 = \varepsilon_0\varepsilon_r A/(d_0 + x)$.

ENCODER RESOLUTION

$$\Delta\theta = \frac{360^\circ}{N_{\text{counts}}}, \qquad N_{\text{counts}} = \begin{cases} N & \text{incremental, one edge per slot} \\ 2N & \text{incremental, } \times 2 \text{ decoding} \\ 4N & \text{incremental, quadrature } \times 4 \\ 2^n & \text{absolute, } n \text{ tracks (one bit per track)} \end{cases}$$

N is the number of slots (pulses per revolution per channel). Resolution is the smallest angle that can be distinguished.

TACHOGENERATOR

$$e = k_t\,\omega, \qquad \omega = \frac{2\pi n}{60}$$

ω in rad/s, k_t in V per rad/s, n in rpm.

SPEED FROM AN ENCODER OR A TOOTHED WHEEL

$$\omega = \frac{2\pi f}{N}, \qquad n_{\text{rpm}} = \frac{60\,f}{N}$$

For an encoder, f is the pulse frequency of one channel and N the pulses per revolution of that channel; with $\times 4$ decoding, use the count rate and $4N$. For a variable-reluctance (magnetic) pickup facing a toothed ferromagnetic wheel, each tooth gives one pulse, so N is the number of teeth: $f = Nn/60$.

RELATIVE DISPLACEMENT FOR BASE MOTION Y = Y SIN ΩT

$$\frac{Z}{Y} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}, \qquad r = \frac{\omega}{\omega_n}$$

Z and Y are amplitudes. For $r \gg 1$, $Z/Y \to 1$ and the relative motion is about 180° out of phase with the base motion (vibrometer).

ACCELEROMETER RESPONSE

$$\frac{Z}{Y\omega^2} = \frac{1}{\omega_n^2} \cdot \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

Z is proportional to the acceleration amplitude $Y\omega^2$ when $r \ll 1$, with static sensitivity $1/\omega_n^2$ (relative displacement per unit acceleration). For $\zeta = 1/\sqrt{2}$ the second factor is $1/\sqrt{1+r^4}$.

PHASE LAG OF THE SEISMIC INSTRUMENT

$$\phi = \tan^{-1}\frac{2\zeta r}{1-r^2}$$

The angle by which the relative displacement lags the input acceleration, taken between 0° and 180° (90° at $r = 1$). For $\zeta \approx 0.7$ and $r < 1$, ϕ is nearly proportional to frequency, so a waveform with several frequency components is delayed but not distorted.

PIEZOELECTRIC CHARGE AND OPEN-CIRCUIT VOLTAGE

$$q = d\,F = d\,m\,a, \qquad V = \frac{q}{C_p + C_c + C_a}$$

d is the charge sensitivity in C/N. C_p is the crystal capacitance, C_c the cable capacitance and C_a the amplifier input capacitance. Cable length changes the voltage sensitivity, so a charge amplifier ($V_o = -q/C_f$) is used to remove that dependence.

VOLTAGE SENSITIVITY OF A CRYSTAL

$$g = \frac{d}{\varepsilon_0 \varepsilon_r}, \qquad V = g\,t\,\frac{F}{A}$$

g is the voltage coefficient in V m/N, t the crystal thickness and F/A the applied stress. This is the open-circuit voltage of the crystal alone, q/C_p with $C_p = \varepsilon_0 \varepsilon_r A/t$.

LOW-FREQUENCY CUT-OFF OF A PIEZOELECTRIC SENSOR CIRCUIT

$$f_L = \frac{1}{2\pi RC}, \qquad f_{L,\text{charge amp}} = \frac{1}{2\pi R_f C_f}$$

For a voltage amplifier, R is the combined leakage and input resistance and C the total capacitance of sensor, cable and amplifier input. With a charge amplifier, the feedback resistor R_f and capacitor C_f set the −3 dB point.

SINUSOIDAL VIBRATION AMPLITUDES

$$x = X\sin\omega t: \qquad v_{\max} = \omega X = 2\pi f X, \qquad a_{\max} = \omega^2 X = 4\pi^2 f^2 X$$

X is the displacement amplitude and f the vibration frequency in Hz.

GAUGE FACTOR

$$G = \frac{\Delta R/R}{\varepsilon} = 1 + 2\nu + \frac{\Delta\rho/\rho}{\varepsilon}$$

ν is Poisson's ratio of the gauge material and $\Delta\rho/\rho$ the fractional change in its resistivity (the piezoresistive effect). For metal-foil gauges $G \approx 2$; semiconductor gauges have a much larger, temperature-sensitive G.

COLUMN LOAD CELL

$$\varepsilon = \frac{\sigma}{E} = \frac{F}{AE}, \qquad \frac{\Delta R}{R} = G\,\varepsilon$$

Axial load F on cross-section A of Young's modulus E, within the elastic limit. A gauge bonded across the column sees the transverse strain $-\nu\varepsilon$.

STRAIN-GAUGE BRIDGE OUTPUT

$$V_o = \begin{cases} \dfrac{V_s\,G\varepsilon}{4}\cdot\dfrac{1}{1+G\varepsilon/2} \approx \dfrac{V_s\,G\varepsilon}{4} & \text{one active arm} \\ \dfrac{V_s\,G\varepsilon}{2} & \text{two active arms, } \pm G\varepsilon \text{ (adjacent)} \\ V_s\,G\varepsilon & \text{four active arms, each } \pm G\varepsilon \\ \dfrac{V_s\,G\varepsilon(1+\nu)}{2} & \text{column: two axial and two transverse gauges} \end{cases}$$

Equal arms, bridge balanced at zero strain, signs of the active arms arranged so their effects add. The half- and full-bridge results with equal and opposite strains are exactly linear. Temperature effects cancel when identical gauges at the same temperature sit in adjacent arms (a dummy gauge does this for one active gauge).

SOLID CIRCULAR SHAFT UNDER TORQUE T

$$\tau = \frac{16\,T}{\pi d^3}, \qquad \varepsilon_{45^\circ} = \pm\frac{\tau}{2G_s} = \pm\frac{8\,T}{\pi d^3 G_s}$$

d is the shaft diameter in metres, T in N m, G_s in Pa. Mechanical power transmitted is $P = T\omega$.

Pressure and flow measurement

ABSOLUTE AND GAUGE PRESSURE

$$p_{abs} = p_{gauge} + p_{atm}$$

Gauge pressure is negative below atmospheric; absolute pressure is never negative.

HYDROSTATIC PRESSURE

$$p = \rho g h$$

SI units. p is the pressure difference between two points a vertical height h apart in a fluid of density ρ at rest.

U-TUBE MANOMETER

$$\Delta p = (\rho_m - \rho_f)\,g\,h$$

Both pressure taps at the same elevation. ρ_m is the manometer liquid, ρ_f the fluid above it in the limbs, h the difference of levels. For a gas, $\rho_f \ll \rho_m$ and $\Delta p \approx \rho_m g h$.

INCLINED-TUBE MANOMETER

$$\Delta p = \rho g L\left(\sin\theta + \frac{a}{A}\right) \;\approx\; \rho g L \sin\theta \quad (A \gg a)$$

L is the reading along the inclined tube of area a, θ its angle to the horizontal, A the area of the well; a gas sits above the liquid (otherwise use $\rho_m - \rho_f$). For $A \gg a$ the vertical height is $h = L\sin\theta$, so the reading is $1/\sin\theta$ times what a vertical tube would show. The magnification rises as θ falls, so it suits small pressures.

WELL-TYPE MANOMETER

$$\Delta p = \rho g h\left(1 + \frac{a}{A}\right)$$

h is the rise read on the vertical tube of area a, A the area of the well. The well level falls by $x = ah/A$; only the tube reading is taken. For $A \gg a$ it reduces to $\rho g h$.

FLAT CLAMPED DIAPHRAGM, CENTRE DEFLECTION

$$y_c = \frac{3\,(1-\nu^2)\,p\,R^4}{16\,E\,t^3}$$

Circular flat diaphragm clamped at its edge, radius R, thickness t, Young's modulus E, Poisson's ratio ν, uniform pressure difference p. Valid while the deflection is small compared with the thickness; the deflection is then linear in p.

MCLEOD GAUGE (SQUARE-LAW READING)

$$p = \frac{a h^2}{V - a h} \approx \frac{a h^2}{V} \quad (a h \ll V)$$

The gas of total trapped volume V at unknown pressure p is compressed into a closed capillary of area a. The mercury in the open reference capillary is raised level with the closed end, so the trapped gas column length equals the level difference h. Boyle's law gives $pV = (p + h)\, ah$ with pressures in mmHg; use consistent units (a in mm², h in mm, V in mm³ gives p in mmHg).

DEAD-WEIGHT TESTER

$$p = \frac{m\, g}{A_p}$$

m is the total mass of the piston and the weights on it, A_p the effective piston area; the oil pressure that floats the piston is applied to the gauge under test. SI units.

CONTINUITY

$$Q = A\, v, \qquad \dot{m} = \rho Q = \rho A v$$

Q volumetric flow rate, v mean velocity, $\dot{m}$ mass flow rate.

BERNOULLI (HORIZONTAL)

$$p_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \tfrac{1}{2}\rho v_2^2$$

Steady, incompressible, frictionless flow along a streamline at the same elevation.

ORIFICE, VENTURI AND NOZZLE FLOW RATE

$$Q = \frac{C_d\, A_2}{\sqrt{1 - \beta^4}} \sqrt{\frac{2\, \Delta p}{\rho}}$$

A_2 is the throat or orifice area, $\beta = d/D$, ρ the fluid density; incompressible fluid. $1/\sqrt{1 - \beta^4}$ is the velocity-of-approach factor, and $K = C_d/\sqrt{1 - \beta^4}$ is called the flow coefficient. Q is proportional to $\sqrt{\Delta p}$. For a gas with an appreciable pressure drop, multiply by an expansibility factor $\varepsilon < 1$.

VARIABLE-HEAD METER WITH A U-TUBE MANOMETER

$$Q = \frac{C_d\, A_2}{\sqrt{1 - \beta^4}} \sqrt{2\, g\, h \left(\frac{\rho_m - \rho}{\rho} \right)}$$

h is the manometer level difference, ρ_m the manometer liquid, ρ the flowing liquid filling the leads; from $\Delta p = (\rho_m - \rho) g h$. The manometer reads the piezometric difference, so the same relation holds for an inclined or vertical meter.

PITOT TUBE

$$v = C_p \sqrt{\frac{2\,(p_0 - p_s)}{\rho}}$$

p_0 stagnation pressure, p_s static pressure, $C_p \approx 1$; incompressible flow (low Mach number for a gas). Gives the local velocity at the tip, not the mean velocity.

ROTAMETER

$$Q = C_d\, A_a \sqrt{\frac{2\, g\, V_b\, (\rho_b - \rho)}{A_b\, \rho}}$$

A_a annular area between float and tube, V_b float volume, A_b float cross-section, ρ_b float density, ρ fluid density. With a small linear taper, and the bore at the zero mark equal to the float diameter, A_a and so Q are nearly proportional to float height, giving a nearly linear scale. The calibration holds only for the fluid density (and viscosity) it was made for.

ELECTROMAGNETIC FLOWMETER

$$e = B\,D\,v, \qquad Q = \frac{\pi D^2}{4}\,v = \frac{\pi D\,e}{4B}$$

Uniform field B; v is the mean velocity for axisymmetric flow; SI units (B in tesla). The emf is linear in flow and, above the minimum conductivity, independent of density, viscosity, temperature and pressure.

ULTRASONIC TRANSIT-TIME

$$\Delta t = t_{up} - t_{down} = \frac{2Lv\cos\theta}{c^2 - v^2\cos^2\theta} \approx \frac{2Lv\cos\theta}{c^2}$$

From $t_{down} = L/(c + v\cos\theta)$ and $t_{up} = L/(c - v\cos\theta)$. Since $v \ll c$ the approximation is used, so $v = c^2\Delta t/(2L\cos\theta)$. It needs a clean fluid, and this form depends on c, which varies with temperature.

TRANSIT-TIME, RECIPROCAL FORM

$$\frac{1}{t_{down}} - \frac{1}{t_{up}} = \frac{2v\cos\theta}{L}$$

Exact, and independent of the speed of sound c, so a meter computing v this way is not affected by temperature changes in c.

ULTRASONIC DOPPLER

$$\Delta f = \frac{2f_0\,v\cos\theta}{c}, \qquad v = \frac{c\,\Delta f}{2f_0\cos\theta}$$

f_0 is the transmitted frequency and θ the angle between the beam and the flow, with transmitter and receiver both at that angle. The beam is reflected by particles or bubbles moving with the fluid, so the fluid must contain scatterers.

TURBINE FLOWMETER

$$f = K\,Q, \qquad Q = \frac{f}{K}$$

f is the pulse frequency, K the K-factor in pulses per unit volume; the total count gives total volume. Linear over its working range; needs clean, low-viscosity fluid (K changes with viscosity), and wears in the bearings.

Temperature measurement

TEMPERATURE SCALE CONVERSIONS

$$T(\mathrm{K}) = t(^\circ\mathrm{C}) + 273.15, \qquad t(^\circ\mathrm{F}) = \tfrac{9}{5}\,t(^\circ\mathrm{C}) + 32$$

A change of 1 K equals a change of 1 °C and 1.8 °F. Use 273 when the question does.

THERMOCOUPLE EMF, QUADRATIC APPROXIMATION

$$E_{AB}(T) = aT + bT^2 \quad (\text{reference at } 0\,^\circ\mathrm{C}), \qquad \frac{dE}{dT} = a + 2bT$$

T in °C, a and b depend on the pair. dE/dT is the thermoelectric sensitivity (Seebeck coefficient of the pair). For hot junction T_1 and cold junction T_2: E = E(T_1) - E(T_2).

NEUTRAL AND INVERSION TEMPERATURES

$$T_n = -\frac{a}{2b}, \qquad T_i = 2T_n - T_c = -\frac{a}{b} - T_c$$

For E = aT + bT^2 with a > 0 and b < 0. The emf is maximum when the hot junction is at T_n (where dE/dT = 0), whatever the cold-junction temperature T_c. It returns to zero at the inversion temperature T_i, which lies as far above T_n as T_c lies below it. With the cold junction at 0 °C, T_i = -a/b = 2T_n.

THERMOPILE

$$E_{pile} = N\,E_{single}$$

N identical thermocouples in series, with all hot junctions at the hot temperature and all cold junctions at the cold one. Output and sensitivity are N times those of one thermocouple. It is used to detect radiant power.

RTD LINEAR LAW

$$R_T = R_0\,(1 + \alpha T)$$

T in °C, α referred to 0 °C. Good over a modest range.

CHANGING THE REFERENCE TEMPERATURE OF A

$$\alpha_{T_1} = \frac{\alpha_0}{1 + \alpha_0 T_1}, \qquad R_T = R_{T_1}\left[1 + \alpha_{T_1}(T - T_1)\right]$$

α_0 is referred to 0 °C and α_{T1} to T_1 (°C). Both describe the same linear law. For example, α_0 = 0.00385 /°C gives α_20 = 0.00385/1.077 = 0.003575 /°C.

CALLENDAR-VAN DUSEN FORM

$$R_T = R_0\left(1 + AT + BT^2\right)\ (T \geq 0\,^\circ\mathrm{C}), \quad R_T = R_0\left[1 + AT + BT^2 + C(T - 100)T^3\right]\ (T < 0\,^\circ\mathrm{C})$$

For industrial platinum A = 3.9083 x 10^-3 /°C, B = -5.775 x 10^-7 /°C^2, C = -4.183 x 10^-12 /°C^4. Sensitivity falls slightly as T rises because B is negative.

LEAD-WIRE ERROR OF A 2-WIRE RTD

$$\Delta T_{\text{2-wire}} = \frac{2R_L}{R_0\,\alpha}$$

R_L is the resistance of each lead. The lead resistance is assumed constant and the linear law is used.

RTD IN A WHEATSTONE BRIDGE

$$V_o = V_s\left[\frac{R_T}{R_T + R_0} - \frac{1}{2}\right] = \frac{V_s\,\alpha T}{2\,(2 + \alpha T)} \approx \frac{V_s\,\alpha T}{4}$$

Bridge with supply V_s, the RTD in one arm and the other three arms equal to R_0, so it balances at 0 °C. The approximation needs αT much less than 2. The exact output is slightly non-linear in T.

NTC THERMISTOR LAW

$$R = R_0\,\exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$

R_0 is the resistance at reference temperature T_0 (kelvin), β is the material constant in kelvin. R falls as T rises.

B FROM TWO READINGS

$$\beta = \frac{\ln(R_1/R_2)}{\dfrac{1}{T_1} - \dfrac{1}{T_2}}$$

T_1, T_2 in kelvin, R_1 and R_2 the resistances at them.

THERMISTOR TEMPERATURE COEFFICIENT

$$\alpha_T = \frac{1}{R}\frac{dR}{dT} = -\frac{\beta}{T^2}$$

Negative for NTC, and its magnitude falls as T rises. At room temperature it is a few per cent per kelvin.

STEINHART-HART EQUATION

$$\frac{1}{T} = A + B\ln R + C\,(\ln R)^3$$

T in kelvin, R in ohms. A, B and C are found from three calibration points. It fits a thermistor more closely over a wide range than the single-β law.

PTAT VOLTAGE

$$\Delta V_{BE} = \frac{kT}{q} \ln n$$

For two matched transistors at current-density ratio n. kT/q is about 25.85 mV at 300 K. The output is linear in absolute temperature.

STEFAN-BOLTZMANN LAW

$$W = \varepsilon \sigma T^4, \qquad \sigma = 5.67 \times 10^{-8}\ \mathrm{W\,m^{-2}K^{-4}}$$

T in kelvin. With surroundings at T_s, the net emission is εσ(T^4 - T_s^4).

EMISSIVITY CORRECTION (TOTAL RADIATION)

$$T = \frac{T_a}{\varepsilon^{1/4}}$$

T_a is the black-body (apparent) temperature the pyrometer shows, T is the true surface temperature in kelvin. Valid when radiation from the surroundings is neglected and ε is the total emissivity.

WIEN'S DISPLACEMENT LAW

$$\lambda_{max}\, T = 2898\ \mu\mathrm{m\,K}$$

The wavelength at which a black body's spectral emission peaks falls as its temperature rises. T in kelvin.

EMISSIVITY CORRECTION (OPTICAL PYROMETER)

$$\frac{1}{T} = \frac{1}{T_a} + \frac{\lambda}{C_2} \ln \varepsilon_\lambda$$

T_a is the brightness temperature read, ε_λ the spectral emissivity at wavelength λ, temperatures in kelvin. Since ln ε_λ < 0, the true temperature is above the reading.

TWO-COLOUR (RATIO) PYROMETER

$$\frac{L_{\lambda_1}}{L_{\lambda_2}} = \frac{\varepsilon_1}{\varepsilon_2} \left(\frac{\lambda_2}{\lambda_1}\right)^5 \exp\left[-\frac{C_2}{T}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)\right]$$

Ratio of spectral radiances at two wavelengths, under the Wien approximation. For a grey body (ε_1 = ε_2) the emissivity cancels, so the temperature follows from the ratio alone.

FIRST-ORDER SENSOR MODEL

$$\tau \frac{dT_m}{dt} + T_m = T_f, \qquad \tau = \frac{mc}{hA}$$

T_m is the sensor temperature, T_f the fluid temperature, m the mass, c the specific heat, h the film coefficient, A the surface area. A small, light sensor has a small τ.

STEP RESPONSE

$$T_m(t) = T_f + (T_0 - T_f)\, e^{-t/\tau}$$

The fluid steps from the initial temperature T_0 to T_f at t = 0. The sensor reaches 63.2 % of the step at t = τ, 90 % at 2.303τ and about 98 % at 4τ.

RAMP RESPONSE (STEADY-STATE LAG)

$$T_f(t) = T_0 + r\,t \;\Rightarrow\; T_f - T_m \to r\,\tau$$

For a fluid temperature rising at a steady rate r (°C/s), after the transient dies away the reading lags the fluid by rτ in temperature, which is a time lag of τ.

Level, pH, conductivity and viscosity

HYDROSTATIC HEAD

$$\Delta P = \rho\, g\, h$$

Open tank, high side to the tank, low side to atmosphere; h is the liquid height above the tapping point. If the transmitter sits a height h_0 below the tapping point (impulse line full of the same liquid), $\Delta P = \rho g(h + h_0)$, and the transmitter is given a zero suppression of $\rho g h_0$ so that empty still reads 4 mA.

BUOYANT FORCE ON A DISPLACER

$$F_b = A\,\rho\, g\, h$$

A is the displacer's uniform cross-sectional area and h the submerged length (partly submerged); the buoyancy of the vapour above the liquid is neglected. Apparent weight $= W - A\rho g h$, where W is the displacer's weight in air.

COAXIAL CAPACITIVE LEVEL PROBE

$$C = \frac{2\pi\varepsilon_0}{\ln(b/a)}\,[L + (\varepsilon_r - 1)\,h]$$

Insulating liquid, $0 \le h \le L$, gas above the liquid taken as $\varepsilon_r = 1$, end (fringing) effects neglected. Capacitance is linear in h; at $h = 0$ it is the empty value $2\pi\varepsilon_0 L/\ln(b/a)$.

ECHO-TIME LEVEL (ULTRASONIC OR RADAR)

$$h = H - \frac{v\,t}{2}$$

t is the round-trip time and v the wave speed in the space above the liquid: speed of sound for ultrasonic, speed of light for radar.

NERNST EQUATION FOR A PH ELECTRODE

$$E = E_0 - \frac{2.303\,R\,T}{F}\,\mathrm{pH}$$

$R = 8.314$ J/(mol K), $F = 96485$ C/mol, T in kelvin. E_0 is the potential at pH 0 and combines the reference and internal potentials. An ideal electrode with its isopotential point at pH 7 and zero offset gives $E = S(7 - \mathrm{pH})$, that is, 0 mV at pH 7.

ELECTRODE SLOPE

$$S = \frac{2.303\,R\,T}{F} \approx 0.1984\,T\ \mathrm{mV\ per\ pH}$$

T in kelvin. At 25 °C (298.15 K) the slope is about 59.16 mV per pH; at 0 °C about 54.2 mV; at 50 °C about 64.1 mV.

CELL CONSTANT AND CONDUCTIVITY

$$K = \frac{l}{A}, \qquad \kappa = K\,G = \frac{K}{R}$$

K in m^{-1}, R the measured cell resistance in ohms, κ in S/m. Unit conversion: 1 S/m $= 10^4$ μS/cm.

CAPILLARY VISCOMETER (POISEUILLE)

$$Q = \frac{\pi r^4\,\Delta P}{8\,\mu\,L} \quad \Rightarrow \quad \mu = \frac{\pi r^4\,\Delta P}{8\,Q\,L}$$

Steady, fully developed laminar flow of a Newtonian liquid in a tube of radius r and length L under pressure drop ΔP; Q is the volume flow rate.

FALLING-SPHERE VISCOMETER (STOKES)

$$v_t = \frac{2\,r^2\,(\rho_s - \rho_f)\,g}{9\,\mu} \quad \Rightarrow \quad \mu = \frac{2\,r^2\,(\rho_s - \rho_f)\,g}{9\,v_t}$$

Sphere radius r and density ρ_s falling at constant terminal speed v_t in an unbounded Newtonian liquid of density ρ_f; valid for small Reynolds number ($\rho_f v_t (2r)/\mu$ below about 1, creeping flow). Near a container wall the sphere falls slower, so a wall correction is needed.

Instrument networks and the 4-20 mA transmitter

MEASURED VALUE TO LOOP CURRENT

$$I = 4 + 16\,\frac{x - \mathrm{LRV}}{\mathrm{URV} - \mathrm{LRV}}\ \mathrm{mA}$$

Linear transmitter, 4 mA at LRV and 20 mA at URV; I in mA.

LOOP CURRENT TO MEASURED VALUE

$$x = \mathrm{LRV} + \frac{I - 4}{16}\,(\mathrm{URV} - \mathrm{LRV})$$

Inverse of the relation above; valid for 4 mA to 20 mA.

PERCENT OF SPAN FROM CURRENT

$$\text{Percent of span} = \frac{I - 4}{16} \times 100$$

0 % at 4 mA, 50 % at 12 mA, 100 % at 20 mA.

RECEIVER VOLTAGE

$$V_R = I\,R_R$$

A 250 Ω receiver turns 4 mA to 20 mA into 1 V to 5 V.

SUPPLY VOLTAGE REQUIREMENT

$$V_s \geq V_{T,\min} + I_{\max}\,(R_{\text{wire}} + R_{\text{receivers}})$$

I_max = 20 mA unless an over-range current is stated; all series resistances are included.

MAXIMUM TOTAL LOOP RESISTANCE

$$R_{L,\max} = \frac{V_s - V_{T,\min}}{I_{\max}}$$

R_L is wire plus receiver resistance in series; with V in volts and I in amperes, R is in ohms.

FLOW FROM DIFFERENTIAL PRESSURE

$$Q = Q_{\max}\sqrt{\frac{\Delta p}{\Delta p_{\max}}}$$

Head-type primary element, turbulent flow, constant fluid density; Q_max is the flow at full-scale differential pressure.

LOOP CURRENT FROM FLOW, NO SQUARE-ROOT EXTRACTOR

$$I = 4 + 16\left(\frac{Q}{Q_{\max}}\right)^2\ \mathrm{mA}$$

Same conditions as the flow law; with a square-root extractor the relation becomes linear, I = 4 + 16 Q/Q_max mA.

Optical sources, detectors, interferometry and spectroscopy

PHOTON ENERGY AND WAVELENGTH

$$E = h\nu = \frac{hc}{\lambda} \quad \Rightarrow \quad \lambda\,(\mu\text{m}) \approx \frac{1.24}{E\,(\text{eV})}$$

The emitted wavelength of an LED is set by the band gap, so $E \approx E_g$. SI: $h = 6.63 \times 10^{-34}$ J s, $c = 3 \times 10^{8}$ m/s, $1\ \text{eV} = 1.6 \times 10^{-19}$ J.

COHERENCE LENGTH AND COHERENCE TIME

$$L_c \approx \frac{\lambda^2}{\Delta\lambda}, \qquad \tau_c = \frac{L_c}{c}$$

$\Delta\lambda$ is the spectral width of the source (order-of-magnitude relation). Clear interference fringes need the path difference to be smaller than L_c.

CUT-OFF WAVELENGTH

$$\lambda_c = \frac{hc}{E_g} \quad \Rightarrow \quad \lambda_c\,(\mu\text{m}) \approx \frac{1.24}{E_g\,(\text{eV})}$$

Light of wavelength longer than λ_c is not absorbed and gives no photocurrent. Silicon ($E_g \approx 1.12$ eV): $\lambda_c \approx 1.1\ \mu$m.

QUANTUM EFFICIENCY

$$\eta = \frac{\text{electrons collected per second}}{\text{photons incident per second}} = \frac{I_p/q}{P/(h\nu)}$$

For a detector without internal gain, $0 \le \eta \le 1$. Avalanche gain is counted separately.

RESPONSIVITY

$$\mathcal{R} = \frac{I_p}{P} = \frac{\eta q \lambda}{hc} \approx \frac{\eta\,\lambda\,(\mu\text{m})}{1.24}\ \text{A/W}, \qquad I_p = M\,\mathcal{R}\,P\ \ (\text{APD})$$

I_p is the photocurrent and P the incident optical power. Applies for $\lambda < \lambda_c$; responsivity falls to zero beyond the cut-off. M is the multiplication factor of an avalanche photodiode ($M = 1$ for an ordinary photodiode).

SQUARE-LAW DETECTION

$$i_p \propto \overline{E_{\text{field}}^2} \propto P_{\text{opt}}, \qquad P_{\text{elec}} = i_p^2 R_L \propto P_{\text{opt}}^2$$

The photocurrent follows optical power, not field amplitude, so the electrical power in the load R_L varies as the square of the optical power. The bar denotes an average over many optical cycles.

FRINGE COUNTING FOR DISPLACEMENT

$$\Delta d = \frac{N\lambda}{2}$$

N fringes counted; λ is the wavelength in the medium of the path; the mirror moves along the beam (Michelson, light passes twice).

PLATE OR GAS CELL IN ONE ARM

$$2(n-1)\,t = N\lambda$$

A transparent plate of index n and thickness t (or a gas cell of length t whose index changes from 1 to n) placed normal to the beam in one arm shifts N fringes. Michelson double pass, surrounding medium of index 1, λ the vacuum wavelength.

NUMERICAL APERTURE AND CRITICAL ANGLE

$$\text{NA} = \sqrt{n_1^2 - n_2^2} = n_0 \sin\theta_{\max} \approx n_1\sqrt{2\Delta}, \qquad \Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \approx \frac{n_1 - n_2}{n_1}, \qquad \sin\theta_c = \frac{n_2}{n_1}$$

Step-index fibre. n_1 core index, n_2 cladding index, n_0 index of the outside medium (1 for air); $\theta_{\max}$ is the acceptance half-angle measured from the fibre axis; θ_c is the critical angle at the core-cladding boundary.

V-NUMBER (NORMALISED FREQUENCY)

$$V = \frac{2\pi a}{\lambda}\,\mathrm{NA}, \qquad \text{single mode if } V < 2.405$$

a is the core radius. Step-index fibre.

FIBRE BRAGG GRATING

$$\lambda_B = 2\, n_{\text{eff}}\, \Lambda$$

n_{eff} is the effective index of the guided mode and Λ the grating period. Strain or temperature changes Λ and n_{eff}, which shifts the reflected wavelength λ_B.

BEER-LAMBERT LAW

$$A = \log_{10}\frac{I_0}{I} = \varepsilon\, c\, l$$

ε is the molar absorptivity at the chosen wavelength (usually L mol^{-1} cm^{-1}), c the concentration (mol/L) and l the path length (cm). Holds for a dilute, non-scattering solution, monochromatic light, and no association or reaction of the absorbing species.

TRANSMITTANCE AND ABSORBANCE

$$T = \frac{I}{I_0}, \qquad A = -\log_{10} T = 2 - \log_{10}(T\%)$$

T as a fraction; $T\%$ is the percentage transmittance.

MAGNETIC-SECTOR ANALYSER

$$qV = \tfrac{1}{2}mv^2, \quad r = \frac{mv}{qB} \;\Rightarrow\; \frac{m}{q} = \frac{B^2 r^2}{2V}, \quad r = \frac{1}{B}\sqrt{\frac{2Vm}{q}}$$

The ion starts from rest, is accelerated through potential V, then moves on a circle of radius r in a uniform magnetic field B perpendicular to its velocity. SI units.

TIME-OF-FLIGHT ANALYSER

$$t = \frac{L}{v} = L\sqrt{\frac{m}{2qV}}$$

The ion starts from rest, is accelerated through V and drifts a field-free length L. Flight time grows as $\sqrt{m/q}$.

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