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Electrical and Electronic Measurements Formula Sheet

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Units, standards, errors and uncertainty

STATIC (ABSOLUTE) ERROR AND CORRECTION

$$\delta A = A_m - A_t, \qquad \delta C = A_t - A_m = -\delta A$$

Sign convention used throughout these notes: error = measured minus true, so a positive error means the instrument reads high. The correction is what must be added to the reading to get the true value. Some texts define the error with the opposite sign; check the convention a question uses.

RELATIVE (FRACTIONAL) ERROR

$$\varepsilon_r = \frac{\delta A}{A_t} = \frac{A_m - A_t}{A_t}, \qquad \%\ \text{error} = 100\,\varepsilon_r$$

Taken relative to the true value unless the question says otherwise. When the error is small, dividing by A_m instead changes the result only in the second order. An accuracy quoted as a percentage of full scale is a different quantity (see limiting errors).

RELATIVE ACCURACY

$$A = 1 - \left|\frac{A_t - A_m}{A_t}\right|, \qquad a = 100A = 100\% - \left|\%\ \text{error}\right|$$

Accuracy is the complement of the relative error: a reading 2% in error is 98% accurate.

STATIC SENSITIVITY AND DEFLECTION FACTOR

$$S = \frac{\Delta q_o}{\Delta q_i}, \qquad \text{deflection factor} = \frac{1}{S} = \frac{\Delta q_i}{\Delta q_o}$$

Change in steady output q_o per change in input q_i; for a linear instrument, the slope of its calibration line. The unit combines output and input units.

VOLTMETER LOADING ERROR

$$V_{\text{read}} = V_s\,\frac{R_V}{R_V + R_s}, \qquad \varepsilon_r = \frac{V_{\text{read}} - V_s}{V_s} = -\frac{R_s}{R_V + R_s}$$

Voltmeter of resistance R_V across a source of open-circuit (Thevenin) voltage V_s and internal (Thevenin) resistance R_s. The error is always negative (the meter reads low) and shrinks as R_V/R_s grows. For a PMMC voltmeter, R_V = (ohms-per-volt sensitivity) $\times$ (range in use), where ohms-per-volt $= 1/I_{\text{fs}}$, the reciprocal of the full-scale current.

AMMETER INSERTION (LOADING) ERROR

$$I_{\text{read}} = I_t\,\frac{R_{\text{th}}}{R_{\text{th}} + R_A}, \qquad \varepsilon_r = \frac{I_{\text{read}} - I_t}{I_t} = -\frac{R_A}{R_{\text{th}} + R_A}$$

Ammeter of resistance R_A inserted in a branch; I_t is the current before insertion and R_{th} the Thevenin resistance of the circuit seen from the break. The ammeter always reads low; the error is small when $R_A \ll R_{\text{th}}$.

LIMITING ERROR OF A COMPONENT OR READING

$$A_a = A_s \pm \delta A, \qquad \varepsilon_r = \frac{\delta A}{A_s}$$

A_s = nominal or specified value, δA = guaranteed bound, A_a = actual value, which can lie anywhere in the band.

FULL-SCALE SPECIFICATION: LIMITING ERROR AT A READING

$$\delta A = \varepsilon_{\text{FS}}\,A_{\text{FS}}, \qquad \varepsilon_r(A) = \frac{\delta A}{A} = \varepsilon_{\text{FS}}\,\frac{A_{\text{FS}}}{A}$$

For an accuracy quoted as a fraction ε_{FS} of the full-scale value A_{FS}. At half scale the relative limiting error is twice the quoted figure; at one-tenth scale, ten times. Hence the range is chosen so that the reading falls in the upper part of the scale. A specification as a percentage of span uses the span (upper minus lower range value) in place of A_{FS}; the two differ only for a range that does not start at zero.

SUM OR DIFFERENCE

$$X = x_1 \pm x_2: \quad \delta X = \pm(\delta x_1 + \delta x_2), \qquad \frac{\delta X}{X} = \pm\left(\frac{x_1}{X}\frac{\delta x_1}{x_1} + \frac{x_2}{X}\frac{\delta x_2}{x_2}\right)$$

Absolute limiting errors add, for a difference as well as a sum. For a difference, X is small when $x_1 \approx x_2$, so the relative error can be very large.

PRODUCT OR QUOTIENT

$$X = x_1 x_2 \text{ or } \frac{x_1}{x_2}: \quad \frac{\delta X}{X} = \pm\left(\frac{\delta x_1}{x_1} + \frac{\delta x_2}{x_2}\right)$$

Relative limiting errors add, for a quotient as well as a product.

POWERS (GENERAL PRODUCT FORM)

$$X = k\,u^p\,v^q\,w^r: \quad \frac{\delta X}{X} = \pm\left(|p|\,\frac{\delta u}{u} + |q|\,\frac{\delta v}{v} + |r|\,\frac{\delta w}{w}\right)$$

Each relative error is weighted by the magnitude of its exponent, whether positive, negative or fractional (k is an exact constant). $P = V^2/R$ gives $2\varepsilon_V + \varepsilon_R$; $Z_0 = \sqrt{L/C}$ gives $\frac{1}{2}\varepsilon_L + \frac{1}{2}\varepsilon_C$.

GENERAL FUNCTION (WORST CASE)

$$\delta X = \pm\sum_i \left|\frac{\partial X}{\partial x_i}\right|\delta x_i, \qquad \frac{\delta X}{X} = \pm\sum_i \left|\frac{x_i}{X}\frac{\partial X}{\partial x_i}\right|\frac{\delta x_i}{x_i}$$

Linearised, so valid for small errors. Use it whenever a variable appears more than once, as in $R_1R_2/(R_1+R_2)$: applying the product and sum rules separately treats the two appearances as independent, which can only overstate the error, never understate it.

ARITHMETIC MEAN AND DEVIATION

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \qquad d_i = x_i - \bar{x}, \qquad \sum_{i=1}^{n} d_i = 0$$

The deviations from the mean always sum to zero, a useful arithmetic check.

AVERAGE DEVIATION

$$\bar{D} = \frac{1}{n}\sum_{i=1}^{n} |d_i|$$

Magnitudes are needed because the signed deviations cancel. For a large normally distributed set, $\bar{D} \approx 0.80\,\sigma$.

STANDARD DEVIATION AND VARIANCE

$$\sigma = \sqrt{\frac{\sum d_i^2}{n}}, \qquad s = \sqrt{\frac{\sum d_i^2}{n-1}}, \qquad \text{variance} = \sigma^2 \text{ (or } s^2)$$

σ (divide by n) describes a whole population or a very large set. s (divide by $n-1$) is the estimate from a finite sample, because one degree of freedom is used up in computing $\bar{x}$; the difference matters only when n is small. Use the one the question asks for.

PROBABLE ERROR

$$r = 0.6745\,\sigma$$

Half of a large set of normally distributed readings lie within $\bar{x} \pm r$. Use the same σ or s that the question specifies.

STANDARD DEVIATION OF THE MEAN

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}, \qquad r_{\bar{x}} = 0.6745\,\sigma_{\bar{x}}$$

Scatter of the mean of n independent readings, each of standard deviation σ. Quadrupling the number of readings halves it; it does nothing to systematic error.

TYPE A STANDARD UNCERTAINTY (MEAN OF REPEATED READINGS)

$$u_A(\bar{x}) = \frac{s}{\sqrt{n}}$$

s is the sample standard deviation (with $n-1$) of the n readings.

TYPE B STANDARD UNCERTAINTY

$$\text{rectangular: } u = \frac{a}{\sqrt{3}}, \qquad \text{triangular: } u = \frac{a}{\sqrt{6}}, \qquad \text{from a certificate: } u = \frac{U}{k}$$

$\pm a$ are the limits. Rectangular when any value within the limits is equally likely (a tolerance with no further information); a display of resolution Δ gives $a = \Delta/2$, so $u = \Delta/(2\sqrt{3})$. A certificate quoting U at coverage factor k gives $u = U/k$.

COMBINED STANDARD UNCERTAINTY

$$u_c(y) = \sqrt{\sum_i c_i^2\, u^2(x_i)}, \qquad c_i = \frac{\partial f}{\partial x_i}$$

For $y = f(x_1, x_2, \dots)$ with uncorrelated inputs. c_i is the sensitivity coefficient of input x_i. The same root-sum-square rule propagates the standard deviations, or the probable errors, of independent quantities: $\sigma_X = \sqrt{\sum_i (\partial X/\partial x_i)^2\, \sigma_i^2}$ and $r_X = \sqrt{\sum_i (\partial X/\partial x_i)^2\, r_i^2}$.

RELATIVE COMBINED UNCERTAINTY FOR A PRODUCT FORM

$$y = k\, x_1^{p_1} x_2^{p_2} \cdots \;\Rightarrow\; \frac{u_c(y)}{|y|} = \sqrt{\sum_i \left(p_i \, \frac{u(x_i)}{x_i} \right)^2}$$

The statistical counterpart of the power rule for limiting errors: relative uncertainties weighted by the exponents, squared, added, square-rooted.

EXPANDED UNCERTAINTY AND THE REPORTED RESULT

$$U = k\, u_c, \qquad y = \bar{y} \pm U$$

$k = 2$ gives about 95% coverage and $k = 3$ about 99.7% when the combined distribution is close to normal with many degrees of freedom. State k with the result.

JOSEPHSON VOLTAGE STANDARD

$$V = \frac{nf}{K_J}, \qquad K_J = \frac{2e}{h} \approx 483\,597.848\ \text{GHz/V}$$

A junction irradiated at microwave frequency f shows constant-voltage steps at integer n, each f/K_J high: about 2.07 μV per GHz. Since 2019 K_J is exact. N junctions in series, each on step n, give $V = Nnf/K_J$; arrays of thousands of junctions give 1 V and 10 V.

QUANTUM HALL RESISTANCE STANDARD

$$R_H = \frac{R_K}{i}, \qquad R_K = \frac{h}{e^2} \approx 25\,812.807\ \Omega$$

In a two-dimensional electron gas at low temperature and high magnetic field, the Hall resistance takes plateau values for integer i, independent of the sample's dimensions and material. The $i = 2$ plateau, $R_K/2 \approx 12\,906.4\ \Omega$, is commonly used.

LEAST-SQUARES STRAIGHT LINE (UNWEIGHTED)

$$b = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2}, \qquad a = \bar{y} - b\,\bar{x}$$

Every point equally reliable. The line passes through the centroid $(\bar{x}, \bar{y})$ and the residuals sum to zero.

SLOPE IN DEVIATION FORM

$$b = \frac{S_{xy}}{S_{xx}}, \qquad S_{xy} = \sum x_i y_i - n\bar{x}\bar{y}, \qquad S_{xx} = \sum x_i^2 - n\bar{x}^2$$

Same slope as above with smaller numbers. $S_{xy} = \sum (x_i - \bar{x})(y_i - \bar{y})$ and $S_{xx} = \sum (x_i - \bar{x})^2$.

LINE THROUGH THE ORIGIN

$$y = bx: \quad b = \frac{\sum x_i y_i}{\sum x_i^2}$$

Only when the model is known to have zero intercept. This line does not in general pass through the centroid.

WEIGHTED LEAST-SQUARES STRAIGHT LINE

$$b = \frac{\sum w_i \sum w_i x_i y_i - \sum w_i x_i \sum w_i y_i}{\sum w_i \sum w_i x_i^2 - \left(\sum w_i x_i\right)^2}, \qquad a = \frac{\sum w_i y_i - b \sum w_i x_i}{\sum w_i}, \qquad w_i = \frac{1}{\sigma_i^2}$$

The line passes through the weighted centroid $(\sum w_i x_i / \sum w_i,\ \sum w_i y_i / \sum w_i)$.

WEIGHTED MEAN

$$\bar{x}_w = \frac{\sum w_i x_i}{\sum w_i}, \qquad \sigma_{\bar{x}_w} = \frac{1}{\sqrt{\sum w_i}}, \qquad w_i = \frac{1}{\sigma_i^2}$$

Combines independent measurements of the same quantity made with different precision. The combined uncertainty is smaller than the smallest individual one; with equal $\sigma_i = \sigma$ it reduces to $\bar{x}$ and $\sigma/\sqrt{n}$.

Bridges, potentiometers and the Q-meter

WHEATSTONE BRIDGE BALANCE

$$\frac{P}{Q} = \frac{R}{S} \quad \Longleftrightarrow \quad PS = QR \quad \Rightarrow \quad R = S\,\frac{P}{Q}$$

Products of opposite arms are equal. The condition does not involve E, G or the source resistance, and it is unchanged if the supply and the galvanometer are interchanged (only the sensitivity changes).

LIMITING ERROR OF A WHEATSTONE MEASUREMENT

$$R = S\,\frac{P}{Q} \;\Rightarrow\; \frac{\delta R}{R} = \pm\left(\frac{\delta P}{P} + \frac{\delta Q}{Q} + \frac{\delta S}{S}\right)$$

Limiting error is a guaranteed (worst-case) bound: for a product or quotient, the relative limiting errors of the factors add, whether a factor multiplies or divides. For example, arms each known to $\pm 0.05\%$ give R to within $\pm 0.15\%$. Any lack of detector sensitivity (the smallest unbalance that can be seen) adds to this.

UNBALANCED BRIDGE: THEVENIN EQUIVALENT AT THE GALVANOMETER TERMINALS

$$V_{\text{th}} = V_B - V_D = E\left(\frac{R}{R+S} - \frac{P}{P+Q}\right), \qquad R_{\text{th}} = \frac{PQ}{P+Q} + \frac{RS}{R+S}$$

Open-circuit voltage of B with respect to D, and the resistance seen from B–D. This simple R_{th} holds for a supply of negligible internal resistance; with a source resistance, find R_{th} by a delta–star conversion instead.

GALVANOMETER CURRENT AND DEFLECTION

$$I_g = \frac{V_{\text{th}}}{R_{\text{th}} + G}, \qquad \theta = S_i\, I_g$$

Current in a galvanometer of resistance G connected across an unbalanced bridge. S_i is the galvanometer's current sensitivity (e.g. mm/μA).

SMALL UNBALANCE, AND THE EQUAL-ARM BRIDGE

$$\Delta V_{\text{th}} \approx E\,\frac{S\,\Delta R}{(R+S)^2}; \qquad P=Q=R=S:\ \ \Delta V_{\text{th}} = \frac{E\,\Delta R}{4R+2\Delta R} \approx \frac{E\,\Delta R}{4R}$$

Change in the open-circuit output when the unknown moves from its balance value R to $R+\Delta R$, with $\Delta R \ll R$ for the approximate forms. For a given R, $S/(R+S)^2$ is largest at $S=R$, so equal arms give the largest open-circuit output per ohm of unbalance. The middle form is exact and shows the output is linear only for small ΔR.

QUARTER, HALF AND FULL BRIDGES (EQUAL ARMS R, SUPPLY E)

$$\text{quarter: } \frac{E\,\Delta R}{4R}, \qquad \text{half: } \frac{E\,\Delta R}{2R}, \qquad \text{full: } \frac{E\,\Delta R}{R}$$

Open-circuit output with one, two or four active arms (e.g. strain gauges), each changing by $\pm\Delta R$. The active arms must be placed so their effects add: two adjacent arms must change in opposite senses, two opposite arms in the same sense. The full bridge, and the half bridge with two adjacent arms changing in opposite senses, are exactly linear. The quarter bridge, and a half bridge with two opposite arms changing in the same sense (exact output $E\,\Delta R/(2R+\Delta R)$), are linear only for small ΔR.

KELVIN DOUBLE BRIDGE BALANCE

$$R = \frac{P}{Q}\,S + \frac{q\,r}{p+q+r}\left(\frac{P}{Q}-\frac{p}{q}\right)$$

P, Q = outer ratio arms; p, q = inner ratio arms (P and p on the side of the unknown R, Q and q on the side of the standard S); r = resistance of the link between R and S. The second term is the error caused by the link, and it has the sign of $(P/Q - p/q)$.

KELVIN DOUBLE BRIDGE WITH MATCHED RATIOS

$$\frac{P}{Q}=\frac{p}{q} \ \Rightarrow\ R=\frac{P}{Q}\,S \quad (\text{independent of } r)$$

In practice the two ratio pairs are ganged so they stay equal, the link is made short and thick so any residual mismatch multiplies a small r, and every ratio arm is made much larger than the lead and contact resistances in series with it. Used for resistances below about 1 Ω, such as shunts and busbar sections.

LOSS-OF-CHARGE METHOD

$$v(t) = V_0\,e^{-t/(RC)} \ \Rightarrow\ R = \frac{t}{C\ln(V_0/V)} = \frac{0.4343\,t}{C\log_{10}(V_0/V)}$$

A capacitor C charged to V_0 discharges through the unknown R; V is the voltage after time t, read on a voltmeter that draws no current (an electrostatic voltmeter). It suits very high resistances, where the decay is slow enough to time accurately; with a small R the voltage collapses too quickly.

LOSS OF CHARGE: CORRECTING FOR THE CAPACITOR'S OWN LEAKAGE

$$R' = \frac{R\,R_1}{R+R_1} \ \Rightarrow\ R = \frac{R_1 R'}{R_1 - R'}$$

R_1 is the leakage resistance of the capacitor itself (together with that of the voltmeter), found by a second test with R disconnected. It is always in parallel with R, so the first test gives $R' < R$.

GENERAL AC BRIDGE BALANCE

$$Z_1 Z_4 = Z_2 Z_3 \qquad \text{or} \qquad Y_1 Y_4 = Y_2 Y_3$$

Products of the impedances of opposite arms are equal. The admittance form is convenient when arms are parallel combinations.

MAGNITUDE AND PHASE CONDITIONS

$$|Z_1|\,|Z_4| = |Z_2|\,|Z_3|, \qquad \theta_1+\theta_4 = \theta_2+\theta_3$$

Both must hold at once. θ is the impedance angle: positive for an inductive arm, negative for a capacitive arm, zero for a pure resistance.

Q OF A COIL AND DISSIPATION FACTOR OF A CAPACITOR

$$Q = \frac{\omega L_s}{R_s}, \qquad D = \tan\delta = \omega C_s R_s = \frac{1}{\omega C_p R_p}$$

Subscript s = series equivalent circuit, p = parallel equivalent circuit; in the parallel model the coil's $Q = R_p/(\omega L_p)$. δ is the loss angle: 90° minus the angle by which the capacitor's current leads its voltage. For small δ the capacitor's power factor $\cos\phi = \sin\delta \approx \tan\delta = D$. For a coil, $D = 1/Q$. From here on Q denotes a quality factor, not a ratio arm of the DC bridges.

SERIES–PARALLEL EQUIVALENTS AT ONE FREQUENCY

$$\text{coil: } R_p = R_s\left(1+Q^2\right),\ L_p = L_s\left(1+\frac{1}{Q^2}\right); \qquad \text{capacitor: } R_p = R_s\left(1+\frac{1}{D^2}\right),\ C_p = \frac{C_s}{1+D^2}$$

Converts between the series model (natural for an arm like $R + j\omega L$) and the parallel model. For $Q \geq 10$, L_p and L_s agree within 1%; for small D, $C_p \approx C_s$.

MAXWELL INDUCTANCE (COMPARISON) BRIDGE

$$L_1 = \frac{R_3}{R_4}L_2, \qquad R_1 = \frac{R_3}{R_4}R_2$$

Arm 2 is a variable standard inductor L_2 with total series resistance R_2 (its own resistance plus any added resistor); arms 3 and 4 are resistors, R_4 opposite the unknown and R_3 opposite the standard. From $(R_1 + j\omega L_1)R_4 = (R_2 + j\omega L_2)R_3$. Frequency-independent; its drawback is the need for a variable standard inductor.

MAXWELL–WIEN (MAXWELL INDUCTANCE–CAPACITANCE) BRIDGE

$$L_1 = R_2 R_3 C_4, \qquad R_1 = \frac{R_2 R_3}{R_4}, \qquad Q = \frac{\omega L_1}{R_1} = \omega C_4 R_4$$

Arm 4 = R_4 in **parallel** with a standard capacitor C_4. From $Z_1 = R_2 R_3 Y_4 = R_2 R_3 (1/R_4 + j\omega C_4)$. The balance equations are free of frequency, and if R_4 and C_4 are the adjusted elements the two conditions are independent. Suited to medium-Q coils (roughly $1 < Q < 10$).

HAY'S BRIDGE

$$L_1 = \frac{R_2 R_3 C_4}{1+\omega^2 C_4^2 R_4^2}, \qquad R_1 = \frac{\omega^2 C_4^2 R_2 R_3 R_4}{1+\omega^2 C_4^2 R_4^2}, \qquad Q = \frac{1}{\omega C_4 R_4}$$

Arm 4 = R_4 in **series** with C_4. From $Z_1 = R_2 R_3/(R_4 - j/(\omega C_4))$. Both L_1 and R_1 depend on ω, so the frequency must be known. Suited to high-Q coils ($Q > 10$).

HAY'S BRIDGE IN TERMS OF Q

$$L_1 = \frac{R_2 R_3 C_4}{1+1/Q^2} \approx R_2 R_3 C_4 \quad (Q > 10)$$

Because $\omega^2 C_4^2 R_4^2 = 1/Q^2$. The approximation overstates L_1 by the factor $1 + 1/Q^2$: 1% at $Q = 10$, 0.25% at $Q = 20$. Use the exact form when a question gives ω and all the element values.

ANDERSON BRIDGE

$$L_1 = C\,\frac{R_3}{R_4}\left[r\left(R_2+R_4\right) + R_2 R_4\right], \qquad R_1 = \frac{R_2 R_3}{R_4}$$

Topology as described above: the series r–C branch is in parallel with R_4, and the detector taps its midpoint. R_1 is the total resistance of the coil's arm: if a variable resistor r_1 is added in series with a coil of resistance R_c to obtain balance, $R_c + r_1 = R_2 R_3/R_4$. The balance is frequency-independent, and r does not appear in the resistance condition.

SCHERING BRIDGE BALANCE

$$C_1 = C_2 \frac{R_4}{R_3}, \qquad r_1 = R_3 \frac{C_4}{C_2}$$

From $Z_1 = Z_2 Z_3 Y_4 = \frac{R_3}{j\omega C_2}\left(\frac{1}{R_4} + j\omega C_4\right)$. Both equations are free of frequency. R_4, the resistor in the arm opposite the unknown, is in the numerator for C_1.

SCHERING BRIDGE DISSIPATION FACTOR

$$D_1 = \tan\delta = \omega C_1 r_1 = \omega C_4 R_4$$

Series-model dissipation factor of the unknown; unlike C_1 and r_1, it contains ω. With R_4 fixed, the C_4 dial can be calibrated directly in D at one frequency, and R_3 is varied for the C_1 balance.

WIEN BRIDGE BALANCE FREQUENCY

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}, \qquad f_0 = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

From the imaginary part of $Z_1 Y_2 = R_3/R_4$: $\omega C_2 R_1 = 1/(\omega C_1 R_2)$.

WIEN BRIDGE RESISTANCE-RATIO CONDITION

$$\frac{R_3}{R_4} = \frac{R_1}{R_2} + \frac{C_2}{C_1}$$

Real part of the same equation. R_3 is the resistor opposite the parallel-RC arm, R_4 the one opposite the series-RC arm.

EQUAL-ELEMENT WIEN BRIDGE

$$R_1 = R_2 = R,\ \ C_1 = C_2 = C: \qquad f_0 = \frac{1}{2\pi RC}, \qquad R_3 = 2R_4$$

With ganged $R_1 = R_2$ and equal fixed capacitors, one dial reads frequency directly. At f_0 the series-RC/parallel-RC divider passes exactly one third of the voltage with zero phase shift, which is why a Wien-bridge oscillator's amplifier needs a gain of 3.

Q-METER PRINCIPLE

$$\omega_0^2 LC = 1, \qquad I_0 = \frac{E}{R}, \qquad Q = \frac{V_C}{E} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$$

At resonance the loop's reactances cancel, so the current is E/R (R = coil resistance) and the capacitor voltage is Q times the injected voltage. Assumes R_{sh} and the capacitor's losses are negligible compared with R, the voltmeter does not load the circuit, and the self-capacitance is neglected.

INDUCTANCE FROM THE RESONANCE SETTING

$$L = \frac{1}{\omega_0^2\,(C + C_d)} \approx \frac{1}{4\pi^2 f_0^2\, C}$$

C_d = the coil's self (distributed) capacitance, which adds to the tuning capacitance as far as the resonant frequency is concerned. Neglect it only when $C \gg C_d$.

CORRECTION FOR THE SHUNT (INSERTION) RESISTANCE

$$Q_m = \frac{\omega_0 L}{R + R_{\text{sh}}} \ \Rightarrow\ Q_{\text{true}} = Q_m\left(1 + \frac{R_{\text{sh}}}{R}\right)$$

The shunt is part of the resonant loop, so the measured Q_m is slightly low. The correction is negligible when $R_{\text{sh}} \ll R$ and matters for coils of very low resistance.

SELF-CAPACITANCE BY THE TWO-FREQUENCY METHOD

$$f_2 = 2f_1: \quad C_d = \frac{C_1 - 4C_2}{3}; \qquad f_2 = nf_1: \quad C_d = \frac{C_1 - n^2 C_2}{n^2 - 1}$$

C_1 and C_2 are the tuning-capacitor settings for resonance at f_1 and at the higher frequency f_2. From $\omega^2 L(C + C_d) = 1$ at both frequencies: $C_1 + C_d = n^2(C_2 + C_d)$.

CORRECTION FOR SELF-CAPACITANCE

$$Q_{\text{true}} = Q_{\text{ind}} \frac{C + C_d}{C}$$

Q_{ind} is the reading taken at tuning setting C. The self-capacitance acts across the coil's terminals, so the indicated Q falls short of the coil's true $\omega L/R$ by the factor $C/(C + C_d)$.

SLIDE-WIRE COMPARISON

$$V_x = V_s \frac{l_x}{l_s}, \qquad V = k\,l, \quad k = I_w\,\rho_l$$

l_s and l_x are the balance lengths for the standard emf V_s and the unknown V_x at the same working current I_w, on a uniform wire of resistance ρ_l per unit length; k is the potential gradient (volts per unit length).

EXTENDING THE RANGE: VOLT-RATIO BOX

$$V = K\,V_p$$

A precision resistive divider presents V/K to the potentiometer (K = ratio, e.g. 10 or 100) and V_p is the potentiometer reading. Unlike the potentiometer itself, the divider draws current from the circuit under test.

CURRENT MEASUREMENT

$$I = \frac{V_p}{R_s}$$

The potentiometer reads the drop V_p across a four-terminal standard resistor R_s carrying the current. This is how ammeters are calibrated.

RESISTANCE BY COMPARISON

$$R_x = R_s \frac{V_x}{V_s}$$

R_x and a standard R_s carry the same current in series, and the potentiometer reads the two drops in quick succession. Suitable for low resistances, because only the potential terminals are involved.

COORDINATE-TYPE AC POTENTIOMETER

$$V = \sqrt{V_1^2 + V_2^2}, \qquad \theta = \tan^{-1}\frac{V_2}{V_1}$$

V_1 = in-phase component, V_2 = quadrature component, both measured against the potentiometer's reference phase; the signs of V_1 and V_2 fix the quadrant of θ. The polar type gives V and θ directly.

Measuring voltage, current, power, energy and power factor

PMMC DEFLECTION

$$T_d = NBAI = GI, \qquad \theta = \frac{G}{K} I$$

N is the number of turns, B the gap flux density, A the coil area and $G = NBA$. With a varying current the inertia of the moving system averages the torque, so θ follows the average (DC) value. The scale is linear, and the pointer tries to deflect backwards if the current reverses.

MOVING-IRON DEFLECTION

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}, \qquad \theta = \frac{I^2}{2K}\frac{dL}{d\theta}$$

Derived from stored energy. Torque is proportional to i^2, so the direction of deflection does not depend on polarity and I is the rms value on AC. The scale is square-law unless the vane is shaped so that $dL/d\theta$ falls as θ increases.

ELECTRODYNAMOMETER DEFLECTING TORQUE (AVERAGE)

$$T_d = I_1 I_2 \cos\phi \, \frac{dM}{d\theta}, \qquad \text{coils in series: } T_d = I^2 \, \frac{dM}{d\theta}$$

I_1 and I_2 are the rms currents in the fixed and moving coils, and ϕ is the angle between them (on DC, use the DC currents with $\cos\phi = 1$). Used as an ammeter or voltmeter, it reads rms and has the same calibration on DC and AC.

RMS AND AVERAGE VALUES

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2 \, dt}, \qquad V_{\text{av}} = \frac{1}{T}\int_0^T |v| \, dt$$

T is one full period. A PMMC reads the *signed* mean $\frac{1}{T}\int_0^T v\,dt$, which is zero for a symmetric AC wave. Do not confuse it with V_{av}.

FORM FACTOR AND CREST (PEAK) FACTOR

$$k_f = \frac{V_{\text{rms}}}{V_{\text{av}}}, \qquad k_c = \frac{V_p}{V_{\text{rms}}}$$

For a sine wave, $k_f = \pi/(2\sqrt{2}) \approx 1.111$ and $k_c = \sqrt{2} \approx 1.414$.

RMS OF A WAVE WITH DC AND HARMONICS

$$V_{\text{rms}} = \sqrt{V_0^2 + V_1^2 + V_2^2 + \cdots}$$

V_0 is the DC component, and $V_1, V_2, \ldots$ are the rms values of components at *different* frequencies, whose cross-products average to zero. For two sinusoids of the same frequency, add them as phasors first.

FULL-WAVE RECTIFIER METER READING

$$V_{\text{read}} = \frac{\pi}{2\sqrt{2}} \, V_{\text{av}} \approx 1.111 \, V_{\text{av}}$$

Applies to an average-responding full-wave meter whose scale is marked in rms of a sine. V_{av} is the mean of $|v|$. The reading is correct only when the wave's $k_f = 1.111$.

HALF-WAVE RECTIFIER METER READING

$$V_{\text{read}} = \frac{\pi}{\sqrt{2}} \, \bar{v}^{+} \approx 2.221 \, \bar{v}^{+}, \qquad \bar{v}^{+} = \frac{1}{T}\int_0^T \max(v, 0) \, dt$$

Applies to an average-responding half-wave meter with its scale marked in rms of a sine. Only the conducting half-cycle contributes. Reversing the leads measures the other half, which gives a different reading on an asymmetric wave.

FORM-FACTOR ERROR OF AN AVERAGE-RESPONDING METER

$$\%\,\text{error} = \frac{1.111 - k_f}{k_f} \times 100$$

Applies to a full-wave average-responding meter calibrated for a sine, with k_f computed from the mean of $|v|$. The error is a percentage of the TRUE rms of the applied wave. Square wave ($k_f = 1$): $+11.1\%$. Triangle ($k_f = 1.155$): -3.8%.

AMMETER SHUNT

$$R_{sh} = \frac{R_m}{m-1}, \qquad m = \frac{I}{I_{fs}}$$

m is the multiplying power: the new full-scale current I divided by the movement's full-scale current. The shunt and the movement share the same voltage.

VOLTMETER MULTIPLIER

$$R_{se} = \frac{V}{I_{fs}} - R_m = (m-1)R_m, \qquad m = \frac{V}{I_{fs}R_m}$$

V is the desired full-scale voltage. The total meter resistance is V/I_{fs}.

VOLTMETER SENSITIVITY AND RESISTANCE

$$S = \frac{1}{I_{fs}} \quad (\Omega/\mathrm{V}), \qquad R_v = S\,V_{\text{range}}$$

The voltmeter's resistance depends on the RANGE selected, not on the reading. A higher sensitivity (smaller I_{fs}) means less loading.

AC SENSITIVITY OF A RECTIFIER VOLTMETER

$$S_{ac} = 0.9\,S_{dc} \ \text{(full-wave)}, \qquad S_{ac} = 0.45\,S_{dc} \ \text{(half-wave)}$$

For a PMMC movement with ideal diodes and a scale marked in rms of a sine, where $S_{dc} = 1/I_{fs}$. A sine of rms value V has a mean of $0.9V$ after full-wave rectification and $0.45V$ after half-wave rectification, so the total resistance for full scale on an AC range V is $0.9V/I_{fs}$ or $0.45V/I_{fs}$.

VOLTMETER LOADING

$$V_{\text{read}} = V_{th}\,\frac{R_v}{R_{th}+R_v}, \qquad \%\,\text{error} = -\frac{R_{th}}{R_{th}+R_v}\times 100$$

V_{th} and R_{th} form the Thevenin equivalent seen from the voltmeter's terminals, and V_{th} is the true (unloaded) voltage. The error is a percentage of the true value and is always negative.

AMMETER INSERTION ERROR

$$I_{\text{read}} = I_{\text{true}}\,\frac{R_{th}}{R_{th}+R_A}, \qquad \%\,\text{error} = -\frac{R_A}{R_{th}+R_A}\times 100$$

R_{th} is the Thevenin resistance of the loop the ammeter is inserted into, and R_A is the ammeter's resistance. The error is a percentage of the true current, and the current is always reduced.

COMPENSATED VOLTAGE PROBE (RC DIVIDER)

$$R_pC_p = R_{in}C_{in}, \qquad \text{attenuation} = \frac{R_p + R_{in}}{R_{in}}$$

C_{in} includes the capacitance of the scope input and the cable. Example: a 9 MΩ tip resistor with a 1 MΩ ‖ 18 pF input gives 10:1 and needs $C_p = 2$ pF. The circuit under test then sees 10 MΩ in parallel with 1.8 pF. When the condition holds, the division ratio is the same at all frequencies.

SCALING THROUGH INSTRUMENT TRANSFORMERS

$$I_1 = K_{CT}\,I_2, \qquad V_1 = K_{PT}\,V_2, \qquad P_1 = K_{CT}K_{PT}\,W$$

K is the nominal ratio, primary:secondary. For example, a 100/5 A CT has $K_{CT} = 20$, and an 11 kV/110 V PT has $K_{PT} = 100$. Ratio and phase-angle errors are neglected here.

DC POWER, VOLTMETER ACROSS THE LOAD

$$VI = P_L + \frac{V^2}{R_v}$$

The ammeter carries the load current plus the voltmeter's current. This connection is preferred for low-voltage, high-current loads.

DC POWER, AMMETER NEXT TO THE LOAD

$$VI = P_L + I^2 R_A$$

The voltmeter reads the load voltage plus the ammeter's drop. This connection is preferred for high-voltage, low-current loads.

SINGLE-PHASE POWER

$$P = VI\cos\phi, \qquad Q = VI\sin\phi, \qquad S = VI, \qquad \bar{S} = \bar{V}\,\bar{I}^{*} = P + jQ$$

V and I are rms values of sinusoids. ϕ is the angle by which the current lags the voltage: for a lagging (inductive) load, $\phi > 0$ and $Q > 0$.

THREE-VOLTMETER METHOD

$$P = \frac{V_1^2 - V_2^2 - V_3^2}{2R}, \qquad \cos\phi = \frac{V_1^2 - V_2^2 - V_3^2}{2V_2V_3}$$

A known non-inductive resistance R is connected in series with the load. V_1 is the voltage across the series combination, V_2 the voltage across R and V_3 the voltage across the load; P and $\cos\phi$ are the load's. It follows from the phasor sum $V_1^2 = V_2^2 + V_3^2 + 2V_2V_3\cos\phi$ with $I = V_2/R$.

THREE-AMMETER METHOD

$$P = \frac{R}{2}\left(I_1^2 - I_2^2 - I_3^2\right), \qquad \cos\phi = \frac{I_1^2 - I_2^2 - I_3^2}{2I_2I_3}$$

A known non-inductive resistance R is connected in parallel with the load. I_1 is the total current, I_2 the current in R and I_3 the load current. It follows from $I_1^2 = I_2^2 + I_3^2 + 2I_2I_3\cos\phi$ with $V = I_2R$.

ACTIVE POWER WITH NON-SINUSOIDAL VOLTAGE AND CURRENT

$$P = V_0I_0 + \sum_{n=1}^{\infty} V_nI_n\cos\phi_n, \qquad \text{power factor} = \frac{P}{V_{\text{rms}}\,I_{\text{rms}}}$$

V_0 and I_0 are the DC components, V_n and I_n the rms values of the nth harmonics, and ϕ_n the angle between them. Products of components at different frequencies average to zero, so a harmonic present in only one of v and i contributes no power. An electrodynamometer wattmeter with negligible PC inductance reads this P. V_{rms} and I_{rms} are the full rms values, including every harmonic.

READING WITH PRESSURE-COIL INDUCTANCE

$$W = VI\cos\beta\,\cos(\phi - \beta), \qquad \beta = \tan^{-1}\frac{\omega L_p}{R_p}$$

Applies to a wattmeter calibrated on DC with a PC resistance of R_p, on a sinusoidal supply. The PC current lags V by β. ϕ is positive for a lagging load and negative for a leading load.

CORRECTION FACTOR FOR PRESSURE-COIL INDUCTANCE

$$P_{\text{true}} = \frac{\cos\phi}{\cos\beta\,\cos(\phi - \beta)}\,W$$

Multiply the reading by this factor. Equivalently, $W/P_{\text{true}} = \cos^2\beta\,(1 + \tan\phi\tan\beta)$.

APPROXIMATE ERROR DUE TO PRESSURE-COIL INDUCTANCE

$$W - P_{\text{true}} \approx VI\sin\phi\,\tan\beta, \qquad \frac{W - P_{\text{true}}}{P_{\text{true}}} \approx \tan\phi\,\tan\beta$$

Valid for small β. A lagging load makes the wattmeter read high, and a leading load makes it read low. The error grows quickly as the power factor falls.

CONNECTION (COIL-LOSS) ERROR

$$\text{PC on supply side: } W = P_L + I^2 R_c, \qquad \text{PC on load side: } W = P_L + \frac{V^2}{R_p}$$

R_c is the CC resistance and R_p the PC-circuit resistance. With the PC on the load side, the CC carries the PC current as well as the load current. Choose the connection whose extra loss is smaller: the load-side PC for high-current, low-voltage loads, and the supply-side PC for low-current, high-voltage loads.

WATTMETER FULL-SCALE VALUE AND MULTIPLYING FACTOR

$$W_{fs} = V_r\, I_r \cos\phi_r, \qquad \text{multiplying factor} = \frac{W_{fs}}{\text{number of scale divisions}}$$

V_r and I_r are the rated (range) voltage and current, and $\cos\phi_r$ is the rated power factor: 1 for a standard wattmeter, and the stated low value (for example 0.2) for a low power-factor wattmeter. The multiplying factor is in watts per division. When the wattmeter is fed through a CT and a PT, multiply further by $K_{CT}K_{PT}$.

TWO-WATTMETER READINGS, BALANCED LOAD

$$W_1 = V_L I_L \cos(30^\circ - \phi), \qquad W_2 = V_L I_L \cos(30^\circ + \phi)$$

For a balanced sinusoidal load, with ϕ the load phase angle (positive for lagging). With phase sequence RYB, current coils in lines R and Y and both pressure coils returned to B, W_1 is the wattmeter in line R.

POWER, REACTIVE POWER AND POWER FACTOR FROM TWO WATTMETERS

$$P = W_1 + W_2, \qquad Q = \sqrt{3}\,(W_1 - W_2), \qquad \tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{W_1 + W_2}$$

$P = W_1 + W_2$ holds for ANY 3-wire load. Q and $\tan\phi$ hold only for a balanced sinusoidal load. A negative reading enters every formula with its sign.

POWER FACTOR FROM THE RATIO OF THE READINGS

$$\cos\phi = \frac{1 + r}{2\sqrt{1 - r + r^2}}, \qquad r = \frac{W_2}{W_1}$$

For a balanced sinusoidal load, with W_1 the larger reading and W_2 entered with its sign, so $-1 \le r \le 1$. It follows from the $\tan\phi$ expression. Checks: $r = 1$ gives 1, $r = 0$ gives 0.5, and $r = -1$ gives 0.

REACTIVE POWER WITH ONE WATTMETER

$$W = V_L I_L \sin\phi, \qquad Q = \sqrt{3}\,W$$

For a balanced sinusoidal 3-wire load, with the current coil in one line and the pressure coil across the other two lines (for sequence RYB: CC in R, PC from Y to B). Reversing the pressure-coil connection or the phase sequence reverses the sign of the reading.

THREE-PHASE POWER, OTHER METHODS

$$P = W_1 + W_2 + W_3 \quad (\text{4-wire}), \qquad P = 3W \quad (\text{balanced, one wattmeter line-to-neutral}), \qquad P = \sqrt{3}\,V_L I_L \cos\phi$$

The one-wattmeter method needs the neutral, or an artificial star point, for its pressure coil. The last expression is for a balanced sinusoidal load. V_L and I_L are line values, and ϕ is the phase angle of each phase's load impedance.

METER CONSTANT AND DISC SPEED

$$n = K\,E, \qquad N_{\text{rpm}} = \frac{K\,P_{\text{kW}}}{60}$$

K is in rev/kWh (imp/kWh for an electronic meter), E in kWh and P in kW. Check the units given: some meters state the constant in kWh per revolution or W·s per revolution instead.

ENERGY-METER ERROR

$$\%\,\text{error} = \frac{E_{\text{reg}} - E_{\text{true}}}{E_{\text{true}}} \times 100, \qquad E_{\text{reg}} = \frac{n}{K}$$

The error is a percentage of the true energy. A positive error means the meter registers more than was consumed (it runs fast).

POWER FROM AN ELECTRONIC METER'S PULSES

$$P_{\text{kW}} = \frac{3600\,n}{K\,t}$$

n is the number of pulses counted in t seconds, and K is in imp/kWh.

POWER-FACTOR METER AND INDIRECT POWER FACTOR

$$\theta = \phi, \qquad \cos\phi = \frac{P}{VI}\ \ (\text{1-phase}), \qquad \cos\phi = \frac{P}{\sqrt{3}\,V_L I_L}\ \ (\text{balanced 3-phase})$$

The indirect forms use wattmeter, voltmeter and ammeter readings. For a balanced three-phase load, the two-wattmeter method gives $\tan\phi$ directly.

Instrument transformers

NOMINAL RATIO

$$K_n = \frac{\text{rated primary current (or voltage)}}{\text{rated secondary current (or voltage)}}$$

Read off the rating plate and fixed for the transformer. For example, 1000/5 A gives $K_n = 200$ and 11000/110 V gives $K_n = 100$. Readings are normally multiplied by K_n.

ACTUAL RATIO (TRANSFORMATION RATIO)

$$R = \frac{I_p}{I_s}\ \ (\text{CT}), \qquad R = \frac{V_p}{V_s}\ \ (\text{PT})$$

The ratio of the actual primary magnitude to the actual secondary magnitude at the operating point. It is not constant: it changes with the primary current or voltage, the burden and the burden power factor.

TURNS RATIO

$$n = \frac{N_s}{N_p}\ \ (\text{CT}), \qquad n = \frac{N_p}{N_s}\ \ (\text{PT})$$

Both are defined so that $n \approx K_n$. A bar (single-turn) primary CT has $N_p = 1$, so $n = N_s$. With turns compensation, n is made slightly smaller than K_n.

RATIO CORRECTION FACTOR

$$\text{RCF} = \frac{R}{K_n}, \qquad \text{true primary value} = R \times (\text{secondary reading}) = \text{RCF} \times K_n \times (\text{secondary reading})$$

An RCF above 1 means the nominal-ratio figure is too low. For an ideal transformer RCF = 1. In terms of RCF, ratio error $= \left(\frac{1}{\text{RCF}} - 1\right) \times 100\%$.

RATIO ERROR (CURRENT ERROR OR VOLTAGE ERROR)

$$\text{ratio error} = \frac{K_n - R}{R} \times 100\% = \frac{K_n I_s - I_p}{I_p} \times 100\%$$

The error in the value obtained with the nominal ratio, as a percentage of the true value. Positive means $K_n\times$ reading is high and negative means it is low. For a PT, replace I with V. If the ratio error is e (as a fraction), then $R = K_n/(1+e)$.

CT SECONDARY EMF AND CORE FLUX

$$E_s = I_s Z_{\text{total}} = 4.44\, f N_s \Phi_m = 4.44\, f N_s B_m A_c$$

Z_{total} is the impedance magnitude of the whole secondary circuit (winding, leads and burden); Φ_m is the peak core flux, B_m the peak flux density and A_c the core cross-section, for sinusoidal flux at frequency f. The flux the CT must carry, and so its exciting current, is set by I_s and the burden impedance, which is why the core flux follows the line current.

CT ACTUAL RATIO

$$R \approx n + \frac{I_e \cos\delta + I_m \sin\delta}{I_s} = n + \frac{I_0 \sin(\delta + \alpha)}{I_s}$$

Valid when $I_0 \ll n I_s$. Since $0 < \delta + \alpha < 180^\circ$, $\sin(\delta+\alpha) > 0$: the correction term is always positive, so $R > n$. An uncompensated CT ($n = K_n$) therefore has $R > K_n$ and a negative ratio error, approximately $-\dfrac{I_0 \sin(\delta+\alpha)}{K_n I_s} \times 100\%$, which means it reads low.

CT PHASE ANGLE

$$\theta_{CT} \approx \frac{I_m \cos\delta - I_e \sin\delta}{n I_s} = \frac{I_0 \cos(\delta+\alpha)}{n I_s} \quad \text{rad}$$

Positive (the reversed secondary current leads I_p) while $\delta + \alpha < 90^\circ$, zero at $\delta + \alpha = 90^\circ$ and negative beyond. Here $n I_s$ is the secondary current referred to the primary.

CT WITH A PURELY RESISTIVE SECONDARY CIRCUIT

$$\delta = 0: \qquad R \approx n + \frac{I_e}{I_s}, \qquad \theta_{CT} \approx \frac{I_m}{n I_s} \quad \text{rad}$$

Use this when the secondary circuit, winding included, is purely resistive. The loss component then sets the ratio error and the magnetising component sets the phase angle.

BURDEN IN OHMS

$$Z_b = \frac{\text{VA}_{\text{rated}}}{I_{s,\text{rated}}^2} \quad \text{(CT)}, \qquad Z_b = \frac{V_{s,\text{rated}}^2}{\text{VA}_{\text{rated}}} \quad \text{(PT)}$$

The burden includes the leads. On a CT the instruments are in series, so their impedances add. On a PT they are in parallel, so their admittances add. Examples: 15 VA on a 5 A CT is 0.6 Ω, 15 VA on a 1 A CT is 15 Ω, and 100 VA on a 110 V PT is 121 Ω.

PT ACTUAL RATIO

$$R \approx n + \frac{\dfrac{I_s}{n}\left(R_P \cos\Delta + X_P \sin\Delta\right) + I_e r_p + I_m x_p}{V_s}$$

Valid when the winding drops are small compared with V_p (the normal case), so the exciting-current components can be taken in phase with and in quadrature to the reversed secondary voltage. At no load ($I_s = 0$) this becomes $R_0 \approx n + (I_e r_p + I_m x_p)/V_s$. The load term is proportional to I_s, so for a burden of fixed power factor the ratio, and hence the ratio error, changes almost linearly with the burden VA.

PT PHASE ANGLE

$$\theta_{PT} \approx \frac{I_m r_p - I_e x_p - \dfrac{I_s}{n}\left(X_P \cos\Delta - R_P \sin\Delta\right)}{n V_s} \quad \text{rad}$$

Sign as on this page: positive when the reversed secondary voltage leads V_p. At no load it is $(I_m r_p - I_e x_p)/(n V_s)$. The load term is negative whenever $\tan\Delta < X_P/R_P$, and in that case adding burden swings the reversed secondary voltage towards lagging V_p. Setting $\theta_{PT} = 0$ gives the burden at which the phase angle vanishes; for a unity-power-factor burden ($\Delta = 0$) that is $I_s = n(I_m r_p - I_e x_p)/X_P$.

ANGLE SEEN BY THE WATTMETER

$$\theta_m = \varphi - \theta_{CT} + \theta_{PT}$$

φ is positive for a lagging load. θ_{CT} and θ_{PT} are positive when the reversed secondary phasor leads its primary. A leading CT phase angle makes a lagging load look closer to unity power factor.

TRUE POWER FROM THE WATTMETER READING

$$P = R_v R_i W \frac{\cos\varphi}{\cos\theta_m}, \qquad \text{correction factor} = \frac{\cos\varphi}{\cos\theta_m}$$

R_v and R_i are the actual PT and CT ratios at the operating point. The same factors multiply the energy registered by an energy meter. If ratio errors are neglected, replace $R_v R_i$ with $K_v K_i$.

ERROR OF THE NOMINAL-RATIO POWER (SMALL ERRORS)

$$\varepsilon_P \approx e_v + e_i + 100(\theta_{CT} - \theta_{PT})\tan\varphi \quad (\%)$$

$\varepsilon_P = (K_v K_i W - P)/P \times 100$. Here e_v and e_i are the PT and CT ratio errors in percent, and the phase angles are in radians. Valid while the ratio errors are small and $(\theta_{CT} - \theta_{PT})\tan\varphi \ll 1$; for a leading load φ, and so $\tan\varphi$, is negative. Because of the $\tan\varphi$ factor, phase-angle errors are negligible near unity power factor and large at low power factor.

Digital voltmeters, multimeters and energy meters

RESOLUTION OF AN N½-DIGIT DISPLAY

$$R = \frac{1}{10^N}$$

N is the number of full digits, so for 3½ digits N = 3 and R = 0.001. R is the smallest step as a fraction of the decade range. The half digit adds overrange, not resolution.

COUNTS OF AN N½-DIGIT DISPLAY

$$\text{counts} = 2 \times 10^N, \qquad \text{maximum display} = 2 \times 10^N - 1$$

3½ digit: 2000 counts, maximum display 1999. 4½ digit: 20000 counts, maximum 19999. A plain N-digit display has 10^N counts.

VALUE OF ONE COUNT (ONE 'DIGIT') ON A RANGE

$$V_{\text{count}} = V_{\text{decade range}} \times 10^{-N} = \frac{V_{\text{labelled range}}}{\text{counts}}$$

This is the place value of the last displayed digit. Use the first form when ranges are named by decade (1 V, 10 V, ...). Use the second when they are named by full count (2 V, 20 V, ... on a 2000-count meter).

SENSITIVITY OF A DVM

$$S = (\text{f.s.})_{\min} \times R$$

$(\text{f.s.})_{\min}$ is the full scale of the lowest range, taken at its decade value. For example, a range that reads up to 199.9 mV counts as 100 mV. S is the smallest change in input the meter can display at all, and it equals the place value of the last digit on the lowest range.

LIMITING ERROR FOR ±(A% OF READING + M COUNTS)

$$\Delta V = \pm\left(\frac{a}{100}\,V_{\text{rdg}} + m\,V_{\text{count}}\right)$$

V_{count} is the value of one count on the range in use. The two terms are added because this is a worst-case bound.

LIMITING ERROR AS A PERCENTAGE OF READING

$$\%\,\epsilon_{\text{rdg}} = \pm\left(a + \frac{100\,m\,V_{\text{count}}}{V_{\text{rdg}}}\right)$$

The count term is fixed in volts, so the percentage error rises as the reading falls.

SINGLE-SLOPE RAMP DVM

$$t = \frac{V_{in}}{S_r}, \qquad N_{\text{count}} = f_c\, t = \frac{f_c\, V_{in}}{S_r}$$

S_r is the ramp slope in V/s and f_c is the clock frequency. Both must be accurate and stable, because nothing cancels them and nothing averages out noise.

DUAL-SLOPE BALANCE

$$\frac{V_{in} T_1}{RC} = \frac{V_{ref} T_2}{RC} \;\Rightarrow\; V_{in} = V_{ref}\,\frac{T_2}{T_1} = V_{ref}\,\frac{N_2}{N_1}$$

Use magnitudes; V_{in} and V_{ref} have opposite polarity. For an input that varies during run-up, V_{in} here is its average over T_1. R, C and f_c cancel as long as they stay constant for one conversion, so the only component value the reading depends on is V_{ref}. The integrator, switches and comparator must still behave ideally (linear and free of offset), and the result is quantised to ±1 count.

PEAK INTEGRATOR OUTPUT

$$|v_o|_{\text{peak}} = \frac{V_{in}\, T_1}{RC}$$

Assumes a dc input and an integrator that starts from 0 V. At full-scale input this must stay inside the op-amp's output swing, which sets the smallest usable RC.

DUAL-SLOPE CONVERSION TIME

$$T_{\text{conv}} = T_1 + T_2 = T_1\left(1 + \frac{V_{in}}{V_{ref}}\right)$$

For a dc input. Add any auto-zero time on top. Integrating DVMs are slow compared with successive-approximation and flash converters, because T_1 is usually one or more periods of the power-line frequency.

NORMAL-MODE REJECTION OF AN INTEGRATING DVM

$$\text{NMRR} = \left|\frac{\pi f T_1}{\sin(\pi f T_1)}\right|, \qquad \text{NMRR}_{\text{dB}} = 20\log_{10}\left|\frac{\pi f T_1}{\sin(\pi f T_1)}\right|$$

Assumes a sinusoidal interference of frequency f added to the input, and a reading equal to the average of the input over T_1. NMRR is the ratio of the peak interference to the worst-case error it causes in that average. It is infinite (ideal rejection) when fT_1 is a whole number, that is, when T_1 is an integer multiple of the interference period.

ADC RESOLUTION (SIZE OF 1 LSB)

$$Q = 1\,\text{LSB} = \frac{V_{FS}}{2^n}$$

V_{FS} is the full-scale input range. Some treatments use $V_{FS}/(2^n - 1)$ because they define the top code as equal to V_{FS}; use whichever definition the question gives. An ideal rounding converter has quantisation error ±½ LSB.

SAR CONVERSION TIME

$$T_{\text{SAR}} = n\, T_{clk}$$

This is fixed and does not depend on V_{in}. Some designs add start or end-of-conversion clock cycles on top.

STAIRCASE (COUNTER) RAMP: WORST-CASE CONVERSION TIME

$$T_{\max} = (2^n - 1)\, T_{clk}$$

The conversion time varies with V_{in}. A full-scale input takes the longest.

FLASH ADC HARDWARE

$$\text{comparators} = 2^n - 1, \qquad \text{ladder resistors} = 2^n$$

All comparisons happen in parallel, so a conversion takes one clock period. An 8-bit flash converter needs 255 comparators.

LOADING ERROR ON THE DC VOLTAGE FUNCTION

$$V_{\text{rdg}} = V_{Th}\,\frac{R_{in}}{R_{in}+R_{Th}}, \qquad \%\,\text{loading error} = -\frac{R_{Th}}{R_{in}+R_{Th}} \times 100$$

The source is modelled as its Thevenin equivalent (V_{Th}, R_{Th}), and R_{in} is the DMM's input resistance. The error is taken relative to the true (unloaded) voltage V_{Th}. It is negative: the meter always reads low.

AVERAGE-RESPONDING METER, CALIBRATED IN RMS OF A SINE WAVE

$$V_{\text{ind}} = \frac{\pi}{2\sqrt{2}}\,\overline{|v|} \approx 1.11\,\overline{|v|}$$

$\overline{|v|}$ is the full-wave rectified average of the actual waveform. The indication equals the true rms only for a sinusoid, because 1.11 is the sine form factor. A half-wave average-responding meter instead multiplies the half-wave average by $\pi/\sqrt{2} \approx 2.22$; it too is correct only for a sine.

WAVEFORM ERROR OF AN AVERAGE-RESPONDING METER

$$\frac{V_{\text{ind}}}{V_{\text{rms}}} = \frac{1.11}{k_f}, \qquad k_f = \frac{V_{\text{rms}}}{\overline{|v|}}$$

V_{rms} is the true rms of the actual waveform, so this error is relative to the true value. k_f is 1.11 for a sine, 1 for a square wave and $2/\sqrt{3} \approx 1.155$ for a triangle wave. So a square wave reads 11.1% high and a triangle wave reads about 3.8% low.

TOTAL RMS OF A WAVEFORM WITH A DC COMPONENT

$$V_{\text{rms}} = \sqrt{V_{dc}^2 + V_{ac,\text{rms}}^2}$$

An ac-coupled true-rms meter shows only $V_{ac,\text{rms}}$. To get the total rms, combine it with the dc reading as shown.

ACTIVE POWER FROM SAMPLES

$$P = \frac{1}{N_s}\sum_{k=0}^{N_s-1} v[k]\,i[k]$$

The N_s samples must span a whole number of cycles; otherwise the reading ripples. It stays correct when harmonics are present, provided the sampling rate satisfies $f_s > 2f_{\max}$.

ENERGY ACCUMULATION

$$E = T_s \sum_{k} v[k]\,i[k]$$

$T_s = 1/f_s$ is the sampling interval. Energy is the running sum of instantaneous power × time.

METER CONSTANT: PULSES AND ENERGY PER PULSE

$$n_p = C \times E_{\text{kWh}}, \qquad \text{energy per pulse} = \frac{1}{C}\,\text{kWh} = \frac{3.6\times10^6}{C}\,\text{J}$$

C is the meter constant in impulses per kWh (imp/kWh). For example, C = 3200 imp/kWh gives 0.3125 Wh (1125 J) per pulse.

PULSE RATE AT CONSTANT POWER

$$f_p = \frac{C\,P_{\text{kW}}}{3600}\ \text{pulses/s}$$

P is the active power in kW and C is in imp/kWh.

PERCENTAGE ERROR OF AN ENERGY METER

$$\%\,\epsilon = \frac{E_{\text{meter}} - E_{\text{true}}}{E_{\text{true}}} \times 100 = \frac{n_{\text{counted}} - n_{\text{expected}}}{n_{\text{expected}}} \times 100$$

Error is taken relative to the true energy. It is positive when the meter over-registers. $n_{\text{expected}} = C \times E_{\text{true}}$ in kWh.

Time, phase and frequency measurement, oscilloscopes and spectrum analysers

FREQUENCY MODE

$$f_x = \frac{N}{T_g}, \qquad \text{resolution } \Delta f = \frac{1}{T_g}$$

N = input cycles counted in the gate time T_g. A 1 s gate reads to 1 Hz and a 10 s gate to 0.1 Hz.

PERIOD MODE (MULTIPLE-PERIOD AVERAGE)

$$T_x = \frac{N}{M f_c}$$

f_c = clock (time-base) frequency and M = number of input periods the gate stays open for ($M = 1$ for a single period). Averaging makes the resolution $1/(M f_c)$.

GATING ERROR (ANY MODE)

$$\varepsilon_g = \pm\frac{1}{N}$$

As a fraction of the reading. The error is ±1 count whatever the input.

GATING ERROR IN FREQUENCY MODE

$$\varepsilon_g = \pm\frac{1}{f_x T_g}$$

Large at low frequencies. A 50 Hz input with a 1 s gate gives $N = 50$, which is an error of ±2%.

GATING ERROR IN PERIOD MODE

$$\varepsilon_g = \pm\frac{1}{M f_c T_x} = \pm\frac{f_x}{M f_c}$$

Small at low frequencies. It grows with the input frequency.

TOTAL LIMITING ERROR

$$\varepsilon = \pm\left(\frac{1}{N} + \left|\frac{\Delta f_c}{f_c}\right|\right)$$

The worst-case (guaranteed) bound as a fraction of the reading. In period and time-interval modes, add the trigger error.

CROSSOVER (MODE-CHANGEOVER) FREQUENCY

$$f_{co} = \sqrt{\frac{M f_c}{T_g}} \qquad (M = 1:\ f_{co} = \sqrt{f_c/T_g})$$

The frequency at which the ±1-count error is the same in both modes, found by setting $1/(f T_g) = f/(M f_c)$. Below f_{co}, measure the period and invert it. Above it, measure frequency directly.

ELECTROSTATIC DEFLECTION ON THE SCREEN

$$D = \frac{L\, l_d\, V_d}{2\, d\, V_a}$$

L is measured from the centre of the plates to the screen. Assumes a uniform field between the plates (no fringing), a small deflection angle and a beam that clears the plates. e/m cancels, so the deflection does not depend on it.

DEFLECTION SENSITIVITY AND DEFLECTION FACTOR

$$S = \frac{D}{V_d} = \frac{L\, l_d}{2\, d\, V_a}\ (\mathrm{m/V}), \qquad G = \frac{1}{S} = \frac{2\, d\, V_a}{L\, l_d}\ (\mathrm{V/m})$$

Convention used here: sensitivity is deflection per volt, and the deflection factor is volts per unit deflection (the V/div knob sets a deflection factor). Doubling V_a halves the sensitivity.

READING THE GRATICULE

$$V_{pp} = n_v \times \frac{\text{V}}{\text{div}} \times k_{probe}, \qquad T = n_h \times \frac{\text{time}}{\text{div}}, \qquad f = \frac{1}{T}$$

n_v and n_h are numbers of divisions. k_{probe} = probe attenuation (10 for a 10:1 probe, unless the scope already corrects for it). For a sine, $V_{rms} = V_{pp}/(2\sqrt{2})$.

PHASE FROM TIME DELAY

$$\phi = 360^\circ \times \frac{\Delta t}{T} = 360^\circ f\,\Delta t$$

Δt is the delay between corresponding points and T the common period. On a CRO, $\Delta t/T$ = (divisions of delay)/(divisions per cycle), so the time/div setting cancels.

FLIP-FLOP (SET–RESET) PHASE DETECTOR

$$V_{avg} = V_H \frac{\phi}{360^\circ}, \qquad 0^\circ \le \phi < 360^\circ$$

The output switches between 0 and V_H. The positive-going zero crossing of the reference sets the flip-flop and the positive-going zero crossing of the other signal resets it, so the duty cycle is $\phi/360^\circ$, where ϕ is the lag of the other signal behind the reference. The reading is unambiguous over the full circle: a 30° lag reads 30° and a 30° lead reads 330°.

XOR PHASE DETECTOR

$$V_{avg} = V_H \frac{\phi}{180^\circ}, \qquad 0^\circ \le \phi \le 180^\circ$$

The output switches between 0 and V_H. Both inputs must be square waves of 50% duty cycle (sinusoids are squared first by comparators). The output is high whenever the inputs differ, which happens twice per cycle. For $180^\circ \le \phi \le 360^\circ$ the output is $V_H(360^\circ - \phi)/180^\circ$, so the detector cannot tell a lag of ϕ from a lead of ϕ.

LISSAJOUS FREQUENCY RATIO

$$\frac{f_y}{f_x} = \frac{n_h}{n_v}$$

f_y is on the Y (vertical) input and f_x on the X (horizontal) input. n_h = points touching a horizontal tangent; n_v = points touching a vertical tangent. For an open figure whose ends are retraced, each free end counts as ½.

LISSAJOUS PHASE (SAME FREQUENCY)

$$\sin\phi = \frac{y_0}{y_m} = \frac{x_0}{x_m}$$

y_0 = the distance from the centre to the point where the ellipse cuts the Y axis (half the chord on the Y axis); y_m = the maximum vertical deflection. The ratio of full chord to full height is the same. Centre the figure first, and assume the X and Y amplifiers add equal phase shifts. This gives the size of φ only, not whether Y leads or lags, and $\sin\phi$ alone cannot separate φ from $180^\circ - \phi$: the quadrants of the major axis decide.

PROBE COMPENSATION CONDITION

$$R_p C_p = R_s C_s$$

R_p and C_p = the probe's series resistance and its trimmer capacitor; R_s and C_s = the scope's input resistance and total shunt capacitance (input plus cable). When compensated, the attenuation $R_s/(R_p + R_s)$ holds at every frequency.

COMPENSATED K:1 PROBE AS SEEN BY THE CIRCUIT

$$R_{in} = R_p + R_s = kR_s, \qquad C_{in} = \frac{C_p C_s}{C_p + C_s} = \frac{C_s}{k}$$

Loading falls by the attenuation factor. For a 10:1 probe, the circuit sees ten times the resistance and one tenth of the capacitance of the bare scope input. With a source resistance $R_{src} \ll R_{in}$, C_{in} forms a low-pass filter that adds a rise time of about $2.2\,R_{src}C_{in}$.

RISE TIME AND BANDWIDTH

$$t_r = \frac{0.35}{BW}$$

10%–90% rise time of a single-pole (first-order) response, from $t_r = 2.2\tau$ and $BW = 1/(2\pi\tau)$. Use 0.35 unless the question gives another constant.

CASCADED RISE TIMES

$$t_{r,\,disp} = \sqrt{t_{r,\,sig}^2 + t_{r,\,scope}^2 + t_{r,\,probe}^2}$$

Root-sum-square rule for cascaded stages with little or no overshoot. It is an approximation (exact for Gaussian responses). To find the true rise time of the signal, subtract the squares.

SAMPLING THEOREM

$$f_s > 2f_{max}$$

For a signal with no components above f_{max}. The inequality is strict: at exactly $2f$, every sample can fall on a zero crossing. In practice a real-time DSO samples well above $2f_{max}$ so that the reconstructed trace is accurate; a common rule of thumb is at least 2.5 samples per cycle of the highest frequency with $\sin x/x$ interpolation and about 10 with linear interpolation.

ALIAS FREQUENCY

$$f_a = \left|f_{in} - k f_s\right|, \quad k = \text{integer nearest to } f_{in}/f_s, \quad 0 \le f_a \le \frac{f_s}{2}$$

Any input above $f_s/2$ that is still inside the analogue bandwidth (with no anti-aliasing filter) is displayed at f_a. For example, a 1.02 MHz sine sampled at 1 MS/s appears as a 20 kHz sine.

RECORD LENGTH, TIME WINDOW AND SAMPLE RATE

$$T_{window} = \frac{N}{f_s} = 10 \times \frac{\text{time}}{\text{div}} \;\Rightarrow\; f_s = \frac{N}{10 \times (\text{time/div})} \le f_{s,max}$$

For a 10-division screen filled by the record. With a fixed memory, a slower time base forces a lower sample rate, which is how aliasing appears at slow sweeps.

VERTICAL RESOLUTION

$$\Delta V = \frac{V_{FS}}{2^n}$$

For an n-bit ADC, taking one step as $V_{FS}/2^n$ (some texts divide by $2^n - 1$; the difference is negligible for large n). An 8-bit ADC gives 256 levels, so one step on a 10 V full-scale range is 39 mV.

FFT FREQUENCY RESOLUTION

$$\Delta f = \frac{f_s}{N} = \frac{1}{T_{window}}$$

The bin spacing of an N-point FFT, which covers 0 to $f_s/2$.

SUPERHETERODYNE TUNING

$$f_{in} = f_{LO} - f_{IF}, \qquad f_{image} = f_{in} + 2f_{IF}$$

For an LO above the input (high-side LO). The image frequency also mixes down to f_{IF}; an input filter or a high first IF keeps it out.

RESOLUTION BANDWIDTH AND NOISE FLOOR

$$\Delta L_{noise} = 10 \log_{10} \frac{RBW_2}{RBW_1} \text{ dB}$$

Change in the displayed noise level when the RBW is changed from RBW_1 to RBW_2 (negative means lower). Displayed noise power is proportional to the RBW, so a 10× narrower RBW lowers the noise floor by 10 dB, while a sinusoid's displayed level stays the same, so small tones emerge from the noise. Two equal tones closer together than about one RBW merge into a single peak.

POWER IN DBM

$$P_{dBm} = 10 \log_{10} \frac{P}{1 \text{ mW}}, \qquad 0 \text{ dBm} \leftrightarrow 0.224 \text{ V rms in } 50\ \Omega$$

Levels are shown relative to 1 mW. Convert to voltage with the system impedance, usually 50 Ω.

HARMONIC DISTORTION FROM A SPECTRUM

$$THD = \frac{\sqrt{V_2^2 + V_3^2 + \cdots}}{V_1}, \qquad \frac{V_n}{V_1} = 10^{(L_n - L_1)/20}$$

V_n = rms value of the nth harmonic; L_n = its level in dB read off the display. This THD is relative to the fundamental unless it is stated as a percentage of the total rms.

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