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GATE CS

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Discrete Mathematics Formula Sheet

INSIDE

- 108 formulas in 6 topics
- When to use each one
- Taken from the free topic notes on iaeacademy.in

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Propositional and first-order logic

IMPLICATION AS A DISJUNCTION

$$p \to q \;\equiv\; \neg p \lor q$$

Use it to remove $\to$ before any manipulation; it holds for all truth values.

BICONDITIONAL

$$p \leftrightarrow q \;\equiv\; (p \to q) \land (q \to p) \;\equiv\; (p \land q) \lor (\neg p \land \neg q)$$

True exactly when p and q have the same truth value; $p \oplus q \equiv \neg(p \leftrightarrow q)$.

COUNTING TRUTH TABLES

$$\text{rows in a truth table} = 2^n, \qquad \text{distinct Boolean functions of } n \text{ variables} = 2^{2^n}$$

For n distinct atoms; the second count is the number of different possible output columns.

IDENTITY, DOMINATION AND COMPLEMENT LAWS

$$p \land \mathrm{T} \equiv p, \quad p \lor \mathrm{F} \equiv p, \quad p \lor \mathrm{T} \equiv \mathrm{T}, \quad p \land \mathrm{F} \equiv \mathrm{F}, \quad p \lor \neg p \equiv \mathrm{T}, \quad p \land \neg p \equiv \mathrm{F}$$

Here T and F stand for the constants true and false.

DE MORGAN'S LAWS

$$\neg(p \land q) \equiv \neg p \lor \neg q, \qquad \neg(p \lor q) \equiv \neg p \land \neg q$$

Negation swaps $\land$ and $\lor$ and negates each part.

DISTRIBUTIVE LAWS

$$p \land (q \lor r) \equiv (p \land q) \lor (p \land r), \qquad p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$$

Each of $\land$ and $\lor$ distributes over the other.

ABSORPTION AND DOUBLE NEGATION

$$p \lor (p \land q) \equiv p, \qquad p \land (p \lor q) \equiv p, \qquad \neg\neg p \equiv p$$

Useful for quickly simplifying a formula.

NEGATION OF AN IMPLICATION

$$\neg(p \to q) \equiv p \land \neg q$$

An implication fails only when the hypothesis holds and the conclusion fails.

MOVING IMPLICATIONS AROUND

$$(p \land q) \to r \equiv p \to (q \to r), \quad (p \to q) \land (p \to r) \equiv p \to (q \land r), \quad (p \to r) \land (q \to r) \equiv (p \lor q) \to r$$

Exportation and the two combining rules; note the hypotheses combine with $\lor$ when the conclusion is shared.

COMBINING IMPLICATIONS WITH OR

$$(p \to q) \lor (p \to r) \equiv p \to (q \lor r), \qquad (p \to r) \lor (q \to r) \equiv (p \land q) \to r$$

Rewrite each side with $\neg a \lor b$ to check them. With a shared conclusion and $\lor$ between the implications, the hypotheses combine with $\land$.

TEST FOR A VALID ARGUMENT

$$\text{valid} \iff (P_1 \land \cdots \land P_n) \to C \text{ is a tautology} \iff P_1 \land \cdots \land P_n \land \neg C \text{ is unsatisfiable}$$

To refute an argument, find one assignment making every premise true and the conclusion false.

NEGATING QUANTIFIERS

$$\neg\forall x\, P(x) \equiv \exists x\, \neg P(x), \qquad \neg\exists x\, P(x) \equiv \forall x\, \neg P(x)$$

Push $\neg$ inside by flipping each quantifier and negating the body; repeat for nested quantifiers.

QUANTIFIERS THAT DISTRIBUTE

$$\forall x\,(P(x) \wedge Q(x)) \equiv \forall x\,P(x) \wedge \forall x\,Q(x), \qquad \exists x\,(P(x) \vee Q(x)) \equiv \exists x\,P(x) \vee \exists x\,Q(x)$$

$\forall$ distributes over $\wedge$ and $\exists$ distributes over $\vee$, as equivalences.

PULLING A QUANTIFIER PAST A FORMULA THAT DOES NOT MENTION X

$$\text{if } x \text{ is not free in } \psi: \quad \forall x\,(P(x) \vee \psi) \equiv \forall x\,P(x) \vee \psi, \qquad \exists x\,(P(x) \wedge \psi) \equiv \exists x\,P(x) \wedge \psi$$

The same holds for $\forall$ with $\wedge$ and $\exists$ with $\vee$. The step $\exists x\,\psi \equiv \psi$ (or $\forall x\,\psi \equiv \psi$) used here relies on the domain being non-empty.

MOVING A QUANTIFIER OUT OF AN ANTECEDENT

$$\text{if } x \text{ is not free in } \psi: \quad \forall x\,(P(x) \to \psi) \equiv \exists x\,P(x) \to \psi, \qquad \exists x\,(P(x) \to \psi) \equiv \forall x\,P(x) \to \psi$$

The quantifier flips when it leaves the antecedent. It keeps its kind when it leaves the consequent: $\forall x\,(\psi \to P(x)) \equiv \psi \to \forall x\,P(x)$ and $\exists x\,(\psi \to P(x)) \equiv \psi \to \exists x\,P(x)$. The $\exists$ forms need a non-empty domain.

EXISTENTIAL OVER AN IMPLICATION

$$\exists x\,(P(x) \to Q(x)) \equiv \forall x\,P(x) \to \exists x\,Q(x)$$

Write the body as $\neg P(x) \vee Q(x)$, distribute $\exists$ over $\vee$, and use $\exists x\,\neg P(x) \equiv \neg\forall x\,P(x)$. By contrast, $\forall x\,(P(x) \to Q(x))$ only implies $\forall x\,P(x) \to \forall x\,Q(x)$.

Sets, relations, functions, partial orders and lattices

SIZE OF THE POWER SET

$$|\mathcal{P}(A)| = 2^{|A|}$$

For a finite set A; each element is either in or out of a subset.

SIZE OF A CARTESIAN PRODUCT

$$|A \times B| = |A|\,|B|$$

Finite sets. A relation from A to B is any subset of A × B, so there are $2^{|A||B|}$ of them.

INCLUSION-EXCLUSION FOR TWO SETS

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Finite sets; this is the basis of Venn-diagram counting.

INCLUSION-EXCLUSION FOR THREE SETS

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Finite sets; add singles, subtract pairs, add the triple.

SIZE OF THE SYMMETRIC DIFFERENCE

$$|A \oplus B| = |A| + |B| - 2\,|A \cap B|$$

Finite sets.

ALL RELATIONS ON AN N-ELEMENT SET

$$2^{n^2}$$

Each of the n^2 ordered pairs is in or out.

REFLEXIVE RELATIONS

$$2^{n(n-1)}$$

The n diagonal pairs are forced; the other $n(n-1)$ are free. Irreflexive relations have the same count.

NEITHER REFLEXIVE NOR IRREFLEXIVE

$$2^{n^2} - 2 \cdot 2^{n(n-1)}$$

For $n \geq 1$, where no relation is both reflexive and irreflexive: subtract the reflexive and the irreflexive relations from all relations.

SYMMETRIC RELATIONS

$$2^{n(n+1)/2}$$

Choose freely for each of the n diagonal pairs and each of the $\binom{n}{2}$ unordered pairs $\{a, b\}$ with $a \neq b$.

ANTISYMMETRIC RELATIONS

$$2^n \cdot 3^{n(n-1)/2}$$

Diagonal pairs are free (2^n); each unordered pair $\{a, b\}$ with $a \neq b$ has 3 choices: none, (a, b) only, (b, a) only.

ASYMMETRIC RELATIONS

$$3^{n(n-1)/2}$$

Diagonal pairs are excluded; each unordered pair has the same 3 choices. Reflexive and antisymmetric relations have the same count (diagonal forced in instead of out).

REFLEXIVE AND SYMMETRIC RELATIONS

$$2^{n(n-1)/2}$$

Diagonal forced to 1; each unordered pair with $a \neq b$ is in or out. Irreflexive and symmetric relations have the same count.

RELATIONS BOTH SYMMETRIC AND ANTISYMMETRIC

$$2^n$$

Such a relation is a subset of the identity relation, so only the diagonal pairs are free.

EQUIVALENCE RELATIONS ON N ELEMENTS

$$B_n = \sum_{k=1}^{n} S(n, k), \qquad B_0, B_1, B_2, B_3, B_4, B_5 = 1, 1, 2, 5, 15, 52$$

B_n (Bell number) counts partitions of an n-element set; $S(n, k)$ (Stirling number of the second kind) counts partitions into exactly k non-empty blocks.

RECURRENCES FOR BELL AND STIRLING NUMBERS

$$B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} B_k, \qquad S(n, k) = k\, S(n-1, k) + S(n-1, k-1)$$

With $S(n, 1) = S(n, n) = 1$ for $n \geq 1$, and $S(n, n-1) = \binom{n}{2}$.

PARTITIONS INTO TWO BLOCKS

$$S(n, 2) = 2^{n-1} - 1$$

For $n \geq 2$: the number of equivalence relations on n elements with exactly two classes.

ALL FUNCTIONS FROM A TO B

$$|B|^{|A|}$$

Each element of A independently picks an image.

INJECTIVE FUNCTIONS

$$\frac{n!}{(n-m)!} = n(n-1)\cdots(n-m+1)$$

$m = |A|$, $n = |B|$, $m \leq n$; when $m > n$ there are none (the product form gives 0).

BIJECTIONS BETWEEN SETS OF EQUAL SIZE

$$n!$$

$|A| = |B| = n$; this is also the number of permutations of an n-set.

SURJECTIVE FUNCTIONS (INCLUSION-EXCLUSION)

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} (n-k)^m = n!\, S(m,n)$$

$m = |A|$, $n = |B|$; zero when $m < n$.

PARTIAL FUNCTIONS FROM A TO B

$$(n+1)^m$$

$m = |A|$, $n = |B|$; each element of A maps to one of the n elements or is left undefined.

SUBSETS OF AN N-SET ORDERED BY INCLUSION

$$\text{Hasse diagram edges} = n\, 2^{n-1}$$

Poset $(\mathcal{P}(A), \subseteq)$ with $|A| = n$: S is covered by $S \cup \{x\}$ for each $x \notin S$.

DIVISIBILITY ON THE DIVISORS OF N

$$N = p_1^{a_1} \cdots p_k^{a_k} \;\Rightarrow\; \text{number of divisors} = \prod_{i=1}^{k} (a_i + 1)$$

Poset of positive divisors under 'divides' (p_i distinct primes); join is lcm and meet is gcd.

HASSE DIAGRAM EDGES FOR THE DIVISORS OF N

$$\sum_{i=1}^{k} a_i \prod_{j \neq i} (a_j + 1)$$

$N = p_1^{a_1} \cdots p_k^{a_k}$; d is covered by $d\,p_i$ whenever $d\,p_i$ divides N. For $N = 12 = 2^2 \cdot 3$: $2 \cdot 2 + 1 \cdot 3 = 7$.

Monoids and groups

NUMBER OF BINARY OPERATIONS ON AN N-ELEMENT SET

$$n^{n^2} \text{ in all}, \qquad n^{n(n+1)/2} \text{ commutative}$$

A binary operation on a set of n elements is a filled $n \times n$ table, each of its n^2 entries chosen from the n elements. A commutative one is fixed by the $n(n+1)/2$ entries on and above the main diagonal. For $n = 2$: 16 in all, 8 commutative.

INVERSE OF A PRODUCT

$$(a * b)^{-1} = b^{-1} * a^{-1}$$

Holds in every group; note the reversed order. If the group is abelian the order does not matter. Also $(a^{-1})^{-1} = a$.

LAGRANGE'S THEOREM

$$|H| \text{ divides } |G|, \qquad |G| = [G : H] \cdot |H|$$

For a finite group G and any subgroup H; $[G : H]$ is the number of distinct left cosets of H in G.

ORDER OF AN ELEMENT DIVIDES THE GROUP ORDER

$$o(a) \text{ divides } |G|, \qquad a^{|G|} = e$$

In every finite group, by Lagrange applied to $\langle a \rangle$. Also $a^k = e \iff o(a) \mid k$.

ORDER OF A POWER

$$o(a^k) = \frac{n}{\gcd(n,k)} \quad \text{when } o(a) = n$$

Holds in any group. In $\mathbb{Z}_n$ the element k has order $n/\gcd(n,k)$. If $G = \langle a \rangle$ has order n, the element a^k generates G exactly when $\gcd(n,k) = 1$.

NUMBER OF GENERATORS OF A CYCLIC GROUP

$$\#\{\text{generators}\} = \phi(n)$$

For a cyclic group of order n, where $\phi(n)$ is Euler's totient (the count of k with $1 \le k \le n$ and $\gcd(k,n) = 1$).

EULER'S TOTIENT

$$\phi(n) = n \prod_{p \mid n} \left(1 - \frac{1}{p}\right)$$

The product runs over the distinct primes p dividing n. For a prime p, $\phi(p) = p - 1$ and $\phi(p^k) = p^k - p^{k-1}$.

SUBGROUPS OF A CYCLIC GROUP

$$\text{one subgroup of each order } d \mid n, \text{ containing } \phi(d) \text{ generators}$$

A cyclic group of order n has exactly one subgroup for each positive divisor d of n, so the number of subgroups equals the number of divisors of n. Every subgroup of a cyclic group is cyclic, and the group has exactly $\phi(d)$ elements of order d for each $d \mid n$.

ORDER IN A DIRECT PRODUCT

$$o\big((a,b)\big) = \operatorname{lcm}\big(o(a), o(b)\big)$$

In $G \times H$ with finite $o(a)$ and $o(b)$. Consequently $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic if and only if $\gcd(m,n) = 1$.

GROUPS OF PRIME ORDER

$$|G| = p \text{ prime } \Rightarrow\ G \cong \mathbb{Z}_p$$

Any non-identity element has order dividing p, so order p, hence generates G. Every non-identity element is a generator, giving $p - 1$ generators.

Graph theory: connectivity, matching and colouring

HANDSHAKING LEMMA

$$\sum_{v} \deg(v) = 2m$$

Holds for every undirected graph (loops count 2). So the number of odd-degree vertices is always even.

MOST EDGES IN A SIMPLE GRAPH

$$m_{\max} = \binom{n}{2} = \frac{n(n-1)}{2}$$

Attained exactly by the complete graph K_n. A k-regular simple graph on n vertices has $m = nk/2$ edges, so it can exist only if nk is even and $0 \le k \le n-1$.

EDGES NEEDED FOR K COMPONENTS

$$m \ge n - k$$

A graph with n vertices and k components has at least $n - k$ edges; so a connected graph has at least $n - 1$ edges. Equality holds exactly for forests.

MOST EDGES IN A DISCONNECTED SIMPLE GRAPH

$$m_{\max}(\text{disconnected}) = \frac{(n-1)(n-2)}{2}$$

Simple graph on $n \geq 2$ vertices with at least 2 components. Attained by K_{n-1} plus one isolated vertex. With exactly k components the maximum is $\binom{n-k+1}{2}$.

VERTEX AND EDGE CONNECTIVITY

$$\kappa(G) \leq \lambda(G) \leq \delta(G)$$

The **vertex connectivity** $\kappa(G)$ is the fewest vertices whose removal disconnects G or leaves a single vertex; the **edge connectivity** $\lambda(G)$ is the fewest edges whose removal disconnects G. $\kappa(K_n) = \lambda(K_n) = n-1$. A connected graph on $n \geq 3$ vertices has a cut vertex exactly when $\kappa = 1$, and a connected graph on $n \geq 2$ vertices has a bridge exactly when $\lambda = 1$.

LEAVES OF A TREE

$$\#\text{leaves} = 2 + \sum_{\deg(v) \geq 3} (\deg(v) - 2)$$

For a tree with $n \geq 2$ vertices. In particular every such tree has at least 2 leaves (vertices of degree 1).

CAYLEY'S FORMULA

$$\tau(K_n) = n^{n-2}$$

Number of labelled trees on n vertices, which equals the number of spanning trees of K_n. Also $\tau(K_{p,q}) = p^{q-1}q^{p-1}$ and $\tau(C_n) = n$.

COMPLEMENT EDGE COUNT

$$m(G) + m(\overline{G}) = \binom{n}{2}$$

For a simple graph G, the complement $\overline{G}$ has the same vertices and exactly the non-edges of G. Degrees satisfy $\deg_{\overline{G}}(v) = n - 1 - \deg_G(v)$.

SELF-COMPLEMENTARY GRAPHS

$$G \cong \overline{G} \Rightarrow m = \frac{n(n-1)}{4}, \quad n \equiv 0 \text{ or } 1 \pmod 4$$

Necessary conditions on the order and size of a simple self-complementary graph. Examples: P_4 (4 vertices, 3 edges) and C_5 (5 vertices, 5 edges).

INDEPENDENT SET AND VERTEX COVER

$$\alpha(G) + \beta(G) = n$$

Holds for every simple (loopless) graph: a set is independent exactly when its complement is a vertex cover.

KÖNIG'S THEOREM

$$\nu(G) \leq \beta(G) \quad \text{and} \quad \nu(G) = \beta(G) \text{ if } G \text{ is bipartite}$$

Every vertex cover needs a distinct vertex for each matching edge, so $\nu \leq \beta$ always; equality is guaranteed only for bipartite graphs. (The triangle has $\nu = 1$, $\beta = 2$.)

HALL'S CONDITION

$$|N(S)| \geq |S| \text{ for all } S \subseteq X$$

A bipartite graph with parts X, Y has a matching covering every vertex of X if and only if this holds, where $N(S)$ is the set of neighbours of S. When $|X| = |Y|$, this is the condition for a perfect matching.

GREEDY COLOURING BOUND

$$\chi(G) \leq \Delta(G) + 1$$

For every graph, where Δ is the maximum degree. By Brooks' theorem, a connected graph that is neither a complete graph nor an odd cycle has $\chi(G) \leq \Delta(G)$.

CHROMATIC POLYNOMIALS

$$P(K_n, k) = k(k-1)\cdots(k-n+1)$$
$$P(T, k) = k(k-1)^{n-1}$$
$$P(C_n, k) = (k-1)^n + (-1)^n(k-1)$$

$P(G, k)$ is the number of proper colourings of G using at most k colours, with the colours and vertices labelled. T is any tree on n vertices; C_n needs $n \geq 3$. Check: $P(C_3, k) = k(k-1)(k-2) = P(K_3, k)$. $\chi(G)$ is the least k with $P(G, k) > 0$.

EDGE COLOURING

$$\Delta(G) \leq \chi'(G) \leq \Delta(G) + 1$$

The **chromatic index** $\chi'(G)$ is the fewest colours that give edges sharing a vertex different colours. The bounds hold for every simple graph (Vizing). A bipartite graph has $\chi' = \Delta$. $\chi'(K_n) = n - 1$ for even n and n for odd n.

EULER'S FORMULA

$$V - E + F = 2$$

For a connected planar graph (drawn without crossings). With k components the right side is $k + 1$.

EDGE BOUND FOR SIMPLE PLANAR GRAPHS

$$E \leq 3V - 6$$

For a simple planar graph with $V \geq 3$. If the graph also has no triangle (for example, it is bipartite), $E \leq 2V - 4$. Reason: in a simple connected planar graph with $V \geq 3$, every face has degree at least 3 (at least 4 with no triangle), and $\sum_f \deg(f) = 2E$. More generally, if every cycle has length at least g, then $E \leq \frac{g}{g-2}(V-2)$.

DIRAC'S THEOREM

$$\delta(G) \geq \frac{n}{2} \Rightarrow G \text{ Hamiltonian}$$

Simple graph with $n \geq 3$. Ore's generalisation: if $\deg(u) + \deg(v) \geq n$ for every pair of non-adjacent u, v, then G is Hamiltonian. Both are sufficient only.

HAMILTONIAN CYCLES IN K_N

$$\#\text{Hamiltonian cycles of } K_n = \frac{(n-1)!}{2}$$

For $n \geq 3$, labelled vertices, counting a cycle and its reverse as the same. $K_{p,q}$ is Hamiltonian exactly when $p = q \geq 2$, and $K_{n,n}$ ($n \geq 2$) has $\frac{n!\,(n-1)!}{2}$ Hamiltonian cycles (1 for $K_{2,2}$, 6 for $K_{3,3}$).

POWERS OF THE ADJACENCY MATRIX

$$(A^k)_{ij} = \text{number of walks of length } k \text{ from } i \text{ to } j$$

For the adjacency matrix A of a graph (directed or undirected; in a multigraph A_{ij} counts parallel edges, and a loop is entered as 1 in A_{ii}). Walks may repeat vertices.

Combinatorics: counting

PERMUTATIONS OF R FROM N DISTINCT OBJECTS

$$P(n,r) = \frac{n!}{(n-r)!}$$

Ordered arrangements, no repetition, $0 \le r \le n$. Also the number of injective functions from an r-set to an n-set.

COMBINATIONS

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \binom{n}{n-r}$$

Unordered selections of r from n distinct objects, no repetition, $0 \le r \le n$. Symmetry: choosing the ones taken is the same as choosing the ones left.

SELECTIONS WITH REPETITION

$$\binom{n+r-1}{r}$$

Unordered choice of r items from n kinds ($n \ge 1$) when each kind may be taken any number of times (multisets of size r).

CIRCULAR ARRANGEMENTS

$$(n-1)!$$

Seating $n \ge 1$ distinct people round a table when rotations count as the same arrangement (reflections counted as different). If reflections are also the same and $n \ge 3$, divide by 2: $(n-1)!/2$.

ARRANGEMENTS WITH IDENTICAL ITEMS (MULTINOMIAL COEFFICIENT)

$$\frac{n!}{n_1!\,n_2!\cdots n_k!}$$

Distinct arrangements of n objects of which n_i are alike of type i, $\sum n_i = n$. It is also the number of ways to split n distinct objects into labelled groups of sizes $n_1, \dots, n_k$.

STARS AND BARS: NON-NEGATIVE SOLUTIONS

$$x_1 + x_2 + \cdots + x_k = n,\ x_i \ge 0 \ \Rightarrow\ \binom{n+k-1}{k-1}$$

Number of integer solutions ($k \ge 1$); equals the number of ways to put n identical objects into k distinct bins, empty bins allowed.

STARS AND BARS: POSITIVE SOLUTIONS

$$x_1 + x_2 + \cdots + x_k = n,\ x_i \ge 1 \ \Rightarrow\ \binom{n-1}{k-1}$$

Every bin non-empty ($n, k \ge 1$). For a lower bound $x_i \ge a_i$, substitute $y_i = x_i - a_i$ and reduce n by $\sum a_i$.

STARS AND BARS WITH AN INEQUALITY

$$x_1 + x_2 + \cdots + x_k \le n,\ x_i \ge 0 \ \Rightarrow\ \binom{n+k}{k}$$

Add a slack variable $x_{k+1} = n - (x_1 + \cdots + x_k) \ge 0$; the equation now has $k+1$ unknowns summing to n.

STIRLING NUMBERS OF THE SECOND KIND

$$S(n,k) = k\,S(n-1,k) + S(n-1,k-1), \quad S(n,n) = 1,\ S(n,1) = 1\ (n \ge 1)$$

$S(n,k)$ = number of ways to split a set of n distinct objects into k non-empty unlabelled blocks. Conventions: $S(0,0) = 1$, $S(n,0) = 0$ for $n \ge 1$, $S(n,k) = 0$ for $k > n$. Also $S(n,2) = 2^{n-1} - 1$. Example values: $S(4,2) = 7$, $S(5,2) = 15$, $S(5,3) = 25$.

PARTITIONS OF AN INTEGER INTO EXACTLY K PARTS

$$p(n,k) = p(n-1,k-1) + p(n-k,k)$$

Number of ways to write n as a sum of exactly k positive integers with order ignored. Boundary values: $p(0,0) = 1$; $p(n,k) = 0$ if $k > n$, or if $n \geq 1$ and $k \leq 0$. $p(n)$ is the total over all k; $p(1), \ldots, p(7) = 1, 2, 3, 5, 7, 11, 15$.

BINOMIAL THEOREM

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

For a non-negative integer n. The coefficient of $x^{n-k}y^k$ is $\binom{n}{k}$; there are $n+1$ terms. In $(x_1 + \cdots + x_k)^n$ the coefficient of $x_1^{n_1} \cdots x_k^{n_k}$, where $n_1 + \cdots + n_k = n$, is $\frac{n!}{n_1! \cdots n_k!}$.

PASCAL'S IDENTITY

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

For $n \geq 1$ and $1 \leq k \leq n$. Split the k-subsets of an n-set by whether they contain a chosen element.

SUM OF A ROW

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n, \qquad \sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0 \ (n \geq 1)$$

Total number of subsets of an n-set; from $x = y = 1$ and $x = 1, y = -1$. Hence even-size subsets and odd-size subsets each number 2^{n-1} for $n \geq 1$.

WEIGHTED ROW SUM

$$\sum_{k=0}^{n} k \binom{n}{k} = n\, 2^{n-1}$$

Follows from $k\binom{n}{k} = n\binom{n-1}{k-1}$ (choose a committee, then its leader).

VANDERMONDE'S IDENTITY

$$\sum_{k=0}^{r} \binom{m}{k} \binom{n}{r-k} = \binom{m+n}{r}$$

Choose r from two groups of sizes m and n by how many come from the first (terms with $k > m$ or $r - k > n$ are 0). The case $m = n = r$ gives $\sum_k \binom{n}{k}^2 = \binom{2n}{n}$.

HOCKEY-STICK IDENTITY

$$\sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$$

For $0 \leq r \leq n$. Sum down a diagonal of Pascal's triangle.

INCLUSION-EXCLUSION

$$\left| \bigcup_{i=1}^{m} A_i \right| = \sum_{i} |A_i| - \sum_{i<j} |A_i \cap A_j| + \sum_{i<j<l} |A_i \cap A_j \cap A_l| - \cdots + (-1)^{m+1} |A_1 \cap \cdots \cap A_m|$$

Counts elements with at least one of m properties; for two sets, $|A \cup B| = |A| + |B| - |A \cap B|$. The number with none of the properties is $|U|$ minus this value.

ONTO FUNCTIONS

$$\#\{\text{onto } f : [n] \to [k]\} = \sum_{j=0}^{k} (-1)^j \binom{k}{j} (k-j)^n$$

Functions from an n-set onto a k-set; equals $k!\, S(n,k)$ and is 0 if $n < k$.

DERANGEMENTS

$$D_n = n!\sum_{k=0}^{n}\frac{(-1)^k}{k!}, \qquad D_n = (n-1)(D_{n-1}+D_{n-2})\ (n \geq 2)$$

Permutations of n objects with no object in its original place. Convention $D_0 = 1$. Also $D_n = n\,D_{n-1} + (-1)^n$, and for $n \geq 1$, D_n is the integer nearest to $n!/e$. $D_1 = 0, D_2 = 1, D_3 = 2, D_4 = 9, D_5 = 44, D_6 = 265$.

PERMUTATIONS WITH EXACTLY K FIXED POINTS

$$\binom{n}{k} D_{n-k}$$

Choose which k objects stay put, then derange the other $n-k$ (for $k = n$ this gives $D_0 = 1$, the identity).

PIGEONHOLE PRINCIPLE (GENERALISED)

$$\text{If } n \text{ objects go into } k \text{ boxes, some box holds at least } \left\lceil \frac{n}{k} \right\rceil .$$

For positive integers n, k. To guarantee at least r objects in one of k boxes you need $k(r-1)+1$ objects. It proves existence only; it does not construct the box.

BELL NUMBERS

$$B_n = \sum_{k=0}^{n} S(n,k), \qquad B_{n+1} = \sum_{k=0}^{n}\binom{n}{k}B_k, \quad B_0, \ldots, B_6 = 1, 1, 2, 5, 15, 52, 203$$

Number of partitions of an n-set into non-empty blocks, which is also the number of equivalence relations on it (each relation corresponds to its set of equivalence classes).

LATTICE PATHS

$$\binom{m+n}{m}$$

Shortest paths from $(0,0)$ to (m,n) using unit steps right and up: choose which m of the $m+n$ steps are right. Forced points are handled by multiplying path counts of each leg.

CATALAN NUMBERS

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1}, \quad C_0, \ldots, C_6 = 1, 1, 2, 5, 14, 42, 132$$

Paths from $(0,0)$ to (n,n) with right steps (along x) and up steps (along y) that never go above the line $y = x$ (touching allowed). Also: balanced bracket strings with n pairs; unlabelled binary trees with n nodes (left and right child distinguished); binary search trees on n distinct keys; full binary trees with $n+1$ leaves; ways to parenthesise a product of $n+1$ factors; triangulations of a convex $(n+2)$-gon.

BINARY STRINGS WITH NO TWO CONSECUTIVE 1S

$$a_n = a_{n-1} + a_{n-2}\ (n \geq 3), \quad a_1 = 2,\ a_2 = 3$$

Condition on the end: a valid string ending in 0 is any valid string of length $n-1$ followed by 0; a valid string ending in 1 must end in 01, after any valid string of length $n-2$. Values: 2, 3, 5, 8, 13, 21, 34, 55 for $n = 1, \ldots, 8$; $a_n = F_{n+2}$ with $F_1 = F_2 = 1$.

K-SUBSETS WITH NO TWO CONSECUTIVE ELEMENTS

$$\binom{n-k+1}{k}$$

Number of k-element subsets of $\{1, 2, \ldots, n\}$ with no two consecutive integers; equivalently binary strings of length n with k ones, no two adjacent (place the k ones in the $n-k+1$ gaps around the $n-k$ zeros).

Recurrence relations and generating functions

CATALAN NUMBERS AND DERANGEMENTS

$$C_n = \frac{1}{n+1}\binom{2n}{n}, \qquad D_n = n!\sum_{i=0}^{n}\frac{(-1)^i}{i!}$$

For $n \geq 0$; for example $C_3 = \frac{1}{4}\binom{6}{3} = 5$ and $D_4 = 24\left(1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24}\right) = 9$.

CHARACTERISTIC EQUATION

$$r^k = c_1 r^{k-1} + c_2 r^{k-2} + \cdots + c_k$$

For $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$; it has degree k, so k roots over the complex numbers counted with multiplicity. Since $c_k \neq 0$, no root is 0.

GENERAL SOLUTION, DISTINCT ROOTS

$$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n + \cdots + \alpha_k r_k^n$$

When all k roots r_i are distinct (real or complex); the constants α_i come from the k initial conditions.

CONTRIBUTION OF A REPEATED ROOT

$$a_n = \left(\alpha_0 + \alpha_1 n + \cdots + \alpha_{m-1} n^{m-1}\right) r^n$$

A root r of multiplicity m contributes this term; add the contributions of all roots.

FIBONACCI AND LUCAS CLOSED FORMS

$$F_n = \frac{\varphi^n - \psi^n}{\sqrt{5}}, \quad L_n = \varphi^n + \psi^n, \quad \varphi = \frac{1+\sqrt{5}}{2},\ \psi = \frac{1-\sqrt{5}}{2}$$

Roots of $r^2 = r + 1$; $F_0 = 0, F_1 = 1$ and $L_0 = 2, L_1 = 1$.

FIRST-ORDER LINEAR RECURRENCE

$$a_n = c\,a_{n-1} + d \;\Rightarrow\; a_n = c^n a_0 + d\,\frac{c^n - 1}{c - 1}\ (c \neq 1), \qquad a_n = a_0 + nd\ (c = 1)$$

For $n \geq 1$ with constants c and d. Example: $a_n = 2a_{n-1} + 1$, $a_0 = 0$ gives $2^n - 1$.

GEOMETRIC SERIES

$$\frac{1}{1-x} = \sum_{n\geq 0} x^n, \qquad \frac{1}{1-ax} = \sum_{n\geq 0} a^n x^n$$

Coefficients $1, 1, 1, \ldots$ and $1, a, a^2, \ldots$; so $1/(1+x)$ gives $(-1)^n$.

LINEAR SEQUENCES

$$\frac{1}{(1-x)^2} = \sum_{n\geq 0}(n+1)x^n, \qquad \frac{x}{(1-x)^2} = \sum_{n\geq 0} n x^n$$

Coefficients $1, 2, 3, \ldots$ and $0, 1, 2, 3, \ldots$.

NEGATIVE BINOMIAL SERIES

$$\frac{1}{(1-x)^k} = \sum_{n\geq 0}\binom{n+k-1}{k-1}x^n, \qquad \frac{1}{(1-ax)^k} = \sum_{n\geq 0}\binom{n+k-1}{k-1}a^n x^n$$

For integer $k \geq 1$; $[x^n](1-x)^{-k}$ counts solutions of $y_1 + \cdots + y_k = n$ in non-negative integers.

FINITE SERIES

$$(1+x)^m = \sum_{k=0}^{m}\binom{m}{k}x^k, \qquad \frac{1-x^{m+1}}{1-x} = 1 + x + \cdots + x^m$$

For integer $m \geq 0$; the second gives a bounded choice of 0 to m items.

FIBONACCI AND LUCAS GENERATING FUNCTIONS

$$\sum_{n\geq 0} F_n x^n = \frac{x}{1-x-x^2}, \qquad \sum_{n\geq 0} L_n x^n = \frac{2-x}{1-x-x^2}$$

With $F_0 = 0, F_1 = 1$ and $L_0 = 2, L_1 = 1$.

NON-NEGATIVE INTEGER SOLUTIONS

$$\#\{(y_1,\dots,y_k) : y_1 + \cdots + y_k = n,\ y_i \geq 0\} = \binom{n+k-1}{k-1}$$

For $k \geq 1$; the coefficient of x^n in $(1-x)^{-k}$, i.e. n identical objects into k distinct boxes. With every $y_i \geq 1$ the count is $\binom{n-1}{k-1}$ (for $n \geq k$; zero if $n < k$).

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