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Random processes, power spectral density and noise

MEAN AND VARIANCE OF A RANDOM VARIABLE

$$\mu_X = E[X], \qquad \sigma_X^2 = E\left[(X-\mu_X)^2\right] = E[X^2] - \mu_X^2$$

Holds for any random variable. $E[X^2]$ is the mean-square value (average power for a signal).

GAUSSIAN DENSITY

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

Fully described by mean μ and variance σ^2.

GAUSSIAN TAIL PROBABILITY

$$P(X>x) = Q\left(\frac{x-\mu}{\sigma}\right), \qquad Q(x) = \frac{1}{2}\operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right)$$

Convention used in these notes: $Q(x) = \frac{1}{2}\operatorname{erfc}(x/\sqrt{2})$. Also $Q(-x) = 1 - Q(x)$ and $Q(0) = 0.5$.

TIME AVERAGE OF A SAMPLE FUNCTION

$$\langle x(t)\rangle = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x(t)\,dt, \qquad \langle x^2(t)\rangle = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x^2(t)\,dt$$

For an ergodic process, $\langle x(t)\rangle = \mu_X$ and $\langle x^2(t)\rangle = R_X(0)$.

AUTOCORRELATION OF A WSS PROCESS (REAL)

$$R_X(\tau) = E\left[X(t+\tau)\,X(t)\right]$$

Depends only on the lag τ when $X(t)$ is WSS. The process is real-valued here.

RANDOM-PHASE SINUSOID

$$X(t) = A\cos(2\pi f_0 t + \Theta),\ \Theta\sim U(0,2\pi)\ \Rightarrow\ R_X(\tau) = \frac{A^2}{2}\cos(2\pi f_0\tau)$$

A and f_0 are constants. A WSS process with zero mean and power $A^2/2$. If Θ is uniform over only $(0,\pi)$, the mean depends on t and the process is not WSS.

WIENER-KHINCHIN THEOREM

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau)\,e^{-j2\pi f\tau}\,d\tau, \qquad R_X(\tau) = \int_{-\infty}^{\infty} S_X(f)\,e^{j2\pi f\tau}\,df$$

Applies to a WSS process. $S_X(f)$ is two-sided, so it covers negative and positive frequencies.

POWER FROM THE PSD

$$P = E[X^2] = R_X(0) = \int_{-\infty}^{\infty} S_X(f)\,df$$

Integrate over both sides of the spectrum. A nonzero mean contributes an impulse $\mu_X^2\,\delta(f)$ at $f=0$.

OUTPUT MEAN

$$\mu_Y = \mu_X\,H(0)$$

$H(0)$ is the DC gain.

OUTPUT PSD

$$S_Y(f) = |H(f)|^2\,S_X(f)$$

Only the magnitude of $H(f)$ matters; phase has no effect on the output PSD.

OUTPUT AUTOCORRELATION

$$R_Y(\tau) = h(\tau) * h(-\tau) * R_X(\tau)$$

The Fourier transform of this is $|H(f)|^2 S_X(f)$ for real $h(t)$.

OUTPUT POWER

$$P_Y = \int_{-\infty}^{\infty} |H(f)|^2 \, S_X(f) \, df$$

Total output mean-square value, including any DC part.

INPUT-OUTPUT CROSS-CORRELATION

$$R_{YX}(\tau) = h(\tau) * R_X(\tau), \qquad S_{YX}(f) = H(f) \, S_X(f)$$

Defined as $R_{YX}(\tau) = E[Y(t+\tau)X(t)]$. For the other order, $R_{XY}(\tau) = h(-\tau) * R_X(\tau)$ and $S_{XY}(f) = H^*(f) S_X(f)$.

SUM OF TWO PROCESSES

$$Z(t) = X(t) + Y(t): \quad R_Z(\tau) = R_X(\tau) + R_Y(\tau), \qquad S_Z(f) = S_X(f) + S_Y(f)$$

Holds when $X(t)$ and $Y(t)$ are jointly WSS and uncorrelated, with at least one of them zero-mean (for example, a signal plus independent zero-mean noise). Otherwise add the cross terms $R_{XY}(\tau) + R_{YX}(\tau)$.

MODULATION BY A RANDOM-PHASE CARRIER

$$Y(t) = X(t)\cos(2\pi f_c t + \Theta): \quad R_Y(\tau) = \frac{1}{2} R_X(\tau)\cos(2\pi f_c \tau), \qquad S_Y(f) = \frac{1}{4}\left[S_X(f - f_c) + S_X(f + f_c)\right]$$

$X(t)$ is WSS and $\Theta \sim U(0, 2\pi)$ is independent of $X(t)$. The output power is $P_Y = R_X(0)/2$.

DIFFERENTIATOR

$$Y(t) = \frac{dX(t)}{dt}: \quad H(f) = j2\pi f, \qquad S_Y(f) = 4\pi^2 f^2 \, S_X(f), \qquad R_Y(\tau) = -\frac{d^2 R_X(\tau)}{d\tau^2}$$

Follows from $S_Y(f) = |H(f)|^2 S_X(f)$.

WHITE NOISE

$$S_W(f) = \frac{N_0}{2}, \qquad R_W(\tau) = \frac{N_0}{2}\,\delta(\tau)$$

Samples at different instants are uncorrelated (independent if the noise is also Gaussian). Total power is infinite, so ideal white noise is a model that is used only after some filtering.

WHITE NOISE THROUGH AN IDEAL LOW-PASS OR BAND-PASS FILTER

$$P_{\text{out}} = \frac{N_0}{2} \cdot |H_0|^2 \cdot 2B = N_0 \, |H_0|^2 \, B$$

H_0 is the passband gain and B is the one-sided bandwidth in Hz (for a low-pass filter, $|f| \le B$; for a band-pass filter, total width B centred on $\pm f_c$).

AUTOCORRELATION OF WHITE NOISE AFTER AN IDEAL LOW-PASS FILTER

$$R_Y(\tau) = N_0 B \ \text{sinc}(2B\tau)$$

Unity gain, passband $|f| \le B$, $\text{sinc}(x) = \dfrac{\sin \pi x}{\pi x}$. $R_Y(\tau)$ is zero at $\tau = k/(2B)$, $k \neq 0$, so samples taken $1/(2B)$ apart are uncorrelated (independent if the noise is Gaussian).

NOISE-EQUIVALENT BANDWIDTH

$$B_N = \frac{\int_0^{\infty} |H(f)|^2 \, df}{|H(f)|^2_{\max}}$$

Width of an ideal filter of the same peak gain that passes the same white-noise power: $P_{\text{out}} = N_0 \, |H|^2_{\max} \, B_N$. The peak is at $f = 0$ for a low-pass filter and at the centre frequency for a band-pass filter; f in Hz.

RC LOW-PASS FILTER WITH WHITE NOISE INPUT

$$|H(f)|^2 = \frac{1}{1+(2\pi fRC)^2}, \quad R_Y(\tau) = \frac{N_0}{4RC}\, e^{-|\tau|/RC}, \quad P_Y = \frac{N_0}{4RC}$$

Here $B_N = \frac{\pi}{2} f_{3\text{dB}} = \frac{1}{4RC}$ with $f_{3\text{dB}} = \frac{1}{2\pi RC}$. The output is coloured noise.

THERMAL NOISE OF A RESISTOR

$$\overline{v_n^2} = 4kTRB, \qquad P_{\text{avail}} = kTB, \qquad \frac{N_0}{2} = \frac{kT}{2}$$

$k = 1.38 \times 10^{-23}$ J/K, T in kelvin, B in Hz. $\overline{v_n^2}$ is the open-circuit mean-square voltage of a resistor R, and P_{avail} is the power it delivers to a matched load. Thermal noise is Gaussian and is white for frequencies well below kT/h (about 6×10^{12} Hz at 290 K).

NARROWBAND NOISE REPRESENTATION

$$n(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

$n(t)$ is zero-mean, WSS and band-pass. $n_c(t)$ and $n_s(t)$ are low-pass, zero-mean and jointly WSS; if $n(t)$ is Gaussian, they are jointly Gaussian.

POWERS OF THE COMPONENTS

$$E[n^2] = E[n_c^2] = E[n_s^2] = \int_{-\infty}^{\infty} S_n(f)\, df$$

For white noise $N_0/2$ through an ideal band-pass filter of width B (gain 1): each of the three powers equals $N_0 B$.

ENVELOPE AND PHASE OF NARROWBAND GAUSSIAN NOISE

$$r = \sqrt{n_c^2 + n_s^2}, \quad f_R(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right),\ r \geq 0; \qquad \psi \sim U(0, 2\pi)$$

For zero-mean Gaussian narrowband noise with $\sigma^2 = E[n^2]$, written as $n(t) = r(t)\cos(2\pi f_c t + \psi(t))$: the envelope is Rayleigh, and the phase is uniform and independent of the envelope. $P(r > a) = \exp\left(-a^2/2\sigma^2\right)$ and $E[r^2] = 2\sigma^2$. If a sinusoid of amplitude A is added, the envelope becomes Rician: close to Rayleigh for $A \ll \sigma$, and close to Gaussian about A for $A \gg \sigma$.

Amplitude and angle modulation, receivers and multiplexing

SINGLE-TONE AM WAVE

$$s(t) = A_c\left[1 + \mu\cos(\omega_m t)\right]\cos(\omega_c t) = A_c\cos\omega_c t + \tfrac{\mu A_c}{2}\cos(\omega_c + \omega_m)t + \tfrac{\mu A_c}{2}\cos(\omega_c - \omega_m)t$$

Carrier plus two sidebands, each of amplitude $\mu A_c/2$. Bandwidth $B = 2f_m$ (for a general message, $B = 2W$ where W is the highest message frequency).

POWER AND EFFICIENCY (SINGLE TONE)

$$P_t = P_c\left(1 + \frac{\mu^2}{2}\right), \quad P_c = \frac{A_c^2}{2R}, \quad P_{SB} = \frac{\mu^2}{2}P_c, \quad \eta = \frac{P_{SB}}{P_t} = \frac{\mu^2}{2+\mu^2}$$

These power relations hold for any μ. Without overmodulation ($\mu \leq 1$) the efficiency is at most $1/3$, reached at $\mu = 1$. Each sideband carries $\mu^2 P_c/4$. In terms of antenna (load) current, $I_t = I_c\sqrt{1 + \mu^2/2}$.

MULTI-TONE AM

$$\mu_t = \sqrt{\mu_1^2 + \mu_2^2 + \cdots}, \qquad P_t = P_c\left(1 + \frac{\mu_t^2}{2}\right)$$

For a message made of tones of different frequencies, each with its own index μ_i. Use μ_t in the power and efficiency formulas. The condition $\mu_1 + \mu_2 + \cdots \leq 1$ is enough to keep the envelope from going negative.

MODULATION INDEX FROM THE WAVEFORM

$$\mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$$

$A_{\max}$ and $A_{\min}$ are the maximum and minimum of the envelope, for a single tone with $\mu \le 1$. For $\mu > 1$ (overmodulation) the envelope is distorted and an envelope detector fails.

ENVELOPE DETECTOR RC CONDITION

$$\frac{1}{f_c} \ll RC \ll \frac{1}{f_m}, \qquad RC \le \frac{\sqrt{1-\mu^2}}{\mu\,\omega_m}$$

The first condition (with f_m replaced by W for a general message) lets the capacitor smooth out the carrier ripple yet follow the envelope. The second is the condition, for a single tone of index $\mu < 1$, that avoids diagonal clipping (the capacitor discharging slower than the envelope falls).

DSB-SC SIGNAL AND POWER

$$s(t) = A_c\, m(t) \cos \omega_c t, \qquad P = \frac{A_c^2\, \overline{m^2}}{2}, \qquad B = 2W$$

Power in a $1\ \Omega$ load, for a zero-mean message. The spectrum has no carrier component. The envelope is $A_c|m(t)|$, not $m(t)$, so an envelope detector cannot be used; DSB-SC needs coherent detection.

COHERENT DETECTION WITH ERRORS

$$v_o(t) = \tfrac{1}{2} A_c\, m(t) \cos\phi \quad (\text{phase error } \phi), \qquad v_o(t) = \tfrac{1}{2} A_c\, m(t) \cos(\Delta\omega\, t) \quad (\text{frequency error})$$

DSB-SC input, local carrier of unit amplitude $\cos(\omega_c t + \phi)$ or $\cos(\omega_c + \Delta\omega)t$, followed by a low-pass filter. The output vanishes at $\phi = 90^\circ$ (quadrature null effect), and a frequency error gives a beating (fading) output. For SSB, a phase error causes phase distortion of the output, and a frequency error shifts every output tone by Δf. A Costas loop or a squaring loop recovers a synchronous carrier for DSB-SC.

SSB SIGNAL (PHASE-SHIFT FORM)

$$s(t) = \tfrac{A_c}{2}\left[m(t)\cos\omega_c t \mp \hat{m}(t)\sin\omega_c t\right], \qquad P = \frac{A_c^2\, \overline{m^2}}{4}, \qquad B = W$$

Upper sign gives USB, lower sign gives LSB. $\hat{m}(t)$ is the Hilbert transform of $m(t)$ and has the same power as $m(t)$. Power in a $1\ \Omega$ load. Coherent detection with a unit-amplitude local carrier gives $\frac{A_c}{4} m(t)$.

PM AND FM DEFINITIONS

$$s_{PM} = A_c \cos\big(\omega_c t + k_p\, m(t)\big), \qquad s_{FM} = A_c \cos\Big(\omega_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau\Big)$$

k_p in rad/V, k_f in Hz/V. Instantaneous frequency: FM $f_i = f_c + k_f\, m(t)$; PM $f_i = f_c + \dfrac{k_p}{2\pi}\dfrac{dm}{dt}$. FM by $m(t)$ is PM by $\int m\, dt$, and PM by $m(t)$ is FM by dm/dt.

DEVIATION AND MODULATION INDEX (TONE $A_M\COS 2\PI F_M T$)

$$\text{FM: } \Delta f = k_f A_m,\ \beta = \frac{k_f A_m}{f_m}; \qquad \text{PM: } \beta = k_p A_m,\ \Delta f = \beta f_m = k_p A_m f_m$$

In FM, Δf depends only on the amplitude of the message and β falls as f_m rises. In PM, β depends only on the amplitude and Δf grows with f_m.

SINGLE-TONE FM SPECTRUM

$$s(t) = A_c \cos(\omega_c t + \beta \sin \omega_m t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos\big[2\pi(f_c + n f_m)t\big], \qquad J_{-n}(\beta) = (-1)^n J_n(\beta)$$

Spectral lines at $f_c \pm n f_m$ with amplitudes $A_c|J_n(\beta)|$. The carrier amplitude $A_c J_0(\beta)$ vanishes at $\beta \approx 2.405,\ 5.520,\ 8.654, \ldots$

POWER OF AN FM WAVE

$$P = \frac{A_c^2}{2} = \frac{A_c^2}{2}\sum_n J_n^2(\beta), \qquad \sum_n J_n^2(\beta) = 1$$

In a $1\ \Omega$ load. Total power is independent of β; modulation only redistributes power from the carrier to the sidebands. Power in the carrier plus k pairs of sidebands is $\frac{A_c^2}{2}\left[J_0^2 + 2\sum_{n=1}^{k} J_n^2\right]$.

CARSON'S RULE

$$B_{FM} \approx 2(\Delta f + f_m) = 2f_m(\beta + 1)$$

An approximation; for a single tone it contains roughly 98 percent of the total power. For $\beta \ll 1$, $B \approx 2f_m$ (narrowband); for $\beta \gg 1$, $B \approx 2\Delta f$ (wideband). For a general message use $B = 2(\Delta f + W) = 2W(D + 1)$, with peak deviation $\Delta f = k_f \max|m|$ and deviation ratio $D = \Delta f / W$.

FIGURE OF MERIT (SINGLE TONE, LARGE INPUT SNR)

$$\left.\frac{\mathrm{SNR}_o}{\mathrm{SNR}_c}\right|_{\text{DSB-SC, SSB}} = 1, \qquad \left.\frac{\mathrm{SNR}_o}{\mathrm{SNR}_c}\right|_{AM} = \frac{\mu^2}{2 + \mu^2}, \qquad \left.\frac{\mathrm{SNR}_o}{\mathrm{SNR}_c}\right|_{FM} = \tfrac{3}{2}\beta^2$$

Coherent detection for DSB-SC and SSB; envelope detection for AM above threshold, with $\mu \le 1$, so its figure of merit is at most $1/3$; FM above threshold without pre-emphasis. For a general zero-mean message of power P: AM gives $k_a^2 P/(1 + k_a^2 P)$ and FM gives $3k_f^2 P/W^2$. FM beats fully modulated ($\mu = 1$) AM when $\frac{3}{2}\beta^2 > \frac{1}{3}$, that is $\beta > \sqrt{2}/3 \approx 0.47$. Above threshold, FM trades bandwidth for noise performance.

PRE-EMPHASIS AND DE-EMPHASIS

$$H_{pe}(f) = 1 + j\frac{f}{f_1}, \qquad H_{de}(f) = \frac{1}{1 + jf/f_1}, \qquad f_1 = \frac{1}{2\pi RC}$$

The noise at an FM discriminator's output has a PSD proportional to f^2, so high-frequency message components suffer most. The transmitter boosts the high frequencies (pre-emphasis) and the receiver cuts them by the same amount (de-emphasis). The overall response to the message is flat, while the high-frequency noise is reduced. A $75\ \mu$s time constant corresponds to $f_1 \approx 2.12$ kHz.

IF AND IMAGE FREQUENCY (HIGH-SIDE LO)

$$f_{LO} = f_s + f_{IF}, \qquad f_{im} = f_s + 2f_{IF}$$

f_s is the wanted station frequency. The image station lies at $f_{LO} + f_{IF}$ and also mixes down to f_{IF}, causing interference. With a low-side LO ($f_{LO} = f_s - f_{IF}$), $f_{im} = f_s - 2f_{IF}$.

IMAGE REJECTION RATIO

$$\alpha = \sqrt{1 + Q^2\rho^2}, \qquad \rho = \frac{f_{im}}{f_s} - \frac{f_s}{f_{im}}$$

For a single tuned circuit of loaded quality factor Q at the RF stage. α is a voltage ratio; in dB it is $20\log_{10}\alpha$. With identical tuned stages before the mixer, their ratios multiply. A larger IF moves the image farther from f_s and improves image rejection; a smaller IF makes adjacent-channel selectivity easier to achieve.

FDM BANDWIDTH

$$B_{total} = \sum_{k=1}^{N} B_k + (N - 1)B_{guard}$$

B_{guard} is the guard band between adjacent channels. For N identical SSB channels of bandwidth W, $B_{total} = NW + (N - 1)B_{guard}$. For DSB-SC or AM channels each $B_k = 2W$.

TDM SAMPLING AND BIT RATE

$$f_{\text{samp}} \ge 2W, \qquad R_{\text{samples}} = N f_{\text{samp}}, \qquad R_b = n\,N f_{\text{samp}}\ (n\text{-bit PCM samples})$$

N channels, each band-limited to W and sampled at f_{samp} (the frame rate), at or above the Nyquist rate $2W$. The minimum (Nyquist) transmission bandwidth is half the pulse rate: $N f_{\text{samp}}/2$ for PAM-TDM and $R_b/2$ for binary PCM-TDM. Add synchronisation bits per frame if given.

Information theory: entropy, source coding and channel capacity

INFORMATION OF A SYMBOL

$$I(x_i) = \log_2 \frac{1}{p_i} = -\log_2 p_i \text{ bits}$$

Always non-negative; a certain event ($p_i = 1$) carries 0 bits.

ENTROPY OF A DMS

$$H(X) = -\sum_{i=1}^{M} p_i \log_2 p_i \text{ bits/symbol}$$

Average information per symbol. Take $0 \log 0 = 0$.

BOUNDS ON ENTROPY

$$0 \le H(X) \le \log_2 M$$

Minimum 0 when one symbol has probability 1; maximum $\log_2 M$ when all M symbols are equiprobable.

BINARY ENTROPY FUNCTION

$$H_b(p) = -p \log_2 p - (1-p) \log_2 (1-p)$$

Entropy of a two-symbol source. $H_b(0.5) = 1$, symmetric about $p = 0.5$, $H_b(0) = H_b(1) = 0$.

EXTENDED SOURCE

$$H(X^n) = n\, H(X)$$

For the nth extension of a DMS (blocks of n independent symbols, M^n block symbols).

JOINT AND CONDITIONAL ENTROPY

$$H(X,Y) = H(X) + H(Y|X) = H(Y) + H(X|Y)$$

Chain rule. If X and Y are independent, $H(X,Y) = H(X) + H(Y)$. The conditional entropy is an average: $H(Y|X) = \sum_x p(x)\, H(Y|X=x)$.

MUTUAL INFORMATION

$$I(X;Y) = H(X) - H(X|Y) = H(Y) - H(Y|X) = H(X) + H(Y) - H(X,Y)$$

Symmetric in X and Y; $I(X;Y) \ge 0$, with equality exactly when X and Y are independent.

MUTUAL INFORMATION FROM PROBABILITIES

$$I(X;Y) = \sum_x \sum_y p(x,y) \log_2 \frac{p(x,y)}{p(x)\,p(y)}$$

Here $p(x,y) = p(x)P(y|x)$ and $p(y) = \sum_x p(x)P(y|x)$. Terms with $p(x,y) = 0$ contribute 0.

AVERAGE CODE LENGTH

$$\overline{L} = \sum_i p_i\, l_i \text{ bits/symbol}$$

Binary code alphabet.

KRAFT INEQUALITY

$$\sum_{i=1}^{M} 2^{-l_i} \le 1$$

A binary prefix code with lengths l_i exists if and only if this holds. Every uniquely decodable binary code also satisfies it, so nothing is gained in length by giving up the prefix property. It tests lengths only; a set of codewords can obey it and still not be prefix-free.

SOURCE CODING THEOREM

$$H(X) \le \overline{L} < H(X) + 1$$

The lower bound holds for every uniquely decodable binary code; an optimal binary prefix code (e.g. Huffman) also meets the upper bound. Using the nth extension, $H(X) \le \overline{L}_n / n < H(X) + 1/n$.

EFFICIENCY AND REDUNDANCY

$$\eta = \frac{H(X)}{\overline{L}}, \qquad \text{redundancy} = 1 - \eta$$

Binary code. For a code alphabet of r symbols, $\eta = \dfrac{H(X)}{\overline{L}\log_2 r}$. A binary code with $\overline{L} = H$ has $\eta = 1$; this needs every p_i to be a power of $\frac{1}{2}$.

CHANNEL CAPACITY

$$C = \max_{P(x)} I(X;Y) \text{ bits per channel use}$$

For a channel used r times per second, the capacity is rC bits/s.

BINARY SYMMETRIC CHANNEL (CROSSOVER PROBABILITY P)

$$C_{\text{BSC}} = 1 - H_b(p)$$

Reached with equiprobable inputs. $C = 1$ at $p = 0$ or $p = 1$, and $C = 0$ at $p = \frac{1}{2}$.

BSC WITH INPUT P(X=1) = A

$$I(X;Y) = H_b\big(\alpha(1-p) + (1-\alpha)p\big) - H_b(p)$$

From $I = H(Y) - H(Y|X)$: $H(Y|X) = H_b(p)$ for every input, and $P(Y=1) = \alpha(1-p) + (1-\alpha)p$. The maximum, at $\alpha = \frac{1}{2}$, is $1 - H_b(p)$.

BINARY ERASURE CHANNEL (ERASURE PROBABILITY E)

$$C_{\text{BEC}} = 1 - \varepsilon$$

Each bit is either received correctly or marked as erased; reached with equiprobable inputs.

BEC WITH INPUT P(X=1) = A

$$I(X;Y) = (1-\varepsilon)\, H_b(\alpha)$$

From $I = H(X) - H(X|Y)$: the input is known unless the bit is erased, so $H(X|Y) = \varepsilon H_b(\alpha)$. The maximum, at $\alpha = \frac{1}{2}$, is $1 - \varepsilon$.

DIFFERENTIAL ENTROPY (CONTINUOUS SOURCE)

$$h(X) = -\int_{-\infty}^{\infty} f(x) \log_2 f(x)\, dx$$

Unlike discrete entropy, it can be negative. Uniform on an interval of length a: $h = \log_2 a$. Gaussian with variance σ^2: $h = \frac{1}{2}\log_2(2\pi e\sigma^2)$, the largest value for any density with that variance.

SHANNON-HARTLEY LAW

$$C = B \log_2\left(1 + \frac{S}{N}\right) \text{ bits/s}$$

Band-limited AWGN channel of bandwidth B Hz. S is the average received signal power and S/N is the power ratio (not dB); $N = N_0 B$ with two-sided noise PSD $N_0/2$.

SNR IN DECIBELS

$$\left(\frac{S}{N}\right)_{\text{dB}} = 10 \log_{10} \frac{S}{N}$$

Convert to a ratio before using in the law: 20 dB means 100, 30 dB means 1000.

INFINITE-BANDWIDTH LIMIT

$$C_\infty = \lim_{B\to\infty} C = \frac{S}{N_0}\log_2 e \approx 1.44\,\frac{S}{N_0}$$

At fixed signal power S, capacity grows with B but saturates, because the noise power $N_0 B$ grows too.

SHANNON LIMIT (CONDITION FOR RELIABLE COMMUNICATION)

$$\frac{E_b}{N_0} \geq \ln 2 \approx 0.693\ (-1.59\ \text{dB})$$

E_b is the energy per bit. With $R = C$ and $S = E_b C$: $\dfrac{E_b}{N_0} = \dfrac{2^{C/B}-1}{C/B}$, which tends to $\ln 2$ as $C/B \to 0$. Below this, no reliable communication is possible at any positive rate.

BANDWIDTH-SNR TRADE-OFF

$$\frac{S}{N} = 2^{C/B} - 1$$

For a target rate C, a smaller bandwidth needs an exponentially larger SNR; a larger bandwidth allows a lower SNR.

PCM, digital modulation and detection

STEP SIZE

$$\Delta = \frac{2m_p}{L} = \frac{2m_p}{2^n}$$

Uniform quantiser with peak-to-peak range $2m_p$ and $L = 2^n$ levels.

QUANTISATION NOISE POWER

$$N_q = \overline{e^2} = \frac{\Delta^2}{12}$$

Error uniform on $(-\Delta/2, \Delta/2)$; valid when the signal is large compared with Δ and does not overload the quantiser.

SQNR FOR A GENERAL SIGNAL

$$\text{SQNR} = \frac{S}{N_q} = \frac{3L^2\,\overline{m^2}}{m_p^2}$$

Here $S = \overline{m^2}$ is the average signal power and m_p the quantiser's peak value.

SQNR, FULL-SCALE SINUSOID

$$\text{SQNR}_{\text{dB}} = 6.02\,n + 1.76\ \text{dB}$$

Sinusoid of amplitude m_p so $S = m_p^2/2$ and SQNR $= 1.5 \cdot 4^n$. Each extra bit adds about 6 dB. For a full-scale uniformly distributed signal ($\overline{m^2} = m_p^2/3$) the SQNR is 4^n, i.e. $6.02n$ dB.

PCM BIT RATE AND MINIMUM BANDWIDTH

$$R_b = n f_s, \qquad B_{\min} = \frac{R_b}{2} = \frac{n f_s}{2}$$

Minimum (ideal Nyquist, sinc pulses) baseband bandwidth for binary PCM; with a raised-cosine pulse of roll-off α it becomes $(1+\alpha)R_b/2$. The first-null bandwidth of rectangular NRZ pulses is R_b.

M-LAW COMPRESSOR

$$y = \text{sgn}(x)\,\frac{\ln(1+\mu|x|)}{\ln(1+\mu)}$$

$\mu = 255$ in the standard North American/Japanese system. In the limit $\mu \to 0$, $y = x$ (no compression).

NO SLOPE OVERLOAD (DM)

$$\left|\frac{dm}{dt}\right|_{\max} \le \frac{\Delta}{T_s} = \Delta f_s$$

For $m(t) = A\sin 2\pi f_m t$ this is $2\pi f_m A \le \Delta f_s$. If violated, the staircase cannot follow the signal (slope overload).

NYQUIST CRITERION FOR ZERO ISI

$$\sum_{k=-\infty}^{\infty} P\left(f - \frac{k}{T}\right) = T$$

Equivalent to $p(kT) = \delta_{k0}$ (normalised so $p(0) = 1$). Smallest bandwidth satisfying it: $1/(2T)$ Hz, with $p(t) = \operatorname{sinc}(t/T)$, where $\operatorname{sinc} x = \sin(\pi x)/(\pi x)$ (ideal, not realisable).

RAISED-COSINE PULSE BANDWIDTH

$$B = \frac{1+\alpha}{2T} = \frac{(1+\alpha)R_s}{2}, \qquad 0 \le \alpha \le 1$$

Baseband (one-sided) bandwidth for symbol rate $R_s = 1/T$. α is the roll-off factor. At $\alpha = 0$ it is the ideal $R_s/2$; at $\alpha = 1$ it is R_s.

MAXIMUM ZERO-ISI SYMBOL RATE

$$R_s \le \frac{2B}{1+\alpha}$$

For a baseband channel of bandwidth B. For a bandpass (double-sideband modulated) channel of bandwidth B_c the limit is $R_s \le B_c/(1+\alpha)$.

BANDWIDTH EFFICIENCY (NULL-TO-NULL, PSK AND QAM ONLY)

$$\eta = \frac{R_b}{B} = \frac{\log_2 M}{2} \ \text{b/s/Hz}$$

With a raised-cosine pulse the passband bandwidth is $(1+\alpha)R_s$, so $\eta = \log_2 M/(1+\alpha)$. Not for FSK, whose bandwidth also depends on the tone spacing.

MINIMUM TONE SPACING FOR ORTHOGONAL BFSK

$$\Delta f_{\min} = \frac{1}{2T_b}\ (\text{coherent}), \qquad \Delta f_{\min} = \frac{1}{T_b}\ (\text{non-coherent})$$

The two tones must be orthogonal over one bit interval for the stated error probability to hold.

MATCHED FILTER

$$h(t) = s(T-t), \qquad \left(\frac{S}{N}\right)_{\max} = \frac{2E}{N_0}, \quad E = \int_0^T s^2(t)\,dt$$

Output sampled at $t = T$. In the frequency domain $H(f) = S^*(f)\,e^{-j2\pi fT}$. The peak SNR depends only on the pulse energy, not its shape. A correlation receiver (multiply by $s(t)$ and integrate over T) gives the same output at the sampling instant $t = T$.

MAP THRESHOLD, TWO LEVELS IN GAUSSIAN NOISE

$$\text{decide } H_1 \text{ if } r > \frac{s_0 + s_1}{2} + \frac{\sigma^2}{s_1 - s_0}\ln\frac{P(H_0)}{P(H_1)}$$

Sample $r = s_i + n$ with $s_1 > s_0$ and noise $n \sim \mathcal{N}(0, \sigma^2)$ under both hypotheses. For antipodal levels $s_1 = +a$, $s_0 = -a$ it is $r_{th} = \dfrac{\sigma^2}{2a}\ln\dfrac{P(H_0)}{P(H_1)}$. For equal priors the threshold is the midpoint (ML). A less likely H_1 moves the threshold toward s_1.

BPSK AND QPSK (GRAY CODED) BIT ERROR PROBABILITY

$$P_b^{\text{BPSK}} = P_b^{\text{QPSK}} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

Here $d^2 = 4E_b$ for BPSK. QPSK is two independent BPSK streams on the I and Q carriers, so its per-bit error probability is the same.

COHERENT BFSK

$$P_b^{\text{BFSK, coh.}} = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Orthogonal tones, $d^2 = 2E_b$. This needs 3 dB more E_b/N_0 than BPSK for the same BER.

COHERENT ASK (ON-OFF), AVERAGE EB

$$P_b^{\text{OOK, coh.}} = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Equal priors, threshold midway between the two received levels. Here E_b is the average bit energy: the "1" has energy $2E_b$, the "0" none. If the same formula is written with the energy E_1 of the "1", use $Q(\sqrt{E_1/(2N_0)})$.

LINK BETWEEN EB/N0 AND SNR

$$\frac{E_b}{N_0} = \frac{S}{N_0 R_b} = \frac{S/N}{R_b/B}$$

S is the received signal power and $N = N_0 B$ the noise power in bandwidth B. In dB: $(E_b/N_0)_{\text{dB}} = \text{SNR}_{\text{dB}} - 10\log_{10}(R_b/B)$.

Error detection and correction

CODE RATE

$$R = \frac{k}{n}$$

Fraction of transmitted bits that carry message information; $0 < R < 1$ for an (n, k) block code, since $n > k$.

DETECTION CAPABILITY

$$s_{max} = d_{min} - 1$$

A code with minimum distance d_{min} is guaranteed to detect every pattern of up to $d_{min} - 1$ bit errors, when it is used for detection only.

CORRECTION CAPABILITY

$$t = \left\lfloor \frac{d_{min} - 1}{2} \right\rfloor$$

Guaranteed number of correctable errors when the receiver decodes to the nearest codeword (the spheres of radius t around codewords do not overlap).

SIMULTANEOUS CORRECTION AND DETECTION

$$d_{min} \geq t + s + 1 \quad (s \geq t)$$

With this distance the decoder corrects every pattern of up to t errors and, at the same time, detects (flags without miscorrecting) every pattern of $t + 1$ up to s errors. Here s is a total number of errors, not extra errors.

ENCODING

$$\mathbf{c} = \mathbf{m}G$$

Row vector $\mathbf{m}$ (length k) times G gives the codeword $\mathbf{c}$ (length n), all operations modulo 2.

SYSTEMATIC FORM

$$G = \begin{bmatrix} I_k & P \end{bmatrix}, \qquad H = \begin{bmatrix} P^T & I_{n-k} \end{bmatrix}$$

Message bits appear unchanged in the first k positions of the codeword; P is $k \times (n-k)$. Over GF(2), $-P^T = P^T$.

PARITY-CHECK CONDITION

$$H\mathbf{c}^T = \mathbf{0}, \qquad GH^T = 0$$

H is $(n-k) \times n$. A word is a codeword exactly when every parity check is satisfied.

SYNDROME

$$\mathbf{s} = H\mathbf{r}^T = H\mathbf{e}^T$$

$\mathbf{s} = \mathbf{0}$ means $\mathbf{r}$ is a valid codeword (no detectable error). A nonzero $\mathbf{s}$ means an error occurred.

HAMMING (SPHERE-PACKING) BOUND

$$2^{n-k} \geq \sum_{i=0}^{t} \binom{n}{i}$$

Any binary (n, k) code that corrects every pattern of up to t errors must satisfy this: the 2^{n-k} syndromes must label every correctable error pattern, including the no-error pattern. A code that meets it with equality is called **perfect**.

NUMBER OF PARITY BITS

$$2^r \geq m + r + 1$$

Smallest r that lets m data bits be protected against any single error: the r check bits must label the $m + r$ error positions plus the no-error case.

HAMMING CODE PARAMETERS

$$n = 2^r - 1, \quad k = n - r = 2^r - 1 - r, \quad d_{min} = 3$$

Full-length Hamming code, $r \geq 2$. Correct one error, or detect two (not both together). Since $1 + n = 2^r$, it meets the Hamming bound with $t = 1$ exactly, so Hamming codes are perfect.

BLOCK DECODING-ERROR PROBABILITY

$$P_E = \sum_{i=t+1}^{n} \binom{n}{i} p^i (1-p)^{n-i} = 1 - \sum_{i=0}^{t} \binom{n}{i} p^i (1-p)^{n-i}$$

Probability that a block has more than t errors. It is exactly the probability of not decoding the block correctly for a decoder that corrects every pattern of up to t errors and no other (for example a Hamming code, $t = 1$); for a decoder that also corrects some heavier patterns it is an upper bound.

UNDETECTED-ERROR PROBABILITY

$$P_u = \sum_{i=d_{min}}^{n} A_i \, p^i (1-p)^{n-i}$$

For a linear code used for detection only: an error goes undetected exactly when the error pattern is itself a nonzero codeword. A_i is the number of codewords of weight i.

CRC COMPUTATION

$$R(x) = \left[x^r M(x)\right] \bmod g(x), \qquad T(x) = x^r M(x) + R(x)$$

Append r zeros to the message, divide by $g(x)$ using XOR (no borrow), and the remainder $R(x)$ replaces the zeros. The transmitted word $T(x)$ is exactly divisible by $g(x)$.

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