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Power Electronics Formula Sheet

INSIDE

- 91 formulas in 5 topics
- When to use each one
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Power semiconductor switches and their gating

TWO-TRANSISTOR MODEL OF THE SCR

$$I_A = \frac{\alpha_2 I_G + I_{CBO1} + I_{CBO2}}{1 - (\alpha_1 + \alpha_2)}$$

The SCR is modelled as a p-n-p transistor Q_1 (common-base current gain α_1) and an n-p-n transistor Q_2 (gain α_2), with the gate current feeding the base of Q_2; I_{CBO1} and I_{CBO2} are their leakage currents. The gains rise with current. As $\alpha_1 + \alpha_2 \to 1$ the anode current is limited only by the external circuit: the device has latched.

LATCHING AND HOLDING CURRENT

$$I_L > I_H$$

Latching current I_L is the minimum anode current that must be reached before the gate pulse is removed for the SCR to stay on. Holding current I_H is the anode current below which a conducting SCR (gate open) turns off. Typically I_L is about two to three times I_H.

THYRISTOR CONDUCTION (AVERAGE) LOSS

$$P_T = V_{T0}\, I_{T(av)} + r_T\, I_{T(rms)}^2$$

Piecewise-linear on-state model $v_T = V_{T0} + r_T\, i_T$: threshold voltage V_{T0} and dynamic resistance r_T, with the average and rms values of the SCR current taken over a full period.

DV/DT (FALSE) TRIGGERING CURRENT

$$i_{J2} = C_{J2}\,\frac{dv_{AK}}{dt}$$

Charging current of the junction capacitance C_{J2} of J_2 (C_{J2} taken as constant). If this current reaches the triggering level, the SCR turns on with no gate signal; this sets the device's critical dv/dt rating.

PULSE GATING: PEAK AND AVERAGE GATE POWER

$$P_{G,peak} \le \frac{P_{G,av(max)}}{f\,T_p}, \qquad f\,T_p \le 1$$

For a rectangular gate pulse train of width T_p at frequency f, the average gate power is $P_{G,peak}\, f\, T_p$. A short pulse may therefore carry a peak power above the average gate power rating $P_{G,av(max)}$, as long as it stays within the device's peak gate power limit.

MINIMUM GATE PULSE WIDTH WITH AN R-L LOAD

$$t_{p,min} = -\frac{L}{R}\,\ln\!\left(1 - \frac{I_L R}{V_s}\right) \approx \frac{L\, I_L}{V_s}$$

A dc supply V_s is switched by the SCR onto a series R-L load, with the current starting from zero. The current $i = \frac{V_s}{R}\left(1 - e^{-Rt/L}\right)$ must reach I_L before the gate pulse ends. The approximation holds when $I_L R \ll V_s$.

R TRIGGERING (HALF-WAVE, AC SUPPLY)

$$\alpha = \sin^{-1}\!\left(\frac{V_{GT} + I_{GT} R}{V_m}\right)$$

Supply $v = V_m \sin \omega t$; α is measured from the positive-going zero crossing of the supply. The SCR fires when the supply reaches the value that drives I_{GT} through the total series gate-circuit resistance R (including the minimum fixed resistor) with V_{GT} across the gate junction (the series diode drop neglected). Since a sine peaks at 90°, the range is about 0° to 90°. Increasing R delays firing.

UJT RELAXATION OSCILLATOR

$$T = RC\,\ln\!\left(\frac{1}{1-\eta}\right), \qquad f = \frac{1}{T}$$

$\eta = R_{B1}/(R_{B1}+R_{B2})$ is the intrinsic stand-off ratio. C charges through R from V_{BB} until v_C reaches the peak-point voltage $V_P = \eta V_{BB} + V_D$; the UJT then fires and C discharges quickly through R_{B1} (or a pulse transformer), giving a gate pulse. This form neglects V_D, the valley voltage V_V and the discharge time.

CAPACITOR COMMUTATION TIME

$$t_c = RC\,\ln 2 \approx 0.693\,RC$$

A dc supply V feeds a purely resistive load R through the SCR. A capacitor C charged to V in the reverse-biasing polarity is connected directly across the conducting SCR (auxiliary switch drop neglected). The SCR turns off and C recharges through R, so the SCR voltage is $v = V - 2Ve^{-t/RC}$. It passes through zero at this time, which is how long the SCR is held reverse biased.

DI/DT LIMIT WITH SERIES INDUCTOR

$$L \geq \frac{V_s}{(di/dt)_{max}}$$

At the instant of turn-on the whole supply V_s appears across the series inductor L (other resistances neglected), so $di/dt = V_s/L$. Conduction starts near the gate and spreads slowly, so a small inductor in series with the SCR protects it from local hot-spot burn-out.

GATE DRIVE POWER (MOSFET OR IGBT)

$$P_G = Q_g\,\Delta V_{GS}\,f_s, \qquad I_{G,avg} = Q_g f_s$$

Q_g is the total gate charge for the gate voltage swing ΔV_{GS} (from the off level to the on level; equal to V_{GS} when the device is turned off at 0 V), and f_s is the switching frequency. P_G is the power drawn from the driver, and $I_{G,avg}$ the average current drawn from the driver supply. With a constant gate current I_G, the time to deliver a charge Q is $t = Q/I_G$.

MOSFET CONDUCTION LOSS

$$P_{cond} = I^2_{D(rms)}\,R_{DS(on)}$$

$I_{D(rms)}$ is the rms drain current taken over the whole switching period, and $R_{DS(on)}$ is taken at the operating junction temperature.

SWITCHING LOSS, CLAMPED INDUCTIVE SWITCHING

$$P_{sw} = \tfrac{1}{2}\,V\,I\,(t_{on} + t_{off})\,f_s$$

The device switches current I against a blocked voltage V with an inductive load clamped by a diode. At turn-on the current ramps linearly with the voltage held at V, then the voltage falls linearly with the current held at I; turn-off is the reverse. t_{on} and t_{off} are the total transition times. The energy per cycle is $\frac{1}{2}VI(t_{on}+t_{off})$, and multiplying by f_s gives the power.

SWITCHING LOSS, RESISTIVE LOAD

$$P_{sw} = \tfrac{1}{6}\,V\,I\,(t_{on} + t_{off})\,f_s$$

With a resistive load the voltage and current ramp linearly at the same time, in opposite directions. The energy per transition is $\int_0^t V\left(1-\frac{\tau}{t}\right) I\,\frac{\tau}{t}\,d\tau = \frac{1}{6}VIt$.

JUNCTION TEMPERATURE

$$T_j = T_a + P\,(\theta_{jc} + \theta_{cs} + \theta_{sa})$$

Steady state. Thermal resistances in °C/W: junction-to-case, case-to-sink and sink-to-ambient; P is the total device loss.

DC-DC converters: buck, boost and buck-boost

VOLT-SECOND BALANCE (INDUCTOR)

$$\int_0^T v_L\,dt = 0 \;\Rightarrow\; V_{L,on}\,DT + V_{L,off}\,(1-D)T = 0$$

In steady state the average inductor voltage over one period is zero. This gives the voltage gain of every converter.

CHARGE BALANCE (CAPACITOR)

$$\overline{i_C} = 0 \;\Rightarrow\; \overline{i_L} = I_o\ (\text{buck}), \qquad \overline{i_D} = I_o\ (\text{boost, buck-boost})$$

In steady state the average capacitor current is zero, so the average current delivered to the load equals the average current of the element feeding the output node: the inductor in the buck, the diode in the boost and buck-boost.

RMS INDUCTOR CURRENT (TRIANGULAR RIPPLE)

$$I_{L,rms} = \sqrt{I_L^2 + \frac{(\Delta i_L)^2}{12}}$$

CCM, I_L = average inductor current, Δi_L = peak-to-peak ripple. Used for rms ratings and copper loss; with negligible ripple $I_{L,rms} \approx I_L$.

BUCK: VOLTAGE GAIN

$$V_o = D\,V_s$$

CCM. Output is positive, same polarity as the input, and $V_o \le V_s$.

BUCK: INDUCTOR CURRENT RIPPLE

$$\Delta i_L = \frac{(V_s - V_o)\,D\,T}{L} = \frac{V_o(1-D)}{Lf}$$

Peak-to-peak ripple, CCM. Average inductor current equals I_o; average input current is $I_s = DI_o$.

BUCK: MAXIMUM RIPPLE

$$\Delta i_L = \frac{V_s\,D(1-D)}{Lf} \;\le\; \Delta i_{L,max} = \frac{V_s}{4Lf} \;\text{ at } D = 0.5$$

For fixed V_s, L and f in CCM. The same result holds for a step-down chopper feeding an inductive load (with or without back emf) when the load resistance is neglected.

BUCK: CAPACITOR VOLTAGE RIPPLE

$$\Delta v_C = \frac{\Delta i_L}{8fC} = \frac{V_o(1-D)}{8LCf^2}$$

Assumes the whole ripple component of the inductor current flows into the capacitor (the load takes only the average).

BUCK: CRITICAL INDUCTANCE

$$L_{crit} = \frac{(1-D)R}{2f}$$

CCM if $L > L_{crit}$ (boundary where $I_{min} = 0$, i.e. $I_o = \Delta i_L/2$).

BOOST: VOLTAGE GAIN

$$V_o = \frac{V_s}{1-D}$$

CCM. Same polarity as input, $V_o \ge V_s$. In the ideal circuit the gain rises without limit as $D \to 1$; in a real converter the inductor and switch resistances make the gain reach a maximum and then fall as $D \to 1$.

BOOST: AVERAGE INDUCTOR (INPUT) CURRENT

$$I_L = I_s = \frac{I_o}{1-D} = \frac{V_s}{(1-D)^2 R}$$

The diode conducts only for $(1-D)T$ and carries I_L, so its average current is $(1-D)I_L = I_o$; the switch average current is DI_L.

BOOST: INDUCTOR CURRENT RIPPLE

$$\Delta i_L = \frac{V_s\, D\, T}{L} = \frac{V_s D}{Lf}$$

CCM, peak-to-peak.

BOOST: CAPACITOR VOLTAGE RIPPLE

$$\Delta v_C = \frac{I_o\, D\, T}{C} = \frac{V_o D}{RCf}$$

During the ON interval the capacitor alone supplies I_o for DT.

BOOST: CRITICAL INDUCTANCE

$$L_{crit} = \frac{D(1-D)^2 R}{2f}$$

CCM if $L > L_{crit}$ (from $I_L = \Delta i_L/2$).

BUCK-BOOST: VOLTAGE GAIN

$$V_o = -\frac{D}{1-D} V_s$$

CCM. Output is negative with respect to the input common; $|V_o| < V_s$ for $D < 0.5$, $|V_o| > V_s$ for $D > 0.5$.

BUCK-BOOST: AVERAGE INDUCTOR CURRENT

$$I_L = \frac{I_o}{1-D} = \frac{V_s D}{(1-D)^2 R}$$

Here $I_o = |V_o|/R$. Diode average current is $(1-D)I_L = I_o$; average input (switch) current is $I_s = DI_L = DI_o/(1-D)$.

BUCK-BOOST: RIPPLES

$$\Delta i_L = \frac{V_s D T}{L}, \qquad \Delta v_C = \frac{|V_o|\, D}{RCf}$$

CCM, peak-to-peak. The capacitor alone feeds the load during the ON interval.

BUCK-BOOST: CRITICAL INDUCTANCE

$$L_{crit} = \frac{(1-D)^2 R}{2f}$$

CCM if $L > L_{crit}$.

DCM: GAIN IN TERMS OF THE DIODE INTERVAL D_2

$$\text{Buck: } \frac{V_o}{V_s} = \frac{D}{D+D_2}; \quad \text{Boost: } \frac{V_o}{V_s} = \frac{D+D_2}{D_2}; \quad \text{Buck-boost: } \left|\frac{V_o}{V_s}\right| = \frac{D}{D_2}$$

From volt-second balance with the inductor voltage zero during $D_3 T = (1-D-D_2)T$. D_2 is the fraction of the period for which the diode conducts.

DCM: GAIN IN TERMS OF D AND K

$$\text{Buck: } \frac{V_o}{V_s} = \frac{2}{1+\sqrt{1+4K/D^2}}; \quad \text{Boost: } \frac{V_o}{V_s} = \frac{1+\sqrt{1+4D^2/K}}{2}; \quad \text{Buck-boost: } \left|\frac{V_o}{V_s}\right| = \frac{D}{\sqrt{K}}$$

Valid for $K < K_{crit}$ (DCM) only. At $K = K_{crit}$ they agree with the CCM gains. For the buck-boost the diode interval is $D_2 = \sqrt{K}$; in every case a DCM answer must satisfy $D + D_2 < 1$.

ARMATURE-VOLTAGE SPEED CONTROL

$$\omega = \frac{DV_s - I_a R_a}{K\phi}$$

Separately excited DC motor fed by a step-down chopper, continuous armature current, constant flux; average values.

REGENERATIVE BRAKING (CLASS B CHOPPER)

$$E_b = (1-D)\,V_s + I_a R_a, \qquad P_{\text{returned}} = (1-D)\,V_s I_a$$

Step-up (Class B) chopper with the machine generating: D is the duty ratio of the switch that shorts the armature, I_a is the magnitude of the continuous armature current flowing from the machine to the supply, and the average voltage across the machine terminals is $(1-D)V_s$. Average values, ripple neglected.

Uncontrolled and controlled rectifiers, AC-DC converters

RIPPLE FACTOR FROM FORM FACTOR

$$RF = \sqrt{\left(\frac{V_{rms}}{V_{dc}}\right)^2 - 1} = \sqrt{FF^2 - 1}$$

Holds for any output waveform; V_{rms} is the total rms including the DC part.

HALF-WAVE RECTIFIER, R LOAD

$$V_{dc} = \frac{V_m}{\pi}, \quad V_{rms} = \frac{V_m}{2}, \quad FF = \frac{\pi}{2} = 1.57, \quad RF = 1.21, \quad PIV = V_m$$

One diode conducts for 180 degrees of each cycle.

FULL-WAVE RECTIFIER (CENTRE-TAP OR BRIDGE), R LOAD

$$V_{dc} = \frac{2V_m}{\pi}, \quad V_{rms} = \frac{V_m}{\sqrt{2}}, \quad FF = \frac{\pi}{2\sqrt{2}} = 1.11, \quad RF = 0.483$$

For the centre-tap type V_m is the peak of each half of the secondary. PIV is V_m for the bridge and $2V_m$ for the centre-tap.

HALF-WAVE RECTIFIER, RL LOAD (NO FREEWHEELING DIODE)

$$V_{dc} = \frac{V_m}{2\pi}(1-\cos\beta), \qquad \sin(\beta-\phi) + \sin\phi\, e^{-\beta/\tan\phi} = 0$$

$\phi = \tan^{-1}(\omega L/R)$. The current is $i = \frac{V_m}{Z}\left[\sin(\omega t - \phi) + \sin\phi\, e^{-\omega t/\tan\phi}\right]$ with $Z = \sqrt{R^2 + (\omega L)^2}$. The current continues after the supply goes negative, up to the extinction angle $\beta > \pi$ where $i = 0$, so the output voltage has a negative portion from π to β.

HALF-WAVE DIODE CHARGING A BATTERY E THROUGH R (NO INDUCTANCE)

$$\theta_1 = \sin^{-1}\frac{E}{V_m}, \qquad I_{dc} = \frac{1}{2\pi R}\left[2V_m\cos\theta_1 - E(\pi - 2\theta_1)\right]$$

The diode conducts only while the instantaneous supply voltage exceeds E, from θ_1 to $\pi - \theta_1$. In a full-wave bridge the same interval repeats in every half cycle, so the average current doubles.

THREE-PHASE HALF-WAVE (3-PULSE) DIODE RECTIFIER

$$V_{dc} = \frac{3\sqrt{3}}{2\pi} V_m = 0.827\, V_m, \quad PIV = \sqrt{3}\, V_m$$

Each diode conducts for 120 degrees; its average current is $I_{dc}/3$ and its rms current is $I_{dc}/\sqrt{3}$. The output ripple frequency is $3f$.

THREE-PHASE BRIDGE (6-PULSE) DIODE RECTIFIER

$$V_{dc} = \frac{3V_{LL,m}}{\pi} = \frac{3\sqrt{3}}{\pi} V_m = 1.35\, V_{LL}$$

The output is made of six line-voltage segments per cycle, so the ripple frequency is $6f$. PIV $= V_{LL,m}$.

BRIDGE CURRENTS WITH CONSTANT I_{DC}

$$I_{D,avg} = \frac{I_{dc}}{3}, \quad I_{D,rms} = \frac{I_{dc}}{\sqrt{3}}, \quad I_{s,rms} = \sqrt{\tfrac{2}{3}}\, I_{dc}, \quad I_{s1,rms} = \frac{\sqrt{6}}{\pi} I_{dc}, \quad PF = \frac{3}{\pi} = 0.955$$

Each diode conducts 120 degrees. The line current has one positive and one negative 120-degree block per cycle, and its fundamental is $0.78\, I_{dc}$.

HALF-WAVE CONTROLLED RECTIFIER, R LOAD

$$V_{dc} = \frac{V_m}{2\pi}(1 + \cos\alpha)$$

Applies to an R load and also to an RL load with a freewheeling diode. Range $0 \le \alpha \le \pi$.

CONTROLLED RECTIFIERS WITH R LOAD: RMS OUTPUT

$$V_{rms} = \frac{V_m}{2}\sqrt{\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)}\ \text{(half-wave)}, \qquad V_{rms} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)}\ \text{(bridge)}$$

Pure R load, so the output follows the supply from α to π in each conducting half cycle. The bridge result holds for both the fully controlled and the half-controlled bridge.

FULLY CONTROLLED BRIDGE, CONTINUOUS CONDUCTION (LARGE L)

$$V_{dc} = \frac{2V_m}{\pi}\cos\alpha$$

Positive for $\alpha < 90^\circ$ (rectification) and negative for $\alpha > 90^\circ$ (inversion, which needs an active load such as a reversed battery or a regenerating machine). With an RLE load and continuous current, $I_{dc} = (V_{dc} - E)/R$.

HALF-CONTROLLED BRIDGE, OR FULLY CONTROLLED BRIDGE WITH R LOAD OR FREEWHEELING DIODE

$$V_{dc} = \frac{V_m}{\pi}(1 + \cos\alpha)$$

The output cannot go negative, so only the rectifying quadrant is available.

FULLY CONTROLLED BRIDGE WITH LARGE L: SOURCE CURRENT

$$I_{s,rms} = I_{dc}, \quad I_{s1,rms} = \frac{2\sqrt{2}}{\pi} I_{dc} = 0.9\, I_{dc}, \quad DPF = \cos\alpha, \quad PF = \frac{2\sqrt{2}}{\pi}\cos\alpha$$

Square-wave source current; each thyristor conducts for 180 degrees, with average current $I_{dc}/2$ and rms current $I_{dc}/\sqrt{2}$. The source current has odd harmonics $n = 3, 5, 7, \ldots$ with $I_n = I_{s1}/n$, and THD $= 48.3\%$. Input powers: $P = \frac{2V_m}{\pi} I_{dc}\cos\alpha = V_{dc} I_{dc}$ and $Q = \frac{2V_m}{\pi} I_{dc}\sin\alpha$.

HALF-CONTROLLED BRIDGE WITH LARGE L: SOURCE CURRENT

$$I_{s,rms} = I_{dc}\sqrt{\frac{\pi - \alpha}{\pi}}, \quad I_{s1,rms} = \frac{2\sqrt{2}}{\pi} I_{dc}\cos\frac{\alpha}{2}, \quad DPF = \cos\frac{\alpha}{2}, \quad PF = \frac{\sqrt{2}\,(1 + \cos\alpha)}{\sqrt{\pi(\pi - \alpha)}}$$

The source current is $+I_{dc}$ from α to π, $-I_{dc}$ from $\pi + \alpha$ to 2π, and zero while the load freewheels. Its displacement factor $\cos(\alpha/2)$ is better than the fully controlled bridge's $\cos\alpha$.

THREE-PHASE FULLY CONTROLLED BRIDGE, CONTINUOUS CONDUCTION

$$V_{dc} = \frac{3V_{LL,m}}{\pi}\cos\alpha = \frac{3\sqrt{3}}{\pi}V_m\cos\alpha = 1.35\,V_{LL}\cos\alpha$$

Negative for $\alpha > 90^\circ$, which is inversion. The load current stays continuous for all α when L is large enough.

THREE-PHASE HALF-CONTROLLED BRIDGE

$$V_{dc} = \frac{3V_{LL,m}}{2\pi}\left(1+\cos\alpha\right)$$

The output never goes negative, so $0 \le \alpha \le 180^\circ$ gives rectification only.

FULLY CONTROLLED BRIDGE, R LOAD, DISCONTINUOUS ($\ALPHA>60^\CIRC$)

$$V_{dc} = \frac{3V_{LL,m}}{\pi}\left[1+\cos(\alpha+60^\circ)\right]$$

Valid for $60^\circ \le \alpha \le 120^\circ$; the output reaches zero at $\alpha = 120^\circ$. For $\alpha \le 60^\circ$ with an R load the output stays positive and the cosine law $\frac{3V_{LL,m}}{\pi}\cos\alpha$ still holds.

THREE-PHASE FULLY CONTROLLED BRIDGE WITH LARGE L: SOURCE CURRENT

$$I_{s,rms} = \sqrt{\tfrac{2}{3}}\,I_{dc}, \quad I_{s1,rms} = \frac{\sqrt{6}}{\pi}I_{dc}, \quad DPF = \cos\alpha, \quad PF = \frac{3}{\pi}\cos\alpha$$

The line current has one positive and one negative 120-degree block per cycle. Its harmonics are of order $n = 6k \pm 1$ $(5, 7, 11, 13, \ldots)$ with $I_n = I_{s1}/n$, and THD $= 31.1\%$. Input powers: $P = \frac{3V_{LL,m}}{\pi}I_{dc}\cos\alpha$ and $Q = \frac{3V_{LL,m}}{\pi}I_{dc}\sin\alpha$.

SINGLE-PHASE FULLY CONTROLLED BRIDGE WITH L_S

$$V_{dc} = \frac{2V_m}{\pi}\cos\alpha - \frac{2\omega L_s}{\pi}I_{dc}, \qquad \cos\alpha - \cos(\alpha+\mu) = \frac{2\omega L_s I_{dc}}{V_m}$$

Large L, continuous conduction. The drop equals $4fL_sI_{dc}$ and does not depend on α.

THREE-PHASE FULLY CONTROLLED BRIDGE WITH L_S

$$V_{dc} = \frac{3V_{LL,m}}{\pi}\cos\alpha - \frac{3\omega L_s}{\pi}I_{dc}, \qquad \cos\alpha - \cos(\alpha+\mu) = \frac{2\omega L_s I_{dc}}{V_{LL,m}}$$

Constant I_{dc}; L_s is per phase. The drop equals $6fL_sI_{dc}$.

VOLTAGE (IMPULSE) COMMUTATION

$$t_c = \frac{C\,V_C}{I_0}$$

An auxiliary thyristor switches a capacitor C, charged to V_C, across the conducting thyristor, reverse-biasing it at once. With the load current I_0 constant during commutation, the capacitor voltage changes linearly from $-V_C$ to zero in time t_c.

CURRENT (RESONANT-PULSE) COMMUTATION

$$I_{cp} = V_C\sqrt{\frac{C}{L}} > I_0, \qquad t_c = \sqrt{LC}\left[\pi - 2\sin^{-1}\frac{I_0}{I_{cp}}\right]$$

An LC circuit drives a resonant current pulse of peak I_{cp} through the thyristor, opposite to its load current I_0, assumed constant during commutation. The thyristor current reaches zero when the pulse reaches I_0. A diode across the thyristor then carries the excess, and its forward drop reverse-biases the thyristor until the pulse falls back to I_0. Commutation needs $I_{cp} > I_0$.

AC VOLTAGE CONTROLLER, R LOAD (SINGLE-PHASE, PHASE CONTROL)

$$V_{o,rms} = V_s\sqrt{\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)}$$

Two antiparallel thyristors (or a triac), fired symmetrically. V_s is the rms supply voltage and α is measured from the voltage zero crossing. Control range $0 \le \alpha \le 180^\circ$. The output has no DC component. Input power factor $= V_{o,rms}/V_s$.

RECTIFIER-FED SEPARATELY EXCITED DC MOTOR

$$V_{dc} = E_b + I_a R_a, \qquad E_b = K\phi\,\omega_m$$

Steady state with continuous armature current. Speed is raised by increasing V_{dc} (reducing α).

Line-current harmonics, power factor and distortion factor

AVERAGE OUTPUT VOLTAGE, SINGLE-PHASE FULLY CONTROLLED BRIDGE

$$V_d = \frac{2\sqrt{2}}{\pi}\,V_s\cos\alpha = 0.9\,V_s\cos\alpha$$

Continuous conduction, zero source inductance.

AVERAGE OUTPUT VOLTAGE, THREE-PHASE FULLY CONTROLLED BRIDGE

$$V_d = \frac{3\sqrt{2}}{\pi}\,V_{LL}\cos\alpha = 1.35\,V_{LL}\cos\alpha$$

Continuous conduction, zero source inductance. At $\alpha = 0$ this is the diode bridge value $V_{d0} = 1.35\,V_{LL}$.

RMS LINE CURRENT, THREE-PHASE BRIDGE

$$I_s = \sqrt{\tfrac{2}{3}}\,I_d = 0.816\,I_d$$

In each cycle a line carries $+I_d$ for 120°, nothing for 60°, $-I_d$ for 120°, and nothing for 60°. With an ideal ripple-free I_d this does not depend on α.

LINE CURRENT OF PHASE A (SIX-PULSE BRIDGE)

$$i_a(t) = \frac{2\sqrt{3}}{\pi} I_d \left[\cos(\omega t-\alpha) - \frac{1}{5}\cos 5(\omega t-\alpha) + \frac{1}{7}\cos 7(\omega t-\alpha) - \frac{1}{11}\cos 11(\omega t-\alpha) + \frac{1}{13}\cos 13(\omega t-\alpha) - \cdots\right]$$

Reference $v_{an} = V_m\cos\omega t$. At $\alpha = 0$ the positive 120° block is centred on the positive peak of v_{an}. For $\alpha > 0$ the whole waveform is the $\alpha = 0$ waveform delayed by α/ω. The peak of the n-th term is $\frac{4I_d}{n\pi}\sin\frac{n\pi}{3}$. Zero overlap, ripple-free I_d.

RMS FUNDAMENTAL LINE CURRENT, THREE-PHASE BRIDGE

$$I_{s1} = \frac{\sqrt{6}}{\pi}\,I_d = 0.78\,I_d$$

The peak of the fundamental is $\frac{2\sqrt{3}}{\pi}I_d$. Divide by $\sqrt{2}$ for the rms value.

RMS OF THE N-TH HARMONIC, THREE-PHASE BRIDGE

$$I_{sn} = \frac{I_{s1}}{n}, \qquad n = 6k \pm 1$$

Six-pulse bridge, zero overlap, ripple-free I_d. Relative to the fundamental the magnitude falls as $1/n$, and it does not depend on α.

LINE CURRENT, SINGLE-PHASE FULLY CONTROLLED BRIDGE

$$i_s(t) = \frac{4I_d}{\pi}\left[\sin(\omega t-\alpha) + \frac{1}{3}\sin 3(\omega t-\alpha) + \frac{1}{5}\sin 5(\omega t-\alpha) + \cdots\right], \qquad I_{sn} = \frac{2\sqrt{2}\,I_d}{n\pi}$$

Reference $v_s = V_m\sin\omega t$. The current is a square wave of $\pm I_d$ delayed by α, so every odd order n is present. Ripple-free I_d, zero overlap. The magnitudes do not depend on α.

LINE CURRENT, SINGLE-PHASE HALF-CONTROLLED BRIDGE

$$I_{sn} = \frac{2\sqrt{2}\,I_d}{n\pi}\left|\cos\frac{n\alpha}{2}\right| \ (n \text{ odd}), \qquad I_s = I_d\sqrt{\frac{\pi-\alpha}{\pi}}$$

Half-controlled bridge (semi-converter), or a full bridge with a freewheeling diode, with ripple-free I_d. The line current is $+I_d$ from α to π, zero while the load freewheels, then $-I_d$ from $\pi+\alpha$ to 2π. Here the harmonic magnitudes **do** depend on α, and the fundamental lags the supply voltage by $\alpha/2$.

DISPLACEMENT FACTOR

$$\mathrm{DPF} = \cos\phi_1$$

ϕ_1 is the phase angle between the supply voltage and the fundamental line current. With zero overlap, $\phi_1 = \alpha$ for a fully controlled bridge and $\phi_1 = \alpha/2$ for a single-phase half-controlled bridge.

DISTORTION FACTOR

$$\mathrm{DF} = \frac{I_{s1}}{I_s}$$

The ratio of the fundamental rms current to the total rms line current. DF ≤ 1.

INPUT POWER FACTOR

$$\mathrm{PF} = \frac{P}{S} = \frac{I_{s1}}{I_s}\cos\phi_1 = \mathrm{DF} \times \mathrm{DPF}$$

Sinusoidal supply voltage. For the three-phase case $P = \sqrt{3}\,V_{LL} I_{s1}\cos\phi_1$ and $S = \sqrt{3}\,V_{LL} I_s$. For the single-phase case $P = V_s I_{s1}\cos\phi_1$ and $S = V_s I_s$.

TOTAL HARMONIC DISTORTION OF THE LINE CURRENT

$$\mathrm{THD} = \frac{\sqrt{I_s^2 - I_{s1}^2}}{I_{s1}} = \sqrt{\frac{1}{\mathrm{DF}^2} - 1}$$

Harmonics are measured relative to the fundamental. Equivalently, $\mathrm{DF} = 1/\sqrt{1 + \mathrm{THD}^2}$.

SIX-PULSE BRIDGE, IDEAL INDUCTIVE LOAD

$$\mathrm{DF} = \frac{3}{\pi} = 0.955, \quad \mathrm{PF} = \frac{3}{\pi}\cos\alpha, \quad \mathrm{THD} = \sqrt{\frac{\pi^2}{9} - 1} \approx 31\%$$

Fully controlled bridge, ripple-free I_d, zero overlap. DF and THD do not change with α. Only DPF does. A diode bridge ($\alpha = 0$) has PF $= 0.955$.

SINGLE-PHASE HALF-CONTROLLED BRIDGE, IDEAL INDUCTIVE LOAD

$$\mathrm{DPF} = \cos\frac{\alpha}{2}, \qquad \mathrm{PF} = \frac{2\sqrt{2}}{\pi}\,\frac{\cos^2(\alpha/2)}{\sqrt{(\pi-\alpha)/\pi}} = \frac{\sqrt{2}\,(1+\cos\alpha)}{\sqrt{\pi(\pi-\alpha)}}$$

Ripple-free I_d, zero overlap. At $\alpha = 0$ this gives PF $= 0.900$, the same as the full bridge. For $\alpha > 0$ its PF is higher than the full bridge's $0.9\cos\alpha$.

POWER AND REACTIVE POWER, THREE-PHASE FULLY CONTROLLED BRIDGE

$$P = V_d I_d = V_{d0} I_d \cos\alpha, \qquad Q_1 = \sqrt{3}\,V_{LL} I_{s1}\sin\alpha = V_{d0} I_d \sin\alpha$$

Zero overlap, lossless converter. Q_1 is the fundamental reactive power. With $V_{d0} = \frac{3\sqrt{2}}{\pi}V_{LL}$, the fundamental apparent power is $\sqrt{P^2 + Q_1^2} = V_{d0} I_d$.

OUTPUT VOLTAGE WITH OVERLAP, THREE-PHASE FULLY CONTROLLED BRIDGE

$$V_d = V_{d0}\cos\alpha - \frac{3\omega L_s}{\pi} I_d = V_{d0}\,\frac{\cos\alpha + \cos(\alpha+\mu)}{2}, \qquad \cos\alpha - \cos(\alpha+\mu) = \frac{\sqrt{2}\,\omega L_s I_d}{V_{LL}}$$

L_s is the source inductance per phase, with ripple-free I_d. For the single-phase fully controlled bridge the matching result is $V_d = 0.9 V_s\cos\alpha - \frac{2\omega L_s}{\pi} I_d$.

Inverters and pulse width modulation

SQUARE-WAVE RMS OUTPUT (FULL-BRIDGE)

$$V_{o,\mathrm{rms}} = V_s$$

Half-bridge: the output is $\pm V_s/2$, so the rms is $V_s/2$.

FOURIER SERIES OF THE FULL-BRIDGE SQUARE WAVE

$$v_o(t) = \sum_{n=1,3,5,\ldots} \frac{4V_s}{n\pi}\sin(n\omega t)$$

Time origin at a rising edge. Only odd harmonics; the nth has peak $4V_s/(n\pi)$. For the half-bridge use $V_s/2$ in place of V_s.

FUNDAMENTAL RMS (FULL-BRIDGE)

$$V_{1,\mathrm{rms}} = \frac{4V_s}{\sqrt{2}\,\pi} = 0.9\,V_s$$

Half-bridge: $0.45\,V_s$.

TOTAL HARMONIC DISTORTION OF THE SQUARE WAVE

$$\mathrm{THD} = \sqrt{\frac{V_{o,\mathrm{rms}}^2 - V_{1,\mathrm{rms}}^2}{V_{1,\mathrm{rms}}^2}} = \sqrt{\frac{\pi^2}{8} - 1} \approx 0.483$$

About 48.3 percent, the same for half- and full-bridge. The harmonic factor of the nth harmonic is $\mathrm{HF}_n = V_{n,\mathrm{rms}}/V_{1,\mathrm{rms}}$.

LOAD CURRENT WITH A SERIES R-L LOAD

$$I_{n,\mathrm{rms}} = \frac{V_{n,\mathrm{rms}}}{\sqrt{R^2 + (n\omega L)^2}}, \qquad P = \sum_n I_{n,\mathrm{rms}}^2 R$$

Steady state. Each harmonic is treated separately, with the impedance at its own frequency $n\omega$. Higher harmonics see a larger reactance, so the current is closer to a sine wave than the voltage. For a lossless inverter the average dc source current is $I_s = P/V_s$.

QUASI-SQUARE RMS AND HARMONICS

$$V_{o,\mathrm{rms}} = V_s\sqrt{\frac{\sigma}{\pi}}, \qquad V_{n,\mathrm{peak}} = \frac{4V_s}{n\pi}\left|\sin\frac{n\sigma}{2}\right| \quad (n\ \text{odd})$$

σ is the width of each output pulse in radians per half-cycle. Even harmonics are absent.

MULTIPLE-PULSE (UNIFORM) PWM RMS

$$V_{o,\mathrm{rms}} = V_s\sqrt{\frac{p\,\delta}{\pi}}$$

Full-bridge output of p equal pulses of height V_s per half-cycle, each δ radians wide ($p\delta \le \pi$). With $p = 1$ and $\delta = \sigma$ this is the quasi-square result.

SIX-STEP 180°: FUNDAMENTAL AND HARMONICS

$$V_{LL1,\mathrm{rms}} = \frac{\sqrt{6}}{\pi}V_s = 0.78V_s, \qquad V_{ph1,\mathrm{rms}} = \frac{\sqrt{2}}{\pi}V_s = 0.45V_s$$

For 180° conduction. The nth harmonic ($n = 6k \pm 1$) has rms $1/n$ of the fundamental, so $V_{LLn,\mathrm{rms}} = 0.78V_s/n$. Triplen harmonics are absent from both phase and line voltages.

LINE-TO-PHASE RELATION

$$V_{LL,\mathrm{rms}} = \sqrt{3}\,V_{ph,\mathrm{rms}}$$

Balanced load. Holds for the total rms and for the fundamental when the phase voltage has no triplen harmonics, as in both conduction modes here.

MODULATION RATIOS

$$m_a = \frac{V_{\mathrm{control}}}{V_{\mathrm{tri}}}, \qquad m_f = \frac{f_c}{f_m}$$

m_a is the amplitude modulation ratio; m_f is the frequency modulation ratio.

FUNDAMENTAL IN THE LINEAR REGION ($M_A\LE1$)

$$V_{1,\text{peak}} = m_a V_s \text{ (full-bridge)}, \qquad V_{ao1,\text{peak}} = m_a \frac{V_s}{2} \text{ (half-bridge or one leg)}$$

Large m_f; the full-bridge result is the same for bipolar and unipolar switching. The fundamental grows linearly with m_a and its frequency is f_m.

THREE-PHASE SPWM LINE VOLTAGE (LINEAR REGION)

$$V_{LL1,\text{rms}} = \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{m_a V_s}{2} = 0.612\, m_a V_s$$

Each leg gives a pole voltage of peak $m_a V_s/2$, and the line voltage is $\sqrt{3}$ times its rms. V_s is the dc bus voltage, $m_a \le 1$, sinusoidal references with no third-harmonic injection.

CSI OUTPUT CURRENT

$$I_{1,\text{rms}} = \frac{4 I_d}{\sqrt{2}\,\pi} = 0.9\, I_d \text{ (single-phase)}, \qquad I_{L1,\text{rms}} = \frac{\sqrt{6}}{\pi} I_d = 0.78\, I_d \text{ (three-phase)}$$

Single-phase: a square wave of current $\pm I_d$. Three-phase: each line current is a 120°-wide block of height I_d, the same shape as the 180°-mode VSI line voltage, with harmonics of order $6k \pm 1$.

V/F FLUX AND TORQUE RELATIONS

$$\Phi \propto \frac{V}{f}, \qquad T_{\max} \propto \left(\frac{V}{f}\right)^2, \qquad N_s = \frac{120 f}{P}$$

Stator impedance drop neglected. Constant V/f gives constant flux and roughly constant breakdown torque below base speed. P is the number of poles and f is in Hz.

SPEED FROM ENCODER PULSE COUNT

$$n_{\text{rpm}} = \frac{60\,\Delta c}{K\,\Delta t}$$

Δc is the count in time Δt seconds and K is the counts per revolution (N, or $4N$ with 4x decoding). Counting pulses in a fixed time suits high speed, while timing between pulses suits low speed.

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