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Networks (Electric Circuits) Formula Sheet

INSIDE

- 166 formulas in 7 topics
- When to use each one
- Taken from the free topic notes on iaeacademy.in

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Circuit laws, sources, and node and mesh analysis

RESISTOR (OHM'S LAW)

$$v = Ri, \qquad p = vi = i^2R = \frac{v^2}{R} \geq 0$$

Linear resistor, passive sign convention. Conductance $G = 1/R$ (siemens), so $i = Gv$.

INDUCTOR

$$v = L\frac{di}{dt}$$

$$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v\, d\tau$$

$$w_L = \frac{1}{2}Li^2$$

Passive sign convention. The stored energy depends only on the present current. In DC steady state $di/dt = 0$, so an inductor behaves as a short circuit.

CAPACITOR

$$i = C\frac{dv}{dt}$$

$$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i\, d\tau$$

$$q = Cv, \qquad w_C = \frac{1}{2}Cv^2 = \frac{q^2}{2C}$$

Passive sign convention. In DC steady state $dv/dt = 0$, so a capacitor behaves as an open circuit.

CONTINUITY OF CAPACITOR VOLTAGE AND INDUCTOR CURRENT

$$v_C(0^+) = v_C(0^-), \qquad i_L(0^+) = i_L(0^-)$$

0^- is the instant just before the switching and 0^+ the instant just after. The rule holds whenever the capacitor current and the inductor voltage stay finite (no impulses). That is guaranteed when the sources are finite and the switching creates no loop made only of capacitors and ideal voltage sources and no cut-set made only of inductors and ideal current sources. If such a loop is formed, the capacitor voltages are forced to jump (solve with charge conservation); if such a cut-set is formed, the inductor currents are forced to jump (solve with flux-linkage conservation). Capacitor current, inductor voltage and resistor quantities CAN jump at $t = 0$.

SERIES AND PARALLEL COMBINATIONS

$$\text{Series: } R = R_1 + R_2, \quad L = L_1 + L_2, \quad \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\text{Parallel: } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}, \quad \frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}, \quad C = C_1 + C_2$$

Inductors without mutual coupling. Two resistors in parallel give $R_1R_2/(R_1 + R_2)$, and two capacitors in series give $C_1C_2/(C_1 + C_2)$. Series capacitors that start uncharged carry equal charge.

COUPLED INDUCTORS (DOT CONVENTION)

$$v_1 = L_1\frac{di_1}{dt} \pm M\frac{di_2}{dt}$$

$$v_2 = L_2\frac{di_2}{dt} \pm M\frac{di_1}{dt}$$

$$M = k\sqrt{L_1L_2}, \quad 0 \leq k \leq 1$$

Each coil uses the passive sign convention for its own v and i. Take **+M** when both currents enter (or both leave) the dotted terminals, and **−M** when one enters a dot and the other leaves one. $k = 1$ means perfect coupling.

COUPLED INDUCTORS IN SERIES AND PARALLEL; STORED ENERGY

$$\text{Series: } L_{eq} = L_1 + L_2 \pm 2M$$

$$\text{Parallel: } L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 \mp 2M}$$

$$\text{Energy: } w = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 \pm M i_1 i_2$$

For L_{eq}: upper sign = aiding (in series, the current enters both dotted terminals in turn; in parallel, the dotted terminals are joined together), lower sign = opposing. For the energy: $+Mi_1i_2$ when i_1 and i_2 both enter (or both leave) the dotted terminals, $-Mi_1i_2$ otherwise. $w \geq 0$ for all currents, which is why $M \leq \sqrt{L_1 L_2}$.

COUPLED COILS WITH A COMMON TERMINAL (T-EQUIVALENT)

$$\text{Like terminals joined: } L_A = L_1 - M, \quad L_B = L_2 - M, \quad L_C = M$$

$$\text{Unlike terminals joined: } L_A = L_1 + M, \quad L_B = L_2 + M, \quad L_C = -M$$

Two coupled coils that share one terminal can be replaced by an uncoupled T network: series arm L_A on coil 1's side, series arm L_B on coil 2's side, and shunt arm L_C to the common terminal. "Like terminals joined" means both dotted ends (or both undotted ends) meet at the common terminal. A negative inductance in the model is only a mathematical element; the behaviour at the three terminals is exact.

CHARGE SHARING BETWEEN TWO CAPACITORS

$$V_f = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

$$W_{\text{lost}} = \frac{C_1 C_2 \, (V_1 - V_2)^2}{2 \, (C_1 + C_2)}$$

Capacitors charged to V_1 and V_2 are connected in parallel, + plate to + plate. If they are joined with opposite polarities, replace V_2 by $-V_2$. The total charge is conserved; the energy W_{lost} is dissipated in the connecting resistance, whatever its value.

SOURCE TRANSFORMATION

$$I_s = \frac{V_s}{R}, \qquad V_s = I_s R \qquad (\text{same } R)$$

V_s in series with R is equivalent at its two terminals to I_s in parallel with the same R. The current-source arrow points toward the terminal that was + on the voltage source. The two are equivalent only at the terminals: the power dissipated inside differs. An ideal source (no R) cannot be transformed. A dependent source can be transformed the same way, provided its controlling variable is not the current or voltage of the resistor R being transformed, because that quantity changes in the transformation.

KIRCHHOFF'S CURRENT LAW

$$\sum_k i_k = 0 \quad (\text{currents leaving a node or any closed surface})$$

Equivalently, total current entering = total current leaving. It also holds for a closed surface enclosing several nodes, which is the basis of the supernode.

KIRCHHOFF'S VOLTAGE LAW

$$\sum_k v_k = 0 \quad (\text{voltage drops around any closed loop})$$

Count a drop as positive when the loop crosses an element from + to −. It holds for any closed path, including one through a current source or across an open gap (the voltage there is then an unknown).

VOLTAGE DIVISION

$$V_k = V\,\frac{R_k}{R_1 + R_2 + \cdots + R_N}$$

Resistors in series carrying the same current, with nothing else drawing current from the tap. For impedances, replace R by Z.

CURRENT DIVISION

$$I_1 = I\,\frac{R_2}{R_1 + R_2}, \qquad I_k = I\,\frac{G_k}{G_1 + G_2 + \cdots + G_N}$$

Branches in parallel across the same voltage. With two branches, the current in one is proportional to the OTHER branch's resistance.

BALANCED WHEATSTONE BRIDGE

$$\frac{R_{AC}}{R_{CB}} = \frac{R_{AD}}{R_{DB}} \;\Rightarrow\; V_C = V_D, \quad I_{CD} = 0$$

A bridge with arms A–C–B and A–D–B and an element between C and D. When the bridge is balanced and the C–D element contains no source, that element carries no current and can be removed or replaced by a short without changing anything else.

CUBE OF TWELVE EQUAL RESISTORS

$$R_{\text{body diagonal}} = \frac{5}{6}R, \qquad R_{\text{face diagonal}} = \frac{3}{4}R, \qquad R_{\text{edge}} = \frac{7}{12}R$$

Resistance between two corners of a cube whose twelve edges are each R: opposite corners of the cube, opposite corners of one face, and the two ends of one edge. Each follows from symmetry: join the corners that are at equal potential, then reduce.

INFINITE LADDER NETWORK

$$R_{in} = \frac{R_1 + \sqrt{R_1^2 + 4R_1R_2}}{2} \qquad \left(R_1 = R_2 = R:\; R_{in} = \frac{1+\sqrt{5}}{2}\,R \approx 1.618R\right)$$

A ladder that starts at the input with a series resistor R_1, then a shunt resistor R_2, repeated without end. Adding one more section does not change the input resistance, so $R_{in} = R_1 + (R_2 \parallel R_{in})$, and the formula is the positive root. If the ladder starts with the shunt arm instead, its input resistance is $R_2 \parallel R_{in}$ with R_{in} from the formula.

DELTA TO STAR

$$R_a = \frac{R_{ab}R_{ca}}{R_{ab} + R_{bc} + R_{ca}}$$
$$R_b = \frac{R_{ab}R_{bc}}{R_{ab} + R_{bc} + R_{ca}}$$
$$R_c = \frac{R_{bc}R_{ca}}{R_{ab} + R_{bc} + R_{ca}}$$

Star arm at a node = product of the two delta resistors touching that node ÷ sum of all three delta resistors.

STAR TO DELTA

$$R_{ab} = \frac{S}{R_c}, \quad R_{bc} = \frac{S}{R_a}, \quad R_{ca} = \frac{S}{R_b}$$
$$S = R_aR_b + R_bR_c + R_cR_a$$

Delta resistor between two nodes = sum of the pairwise products of the star arms ÷ the star arm at the OPPOSITE (third) node. Equivalently, $R_{ab} = R_a + R_b + R_aR_b/R_c$.

BALANCED (EQUAL-ARM) CASE

$$R_Y = \frac{R_\Delta}{3}, \qquad R_\Delta = 3R_Y$$

All three arms equal. The same relation holds for equal impedances at a single frequency, for example a balanced three-phase load.

NODAL EQUATIONS BY INSPECTION

$$[G][V] = [I]$$

$$G_{kk} = \text{sum of conductances connected to node } k$$

$$G_{kj} = -(\text{sum of conductances between nodes } k \text{ and } j)$$

Valid for circuits with only resistors and independent current sources. I_k = net source current injected INTO node k. The matrix is symmetric; a dependent source breaks the symmetry and has to be added by hand.

MESH EQUATIONS BY INSPECTION

$$[R][I] = [V]$$

$$R_{kk} = \text{sum of resistances in mesh } k$$

$$R_{kj} = -(\text{sum of resistances shared by meshes } k \text{ and } j)$$

Valid when all mesh currents are clockwise and the circuit has only resistors and independent voltage sources. V_k = algebraic sum of the source voltages driving current in mesh k's direction (rises positive). The matrix is symmetric unless dependent sources are present.

NUMBER OF KCL OR KVL EQUATIONS TO WRITE

$$N_{\text{nodal}} = (n-1) - n_V$$

$$N_{\text{mesh}} = (b-n+1) - n_I$$

n = nodes and b = branches of a connected circuit, counting every element as a separate branch. n_V = voltage sources (independent or dependent); each fixes a node voltage or supplies a supernode constraint, so one KCL equation fewer is needed. n_I = current sources (independent or dependent); each fixes a mesh current or supplies a supermesh constraint. A dependent source's constraint is still expressed in the unknowns, so it has to be substituted, not just read off. Assumes no loop made only of voltage sources and no cut-set made only of current sources. Choose the method that needs fewer equations.

POWER, ENERGY AND POWER BALANCE

$$p = vi, \qquad W = \int_{t_1}^{t_2} p\,dt, \qquad \sum_{\text{all elements}} p_{\text{absorbed}} = 0$$

Passive sign convention. $1\,\text{kWh} = 3.6 \times 10^6$ J.

SAFE CURRENT AND VOLTAGE OF A RATED RESISTOR

$$I_{\max} = \sqrt{\frac{P_{\text{rated}}}{R}}, \qquad V_{\max} = \sqrt{P_{\text{rated}}\,R}$$

The power rating is the largest power a component can dissipate continuously without overheating. In series, the smallest $I_{\max}$ limits the whole string; in parallel, the smallest $V_{\max}$ limits.

EFFICIENCY

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{loss}}}$$

Losses are the power dissipated in resistances, for example I^2R in conductors and in a source's internal resistance.

PRACTICAL SOURCE FEEDING A RESISTIVE LOAD

$$I = \frac{V_s}{R_s + R_L}, \qquad P_L = \frac{V_s^2 R_L}{(R_s + R_L)^2}$$
$$\eta = \frac{R_L}{R_s + R_L}$$

η is the fraction of the power generated by the ideal part, $V_s I$, that reaches R_L. It approaches 100% as $R_L \gg R_s$.

MAXIMUM POWER FROM A PRACTICAL SOURCE

$$R_L = R_s \;\Rightarrow\; P_{L,\max} = \frac{V_s^2}{4R_s}, \qquad \eta = 50\%$$

Fixed V_s and R_s with a variable resistive load. Maximum power transfer and maximum efficiency happen at different operating points. If instead R_L is fixed and R_s can be chosen, $P_L = V_s^2 R_L/(R_s + R_L)^2$ is largest at $R_s = 0$, not at $R_s = R_L$: the matching condition applies only when the load is the variable.

BATTERY WITH INTERNAL RESISTANCE

$$V_t = E - Ir \quad \text{(discharging)}$$
$$V_t = E + Ir \quad \text{(charging)}$$

E = EMF (the open-circuit voltage, which depends on the state of charge), r = internal resistance, I = magnitude of the current. Short-circuit current is E/r. Stored energy ≈ nominal voltage × capacity in Ah, which gives Wh.

SOLAR CELL (IDEAL SINGLE-DIODE MODEL)

$$I = I_{ph} - I_0\left(e^{V/(nV_T)} - 1\right)$$
$$I_{sc} = I_{ph}$$
$$V_{oc} = nV_T \ln\left(1 + \frac{I_{ph}}{I_0}\right)$$

Generator convention: I is the current delivered out of the + terminal. I_{ph} is the photocurrent (proportional to irradiance), I_0 the diode saturation current, n the ideality factor, and $V_T = kT/q \approx 25.9$ mV at 300 K. Series and shunt resistances are neglected.

TWIGS AND LINKS OF A CONNECTED GRAPH

$$\text{twigs} = n - 1, \qquad \text{links} = b - n + 1$$

For a graph with p separate parts: twigs $= n - p$ and links $= b - n + p$.

INDEPENDENT EQUATIONS

$$N_{\text{KCL}} = n - 1, \qquad N_{\text{KVL}} = b - n + 1$$

Connected graph. For a planar graph, $b - n + 1$ is also the number of meshes (windows).

Network theorems

POWER AND RMS VALUE WITH COMPONENTS AT DIFFERENT FREQUENCIES

$$P = I_0^2 R + \sum_k I_k^2 R, \qquad I_{\text{rms}} = \sqrt{I_0^2 + \sum_k I_k^2}$$

I_0 = DC component; I_k = rms value of the component at frequency ω_k, with all the ω_k non-zero and distinct. Here average powers do add, because the cross terms average to zero over time. Two components at the same frequency do not: combine them as phasors first. Example: $i = 3 + 4\sqrt{2}\sin\omega t$ A gives $I_{\text{rms}} = \sqrt{9 + 16} = 5$ A.

THEVENIN–NORTON RELATIONS

$$V_{th} = V_{oc}, \qquad I_N = I_{sc}$$
$$Z_{th} = Z_N = \frac{V_{oc}}{I_{sc}}, \qquad V_{th} = I_N Z_{th}$$

$V_{oc} = V_a - V_b$ with a–b open; I_{sc} flows from a to b through a short across a–b. The ratio cannot be used when both are zero. That is always the case for a network with no independent source, and it can also happen when the sources inside produce nothing at a–b (a balanced bridge, for example). Then use the test-source method.

SOURCE TRANSFORMATION

$$V_s \text{ in series with } Z \;\leftrightarrow\; I_s = \frac{V_s}{Z} \text{ in parallel with } Z$$

The arrow of I_s points towards the terminal on the $+$ side of V_s. The two forms are equivalent only as seen from the terminals: the current and power inside Z differ. A dependent source can be transformed the same way, provided its controlling variable is not inside the Z being moved. An ideal voltage source ($Z = 0$) or an ideal current source ($Z \to \infty$) cannot be transformed.

TEST-SOURCE METHOD

$$Z_{th} = \left.\frac{V_t}{I_t}\right|_{\text{independent sources off}}$$

Short the independent voltage sources, open the independent current sources, and leave every dependent source active. Apply V_t across a–b ($+$ at a) and find the current I_t entering terminal a from the test source (or inject I_t into a and find V_t). This works whenever the equivalent exists, and it is the method to use when the network has no independent source.

MAXIMUM POWER TRANSFER, RESISTIVE LOAD

$$R_L = R_{th}, \qquad P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{I_N^2 R_{th}}{4}$$

$R_{th} > 0$ is fixed and R_L varies. If the network has dependent sources, use the R_{th} found with them active.

LOAD POWER FOR ANY LOAD RESISTANCE

$$P_L = \frac{V_{th}^2 R_L}{(R_{th} + R_L)^2}$$

P_L rises for $R_L < R_{th}$ and falls for $R_L > R_{th}$. So if R_L is restricted to a range that excludes R_{th}, the best choice is the allowed value nearest R_{th}.

EFFICIENCY OF THE THEVENIN EQUIVALENT

$$\eta = \frac{R_L}{R_L + R_{th}}, \qquad \eta = 50\% \text{ at } R_L = R_{th}$$

This is the efficiency of the Thevenin equivalent circuit only. The power drawn from the sources inside the original network is different, so the real efficiency at matching is generally not 50%.

AC MAXIMUM POWER TRANSFER, CONJUGATE MATCH

$$Z_L = Z_{th}^* = R_{th} - jX_{th}, \qquad P_{max} = \frac{|V_{th}|^2}{4R_{th}}$$

Use this when both R_L and X_L are free (and $R_{th} > 0$). The reactances cancel, so the loop is resonant. $|V_{th}|$ is an rms value; if the phasor is a peak value V_m, then $P_{max} = |V_m|^2/(8R_{th})$.

AC SOURCE, PURELY RESISTIVE LOAD

$$R_L = |Z_{th}| = \sqrt{R_{th}^2 + X_{th}^2}$$
$$P_{max} = \frac{|V_{th}|^2}{2\left(R_{th} + |Z_{th}|\right)}$$

Use this when the load must be a pure resistor ($X_L = 0$). Choosing $R_L = R_{th}$ here is a common error.

FIXED LOAD REACTANCE, VARIABLE LOAD RESISTANCE

$$R_L = \sqrt{R_{th}^2 + (X_{th} + X_L)^2}$$

$$P_{max} = \frac{|V_{th}|^2}{2\,(R_{th} + R_L)}$$

X_L is fixed and R_L varies; P_{max} is evaluated at that R_L. The pure-resistor case is $X_L = 0$, which gives back $R_L = |Z_{th}|$.

FIXED LOAD ANGLE, VARIABLE MAGNITUDE

$$|Z_L| = |Z_{th}|$$

$$P_{max} = \frac{|V_{th}|^2 \cos\theta_L}{2|Z_{th}|\,[1 + \cos(\theta_{th} - \theta_L)]}$$

Use this for a load $Z_L = |Z_L|\angle\theta_L$ whose angle θ_L is fixed (a fixed power factor) and whose magnitude alone varies; θ_{th} is the angle of Z_{th}. Setting $\theta_L = 0$ gives back the pure-resistor result, and $\theta_L = -\theta_{th}$ gives back the conjugate match.

RECIPROCITY CONDITIONS FOR A TWO-PORT

$$z_{12} = z_{21}, \quad y_{12} = y_{21}, \quad AD - BC = 1$$

$$h_{12} = -h_{21}, \quad g_{12} = -g_{21}$$

Convention: I_1 and I_2 both flow INTO their ports, and ABCD is defined by $V_1 = AV_2 - BI_2$, $I_1 = CV_2 - DI_2$. Where the parameter sets exist, any one condition implies the others. Symmetry ($z_{11} = z_{22}$, $A = D$) is a separate, stronger property.

TELLEGEN'S THEOREM

$$\sum_{k=1}^{b} v_k\, i_k = 0, \qquad \sum_{k=1}^{b} v'_k\, i''_k = 0$$

The sums run over all b branches. The first form is conservation of power: the power delivered by the sources equals the power absorbed by everything else. The second form pairs voltages from one network with currents from another that has the same graph and the same reference directions. It also holds for one network at two different instants.

MILLMAN'S THEOREM

$$V_{ab} = \frac{\sum_k V_k Y_k + \sum_j I_j}{\sum_k Y_k}, \qquad Z_{eq} = \frac{1}{\sum_k Y_k}$$

I_j = ideal current sources between the two nodes, directed into a; they add nothing to the denominator. A passive branch enters with $V_k = 0$. An ideal voltage source with $Z_k = 0$ cannot be included, because it fixes V_{ab} directly.

MILLER'S THEOREM

$$Z_1 = \frac{Z}{1 - K}, \qquad Z_2 = \frac{Z}{1 - 1/K} = \frac{ZK}{K - 1}$$

$$K = \frac{V_2}{V_1}$$

Z is removed and replaced by Z_1 from node 1 to ground and Z_2 from node 2 to ground. K must be the gain with Z connected. For $K > 1$, Z_1 is negative; for $0 < K < 1$ (a follower), Z_2 is negative.

MILLER CAPACITANCE

$$C_1 = C\,(1 - K), \qquad C_2 = C\left(1 - \frac{1}{K}\right)$$

This is the capacitor version (impedance $1/(j\omega C)$). For an inverting stage, $K = -A$ with $A > 0$. Then $C_1 = C(1 + A)$ is large (the Miller effect), and $C_2 \approx C$ when $A \gg 1$. A bridging resistor R becomes $R/(1 - K)$ at the input.

COMPENSATION THEOREM

$$V_c = I\,\Delta Z, \qquad \Delta I_{\text{branch}} = -\frac{I\,\Delta Z}{Z_{th} + Z + \Delta Z}$$

I = original branch current. Z_{th} = impedance seen by the branch looking into the rest of the network, with independent sources off. The minus sign says that increasing the impedance reduces the branch current. A typical use is the error caused by inserting an ammeter of resistance ΔZ.

Transients in RL, RC and RLC circuits

CONTINUITY OF THE STATE VARIABLES

$$v_C(0^+) = v_C(0^-), \qquad i_L(0^+) = i_L(0^-)$$

Holds whenever i_C and v_L contain no impulse. It can fail in two ways: a source that itself contains an impulse (such as $\delta(t)$), or a switching that creates a loop of capacitors and ideal voltage sources whose voltages do not already satisfy KVL, or a cut-set of inductors and ideal current sources whose currents do not already satisfy KCL (see the tip below). Put in terms of charge and flux linkage: $q_C = Cv_C$ and $\lambda_L = Li_L$ are continuous.

FIRST DERIVATIVES AT 0+

$$\left.\frac{dv_C}{dt}\right|_{0^+} = \frac{i_C(0^+)}{C}, \qquad \left.\frac{di_L}{dt}\right|_{0^+} = \frac{v_L(0^+)}{L}$$

Find $i_C(0^+)$ and $v_L(0^+)$ from the $t = 0^+$ circuit, using KCL and KVL with capacitors replaced by voltage sources and inductors by current sources. A second-order circuit needs this as its second initial condition. For the derivative of a resistor quantity, differentiate its KVL or KCL equation and substitute these values.

TIME CONSTANT

$$\tau = R_{th}\,C \quad (\text{RC}), \qquad \tau = \frac{L}{R_{th}} \quad (\text{RL})$$

R_{th} is the resistance seen from the capacitor or inductor terminals with independent voltage sources shorted and independent current sources opened. Keep dependent sources and find R_{th} with a test source, $R_{th} = v_{test}/i_{test}$. Use the circuit that exists after the switching.

COMPLETE RESPONSE OF A FIRST-ORDER CIRCUIT WITH DC SOURCES

$$x(t) = x(\infty) + \left[x(t_0^+) - x(\infty)\right] e^{-(t-t_0)/\tau}, \qquad t \geq t_0$$

Valid for any voltage or current in a first-order circuit with constant sources. t_0 is the switching instant (usually 0). Use the 0^+ value, not the 0^- value, for anything other than v_C and i_L.

SERIES RC ON A DC SOURCE

$$v_C(t) = V_s + (V_0 - V_s)\,e^{-t/RC}, \qquad i(t) = \frac{V_s - V_0}{R}\,e^{-t/RC}$$

DC source V_s switched on at $t = 0$ with $v_C(0^-) = V_0$. i flows from the source into the + terminal of the capacitor. $V_0 = 0$ gives charging, $V_s(1 - e^{-t/RC})$; $V_s = 0$ gives discharge, $V_0 e^{-t/RC}$.

SERIES RL ON A DC SOURCE

$$i_L(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-Rt/L}, \qquad v_L(t) = (V_s - RI_0)\,e^{-Rt/L}$$

DC source V_s switched on at $t = 0$ with $i_L(0^-) = I_0$. $I_0 = 0$ gives build-up, $\frac{V_s}{R}(1 - e^{-Rt/L})$; $V_s = 0$ gives decay, $I_0 e^{-Rt/L}$. v_L jumps to $V_s - RI_0$ at $t = 0^+$; if the inductor was in DC steady state before switching, $v_L(0^-) = 0$.

INITIAL SLOPE AND TIME TO REACH A VALUE

$$\left.\frac{dx}{dt}\right|_{t_0^+} = \frac{x(\infty) - x(t_0^+)}{\tau}, \qquad t - t_0 = \tau \ln \frac{x(t_0^+) - x(\infty)}{x(t) - x(\infty)}$$

The tangent at the start reaches the final value after exactly one time constant. The second form gives the instant at which x reaches a stated value between $x(t_0^+)$ and $x(\infty)$.

RISE TIME AND HALF-CHANGE TIME

$$t_{10\text{-}90} = \tau \ln 9 \approx 2.2\,\tau, \qquad t_{50} = \tau \ln 2 \approx 0.693\,\tau$$

For a first-order step response. The rise time is the time to go from 10 to 90 percent of the change; t_{50} is the time for half the change.

SERIES RLC: CHARACTERISTIC EQUATION

$$s^2 + \frac{R}{L}\,s + \frac{1}{LC} = 0, \qquad \alpha = \frac{R}{2L}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

For the source-free series loop (a voltage source is replaced by a short). α is the damping coefficient (neper frequency, in s^{-1}) and ω_0 is the undamped natural frequency (rad/s).

PARALLEL RLC: CHARACTERISTIC EQUATION

$$s^2 + \frac{1}{RC}\,s + \frac{1}{LC} = 0, \qquad \alpha = \frac{1}{2RC}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

For the source-free parallel circuit (a current source is replaced by an open). It has the same ω_0 as the series case but a different α: here a larger R means less damping.

ROOTS, DAMPING RATIO AND DAMPED FREQUENCY

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}, \qquad \zeta = \frac{\alpha}{\omega_0}, \qquad \omega_d = \sqrt{\omega_0^2 - \alpha^2} = \omega_0\sqrt{1 - \zeta^2}$$

Both cases have the form $s^2 + 2\alpha s + \omega_0^2 = 0$. ω_d is real only when $\alpha < \omega_0$. The roots multiply to $s_1 s_2 = \omega_0^2$ and add to -2α.

CRITICAL RESISTANCE

$$R_{c,\text{series}} = 2\sqrt{\frac{L}{C}}, \qquad R_{c,\text{parallel}} = \frac{1}{2}\sqrt{\frac{L}{C}}$$

A series RLC is overdamped for $R > 2\sqrt{L/C}$ and underdamped for $R < 2\sqrt{L/C}$. A parallel RLC is the other way round: overdamped for $R < \tfrac{1}{2}\sqrt{L/C}$, underdamped for $R > \tfrac{1}{2}\sqrt{L/C}$.

SERIES RLC SWITCHED ONTO A DC SOURCE: THE CURRENT

$$\text{overdamped:} \qquad i(t) = \frac{V_s - V_0}{L(s_1 - s_2)}\left(e^{s_1 t} - e^{s_2 t}\right)$$

$$\text{critically damped:} \qquad i(t) = \frac{V_s - V_0}{L}\,t\,e^{-\alpha t}$$

$$\text{underdamped:} \qquad i(t) = \frac{V_s - V_0}{\omega_d L}\,e^{-\alpha t}\sin \omega_d t$$

DC source V_s applied at $t = 0$, with $i(0^-) = 0$ and $v_C(0^-) = V_0$; i flows from the source into the + terminal of C. All three cases follow from $i(0^+) = 0$ and $\frac{di}{dt}(0^+) = \frac{V_s - V_0}{L}$. With $R = 0$: $i(t) = (V_s - V_0)\sqrt{C/L}\,\sin \omega_0 t$.

SERIES RLC SWITCHED ONTO A DC SOURCE: THE CAPACITOR VOLTAGE

$$\begin{aligned}
&\text{overdamped:} && v_C(t) = V_s + (V_s - V_0)\,\frac{s_2 e^{s_1 t} - s_1 e^{s_2 t}}{s_1 - s_2} \\
&\text{critically damped:} && v_C(t) = V_s - (V_s - V_0)(1 + \alpha t)\,e^{-\alpha t} \\
&\text{underdamped:} && v_C(t) = V_s - (V_s - V_0)\,e^{-\alpha t}\left(\cos\omega_d t + \frac{\alpha}{\omega_d}\sin\omega_d t\right)
\end{aligned}$$

Same conditions as the current formulas above. Each form satisfies $v_C(0^+) = V_0$ and $\frac{dv_C}{dt}(0^+) = i(0^+)/C = 0$, and $i = C\,\frac{dv_C}{dt}$ gives back the matching current. With $R = 0$: $v_C(t) = V_s - (V_s - V_0)\cos\omega_0 t$.

UNDERDAMPED CAPACITOR VOLTAGE: FIRST PEAK AND OVERSHOOT

$$t_p = \frac{\pi}{\omega_d}, \qquad v_C(t_p) = V_s + (V_s - V_0)\,e^{-\alpha\pi/\omega_d}, \qquad \frac{v_C(t_p) - V_s}{V_s - V_0} = e^{-\alpha\pi/\omega_d} = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

Same conditions, with $V_s > V_0$, so the first extremum is the maximum (for $V_s < V_0$ the same instant gives the lowest point, below V_s). The extrema of v_C fall at $t = k\pi/\omega_d$ ($k = 1, 2, \ldots$), the instants where $i = 0$. Successive maxima are one damped period $2\pi/\omega_d$ apart, and the overshoot above V_s shrinks by the factor $e^{-2\pi\alpha/\omega_d}$ from one maximum to the next. The fractional overshoot is the peak-overshoot formula of the standard second-order system.

LC CIRCUIT (UNDAMPED OSCILLATION)

$$v_C(t) = V_0\cos\omega_0 t, \qquad i(t) = V_0\sqrt{\frac{C}{L}}\,\sin\omega_0 t, \qquad I_{\max} = V_0\sqrt{\frac{C}{L}}$$

A capacitor charged to V_0 is connected across an uncharged inductor at $t = 0$; i is the current leaving the + terminal of C and entering L. The energy swings between $\frac{1}{2}CV_0^2$ and $\frac{1}{2}LI_{\max}^2$ every quarter period. An uncharged LC switched onto a DC source V_s gives $v_C = V_s(1 - \cos\omega_0 t)$, which peaks at $2V_s$.

INDUCTOR WITH INITIAL CURRENT

$$V_L(s) = sL\,I_L(s) - L\,i_L(0^-) \qquad \Longleftrightarrow \qquad I_L(s) = \frac{V_L(s)}{sL} + \frac{i_L(0^-)}{s}$$

Series model: impedance sL in series with a voltage source $L\,i_L(0^-)$ that drives current in the direction of i_L (its + terminal is on the side where i_L leaves). Parallel model: impedance sL in parallel with a current source $i_L(0^-)/s$ pointing in the direction of i_L.

CAPACITOR WITH INITIAL VOLTAGE

$$V_C(s) = \frac{I_C(s)}{sC} + \frac{v_C(0^-)}{s} \qquad \Longleftrightarrow \qquad I_C(s) = sC\,V_C(s) - C\,v_C(0^-)$$

Series model: impedance $\frac{1}{sC}$ in series with a voltage source $v_C(0^-)/s$ of the same polarity as v_C. Parallel model: impedance $\frac{1}{sC}$ in parallel with a current source $C\,v_C(0^-)$ whose arrow points toward the + terminal of v_C (the charged capacitor pushes current out of its + terminal).

INITIAL- AND FINAL-VALUE THEOREMS

$$x(0^+) = \lim_{s\to\infty} sX(s), \qquad x(\infty) = \lim_{s\to 0} sX(s)$$

For a rational $X(s)$, the initial-value theorem needs $X(s)$ to be strictly proper; otherwise $x(t)$ has an impulse at $t = 0$. The final-value theorem is valid only when every pole of $sX(s)$ lies in the open left half-plane. It gives a wrong answer for a sinusoid or a growing response.

NETWORK FUNCTION, IMPULSE RESPONSE AND STEP RESPONSE

$$H(s) = \left.\frac{Y(s)}{X(s)}\right|_{\text{zero initial state}}, \qquad h(t) = \mathcal{L}^{-1}\{H(s)\}, \qquad y_{\text{step}}(t) = \mathcal{L}^{-1}\left\{\frac{H(s)}{s}\right\}, \qquad h(t) = \frac{d\,y_{\text{step}}(t)}{dt}$$

Impulse and step responses are defined with every initial condition zero. The zero-state response to any input is $\mathcal{L}^{-1}\{H(s)X(s)\}$, the convolution of $h(t)$ with the input. The complete response adds the zero-input response produced by the initial-condition sources. Every pole of $H(s)$ is a natural frequency of the circuit; a natural frequency can be missing from a particular $H(s)$ when it cancels against a zero, but it still appears in the zero-input response.

DELAYED INPUTS AND RECTANGULAR PULSES

$$f(t-T)\,u(t-T) \;\leftrightarrow\; e^{-sT}F(s), \qquad x(t) = V\,[u(t) - u(t-T)] \;\Rightarrow\; y(t) = V\,[y_{\text{step}}(t) - y_{\text{step}}(t-T)]$$

Time-shift property, with $T > 0$. A pulse of height V and width T applied at zero initial state gives, by linearity and time invariance, the step response minus a copy delayed by T (each copy is zero before its own step). For the series RC with output v_C: $v_C = V(1 - e^{-t/RC})$ for $0 \le t \le T$, and $v_C = V(1 - e^{-T/RC})\,e^{-(t-T)/RC}$ for $t \ge T$.

IMPULSES CHANGE THE STATE INSTANTLY

$$v_C(0^+) = v_C(0^-) + \frac{1}{C}\int_{0^-}^{0^+} i_C\,dt, \qquad i_L(0^+) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{0^+} v_L\,dt$$

A current impulse $Q\,\delta(t)$ into a capacitor raises its voltage by Q/C. A voltage impulse $\Lambda\,\delta(t)$ across an inductor raises its current by Λ/L. When the impulse is the only source, the circuit is source-free after 0^+, so the impulse response is the zero-input response from these new initial values.

FIRST-ORDER CIRCUIT WITH ANY SOURCE

$$x(t) = x_{ss}(t) + \left[x(0^+) - x_{ss}(0^+)\right] e^{-t/\tau}$$

$x_{ss}(t)$ is the steady-state response (from phasors, for a sinusoid). This generalises the DC formula, where $x_{ss} = x(\infty)$. The transient term vanishes when $x_{ss}(0^+) = x(0^+)$.

SERIES RL SWITCHED ONTO A SINUSOID

$$i(t) = \frac{V_m}{|Z|}\left[\sin(\omega t + \theta - \phi) - \sin(\theta - \phi)\,e^{-Rt/L}\right], \quad |Z| = \sqrt{R^2 + \omega^2 L^2}, \quad \phi = \tan^{-1}\frac{\omega L}{R}$$

With $i(0^-) = 0$. The steady-state current lags the voltage by ϕ. The exponential (DC offset) term starts at $-\frac{V_m}{|Z|}\sin(\theta - \phi)$.

SERIES RC SWITCHED ONTO A SINUSOID

$$v_C(t) = \frac{V_m}{\omega C|Z|}\left[-\cos(\omega t + \theta + \phi) + \cos(\theta + \phi)\,e^{-t/RC}\right], \quad |Z| = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}, \quad \phi = \tan^{-1}\frac{1}{\omega RC}$$

With $v_C(0^-) = 0$. The steady-state current $\frac{V_m}{|Z|}\sin(\omega t + \theta + \phi)$ leads the voltage by ϕ. The current's own transient term is $-\frac{V_m}{|Z|}\,\frac{\cos(\theta+\phi)}{\omega RC}\,e^{-t/RC}$, which gives $i(0^+) = \frac{V_m \sin\theta}{R}$.

SWITCHING ANGLE FOR NO TRANSIENT

$$\text{RL: } \theta = \phi = \tan^{-1}\frac{\omega L}{R}, \qquad \text{RC: } \theta = 90^\circ - \phi = \tan^{-1}(\omega RC) \qquad (\text{or } 180^\circ \text{ more})$$

Both come from one rule: there is no transient when the steady-state value of the state variable (i_L or v_C) at the switching instant equals its actual initial value, which is zero here. For RL, switch when the steady-state current passes through zero; for RC, switch when the steady-state capacitor voltage does.

WORST-CASE SWITCHING FOR RL

$$\theta - \phi = \pm 90^\circ \;\Rightarrow\; \text{offset} = \mp\frac{V_m}{|Z|}\,e^{-Rt/L}, \qquad i_{\text{peak}} \to \frac{2V_m}{|Z|} \text{ when } \omega L \gg R$$

The transient is largest when the switch closes at a peak of the steady-state current. About half a cycle later, the total current's first peak approaches twice the steady-state amplitude, provided the time constant is long compared with the period.

STORED ENERGY

$$w_C = \frac{1}{2}Cv_C^2, \qquad w_L = \frac{1}{2}Li_L^2$$

These depend only on the present v_C or i_L, which is why those two quantities are the continuous ones.

CHARGING A CAPACITOR FROM A DC SOURCE THROUGH A RESISTOR

$$W_{\text{source}} = CV_s(V_s - V_0), \qquad \Delta W_C = \frac{1}{2}C(V_s^2 - V_0^2), \qquad W_R = \frac{1}{2}C(V_s - V_0)^2$$

From $v_C = V_0$ to the final value V_s, through a resistor of any value. Starting uncharged, the source delivers CV_s^2 and exactly half of it is dissipated. If $V_0 > V_s$ the capacitor discharges toward V_s, and the same W_R holds.

ENERGY DISSIPATED IN AN EXPONENTIAL DECAY

$$i(t) = I_0 e^{-t/\tau} \;\Rightarrow\; W_R(0 \to t) = \frac{1}{2}I_0^2 R\,\tau\left(1 - e^{-2t/\tau}\right)$$

The power i^2R decays with time constant $\tau/2$. For an RL decay ($\tau = L/R$) the total is $\frac{1}{2}LI_0^2$, and for an RC discharge from V_0 it is $\frac{1}{2}CV_0^2$. After one time constant, $1 - e^{-2} = 0.865$ of the stored energy has been dissipated.

CHARGE SHARING BETWEEN TWO CAPACITORS

$$V_f = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}, \qquad W_{\text{lost}} = \frac{1}{2}\,\frac{C_1C_2}{C_1 + C_2}\,(V_1 - V_2)^2$$

For C_1 at V_1 and C_2 at V_2 connected in parallel, through a switch or a resistor. V_1 and V_2 are measured across the same pair of joined terminals, so a capacitor connected with reversed polarity enters with a negative voltage. The loss does not depend on the connecting resistance R, which only sets the time constant $R\,\frac{C_1C_2}{C_1+C_2}$.

Sinusoidal steady state, AC power and resonance

AVERAGE VALUE

$$V_{\text{avg}} = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)\,dt$$

Any periodic waveform; integrate over any one full period.

RMS VALUE

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v^2(t)\,dt}$$

Any periodic waveform. Square first, then average, then take the root.

SINUSOID

$$V_{\text{rms}} = \frac{V_m}{\sqrt{2}} \approx 0.707\,V_m, \qquad V_{\text{avg, half cycle}} = \frac{2V_m}{\pi} \approx 0.637\,V_m$$

$v = V_m \sin(\omega t + \theta)$. The full-cycle average is zero; the rms value does not depend on the phase θ.

FORM FACTOR AND CREST (PEAK) FACTOR

$$k_f = \frac{V_{\text{rms}}}{V_{\text{avg}}}, \qquad k_c = \frac{V_{\text{peak}}}{V_{\text{rms}}}$$

V_{avg} is the half-cycle average for a waveform whose full-cycle average is zero; V_{peak} is the largest absolute value. For a sine, $k_f = \pi/(2\sqrt{2}) \approx 1.11$ and $k_c = \sqrt{2} \approx 1.414$.

RMS OF A SUM OF DIFFERENT FREQUENCIES

$$V_{\text{rms}} = \sqrt{V_0^2 + \sum_n V_{n,\text{rms}}^2} = \sqrt{V_0^2 + \sum_n \frac{V_{mn}^2}{2}}$$

Only when the components have **different** frequencies (dc counts as frequency zero): products of different frequencies average to zero over the common period. Add same-frequency terms as phasors first.

ELEMENT IMPEDANCES

$$Z_R = R, \qquad Z_L = j\omega L, \qquad Z_C = \frac{1}{j\omega C} = -\frac{j}{\omega C}$$

Inductor: voltage leads current by 90°. Capacitor: current leads voltage by 90°. Resistor: in phase.

IMPEDANCE

$$Z = R + jX = |Z|\angle\phi, \qquad |Z| = \sqrt{R^2 + X^2}, \qquad \phi = \tan^{-1}\frac{X}{R}$$

$X > 0$: inductive, current lags. $X < 0$: capacitive, current leads. ϕ is the angle by which voltage leads current. The arctangent form assumes $R > 0$ (a passive impedance); for $R = 0$, $\phi = +90^\circ$ or -90° with the sign of X.

ADMITTANCE

$$Y = \frac{1}{Z} = G + jB, \qquad G = \frac{R}{R^2 + X^2}, \qquad B = \frac{-X}{R^2 + X^2}$$

$G = 1/R$ only when $X = 0$. Susceptances: capacitor $B_C = \omega C > 0$, inductor $B_L = -1/(\omega L) < 0$.

COMBINATIONS AND DIVIDERS

$$Z_{\text{series}} = \sum_k Z_k, \qquad Y_{\text{parallel}} = \sum_k Y_k, \qquad V_k = V\frac{Z_k}{\sum Z}, \qquad I_1 = I\frac{Z_2}{Z_1 + Z_2}$$

Same as dc, with complex numbers: the voltage divider is for elements in series, the current divider for two parallel branches. Magnitudes add only when the phasors are in phase.

ACTIVE, REACTIVE AND APPARENT POWER

$$P = |V||I|\cos\phi\ (\text{W}), \qquad Q = |V||I|\sin\phi\ (\text{var}), \qquad |S| = |V||I| = \sqrt{P^2 + Q^2}\ (\text{VA})$$

rms values. P is the average of $p(t)$. Q is the amplitude of the part of $p(t)$ that flows back and forth between the source and the fields of L and C, with zero average.

COMPLEX POWER

$$S = VI^* = P + jQ = |I|^2 Z = \frac{|V|^2}{Z^*}$$

rms phasors, power absorbed by the load. With **peak** phasors, $S = \frac{1}{2}V_m I_m^*$. The conjugate goes on the current; writing V^*I instead flips the sign of Q.

POWER IN EACH ELEMENT

$$P_R = |I|^2 R = \frac{|V_R|^2}{R}, \qquad Q_L = |I|^2 X_L = \frac{|V_L|^2}{X_L} > 0, \qquad Q_C = -|I|^2 X_C = -\frac{|V_C|^2}{X_C} < 0$$

$X_L = \omega L$ and $X_C = 1/(\omega C)$ are positive magnitudes. An inductor absorbs reactive power, a capacitor delivers it, and neither absorbs average power.

POWER FACTOR

$$\text{pf} = \cos\phi = \frac{P}{|S|} = \frac{R}{|Z|}$$

Lagging when current lags voltage ($\phi > 0$, $Q > 0$, inductive); **leading** when current leads ($\phi < 0$, $Q < 0$, capacitive); unity at $\phi = 0$. Always state lagging or leading with the number.

CONSERVATION OF COMPLEX POWER

$$\sum_{\text{all elements}} S_k = 0 \ \Rightarrow\ \sum P_{\text{delivered}} = \sum P_{\text{absorbed}}, \qquad \sum Q_{\text{delivered}} = \sum Q_{\text{absorbed}}$$

Every element taken with the passive sign convention. P and Q of loads add whether they are in series or parallel; apparent powers $|S|$ do NOT add in general (only when every load has the same power-factor angle).

SHUNT CAPACITOR FOR PF CORRECTION

$$Q_C = P(\tan\phi_1 - \tan\phi_2), \qquad C = \frac{P(\tan\phi_1 - \tan\phi_2)}{\omega\,|V|^2}$$

ϕ_1, ϕ_2 = pf angles before and after (lagging taken positive), $|V|$ = rms voltage across the capacitor, P in W. For a final **leading** pf take ϕ_2 negative.

AVERAGE POWER WITH HARMONICS

$$P = V_0 I_0 + \sum_{n=1}^{\infty} V_n I_n \cos\phi_n$$

V_n, I_n = rms values of the n-th harmonic; ϕ_n = angle by which V_n leads I_n. A harmonic present in only one of v, i contributes nothing. Power factor $= P/(V_{\text{rms}} I_{\text{rms}})$ using the full rms values. If v is a pure sinusoid and only i is distorted, this becomes $\text{pf} = (I_1/I_{\text{rms}})\cos\phi_1$: distortion factor times displacement factor.

LOAD POWER

$$P_L = \frac{|V_{Th}|^2\, R_L}{(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2}$$

V_{Th} rms. Start every restricted-load problem from this expression.

CONJUGATE MATCH

$$Z_L = Z_{Th}^{*} \;\Rightarrow\; P_{L,\max} = \frac{|V_{Th}|^2}{4R_{Th}}$$

rms V_{Th} (with a peak value $V_{Th,m}$ it is $V_{Th,m}^2/(8R_{Th})$). The Thevenin source then delivers twice this power, so the efficiency measured against the Thevenin equivalent is 50%.

PURELY RESISTIVE LOAD

$$R_L = |Z_{Th}| = \sqrt{R_{Th}^2 + X_{Th}^2}$$

When the load cannot have reactance. Gives less power than the conjugate match unless $X_{Th} = 0$.

RESONANT FREQUENCY

$$\omega_0 = \frac{1}{\sqrt{LC}}, \qquad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Ideal series RLC and ideal parallel RLC (frequency-independent R). The value of R does not shift it.

QUALITY FACTOR, SERIES RLC

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

R is the total series resistance, including the coil's own resistance and the source's internal resistance.

VOLTAGE MAGNIFICATION (SERIES)

$$|V_L| = |V_C| = Q\,|V| \quad \text{at } \omega_0$$

The L and C voltages are 180° apart and cancel, but each is Q times the supply voltage.

HALF-POWER FREQUENCIES (SERIES)

$$\omega_{1,2} = \mp\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}, \qquad \omega_0 = \sqrt{\omega_1\omega_2}$$

Constant-amplitude voltage source. At ω_1 and ω_2: $|I| = I_{\max}/\sqrt{2}$, $P = P_{\max}/2$, $|Z| = \sqrt{2}R$, impedance angle -45° at ω_1 (capacitive) and $+45^\circ$ at ω_2 (inductive). ω_0 is their exact geometric mean.

BANDWIDTH (SERIES)

$$\mathrm{BW} = \omega_2 - \omega_1 = \frac{R}{L}\ \mathrm{rad/s} = \frac{\omega_0}{Q}, \qquad \text{in Hz: } \frac{R}{2\pi L} = \frac{f_0}{Q}$$

Exact for any Q. For high Q (roughly $Q \geq 10$), $\omega_{1,2} \approx \omega_0 \mp \mathrm{BW}/2$; the arithmetic symmetry about ω_0 is only approximate, the geometric one is exact.

QUALITY FACTOR AND BANDWIDTH, PARALLEL RLC

$$Q = \frac{R}{\omega_0 L} = \omega_0 C R = R\sqrt{\frac{C}{L}}, \qquad \mathrm{BW} = \frac{1}{RC}\ \mathrm{rad/s} = \frac{\omega_0}{Q}$$

Ideal parallel RLC. Current magnification: at ω_0, $|I_L| = |I_C| = Q\,|I_{\text{source}}|$. A current source with internal resistance R_s puts R_s in parallel with R: use $R \parallel R_s$ in Q and BW.

SERIES-PARALLEL EQUIVALENT AT ONE FREQUENCY

$$R_p = \frac{R_s^2 + X_s^2}{R_s} = R_s(1 + Q^2), \qquad X_p = \frac{R_s^2 + X_s^2}{X_s} = X_s\left(1 + \frac{1}{Q^2}\right), \qquad Q = \frac{|X_s|}{R_s} = \frac{R_p}{|X_p|}$$

A series $R_s + jX_s$ and a parallel combination of R_p and jX_p have the same impedance at the frequency where the values are computed, and only there. For $Q \gg 1$, $R_p \approx Q^2 R_s$ and $X_p \approx X_s$, which is where the tank result $Z_D \approx Q^2 R$ below comes from.

PRACTICAL TANK (COIL R, L IN PARALLEL WITH C)

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}, \qquad Z_D = \frac{L}{CR}$$

ω_0 = frequency of unity pf; Z_D = dynamic impedance (purely resistive) at that frequency. A real ω_0 exists only if $R < \sqrt{L/C}$. Exactly, $Z_D = R(1 + Q^2)$ with the coil $Q = \omega_0 L/R$ taken at this ω_0; for $Q \gg 1$, $\omega_0 \approx 1/\sqrt{LC}$ and $Z_D \approx Q^2 R$.

FIRST-ORDER LOW-PASS

$$H(j\omega) = \frac{1}{1 + j\omega/\omega_c}, \qquad \omega_c = \frac{1}{RC}\ (\text{series R, output across C}) \ \text{ or } \ \frac{R}{L}\ (\text{series L, output across R})$$

At ω_c: $|H| = 1/\sqrt{2}$ (-3 dB), phase -45°. Asymptotic roll-off -20 dB/decade above ω_c.

FIRST-ORDER HIGH-PASS

$$H(j\omega) = \frac{j\omega/\omega_c}{1 + j\omega/\omega_c}, \qquad \omega_c = \frac{1}{RC}\ (\text{series C, output across R}) \ \text{ or } \ \frac{R}{L}\ (\text{series R, output across L})$$

At ω_c: $|H| = 1/\sqrt{2}$, phase $+45^\circ$. Asymptotic slope $+20$ dB/decade below ω_c.

STANDARD SECOND-ORDER FORMS

$$H_{LP}(s) = \frac{\omega_0^2}{s^2 + (\omega_0/Q)s + \omega_0^2}$$

$$H_{HP}(s) = \frac{s^2}{s^2 + (\omega_0/Q)s + \omega_0^2}$$

$$H_{BP}(s) = \frac{(\omega_0/Q)\,s}{s^2 + (\omega_0/Q)s + \omega_0^2}$$

$$H_{BS}(s) = \frac{s^2 + \omega_0^2}{s^2 + (\omega_0/Q)s + \omega_0^2}$$

Match the denominator to read ω_0 and Q directly: series RLC has $\omega_0/Q = R/L$, parallel RLC has $\omega_0/Q = 1/(RC)$. Damping ratio $\zeta = 1/(2Q)$. The band-pass and band-stop bandwidth is ω_0/Q. Asymptotic slopes: low-pass -40 dB/decade above ω_0, high-pass $+40$ dB/decade below it, band-pass ± 20 dB/decade on its two skirts.

Magnetically coupled circuits

TERMINAL EQUATIONS OF TWO COUPLED COILS (TIME DOMAIN)

$$v_1 = L_1\frac{di_1}{dt} \pm M\frac{di_2}{dt}, \qquad v_2 = L_2\frac{di_2}{dt} \pm M\frac{di_1}{dt}$$

Each coil's voltage follows the passive sign convention for that coil (+ at the terminal where its own current enters). Use $+M$ when both currents enter (or both leave) their dotted terminals and $-M$ when one enters and the other leaves.

PHASOR (SINUSOIDAL STEADY-STATE) FORM

$$V_1 = j\omega L_1 I_1 \pm j\omega M I_2, \qquad V_2 = j\omega L_2 I_2 \pm j\omega M I_1$$

Same sign rule as the time-domain equations. ωM is the mutual reactance in ohms.

COUPLING COEFFICIENT

$$k = \frac{M}{\sqrt{L_1 L_2}}, \qquad 0 \le k \le 1 \;\Rightarrow\; M \le \sqrt{L_1 L_2}$$

$k = 0$: no coupling; $k = 1$: perfect coupling (all the flux of each coil links the other). Coils wound on a common iron core are typically tightly coupled (k close to 1); air-cored coils are usually more loosely coupled.

SERIES CONNECTION

$$L_{\text{eq}} = L_1 + L_2 \pm 2M$$

$+2M$ for series-aiding (the junction joins an undotted terminal to a dotted one); $-2M$ for series-opposing (dotted to dotted, or undotted to undotted). Both values are non-negative because $k \le 1$.

THREE COUPLED COILS IN SERIES

$$L_{\text{eq}} = L_1 + L_2 + L_3 \pm 2M_{12} \pm 2M_{23} \pm 2M_{13}$$

The same current flows through all three coils. Decide each pair separately with that pair's own marks: $+2M_{ij}$ if the current enters both of the pair's marked terminals (or leaves both), $-2M_{ij}$ otherwise.

MUTUAL INDUCTANCE FROM TWO SERIES MEASUREMENTS

$$M = \frac{L_{\text{aid}} - L_{\text{opp}}}{4}, \qquad L_1 + L_2 = \frac{L_{\text{aid}} + L_{\text{opp}}}{2}$$

Measure the series pair, reverse one coil and measure again. The larger reading is always the aiding one.

PARALLEL CONNECTION

$$L_{\text{eq}} = \frac{L_1 L_2 - M^2}{L_1 + L_2 \mp 2M}$$

$-2M$ in the denominator when like terminals are joined (both dots on the same node: parallel-aiding); $+2M$ when unlike terminals are joined (parallel-opposing). Coil resistances neglected. With $k = 1$ the numerator is zero, so $L_{\text{eq}} = 0$, except for two equal coils ($L_1 = L_2$) in parallel-aiding, where the formula reads $0/0$ and $L_{\text{eq}} = L_1$.

ENERGY STORED IN TWO COUPLED COILS

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2$$

Instantaneous currents. $+M$ when both currents enter (or both leave) their dotted terminals, $-M$ otherwise: the same rule as for the voltages.

T-EQUIVALENT OF COUPLED COILS

$$\text{dots on the same side:} \quad L_a = L_1 - M,\ L_b = L_2 - M,\ L_c = +M$$
$$\text{dots on opposite sides:} \quad L_a = L_1 + M,\ L_b = L_2 + M,\ L_c = -M$$

L_a and L_b are the series arms on the coil-1 and coil-2 sides, L_c the shunt arm to the common terminal. Valid only when the two coils share a common terminal (or their lower terminals are joined in the circuit), because the T adds that connection. "Same side" means both dots away from the common terminal or both on it. A negative inductance in the model is acceptable for analysis, although it is not a physical element.

INPUT IMPEDANCE OF A LINEAR TRANSFORMER (REFLECTED IMPEDANCE)

$$Z_{\text{in}} = Z_{11} + Z_r, \qquad Z_r = \frac{\omega^2 M^2}{Z_{22}}$$

Z_{22} is the whole secondary loop impedance, load included. Z_r does not depend on the dot positions (M enters squared), and its reactance has the opposite sign to that of Z_{22}: an inductive secondary reflects as a capacitive reactance.

INPUT INDUCTANCE WITH THE SECONDARY SHORT-CIRCUITED

$$L_{\text{sc}} = L_1 - \frac{M^2}{L_2} = L_1\left(1 - k^2\right)$$

Lossless coils, from Z_r with $Z_{22} = j\omega L_2$. With the secondary open, the input is simply $j\omega L_1$; comparing the two measurements gives k.

IDEAL TRANSFORMER: VOLTAGE AND CURRENT RATIOS

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = n, \qquad \frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{n}$$

With the convention above. If one dot is moved to the other end of its winding while the voltage and current references stay on the same physical terminals, both ratios change sign: $V_1/V_2 = -n$ and $I_1/I_2 = -1/n$. In the two-port convention (I_2 entering the secondary dot) the current relation reads $I_1 = -I_2/n$.

IMPEDANCE REFLECTION THROUGH AN IDEAL TRANSFORMER

$$Z_{\text{in}} = \frac{V_1}{I_1} = n^2 Z_L$$

Z_L connected across the secondary; independent of the dot positions. In the other direction, an impedance on the primary side appears on the secondary side as Z/n^2.

POWER CONSERVATION IN AN IDEAL TRANSFORMER

$$V_1 I_1^* = V_2 I_2^*$$

It neither stores nor dissipates energy: complex power in equals complex power out (convention above).

MAXIMUM POWER TRANSFER THROUGH AN IDEAL TRANSFORMER

$$n^2 R_L = R_s \ \Rightarrow\ n = \sqrt{\frac{R_s}{R_L}}$$

Resistive source R_s on the primary, resistive load R_L on the secondary, $n = N_1/N_2$. The load then receives $|V_s|^2/(4R_s)$, with V_s the rms source voltage.

Two-port networks

Z (OPEN-CIRCUIT IMPEDANCE) PARAMETERS

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}, \qquad \begin{aligned} z_{11} &= \left.\frac{V_1}{I_1}\right|_{I_2=0}, & z_{12} &= \left.\frac{V_1}{I_2}\right|_{I_1=0} \\ z_{21} &= \left.\frac{V_2}{I_1}\right|_{I_2=0}, & z_{22} &= \left.\frac{V_2}{I_2}\right|_{I_1=0} \end{aligned}$$

All four are in ohms, and each is measured with the other port open. z_{11} and z_{22} are driving-point (input and output) impedances. z_{12} and z_{21} are transfer impedances.

Y (SHORT-CIRCUIT ADMITTANCE) PARAMETERS

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \qquad \begin{aligned} y_{11} &= \left.\frac{I_1}{V_1}\right|_{V_2=0}, & y_{12} &= \left.\frac{I_1}{V_2}\right|_{V_1=0} \\ y_{21} &= \left.\frac{I_2}{V_1}\right|_{V_2=0}, & y_{22} &= \left.\frac{I_2}{V_2}\right|_{V_1=0} \end{aligned}$$

All four are in siemens, and each is measured with the other port shorted. $[y] = [z]^{-1}$ whenever both exist. In general $y_{11} \neq 1/z_{11}$, because the two are measured under different terminations.

H (HYBRID) PARAMETERS

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}, \qquad \begin{aligned} h_{11} &= \left.\frac{V_1}{I_1}\right|_{V_2=0}, & h_{12} &= \left.\frac{V_1}{V_2}\right|_{I_1=0} \\ h_{21} &= \left.\frac{I_2}{I_1}\right|_{V_2=0}, & h_{22} &= \left.\frac{I_2}{V_2}\right|_{I_1=0} \end{aligned}$$

h_{11} is the short-circuit input impedance (ohms), h_{12} the open-circuit reverse voltage gain, h_{21} the short-circuit forward current gain, and h_{22} the open-circuit output admittance (siemens). Port 2 is shorted for h_{11} and h_{21}; port 1 is open for h_{12} and h_{22}.

G (INVERSE HYBRID) PARAMETERS

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}, \qquad \begin{aligned} g_{11} &= \left.\frac{I_1}{V_1}\right|_{I_2=0}, & g_{12} &= \left.\frac{I_1}{I_2}\right|_{V_1=0} \\ g_{21} &= \left.\frac{V_2}{V_1}\right|_{I_2=0}, & g_{22} &= \left.\frac{V_2}{I_2}\right|_{V_1=0} \end{aligned}$$

$[g] = [h]^{-1}$ whenever both exist. g_{11} is the open-circuit input admittance (siemens), g_{12} the short-circuit reverse current gain, g_{21} the open-circuit forward voltage gain, and g_{22} the short-circuit output impedance (ohms).

ABCD (TRANSMISSION) PARAMETERS

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}, \qquad \begin{aligned} A &= \left.\frac{V_1}{V_2}\right|_{I_2=0}, & B &= -\left.\frac{V_1}{I_2}\right|_{V_2=0} \\ C &= \left.\frac{I_1}{V_2}\right|_{I_2=0}, & D &= -\left.\frac{I_1}{I_2}\right|_{V_2=0} \end{aligned}$$

Written out, $V_1 = AV_2 - BI_2$ and $I_1 = CV_2 - DI_2$. A and D are dimensionless, B is in ohms and C in siemens. A and C are measured with port 2 open, B and D with port 2 shorted. The minus sign is there because $-I_2$ is the current that leaves port 2 and enters whatever is connected to it.

CONDITION FOR RECIPROCITY

$$z_{12} = z_{21} \iff y_{12} = y_{21} \iff h_{12} = -h_{21} \iff g_{12} = -g_{21} \iff AD - BC = 1 \iff A'D' - B'C' = 1$$

Any one of these implies all the others (for the sets that exist). A reciprocal two-port has only three independent parameters.

CONDITION FOR SYMMETRY

$$z_{11} = z_{22} \iff y_{11} = y_{22} \iff A = D \iff \Delta_h = h_{11}h_{22} - h_{12}h_{21} = 1 \iff \Delta_g = 1$$

These express electrical symmetry of the two ports, and they hold whether or not the network is reciprocal. For a network that is also reciprocal, only two parameters are independent (for example z_{11} and z_{12}). Ports that can be swapped with no change at all need both $z_{11} = z_{22}$ and $z_{12} = z_{21}$.

DETERMINANTS AND QUICK IDENTITIES

$$\Delta_z = z_{11}z_{22} - z_{12}z_{21}, \qquad \Delta_y = \frac{1}{\Delta_z}, \qquad \Delta_T = AD - BC = \frac{z_{12}}{z_{21}} = \frac{y_{12}}{y_{21}} = -\frac{h_{12}}{h_{21}}$$

Use Δ_T for a quick reciprocity check from any set. The determinant of a product is the product of the determinants, so after multiplying the ABCD matrices of reciprocal sections (built from R, L, C, coupled coils and ideal transformers), Δ_T must still equal 1. That catches arithmetic slips.

Z AND ABCD

$$A = \frac{z_{11}}{z_{21}},\ B = \frac{\Delta_z}{z_{21}},\ C = \frac{1}{z_{21}},\ D = \frac{z_{22}}{z_{21}} \qquad z_{11} = \frac{A}{C},\ z_{12} = \frac{\Delta_T}{C},\ z_{21} = \frac{1}{C},\ z_{22} = \frac{D}{C}$$

$[z]$ exists only if $C \neq 0$; $[T]$ exists only if $z_{21} \neq 0$.

Y AND ABCD

$$A = -\frac{y_{22}}{y_{21}},\ B = -\frac{1}{y_{21}},\ C = -\frac{\Delta_y}{y_{21}},\ D = -\frac{y_{11}}{y_{21}} \qquad y_{11} = \frac{D}{B},\ y_{12} = -\frac{\Delta_T}{B},\ y_{21} = -\frac{1}{B},\ y_{22} = \frac{A}{B}$$

$[y]$ exists only if $B \neq 0$; $[T]$ exists only if $y_{21} \neq 0$.

H FROM Z AND FROM Y

$$h_{11} = \frac{\Delta_z}{z_{22}} = \frac{1}{y_{11}}, \quad h_{12} = \frac{z_{12}}{z_{22}} = -\frac{y_{12}}{y_{11}}, \quad h_{21} = -\frac{z_{21}}{z_{22}} = \frac{y_{21}}{y_{11}}, \quad h_{22} = \frac{1}{z_{22}} = \frac{\Delta_y}{y_{11}}$$

Note the pairs that are true reciprocals, each pair measured under the same termination: $h_{11} = 1/y_{11}$ (port 2 shorted), $h_{22} = 1/z_{22}$ (port 1 open), $g_{11} = 1/z_{11}$ (port 2 open) and $g_{22} = 1/y_{22}$ (port 1 shorted).

G FROM Z, FROM Y AND FROM ABCD

$$g_{11} = \frac{1}{z_{11}} = \frac{\Delta_y}{y_{22}} = \frac{C}{A}, \qquad g_{12} = -\frac{z_{12}}{z_{11}} = \frac{y_{12}}{y_{22}} = -\frac{\Delta_T}{A}$$
$$g_{21} = \frac{z_{21}}{z_{11}} = -\frac{y_{21}}{y_{22}} = \frac{1}{A}, \qquad g_{22} = \frac{\Delta_z}{z_{11}} = \frac{1}{y_{22}} = \frac{B}{A}$$

$g_{21} = 1/A$ is the open-circuit forward voltage gain. $[g]$ exists only if $z_{11} \neq 0$ (equivalently $A \neq 0$).

T NETWORK AND T EQUIVALENT

$$[z] = \begin{bmatrix} Z_a + Z_c & Z_c \\ Z_c & Z_b + Z_c \end{bmatrix} \qquad Z_a = z_{11} - z_{12}, \quad Z_b = z_{22} - z_{12}, \quad Z_c = z_{12} = z_{21}$$

Z_a is the series arm at port 1, Z_b the series arm at port 2, and Z_c the shunt arm to the common terminal. Every reciprocal two-port that has a $[z]$ has this T equivalent, though an arm may come out negative (not realisable with positive elements). The T joins the two lower terminals, so it is equivalent only in its port voltages and currents.

PI NETWORK AND PI EQUIVALENT

$$[y] = \begin{bmatrix} Y_a + Y_c & -Y_c \\ -Y_c & Y_b + Y_c \end{bmatrix} \qquad Y_a = y_{11} + y_{12}, \quad Y_b = y_{22} + y_{12}, \quad Y_c = -y_{12} = -y_{21}$$

Y_a is the shunt arm at port 1, Y_b the shunt arm at port 2, and Y_c the series arm between the upper terminals. Every reciprocal two-port that has a $[y]$ has this π equivalent, with the same caveats as the T.

COUPLED COILS

$$[z] = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix}, \qquad \text{T equivalent: } L_1 - M,\ L_2 - M \text{ (series arms)},\ M \text{ (shunt arm)}$$

Coil L_1 across port 1 and coil L_2 across port 2, mutual inductance M, with both dots at the upper terminals, so both port currents enter dotted terminals. Moving one dot to its lower terminal changes M to $-M$ everywhere. In the s-domain replace $j\omega$ by s. The T equivalent (which may need a negative inductance) holds only if the two lower terminals may be joined.

SYMMETRIC LATTICE

$$z_{11} = z_{22} = \frac{Z_b + Z_a}{2}, \qquad z_{12} = z_{21} = \frac{Z_b - Z_a}{2}$$

Series arms Z_a join upper terminal to upper terminal and lower terminal to lower terminal; cross arms Z_b join each upper terminal to the lower terminal of the other port. Its transfer impedance changes sign when $Z_a > Z_b$.

PARAMETER ADDITION

series: $[z] = [z_a] + [z_b]$, parallel: $[y] = [y_a] + [y_b]$, series-parallel: $[h] = [h_a] + [h_b]$, parallel-series: $[g] = [g_a] + [g_b]$

Valid only when each sub-network still satisfies the port condition after the connection (see the validity note below).

CASCADE CONNECTION

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}, \qquad n \text{ identical sections: } [T]^n$$

N_a is the section nearer the source; keep the order. Always valid, because the current $-I_2$ leaving N_a is exactly the current entering N_b. That is why the minus sign is built into the ABCD definition.

INPUT IMPEDANCE OF A TERMINATED TWO-PORT

$$Z_{in} = \frac{V_1}{I_1} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L} = \frac{AZ_L + B}{CZ_L + D} = h_{11} - \frac{h_{12}h_{21}Z_L}{1 + h_{22}Z_L}, \qquad Y_{in} = y_{11} - \frac{y_{12}y_{21}}{y_{22} + Y_L}$$

Port 2 terminated in Z_L (so $V_2 = -I_2 Z_L$). As $Z_L \to \infty$ this becomes $z_{11} = A/C$, the open-circuit input impedance; at $Z_L = 0$ it becomes $1/y_{11} = h_{11} = B/D$, the short-circuit input impedance.

VOLTAGE AND CURRENT GAIN WITH A LOAD

$$\frac{V_2}{V_1} = \frac{z_{21}Z_L}{z_{11}Z_L + \Delta_z} = \frac{Z_L}{AZ_L + B} = \frac{-y_{21}}{y_{22} + Y_L}, \qquad \frac{I_2}{I_1} = -\frac{z_{21}}{z_{22} + Z_L} = \frac{-1}{CZ_L + D} = \frac{h_{21}}{1 + h_{22}Z_L}$$

Same termination, with port 1 driven directly by V_1. The current delivered to the load is $-I_2$, so the load-current gain is $-I_2/I_1$. With the output open, $V_2/V_1 = 1/A = z_{21}/z_{11}$.

THEVENIN EQUIVALENT AT PORT 2

$$V_{th} = \frac{z_{21}V_s}{z_{11} + Z_s} = \frac{V_s}{A + CZ_s}, \qquad Z_{th} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_s} = \frac{DZ_s + B}{CZ_s + A}$$

Port 1 driven by V_s in series with Z_s. For Z_{th}, set V_s to zero, which leaves Z_s across port 1. With V_s and Z_s fixed, the load receives maximum power when $Z_L = Z_{th}^*$.

OPEN- AND SHORT-CIRCUIT IMPEDANCES

$$Z_{oc1} = z_{11} = \frac{A}{C}, \quad Z_{sc1} = \frac{1}{y_{11}} = \frac{B}{D}, \quad Z_{oc2} = z_{22} = \frac{D}{C}, \quad Z_{sc2} = \frac{1}{y_{22}} = \frac{B}{A}$$

Each is measured at one port with the other port open (oc) or shorted (sc). For any two-port, $Z_{oc1}/Z_{sc1} = Z_{oc2}/Z_{sc2} = AD/(BC)$. So only three of the four are independent.

IMAGE IMPEDANCES

$$Z_{i1} = \sqrt{Z_{oc1} Z_{sc1}} = \sqrt{\frac{AB}{CD}}, \qquad Z_{i2} = \sqrt{Z_{oc2} Z_{sc2}} = \sqrt{\frac{BD}{AC}}, \qquad \text{symmetric } (A = D):\ Z_0 = \sqrt{\frac{B}{C}}$$

The image impedances are the pair for which terminating port 2 in Z_{i2} makes the input impedance at port 1 equal to Z_{i1}, and terminating port 1 in Z_{i1} makes the impedance at port 2 equal to Z_{i2}. For a symmetric network both equal the characteristic impedance Z_0: a load Z_0 is reproduced at the input. For a reciprocal network the image transfer constant θ satisfies $\cosh\theta = \sqrt{AD}$, which becomes $\cosh\theta = A$ when the network is also symmetric.

Balanced three-phase circuits

BALANCED POSITIVE-SEQUENCE (ABC) SET

$$V_{an} = V_p \angle 0^\circ, \qquad V_{bn} = V_p \angle -120^\circ, \qquad V_{cn} = V_p \angle +120^\circ$$

Phase b lags phase a by 120° and phase c lags phase b by 120°. For negative sequence (acb) the angles of V_{bn} and V_{cn} are interchanged: $V_{bn} = V_p \angle +120^\circ$, $V_{cn} = V_p \angle -120^\circ$.

OPERATOR A AND THE ZERO SUM OF A BALANCED SET

$$a = 1\angle 120^\circ = -\frac{1}{2} + j\frac{\sqrt{3}}{2}, \qquad 1 + a + a^2 = 0$$

For abc: $V_{bn} = a^2 V_{an}$ and $V_{cn} = a V_{an}$. Hence any balanced set sums to zero, $V_{an} + V_{bn} + V_{cn} = 0$, and the same holds for a balanced set of currents.

STAR: LINE AND PHASE RELATIONS (ABC)

$$V_{ab} = \sqrt{3}\, V_{an} \angle +30^\circ, \qquad V_L = \sqrt{3}\, V_{ph}, \qquad I_L = I_{ph}$$

Each line voltage leads the phase voltage of its first subscript by 30° (V_{bc} leads V_{bn}, V_{ca} leads V_{cn}). For negative sequence the magnitudes are unchanged but $V_{ab} = \sqrt{3}\, V_{an} \angle -30^\circ$.

DELTA: LINE AND PHASE RELATIONS (ABC)

$$I_a = \sqrt{3}\, I_{ab} \angle -30^\circ, \qquad I_L = \sqrt{3}\, I_{ph}, \qquad V_L = V_{ph}$$

Phase currents I_{ab}, I_{bc}, I_{ca} are taken in the direction of their subscripts; each line current lags, by 30°, the phase current leaving that line's corner (I_b lags I_{bc}, I_c lags I_{ca}). For negative sequence, $I_a = \sqrt{3}\, I_{ab} \angle +30^\circ$.

BALANCED STAR-DELTA CONVERSION

$$Z_Y = \frac{Z_\Delta}{3}, \qquad Z_\Delta = 3\, Z_Y$$

Identical impedances only. The division is by a real number, so the impedance angle ϕ, and hence the power factor, is unchanged.

THREE-PHASE ACTIVE POWER

$$P = 3\, V_{ph} I_{ph} \cos\phi = \sqrt{3}\, V_L I_L \cos\phi = 3\, I_{ph}^2 R_{ph}$$

Balanced load, star or delta; rms values; ϕ = angle of the per-phase impedance. Pair the delta resistance with the delta phase current, or the star-equivalent resistance with the line current.

THREE-PHASE REACTIVE POWER

$$Q = 3\, V_{ph} I_{ph} \sin\phi = \sqrt{3}\, V_L I_L \sin\phi = 3\, I_{ph}^2 X_{ph}$$

$Q > 0$ (absorbed) for an inductive, lagging load; $Q < 0$ for a capacitive, leading load.

COMPLEX AND APPARENT POWER, POWER FACTOR

$$S = 3\,V_{ph} I_{ph}^{*} = P + jQ, \qquad |S| = \sqrt{3}\,V_L I_L = \sqrt{P^2 + Q^2}, \qquad \text{pf} = \cos\phi = \frac{P}{|S|}$$

V_{ph} and I_{ph} are the phasors of the same phase and * is the complex conjugate. $|S|$ is in VA (or kVA); a three-phase kVA figure means this total for all three phases, not the per-phase value.

INSTANTANEOUS POWER OF A BALANCED LOAD

$$p(t) = v_a i_a + v_b i_b + v_c i_c = 3\,V_{ph} I_{ph} \cos\phi \quad (\text{constant})$$

Balanced sinusoidal voltages and a balanced load; V_{ph}, I_{ph} rms. The total does not vary with time and equals the average power P.

TWO-WATTMETER SUM (ANY THREE-WIRE LOAD)

$$P = W_1 + W_2$$

Readings are taken with their signs. Valid for balanced and unbalanced three-wire loads.

WATTMETER READINGS FOR A BALANCED LOAD

$$W_1 = V_L I_L \cos(30^\circ - \phi), \qquad W_2 = V_L I_L \cos(30^\circ + \phi)$$

Balanced sinusoidal system. The current coils, in lines a and c, carry the line currents towards the load; the pressure coils measure V_{ab} and V_{cb} (each coil's $\pm$ terminal on its own current-coil line, the other end on line b). For abc sequence the meter in line c reads W_1 and the meter in line a reads W_2. The formulas hold for either sign of ϕ ($\phi > 0$ lagging, $\phi < 0$ leading), so a leading load, or a reversed sequence, swaps which meter reads more.

READINGS IN TERMS OF P AND Q (BALANCED LOAD)

$$W_1 = \frac{P}{2} + \frac{Q}{2\sqrt{3}}, \qquad W_2 = \frac{P}{2} - \frac{Q}{2\sqrt{3}}$$

The same pair written with $P = \sqrt{3}\,V_L I_L \cos\phi$ and $Q = \sqrt{3}\,V_L I_L \sin\phi$; handy for predicting the readings when P and Q (or P and the power factor) are given. $W_2 < 0$ exactly when $Q > \sqrt{3}\,P$, i.e. $\phi > 60^\circ$.

POWER-FACTOR ANGLE AND REACTIVE POWER FROM THE READINGS

$$\tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2}, \qquad Q = \sqrt{3}\,(W_1 - W_2)$$

Balanced load only. Follows from $W_1 + W_2 = \sqrt{3}\,V_L I_L \cos\phi$ and $W_1 - W_2 = V_L I_L \sin\phi$. The two readings alone fix $|\phi|$; whether the load lags or leads needs either the nature of the load or knowledge of which physical meter reads W_1, which depends on the connection and the phase sequence.

POWER FACTOR DIRECTLY FROM THE READINGS

$$\cos\phi = \frac{W_1 + W_2}{2\sqrt{W_1^2 - W_1 W_2 + W_2^2}}$$

Balanced load; the same content as the $\tan\phi$ form. Insert a negative reading with its sign.

THREE-WATTMETER AND ONE-WATTMETER METHODS

$$P = W_a + W_b + W_c \quad (\text{four-wire, any load}), \qquad P = 3\,W \quad (\text{balanced load, one meter})$$

Three-wattmeter method: one meter per line, each pressure coil from its line to the neutral, so each meter reads one phase power; valid for balanced and unbalanced loads. One-wattmeter method: for a balanced load, a single meter with its current coil in one line and its pressure coil from that line to the neutral (or to an artificial neutral formed by three equal impedances, one of them the pressure coil) reads one-third of the total.

CAPACITOR KVAR REQUIRED

$$Q_C = P\,(\tan\phi_1 - \tan\phi_2)$$

Total for all three phases; ϕ_1 is the original angle and ϕ_2 the target angle, both lagging. For a unity target, Q_C equals the original Q.

CAPACITANCE PER PHASE OF THE BANK

$$C_Y = \frac{Q_C}{\omega V_L^2}, \qquad C_\Delta = \frac{Q_C}{3\,\omega V_L^2} = \frac{C_Y}{3}$$

V_L = line voltage (rms), $\omega = 2\pi f$. Each star capacitor sees $V_L/\sqrt{3}$ and supplies $Q_C/3 = \omega C_Y V_L^2/3$; each delta capacitor sees the full V_L and supplies $Q_C/3 = \omega C_\Delta V_L^2$.

NEUTRAL-SHIFT VOLTAGE (UNBALANCED STAR, NO NEUTRAL)

$$V_{N'N} = \frac{Y_a V_{an} + Y_b V_{bn} + Y_c V_{cn}}{Y_a + Y_b + Y_c}, \qquad Y_k = \frac{1}{Z_k}$$

Nodal equation at N', with the source phase voltages measured from N; any line impedance is included in Z_k. It is zero when $Y_a = Y_b = Y_c$, which recovers the balanced case. If N' and N are joined through a neutral impedance Z_n, add $Y_n = 1/Z_n$ to the denominator.

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