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Power Systems Formula Sheet

INSIDE

- 98 formulas in 6 topics
- When to use each one
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HYDRO POWER OUTPUT

$$P_{\text{hydro}} = \rho\, g\, Q\, H\, \eta$$

$\rho = 1000\ \text{kg/m}^3$, $g = 9.81\ \text{m/s}^2$, Q in m³/s, H net head in m, η overall efficiency; result in W.

LOAD FACTOR AND CAPACITY FACTOR

$$\text{Load factor} = \frac{\text{average load}}{\text{peak load}}, \qquad \text{Plant capacity factor} = \frac{\text{energy generated}}{\text{rated capacity} \times \text{time}}$$

Both over the same period; each is at most 1.

DEMAND FACTOR AND DIVERSITY FACTOR

$$\text{Demand factor} = \frac{\text{maximum demand}}{\text{connected load}} \le 1, \qquad \text{Diversity factor} = \frac{\sum \text{individual maximum demands}}{\text{maximum demand of the group}} \ge 1$$

Individual maximum demands occur at different times, so their sum is at least the group's maximum demand. A higher diversity factor means less plant capacity is needed to supply the same consumers.

INDUCTANCE PER PHASE OF A TRANSPOSED THREE-PHASE LINE

$$L = 2 \times 10^{-7} \ln\left(\frac{D_m}{D_s}\right)\ \text{H/m}$$

D_m is the GMD between phases, D_s the GMR of one phase conductor (self GMD). For a three-phase line, $D_m = \sqrt[3]{D_{ab} D_{bc} D_{ca}}$. For a bundled phase, D_s is the bundle GMR from the table below.

CAPACITANCE TO NEUTRAL PER PHASE

$$C = \frac{2\pi\varepsilon_0}{\ln(D_m/r)}\ \text{F/m}, \qquad \varepsilon_0 = 8.854 \times 10^{-12}\ \text{F/m}$$

Transposed line, ground effect neglected; D_m is the same GMD as for inductance. The capacitance uses the actual radius r (or the bundle's equivalent radius built with r, not r'), because it depends on the surface charge, not on internal flux.

SINGLE-PHASE TWO-WIRE LINE (RADIUS R, SPACING D)

$$L_{\text{loop}} = 4 \times 10^{-7} \ln\left(\frac{D}{r'}\right)\ \text{H/m}, \qquad C_{ab} = \frac{\pi\varepsilon_0}{\ln(D/r)}\ \text{F/m}$$

The loop inductance is twice the inductance of one conductor. The line-to-line capacitance is half the capacitance of either conductor to neutral, $C_n = 2\pi\varepsilon_0 / \ln(D/r)$. Valid for $D \gg r$.

PROPERTIES OF ABCD CONSTANTS

$$AD - BC = 1, \qquad A = D\ \text{(symmetrical line)}$$

$AD - BC = 1$ holds for any passive, linear, bilateral two-port, such as a transmission line. $A = D$ holds in addition when the two-port is symmetrical; the short, nominal-π, nominal-T and long-line models are all symmetrical.

PROPAGATION CONSTANT AND CHARACTERISTIC IMPEDANCE

$$\gamma = \sqrt{zy} = \alpha + j\beta, \qquad Z_c = \sqrt{z/y}$$

z and y are the series impedance and shunt admittance per unit length.

VOLTAGE REGULATION

$$\text{VR} = \frac{|V_{R,\text{NL}}| - |V_{R,\text{FL}}|}{|V_{R,\text{FL}}|}, \qquad |V_{R,\text{NL}}| = \frac{|V_S|}{|A|}$$

V_S held constant; magnitudes per phase (or line-to-line throughout). Multiply by 100 for per cent. For a short line $|V_{R,\text{NL}}| = |V_S|$. Regulation can be negative, for example with a leading-power-factor load. Efficiency is P_R / P_S at the same load.

RECEIVING-END POWER FROM ABCD CONSTANTS

$$P_R = \frac{|V_S||V_R|}{|B|}\cos(\theta_B - \delta) - \frac{|A||V_R|^2}{|B|}\cos(\theta_B - \theta_A)$$
$$Q_R = \frac{|V_S||V_R|}{|B|}\sin(\theta_B - \delta) - \frac{|A||V_R|^2}{|B|}\sin(\theta_B - \theta_A)$$

$A = |A|\angle\theta_A$, $B = |B|\angle\theta_B$, $V_R = |V_R|\angle 0°$, $V_S = |V_S|\angle\delta$. With line-to-line voltages the result is three-phase power; with line-to-neutral voltages it is power per phase. For a lossless short line ($A = 1$, $B = jX$) it reduces to $P = |V_S||V_R|\sin\delta / X$.

LOSSLESS LINE: SURGE IMPEDANCE AND SIL

$$Z_c = \sqrt{\frac{L}{C}}, \qquad \beta = \omega\sqrt{LC}, \qquad \text{SIL} = \frac{V_L^2}{Z_c}$$

L, C per unit length; V_L is the line-to-line voltage; SIL is the total three-phase power at which the line draws zero net reactive power. At SIL the voltage magnitude is the same all along the line. Below SIL the line produces net vars and the receiving voltage tends to rise; above SIL it absorbs vars and the voltage sags.

FERRANTI EFFECT (LOSSLESS, OPEN RECEIVING END)

$$V_{R,\text{open}} = \frac{V_S}{\cos\beta l} \approx V_S\left(1 + \frac{(\beta l)^2}{2}\right)$$

The receiving-end voltage exceeds the sending-end voltage because the line charging current flows through the series inductance. The approximation holds for small βl; it equals $V_S(1 + \omega^2 LCl^2/2)$ with L, C per unit length, so the rise grows roughly with the square of the length.

POWER-ANGLE RELATION (LOSSLESS LINE)

$$P = \frac{|V_S||V_R|}{X}\sin\delta$$

X is the series reactance of the line; δ is the angle by which V_S leads V_R. Maximum power at $\delta = 90°$ is $|V_S||V_R|/X$ (steady-state limit). Use line-to-line voltages for three-phase power and line-to-neutral voltages for power per phase. For a long lossless line, replace X by $|B| = Z_c\sin\beta l$.

CAPACITANCE AND INSULATION RESISTANCE OF A SINGLE-CORE CABLE

$$C = \frac{2\pi\varepsilon_0\varepsilon_r\, l}{\ln(R/r)}, \qquad R_{\text{ins}} = \frac{\rho}{2\pi l}\ln\frac{R}{r}$$

The insulation resistance is radial, so it is inversely proportional to the cable length l.

DIELECTRIC STRESS IN A CABLE

$$g(x) = \frac{V}{x\,\ln(R/r)}, \qquad g_{\max} = \frac{V}{r\,\ln(R/r)}$$

V is the voltage between conductor and sheath (use rms for rms stress), $r \le x \le R$. Stress is greatest at the conductor surface and least at the sheath.

THREE-CORE CABLE: CAPACITANCE PER PHASE

$$C_n = C_s + 3C_c, \qquad C_1 = 3C_s, \quad C_2 = C_s + 2C_c \;\Rightarrow\; C_n = \frac{3C_2}{2} - \frac{C_1}{6}$$

C_s is the core-to-sheath and C_c the core-to-core capacitance. C_1: all three cores bunched, measured against the sheath. C_2: two cores bunched with the sheath, measured against the third core. C_n is the capacitance of each core to neutral; the charging current per phase is $\omega C_n V_{ph}$.

SERIES COMPENSATION

$$X_{\text{eff}} = X_L - X_C = X_L(1 - k), \qquad k = \frac{X_C}{X_L}$$

k is the degree of series compensation. Then $P_{\max} = |V_S||V_R|/\big(X_L(1-k)\big)$, raised by the factor $1/(1-k)$. Voltage drop and regulation also improve.

REACTIVE POWER OF A SHUNT ELEMENT

$$Q_{\text{sh}} = \frac{V^2}{X_{\text{sh}}}$$

A shunt capacitor supplies Q and a shunt reactor absorbs it. Use V and Q on a consistent basis: per phase with V_{ph}, or three-phase with the line-to-line voltage and star-connected X_{sh} per phase. A capacitor's output falls as V^2, so it gives least support when the voltage is low.

VOLTAGE ACROSS UNITS OF A THREE-UNIT STRING

$$V_2 = V_1(1+k), \qquad V_3 = V_1\left(1+3k+k^2\right)$$

Units numbered from the cross-arm end; the total string voltage is $V_1 + V_2 + V_3$ (phase-to-earth), and V_3 is the largest.

STRING EFFICIENCY

$$\text{String efficiency} = \frac{\text{voltage across the whole string}}{n \times \text{voltage across the unit nearest the conductor}}$$

It is at most 100%, reached only if $k = 0$, and it falls as n increases. Capacitance grading (units of different capacitance), a guard ring (static shielding) and longer cross-arms (smaller k) all improve it.

APPROXIMATE VOLTAGE DROP OF A SECTION (PER PHASE)

$$\Delta V \approx I\left(R\cos\phi + X\sin\phi\right)$$

Lagging load of power factor $\cos\phi$ and current I; R and X are the per-phase resistance and reactance of the section. Valid when the angle between the two end voltages is small. A leading load makes the $X\sin\phi$ term negative.

UNIFORMLY LOADED DISTRIBUTOR FED AT ONE END

$$\Delta V = \frac{I_{\text{tot}}R_{\text{tot}}}{2}, \qquad P_{\text{loss}} = \frac{I_{\text{tot}}^2 R_{\text{tot}}}{3}$$

Load current per unit length constant along the whole length, at unity power factor (or DC). I_{tot} is the total load current and R_{tot} the distributor resistance (the loop resistance of both conductors for a two-wire DC or single-phase distributor). The drop is half that of the same total current concentrated at the far end.

UNIFORMLY LOADED DISTRIBUTOR FED AT BOTH ENDS WITH EQUAL VOLTAGES

$$\Delta V_{\max} = \frac{I_{\text{tot}}R_{\text{tot}}}{8}$$

Same symbols and conditions as above. The minimum voltage is at the mid-point.

Per-unit system, bus admittance matrix and load flow

BASE CURRENT AND BASE IMPEDANCE

$$I_{base} = \frac{S_{base}\times 10^3}{\sqrt{3}\,V_{base}}\ \text{A}, \qquad Z_{base} = \frac{V_{base}^2}{S_{base}}\ \Omega$$

Three-phase MVA base S_{base} in MVA and line-to-line kV base V_{base} in kV; the impedance is per phase of the equivalent star.

PER-UNIT IMPEDANCE

$$Z_{pu} = \frac{Z_\Omega}{Z_{base}} = Z_\Omega\,\frac{S_{base}}{V_{base}^2}$$

Per-unit voltage, current and power follow in the same way: $V_{pu} = V/V_{base}$, $I_{pu} = I/I_{base}$, $S_{pu} = S/S_{base}$.

CHANGE OF BASE

$$Z_{pu,new} = Z_{pu,old} \times \frac{S_{base,new}}{S_{base,old}} \times \left(\frac{V_{base,old}}{V_{base,new}}\right)^2$$

Used when an equipment rating (given on its own MVA and kV) must be moved to the system base; the rated MVA and kV are the old base.

BASE VOLTAGE ACROSS A TRANSFORMER

$$\frac{V_{base,1}}{V_{base,2}} = \frac{V_{rated,1}}{V_{rated,2}}$$

Line-to-line rated voltages of the transformer; S_{base} is unchanged across it.

COMPLEX POWER INJECTED AT BUS I

$$S_i = P_i + jQ_i = V_i\, I_i^*, \qquad I_i = \sum_{k=1}^{n} Y_{ik} V_k$$

Phasors in per unit; Y_{ik} are elements of the bus admittance matrix.

REAL AND REACTIVE POWER EQUATIONS

$$P_i = |V_i| \sum_{k=1}^{n} |V_k| \left(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}\right)$$

$$Q_i = |V_i| \sum_{k=1}^{n} |V_k| \left(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}\right)$$

Here $Y_{ik} = G_{ik} + jB_{ik}$ and $\delta_{ik} = \theta_i - \theta_k$. The equations are non-linear, so they are solved by iteration.

ELEMENTS OF THE BUS ADMITTANCE MATRIX

$$Y_{ii} = y_{i0} + \sum_{k\neq i} y_{ik}, \qquad Y_{ik} = Y_{ki} = -y_{ik}$$

y_{i0} is the total shunt admittance from bus i to ground; y_{ik} is the series admittance between i and k. Valid with no mutual coupling and nominal-ratio transformers.

GAUSS-SEIDEL UPDATE (PQ BUS)

$$V_i^{(r+1)} = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{\left(V_i^{(r)}\right)^*} - \sum_{k=1}^{i-1} Y_{ik} V_k^{(r+1)} - \sum_{k=i+1}^{n} Y_{ik} V_k^{(r)} \right]$$

P_i and Q_i are net injections in per unit. Buses already updated in this iteration use their new values.

NEWTON-RAPHSON CORRECTION EQUATION

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta|V| \end{bmatrix}, \quad J_1 = \frac{\partial P}{\partial \theta},\ J_2 = \frac{\partial P}{\partial |V|},\ J_3 = \frac{\partial Q}{\partial \theta},\ J_4 = \frac{\partial Q}{\partial |V|}$$

Mismatch $\Delta P = P_{spec} - P_{calc}$, $\Delta Q = Q_{spec} - Q_{calc}$; update $\theta \leftarrow \theta + \Delta\theta$, $|V| \leftarrow |V| + \Delta|V|$ and repeat.

SIZE OF THE JACOBIAN

$$\text{order} = (n-1) + (n-1-m) = 2n - m - 2$$

n buses, one slack, m PV buses (polar form). J_1 is $(n-1)\times(n-1)$, J_4 is $(n-1-m)\times(n-1-m)$, J_2 is $(n-1)\times(n-1-m)$ and J_3 is $(n-1-m)\times(n-1)$.

DC LOAD FLOW

$$P_{ik} = \frac{\theta_i - \theta_k}{x_{ik}}, \qquad \mathbf{P} = B'\boldsymbol{\theta}$$

Assumes $|V| = 1$ pu at all buses, $r \ll x$, small angle differences, so $\sin\delta \approx \delta$. Angles in radians. B' is the same matrix as in the fast decoupled method (from $1/x$, slack row and column removed), and $\mathbf{P}$ holds the net real-power injections of the non-slack buses. It gives real power flows only, and no losses.

Economic dispatch, voltage and frequency control, power factor correction

QUADRATIC COST AND INCREMENTAL COST

$$C_i = a_i + b_i P_i + c_i P_i^2, \qquad \frac{dC_i}{dP_i} = b_i + 2c_i P_i$$

Cost in Rs/h, P_i in MW; incremental cost is in Rs/MWh and is linear in P_i.

EQUAL INCREMENTAL COST CRITERION (NO LOSSES)

$$\frac{dC_i}{dP_i} = \lambda \text{ (all units inside limits)}, \qquad \sum_i P_i = P_D$$

A unit held at $P_{i,\max}$ has $dC_i/dP_i \le \lambda$; a unit held at $P_{i,\min}$ has $dC_i/dP_i \ge \lambda$. λ is the system incremental cost in Rs/MWh.

CLOSED FORM FOR QUADRATIC COSTS

$$P_i = \frac{\lambda - b_i}{2c_i}, \qquad \lambda = \frac{P_D + \sum_i \frac{b_i}{2c_i}}{\sum_i \frac{1}{2c_i}}$$

Valid while every unit is inside its limits. The fixed term a_i does not affect the dispatch.

B-COEFFICIENT LOSS FORMULA

$$P_L = \sum_i \sum_j P_i B_{ij} P_j, \qquad \frac{\partial P_L}{\partial P_i} = 2\sum_j B_{ij} P_j$$

Symmetric B_{ij}. For two units: $P_L = B_{11}P_1^2 + 2B_{12}P_1P_2 + B_{22}P_2^2$. $\partial P_L/\partial P_i$ is the incremental transmission loss of unit i.

COORDINATION EQUATIONS

$$\frac{dC_i}{dP_i} + \lambda\frac{\partial P_L}{\partial P_i} = \lambda, \qquad \sum_i P_i = P_D + P_L$$

Holds for units inside limits. Here λ is the incremental cost of power delivered at the load.

PENALTY FACTOR FORM

$$L_i \frac{dC_i}{dP_i} = \lambda, \qquad L_i = \frac{1}{1 - \frac{\partial P_L}{\partial P_i}} \approx 1 + \frac{\partial P_L}{\partial P_i}$$

Every unit inside limits has the same product of penalty factor and incremental cost. The approximation holds when $\partial P_L/\partial P_i$ is small. A unit with larger $\partial P_L/\partial P_i$ has a larger L_i and so a lower optimal IC.

GOVERNOR DROOP

$$\Delta P_i = -\frac{P_{rated,i}}{R_i}\frac{\Delta f}{f_0}$$

R_i in per unit on the unit's own rating; ΔP_i is the change in generator output in steady state with the speed changer setting fixed.

LOAD SHARING OF PARALLEL UNITS

$$\Delta P_i \;\propto\; \frac{P_{rated,i}}{R_i}$$

All units settle at one common frequency. A smaller R or a larger rating takes a larger share of a load change, as long as no unit reaches its limit.

STEADY-STATE FREQUENCY CHANGE FOR A LOAD CHANGE

$$\Delta f = -\frac{\Delta P_D}{\displaystyle\sum_i \frac{P_{rated,i}}{R_i f_0} + D}$$

Free governor action, no supplementary control. ΔP_D is the load increase in MW and D is load damping in MW/Hz (set $D = 0$ if not given).

AREA FREQUENCY RESPONSE CHARACTERISTIC

$$\beta = D + \sum_i \frac{1}{R_i}\ \ (\mathrm{MW/Hz}), \qquad \Delta f = -\frac{\Delta P_D}{\beta}$$

For one isolated area with governor action only (no integral control). $1/R_i$ is the governor response of unit i and D the load damping.

TWO AREAS, GOVERNOR ACTION ONLY

$$\Delta f = -\frac{\Delta P_{D1} + \Delta P_{D2}}{\beta_1 + \beta_2}, \qquad \Delta P_{12} = \frac{\beta_1 \Delta P_{D2} - \beta_2 \Delta P_{D1}}{\beta_1 + \beta_2}$$

Steady state with no supplementary control; ΔP_{12} is the change in flow from area 1 to area 2. For a load step in area 1 only, $\Delta P_{12} = -\beta_2 \Delta P_{D1}/(\beta_1 + \beta_2)$: area 2 sends part of the step over the tie line.

AREA CONTROL ERROR

$$\mathrm{ACE} = \Delta P_{tie} + B_f\, \Delta f$$

ΔP_{tie} is the tie-line flow deviation (positive out of the area) and $B_f > 0$ is the frequency bias in MW/Hz, commonly set equal to the area's β. The integral controller changes the set point as $\Delta P_{ref} = -K \int \mathrm{ACE}\, dt$: negative ACE raises generation.

APPROXIMATE VOLTAGE DROP ON A LINE

$$\Delta V \approx \frac{RP + XQ}{V}$$

Per phase or per unit consistently; P and Q are the real and lagging reactive power received, V the receiving-end voltage; valid for a small angle across the line. In a mainly inductive line ($X \gg R$) the drop depends mostly on Q.

VOLTAGE RISE FROM REACTIVE INJECTION

$$\Delta V \approx \frac{X\, \Delta Q_{inj}}{V}, \qquad \frac{\Delta V}{V} \approx \frac{\Delta Q_{inj}}{S_{sc}}$$

Reactive power ΔQ_{inj} injected (for example by a shunt capacitor) at a bus whose Thevenin source reactance is X; S_{sc} is the three-phase short-circuit level at that bus (per-unit form, X dominant).

OFF-NOMINAL TAP SETTING FOR A LINE

$$t_s^2 = \frac{V_2/V_1}{1 - \dfrac{RP + XQ}{V_1 V_2}}, \qquad t_s t_r = 1$$

Tap-changing transformers at both ends of a line of impedance $R + jX$. V_1 and V_2 are the nominal voltages at the two ends referred to the line side, P and Q are the power received at end 2, and the angle across the line is small. With $V_1 = V_2 = V$: $t_s^2 = 1/\left(1 - (RP + XQ)/V^2\right)$.

CAPACITOR RATING

$$Q_c = P\left(\tan\phi_1 - \tan\phi_2\right), \qquad \phi = \cos^{-1}(\text{pf})$$

Total three-phase kVAr for total real power P in kW. The rating is the reactive power the bank delivers at its rated voltage.

CAPACITANCE FROM THE KVAR RATING

$$C_Y = \frac{Q_c}{\omega V_L^2}\ (\text{star, per phase}), \qquad C_\Delta = \frac{Q_c}{3\,\omega V_L^2}\ (\text{delta, per phase})$$

Q_c is the three-phase VAr, V_L the line voltage in V, $\omega = 2\pi f$, and C comes out in F. Note $C_\Delta = C_Y/3$.

EFFECT ON LINE CURRENT AND LOSS

$$\frac{I_2}{I_1} = \frac{\cos\phi_1}{\cos\phi_2}, \qquad \frac{\text{loss}_2}{\text{loss}_1} = \left(\frac{\cos\phi_1}{\cos\phi_2}\right)^2$$

At constant P and constant load voltage, the source-side current falls when $\cos\phi_2 > \cos\phi_1$, and feeder I^2R loss falls with its square.

MOST ECONOMICAL POWER FACTOR

$$\sin\phi_2 = \frac{y}{x}$$

Constant P; demand charge x per kVA of maximum demand per year and capacitor annual cost y per kVAr per year (both linear). It minimises $xP/\cos\phi_2 + yP(\tan\phi_1 - \tan\phi_2)$.

Symmetrical components and fault analysis

THE A-OPERATOR

$$a = 1\angle 120^\circ = -\tfrac{1}{2} + j\tfrac{\sqrt{3}}{2}, \qquad a^2 = 1\angle 240^\circ = -\tfrac{1}{2} - j\tfrac{\sqrt{3}}{2}, \qquad a^3 = 1, \qquad 1 + a + a^2 = 0$$

Multiplying a phasor by a rotates it by 120° anticlockwise without changing its magnitude. Also $a^2 - a = -j\sqrt{3}$.

SYNTHESIS (PHASE FROM SEQUENCE)

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

Same form for currents. Here $V_b = V_{a0} + a^2 V_{a1} + aV_{a2}$ and $V_c = V_{a0} + aV_{a1} + a^2 V_{a2}$.

ANALYSIS (SEQUENCE FROM PHASE)

$$V_{a0} = \tfrac{1}{3}(V_a + V_b + V_c), \quad V_{a1} = \tfrac{1}{3}(V_a + aV_b + a^2 V_c), \quad V_{a2} = \tfrac{1}{3}(V_a + a^2 V_b + aV_c)$$

Same form for currents. The inverse matrix is $\frac{1}{3}$ times the complex conjugate of the synthesis matrix.

NEUTRAL CURRENT

$$I_n = I_a + I_b + I_c = 3I_{a0}$$

Only the zero-sequence component flows in the neutral (or earth) return path. If there is no neutral path, $I_{a0} = 0$ in the line currents. So the line currents of a three-wire system (delta or ungrounded star) contain no zero-sequence component, although zero-sequence current can still circulate inside a delta winding.

THREE-PHASE COMPLEX POWER IN SEQUENCE COMPONENTS

$$S = V_a I_a^* + V_b I_b^* + V_c I_c^* = 3\left(V_{a0} I_{a0}^* + V_{a1} I_{a1}^* + V_{a2} I_{a2}^*\right)$$

The factor 3 appears because the transformation used here (with $\frac{1}{3}$ in the analysis equations) is not power-invariant. There are no cross terms between different sequences.

SEQUENCE NETWORK EQUATIONS AT THE FAULT BUS

$$V_{a1} = E_a - Z_1 I_{a1}, \qquad V_{a2} = -Z_2 I_{a2}, \qquad V_{a0} = -Z_0 I_{a0}$$

For a system that is balanced before the fault, with balanced sources, only the positive-sequence network has a source: E_a, the pre-fault voltage of the reference phase a at the fault bus. The negative and zero networks are passive. Z_1, Z_2, Z_0 are the Thevenin impedances seen from the fault point, and I_{a1}, I_{a2}, I_{a0} are the sequence currents flowing out of the system into the fault.

NEUTRAL IMPEDANCE IN THE ZERO-SEQUENCE NETWORK

$$Z_0 = Z_{0,\text{machine}} + 3Z_n$$

A neutral impedance Z_n carries $3I_{a0}$, so it appears as $3Z_n$ in the single-phase zero-sequence network. It does not appear in the positive or negative networks.

THREE-PHASE FAULT CURRENT (PER UNIT)

$$I_f = \frac{E}{Z_1 + Z_f}$$

Z_1 is the positive-sequence Thevenin impedance at the fault bus, Z_f the impedance in each phase of the fault. For a bolted fault $Z_f = 0$.

BASE CURRENT

$$I_{base} = \frac{S_{base}}{\sqrt{3}\, V_{base}}, \qquad I_f(\text{amperes}) = I_f(\text{pu}) \times I_{base}$$

S_{base} in VA for the three-phase rating and V_{base} the line-to-line voltage in volts, giving I_{base} in amperes.

FAULT MVA (SHORT-CIRCUIT CAPACITY)

$$\text{Fault MVA} = \frac{S_{base}\,(\text{MVA})}{Z_1\,(\text{pu})} = \sqrt{3}\, V_{LL}\, I_f \times 10^{-6}$$

For a bolted three-phase fault with $E = 1$ pu. In per unit it equals $1/Z_1$. S_{base} is in MVA, V_{LL} is the base (rated) line-to-line voltage in volts and I_f is the fault current in amperes.

CHANGE OF BASE

$$Z_{pu,new} = Z_{pu,old}\, \frac{S_{base,new}}{S_{base,old}} \left(\frac{V_{base,old}}{V_{base,new}} \right)^2$$

Convert every element to the common MVA base, and to the common voltage base in each voltage zone, before combining.

SHORT CIRCUIT OF A SERIES R-L CIRCUIT (DC OFFSET)

$$i(t) = \frac{V_m}{|Z|} \left[\sin(\omega t + \alpha - \theta) - \sin(\alpha - \theta)\, e^{-Rt/L} \right], \qquad |Z| = \sqrt{R^2 + \omega^2 L^2}, \quad \theta = \tan^{-1} \frac{\omega L}{R}$$

A voltage $v = V_m \sin(\omega t + \alpha)$ drives a series R-L circuit that carries no current at $t = 0$. The angle α sets the instant of the fault on the voltage wave. There is no DC offset when $\alpha = \theta$. The DC offset is largest when $\alpha - \theta = \pm 90°$. Then, if R is small compared with ωL, the first current peak approaches $2V_m/|Z|$, twice the steady-state peak. This model takes a constant reactance and ignores the machine's change from sub-transient to synchronous reactance.

SINGLE LINE-TO-GROUND FAULT (PHASE A)

$$I_{a1} = I_{a2} = I_{a0} = \frac{E}{Z_1 + Z_2 + Z_0 + 3Z_f}, \qquad I_f = I_a = 3I_{a0}$$

Fault conditions: $I_b = I_c = 0$ and $V_a = Z_f I_a$. The fault current through the ground is I_a.

LINE-TO-LINE FAULT (PHASES B AND C)

$$I_{a1} = -I_{a2} = \frac{E}{Z_1 + Z_2 + Z_f}, \quad I_{a0} = 0, \quad I_b = -I_c = -j\sqrt{3}\, I_{a1}$$

Z_f is the impedance connected between b and c. $I_a = 0$ and $|I_b| = |I_c| = \sqrt{3}\,|I_{a1}|$. No current flows to ground.

DOUBLE LINE-TO-GROUND FAULT (PHASES B AND C TO GROUND)

$$I_{a1} = \frac{E}{Z_1 + \dfrac{Z_2\,(Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f}}, \quad I_{a2} = -I_{a1}\frac{Z_0 + 3Z_f}{Z_2 + Z_0 + 3Z_f}, \quad I_{a0} = -I_{a1}\frac{Z_2}{Z_2 + Z_0 + 3Z_f}$$

Z_f is the impedance in the common earth path of both phases. $I_a = 0$ and the current into the ground is $I_b + I_c = 3I_{a0}$.

THREE-PHASE FAULT USING Z_BUS

$$I_f = \frac{V_k(0)}{Z_{kk} + Z_f}, \qquad V_i(F) = V_i(0) - Z_{ik}\,I_f$$

$V_k(0)$ is the pre-fault voltage at bus k, $V_i(F)$ the post-fault voltage at bus i, and Z_{bus} the positive-sequence matrix.

SEQUENCE VOLTAGES AT BUS I FOR AN UNSYMMETRICAL FAULT AT BUS K

$$V_i^{(1)} = V_i(0) - Z_{ik}^{(1)} I_k^{(1)}, \qquad V_i^{(2)} = -Z_{ik}^{(2)} I_k^{(2)}, \qquad V_i^{(0)} = -Z_{ik}^{(0)} I_k^{(0)}$$

Find the sequence fault currents $I_k^{(1)}, I_k^{(2)}, I_k^{(0)}$ from the formulas of the previous section with $Z_1 = Z_{kk}^{(1)}$, $Z_2 = Z_{kk}^{(2)}$, $Z_0 = Z_{kk}^{(0)}$ and $E = V_k(0)$. Each sequence voltage is updated with its own matrix, then combined into phase voltages with the synthesis matrix. If a star-delta transformer lies between buses i and k, shift the positive and negative sequence values by its phase shift before combining.

ADDING A BRANCH TO EXISTING BUSES (KRON REDUCTION)

$$Z_{ij,\text{new}} = Z_{ij} - \frac{Z_{ip}\,Z_{pj}}{Z_{pp} + Z_b} \qquad \text{(existing bus } p \text{ to reference)}$$

$$Z_{ij,\text{new}} = Z_{ij} - \frac{(Z_{ip} - Z_{iq})(Z_{pj} - Z_{qj})}{Z_{pp} + Z_{qq} - 2Z_{pq} + Z_b} \qquad \text{(between existing buses } p \text{ and } q\text{)}$$

Apply to every element Z_{ij} of the existing matrix. Both results come from adding a temporary extra row and column and eliminating it.

Protection and circuit breakers

RELAY (SECONDARY) QUANTITIES

$$I_{\text{sec}} = \frac{I_{\text{pri}}}{N_{CT}}, \qquad V_{\text{sec}} = \frac{V_{\text{pri}}}{N_{VT}}, \qquad Z_{\text{sec}} = Z_{\text{pri}}\,\frac{N_{CT}}{N_{VT}}$$

N_CT = CT ratio and N_VT = VT ratio, each as primary over secondary. Z_pri is the impedance in primary ohms, from the primary voltage and current.

PLUG SETTING MULTIPLIER

$$\text{PSM} = \frac{\text{relay current}}{\text{pickup current}} = \frac{I_{\text{fault}}/N_{CT}}{\text{PS} \times I_{\text{CT,rated sec}}}$$

Pickup current = plug setting (as a fraction) times the CT rated secondary current. The relay operates only if PSM > 1.

IDMT OPERATING TIME (IEC STANDARD CURVES)

$$t = \text{TMS} \cdot \frac{k}{(\text{PSM})^{\alpha} - 1}$$

k and alpha depend on the characteristic (table below). Time is in seconds, and the TMS scales the TMS = 1 curve linearly. If a problem gives its own curve, or a table of operating times against PSM for TMS = 1, read the time from it and multiply by the TMS.

PERCENTAGE DIFFERENTIAL RELAY

$$I_{op} = |I_1 - I_2|, \qquad I_r = \frac{|I_1 + I_2|}{2}, \qquad \text{trip if } I_{op} > K\, I_r$$

I_1 and I_2 are the CT secondary current phasors at the two ends, both taken in the same sense through the zone. K is the slope (bias) setting, for example 0.2 for 20 %. A small minimum pickup is usually added.

CT RATIO BALANCE, DELTA-STAR TRANSFORMER

$$\frac{I_{L,\Delta}}{N_{CT,\Delta\ \text{side}}} = \sqrt{3}\,\frac{I_{L,Y}}{N_{CT,Y\ \text{side}}}$$

I_L,Delta = line current on the power transformer's delta-winding side, whose CTs are in star. I_L,Y = line current on the star-winding side, whose CTs are in delta. N = ratio of each CT set, primary over secondary. Both sides refer to the same through load.

MHO CHARACTERISTIC

$$|Z - \tfrac{1}{2}Z_r e^{j\tau}| < \tfrac{1}{2}Z_r, \qquad Z_{\text{reach}}(\theta) = Z_r\cos(\theta - \tau)$$

Z_r = relay reach (the diameter) along the maximum-torque angle tau. theta = angle of the measured impedance. The reach shrinks as the impedance angle moves away from tau. j is the imaginary unit.

UNIVERSAL RELAY TORQUE EQUATION

$$T = K_1 I^2 + K_2 V^2 + K_3 V I\cos(\theta - \tau) + K_4$$

$$\text{Impedance: } K_1 I^2 - K_2 V^2 > 0 \;\Rightarrow\; |Z| < \sqrt{K_1/K_2}$$

$$\text{Reactance } (\tau = 90^\circ)\text{: } K_1 I^2 - K_3 V I\sin\theta > 0 \;\Rightarrow\; X < K_1/K_3$$

$$\text{Mho: } K_3 V I\cos(\theta - \tau) - K_2 V^2 > 0 \;\Rightarrow\; |Z| < \tfrac{K_3}{K_2}\cos(\theta - \tau)$$

theta = angle by which I lags V, so Z = V/I at angle theta. K_4 (spring restraint) is neglected in the three special cases. Here a negative constant restrains and a positive one operates.

IMPEDANCE SEEN FOR PHASE AND EARTH FAULTS

$$\frac{V_b - V_c}{I_b - I_c} = xZ_1, \qquad \frac{V_a}{I_a + k_0 I_{\text{res}}} = xZ_1, \quad k_0 = \frac{Z_0 - Z_1}{3Z_1}, \quad I_{\text{res}} = I_a + I_b + I_c = 3I_0$$

x = distance to a bolted fault, Z_1 and Z_0 = positive- and zero-sequence line impedance per unit length, phasor quantities measured at the relay. The phase pair (b, c) works for any fault involving those two phases. The earth-fault element on phase a needs the residual compensation k_0, or it measures the wrong distance.

RESTRIKING VOLTAGE (UNDAMPED LC)

$$v(t) = V_m(1 - \cos\omega_n t), \quad \omega_n = \frac{1}{\sqrt{LC}}, \quad \text{peak} = 2V_m, \quad \text{RRRV}_{\max} = V_m\omega_n$$

Assumes resistance is neglected and the system voltage is constant during the transient (f_n is much higher than the supply frequency). V_m = system voltage across the pole at current zero; for a purely inductive fault circuit this is the peak phase voltage. L and C = inductance and capacitance of the circuit seen from the breaker. The peak occurs at t = pi/omega_n and the maximum RRRV at t = pi/(2 omega_n). The average RRRV up to the peak is 2V_m/(pi/omega_n) = 4 V_m f_n.

CRITICAL DAMPING RESISTANCE

$$R_{\text{crit}} = \frac{1}{2}\sqrt{\frac{L}{C}}$$

For the source inductance L in series, with a resistor R across the contacts in parallel with the capacitance C. If R <= R_crit, the restriking voltage is non-oscillatory.

BREAKING AND MAKING CAPACITY (THREE-PHASE)

$$\text{Breaking capacity (MVA)} = \sqrt{3}\, V_L\, I_b, \qquad \text{Making current (peak)} \approx 2.55\, I_b$$

V_L = rated line voltage in kV. I_b = rated symmetrical (rms) breaking current in kA. Breaking capacity is in MVA. The making current is the peak of the first current loop, about 1.8 x sqrt(2) = 2.55 times the symmetrical rms breaking current. If the current at contact separation has a dc component I_dc, the asymmetrical rms breaking current is sqrt(I_b^2 + I_dc^2).

Power system stability and the equal area criterion

SWING EQUATION (PER UNIT)

$$\frac{H}{\pi f}\,\frac{d^2\delta}{dt^2} = P_m - P_e = P_a$$

δ in electrical radians, t in seconds, f the system frequency in Hz, P_m and P_e in per unit on the same base as H; P_a is the accelerating power. Equivalent form: $\frac{2H}{\omega_s}\ddot{\delta} = P_m - P_e$ with $\omega_s = 2\pi f$.

SWING EQUATION WITH THE ANGLE IN ELECTRICAL DEGREES

$$\frac{H}{180 f}\,\frac{d^2\delta}{dt^2} = P_m - P_e$$

Same equation with δ in electrical degrees (since π rad $= 180^\circ$). Do not mix the two forms.

INERTIA CONSTANT

$$H = \frac{\text{kinetic energy stored at synchronous speed (MJ)}}{\text{machine rating } S_{\text{machine}}\ \text{(MVA)}} = \frac{\frac{1}{2} J\, \omega_{sm}^2}{S_{\text{machine}}}$$

Unit: seconds (MJ per MVA). In the second form J is in kg m^2, ω_{sm} the synchronous mechanical speed in rad/s and S_{machine} in VA. Stored energy is $H\, S_{\text{machine}}$ megajoules when S is in MVA.

ELECTRICAL AND MECHANICAL ANGLE

$$\theta_e = \frac{p}{2}\,\theta_m, \qquad \omega_e = \frac{p}{2}\,\omega_m$$

p is the number of poles. The swing equation above uses the electrical angle.

CHANGE OF BASE

$$H_{\text{new}} = H_{\text{old}}\,\frac{S_{\text{old}}}{S_{\text{new}}}$$

The stored energy in MJ is fixed; only the MVA it is divided by changes.

EQUIVALENT INERTIA OF MACHINES SWINGING TOGETHER

$$H_{eq} = \frac{H_1 S_1 + H_2 S_2 + \cdots}{S_{\text{base}}}$$

Applies to machines that swing coherently (same δ and speed), each H_i on its own rating S_i. The stored energies in MJ add.

POWER-ANGLE EQUATION

$$P_e = P_{\max} \sin\delta, \qquad P_{\max} = \frac{E\,V}{X}$$

E and V are the voltage magnitudes at the two ends, X the total series reactance between them; per unit values give P_e in per unit.

INITIAL ANGLE

$$\delta_0 = \sin^{-1}\left(\frac{P_m}{P_{\max}}\right)$$

Of the two angles with this sine, the stable operating point is the one below 90°.

SYNCHRONISING POWER COEFFICIENT

$$P_s = \left.\frac{dP_e}{d\delta}\right|_{\delta_0} = P_{\max}\cos\delta_0$$

Positive for $\delta_0 < 90^\circ$ (stable), zero at 90°, negative beyond. Units: per unit power per electrical radian.

FREQUENCY OF SMALL OSCILLATION

$$\omega_n = \sqrt{\frac{\pi f\, P_s}{H}}\ \text{rad/s}, \qquad f_n = \frac{\omega_n}{2\pi}\ \text{Hz}$$

Undamped natural frequency, from linearising the swing equation about δ_0 with $\Delta\delta$ in radians: $\frac{H}{\pi f}\Delta\ddot{\delta} = -P_s\Delta\delta$.

STEADY-STATE STABILITY LIMIT

$$P_{\text{limit}} = P_{\max} \ \text{ at } \delta = 90^\circ$$

Reached when load is increased slowly: beyond $\delta = 90^\circ$ the coefficient P_s becomes negative and the machine falls out of step.

EQUAL AREA CONDITION

$$\int_{\delta_0}^{\delta_m} (P_m - P_e)\, d\delta = 0 \quad\Longleftrightarrow\quad A_{\text{acc}} = A_{\text{dec}}$$

δ_m is the maximum swing angle. Valid for the classical model (constant P_m, no damping).

SUDDEN LOAD (OR P_M) CHANGE, SAME CURVE

$$P_{m1}\,(\delta_m - \delta_0) + P_{\max}\,(\cos\delta_m - \cos\delta_0) = 0$$

P_m jumps from P_{m0} to P_{m1} with the network unchanged, $\delta_0 = \sin^{-1}(P_{m0}/P_{\max})$; angles in radians where they multiply P_{m1}. Stable only if $\delta_m \le \pi - \delta_1$ with $\delta_1 = \sin^{-1}(P_{m1}/P_{\max})$.

POWER-CURVE RATIOS DURING AND AFTER A FAULT

$$r_1 = \frac{X_{\text{pre}}}{X_{\text{during}}}, \qquad r_2 = \frac{X_{\text{pre}}}{X_{\text{post}}}$$

With the same E and V, each curve's peak is EV/X, so the during-fault and post-fault peaks are $r_1 P_{\max}$ and $r_2 P_{\max}$, where X is the transfer reactance between the machine EMF and the infinite bus in each state.

CRITICAL CLEARING ANGLE, ZERO POWER DURING THE FAULT AND UNCHANGED NETWORK AFTER CLEARING

$$\cos\delta_c = \frac{P_m}{P_{\max}}\,(\pi - 2\delta_0) - \cos\delta_0$$

$r_1 = 0$, $r_2 = 1$, so $\delta_{\max} = \pi - \delta_0$; δ_0 in radians. Typical case: three-phase fault at the machine terminals cleared without a line being lost.

CRITICAL CLEARING TIME, ZERO POWER DURING THE FAULT

$$t_c = \sqrt{\frac{2H\,(\delta_c - \delta_0)}{\pi f\, P_m}}$$

Valid only when $P_e = 0$ during the fault ($r_1 = 0$) and the rotor starts at synchronous speed, so acceleration is constant; angles in radians, H in seconds on the same base as P_m. For $r_1 \neq 0$ the swing equation has to be integrated step by step.

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